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Geostrophic Adjustment on the equatorial beta-plane revisited

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Abstract

The process of geostrophic adjustment of localized large-scale pressure anomalies in the standard adiabatic shallow-water model on the equatorial beta-plane is revisited, and it is shown that the standard scenario of generation of westward-moving Rossby and eastward-moving Kelvin waves, which underlies the classical Gill theory of tropical circulation due to a localized heating, is not unique. Depending on strength and aspect ratio of the initial perturbation, the response to the initial perturbation in the western sector can be dominated by inertia-gravity waves. The adjustment in the diabatic moist-convective shallow water model can be totally different, and produces, depending on parameters, either Gill-like response, or eastward-moving coherent dipolar structures of the type of equatorial modons, which do not appear in the “dry” adjustment, or vortices traveling, respectively, North-West in the Northern, and South-West, in the Southern hemispheres.

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The famous Gill theory of the response of the tropical atmosphere to localised large-scale heating [1], which consists in generation of Rossby wave like and Kelvin wave like responses at the western and eastern sides of the source, respectively, is widely known. It has become a folklore in tropical meteorology and climatology, and is ubiquitously evoked in the literature for explanation of observations and numerical simulations. The Gill mechanism is intrinsically related to the process of geostrophic adjustment in tropics, i.e. relaxation of large-scale perturbations by emission of equatorial waves, because the heating source, which was represented in original papers [1, 2] and subsequent studies, e.g. [3], as a mass-sink in the shallow-water model of the atmosphere, can be considered as permanently regenerating a localised negative pressure anomaly. The notion of geostrophic adjustment, borrowed from the theory of relaxation of large-scale perturbations in mid-latitudes, e.g. [4], has a specific meaning in the tropics as, due to the absence of the constant part of the Coriolis parameter at the Equator, the geostrophic balance is well-defined only for the zonal velocity. Yet, the equatorial Rossby waves are approximately geostrophic, as was noticed already in the pioneering paper by Matsuno [5], and the geostrophic ansatz for both components of velocity allows to recover Rossby waves from the linearized potential vorticity equation [6]. The zonal velocity of the equatorial Kelvin waves is in geostrophic equilibrium, as well. Such equilibrium arises naturally for equatorial motions with a strong disparity between meridional (smaller) and zonal (larger) scales, [7], Sect. 5.5.4. This scaling was implicitly used in [1], by imposing the equatorial geostrophic balance. Non-linear theory of equatorial geostrophic adjustment under this scaling was developed in shallow-water model on the equatorial beta-plane, and confirmed by numerical simulations in [8], with the results fully consistent with the Gill mechanism.

Yet, the large-scale perturbations with typical scales of the order of the equatorial deformation radius, could have comparable zonal and meridional scales, and a question arises how the scenario of geostrophic adjustment changes in this case. This question, as to our knowledge, was not addressed in the literature, except for the paper by Matsuda and Takayama [9], where numerical simulations of the adjustment on the sphere revealed different scenarios in the equatorial region, depending on the parameters of initial perturbations. The dominant heating source in the equatorial atmosphere is the moist convection. Therefore, another question arises on changes of the equatorial adjustment in a moist-convective environment, when the evolution of moisture and moist convection, with related latent heat release, are incorporated in the model. The answers to these questions are given below. We show, first, that the “dry” geostrophic adjustment is strongly sensitive to the aspect ratio of the initial perturbations, and can be dominated by large-scale westward-

propagating inertia-gravity waves, instead of Rossby waves in the classical scenario. Second, we show that the relaxation of pressure anomalies in the moist-convective environment may follow the Gill scenario, but may also produce eastward-moving cyclonic dipoles, and vortices detaching from the Equator, which does not happen in the “dry” case.

Rotating shallow water (RSW) on the equatorial beta-plane is the archetype model for atmospheric dynamics in tropics [1, 5]. We are using this model, which will be understood as vertically averaged primitive equations in pseudo-height pressure coordinates [7]. The model can be extended to include water vapor with condensation and related latent heat release [10], as well as surface evaporation. Liquid water, precipitation, vaporization and related cooling can be also incorporated in the model, as well as vertical structure, see [11] and references therein, but below we will be using the simplest one-layer version of such moist-convective RSW (mcRSW).

The equations of the “dry” adiabatic one-layer RSW model in the equatorial beta-plane with no topography, and no dissipation read:

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \beta y \hat{\mathbf{z}} \wedge \mathbf{v} + g \nabla h = 0, \\ \partial_t h + \nabla \cdot (\mathbf{v} h) = 0, \end{cases} \quad (1)$$

where $\nabla = (\partial_x, \partial_y)$, $\mathbf{v} = (u, v)$, u is zonal and v - meridional components of velocity, h is geopotential height (thickness), β is the meridional gradient of the Coriolis parameter, and $\hat{\mathbf{z}}$ is a unit vertical vector. In the simplest version the equations of the diabatic mcRSW [10] read:

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \beta y \hat{\mathbf{z}} \wedge \mathbf{v} = -g \nabla h, \\ \partial_t h + \nabla \cdot (\mathbf{v} h) = -\gamma P, \\ \partial_t Q + \nabla \cdot (Q \mathbf{v}) = -P + E. \end{cases} \quad (2)$$

Here Q is a bulk amount of water vapor in the air column, γ is a parameter depending on the underlying stratification, P is the condensation sink, and E is the evaporation source of moisture, which are parameterised as follows:

$$P = \frac{Q - Q^s}{\tau} \mathcal{H}(Q - Q^s), \quad E = \alpha |\mathbf{v}| (Q^s - Q) \mathcal{H}(Q^s - Q). \quad (3)$$

Q^s is a saturation moisture threshold, τ is relaxation time, α is a parameter regulating evaporation, and $\mathcal{H}(\dots)$ denotes the Heaviside (step-)function.

Notice that, similar to the Gill’s theory, convective heating acts as a forcing in the mass conservation equation, although here it is not *ad hoc*, but is determined by the evolution of moisture,

which, in turn, is advected by the velocity field, and is conserved in the air column, unless the condensation and evaporation are switched on.

Below we present the results of numerical simulations, first of the “dry”, and then of the moist-convective (MC) RSW equations, initialized with localized large-scale pressure anomalies $h = H_i(x, y)$ of the so-called alpha-Gaussian shape [12], symmetric with respect to the equator. The circular anomaly $H_i(r)$ normalized by the mean thickness is given by the formula $H_i(r) = 1 + \varepsilon \frac{2^{1/s}}{s} \Gamma\left(\frac{1}{s} + \frac{1}{2}\right) G\left(\frac{r^s}{2}, \frac{1}{s} + \frac{1}{2}\right)$, where $r = \sqrt{x^2 + y^2}$, $G(r, a) = \frac{1}{\Gamma(a)} \int_r^a e^{-t} t^{a-1} dt$, and Γ denotes the gamma-function. The parameters ε (which will be negative, as we concentrate on depressions) and s determine the amplitude and the steepness of the pressure anomaly, the maximum value of the pressure deviation $\Delta H/H$ depending on both. By stretching the zonal coordinate $x \rightarrow a^{-1}x$ in $H_i(x, y)$ we get a perturbation with the zonal to meridional aspect ratio a .

Numerical simulations were performed with high-resolution well-balanced finite-volume scheme [13] in a rectangular domain with sponges at the boundaries. The sponges allow to mostly evacuate short inertia-gravity waves, which are produced at initial stages of adjustment. No explicit dissipation was added and a second-order scheme [13] was used. The domain is symmetric with respect to the equator and wide enough, in order to minimize the influence of the boundaries: $28L_d \times 28L_d$, where $L_d = (gH)^{1/4}/\beta^{1/2}$ is the equatorial deformation radius. Maximal spatial resolution was 1200×1200 in the dry simulations, and 1400×1400 in MC simulations. The natural units of length and time are L_d and $1/\beta L_d$, respectively. Their numerical values depend on the interpretation of g (full, or reduced gravity) and H , the non-perturbed thickness (full, or equivalent depth), e.g. for $g = 9.8m/s^2$, $H \approx 10km$, $L_d \approx 3000km$ and $1/\beta L_d \approx 5h$. As usual in test cases [10], we start the MC simulations with a uniform initial moisture distribution Q_i close to saturation: $Q_i = Q^s - 0.01$ with $Q^s = 0.9$, a short relaxation time τ equal to several time-steps of the numerical scheme, $\gamma = 1$, and $\alpha = \mathcal{O}(10^{-1})$.

The main result of experiments on the “dry” adjustment is a discovery of a significant difference between the evolution of pressure anomalies with large and order-one aspect ratios. A comparison of adjustments of circular and elongated negative pressure anomalies with $s = 4$ and $\Delta H/H = 0.1$, as seen in relative vorticity and velocity fields, is presented in Fig. 1. As follows from the left column, the adjustment of a circular pressure anomaly in the *eastern* sector is similar to the standard Gill-like scenario, cf. [8], and is dominated by a Kelvin wave (KW), with characteristic velocity pattern although a signal corresponding to the eastward-propagating inertia-gravity wave (EIGW) is also visible. However, it is totally different in the *western* sector, where a typical signal

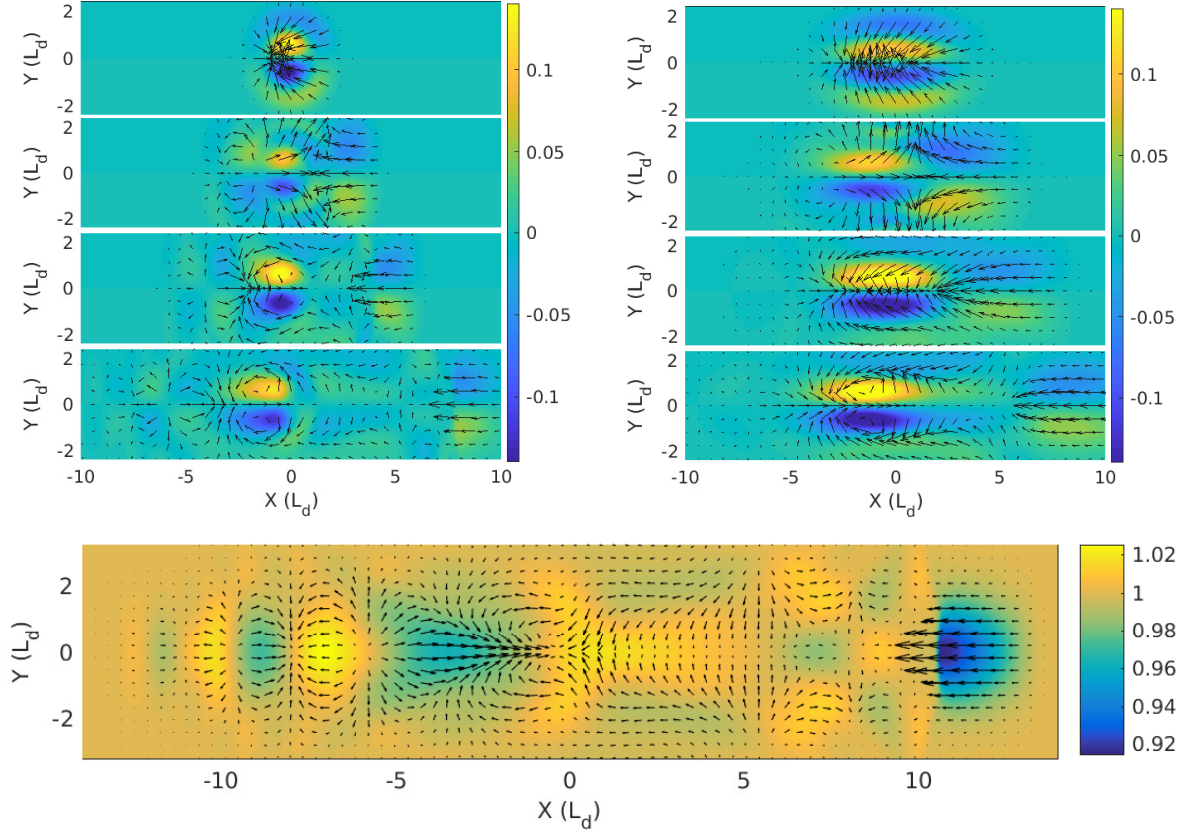


FIG. 1. *Upper left panel:* “Dry” adjustment of a circular pressure anomaly. Relative vorticity (colours), and velocity (arrows): $t = 0, 1, 3, 5, 9 [1/\beta L_d]$ from top to bottom. *Upper right panel:* Same as in the left panel, but for a zonally elongated initial anomaly with aspect ratio $a = 2.5$. *Bottom panel:* Snapshot of adjustment of the circular anomaly at a later time $t = 12 [1/\beta L_d]$.

corresponding to the faster westward-propagating inertia-gravity wave (WIGW) with the structure of lowest meridional mode is clearly seen in the velocity field, in front of a slower propagating Rossby wave (RW) signal. This signal is even clearer identified at a later time $t = 12 [1/\beta L_d]$, as follows from the lowermost panel and also Fig. 2. In comparison, the evolution of the elongated pressure anomaly is closer to the standard scenario with a well-pronounced RW structure, and less pronounced, although still present, WIGW structure (right panels). At large aspect ratios $a \geq 5$ the standard scenario of adjustment described in [8] is reproduced. The evolution of pressure (thickness) confirms these conclusions (not shown). The velocity patterns of lowest meridional modes of all equatorial waves (KW, RW, EIGW and WIGW), which are used for identifications, can be found e.g. in Zeitlin [7], Sect. 4.4.

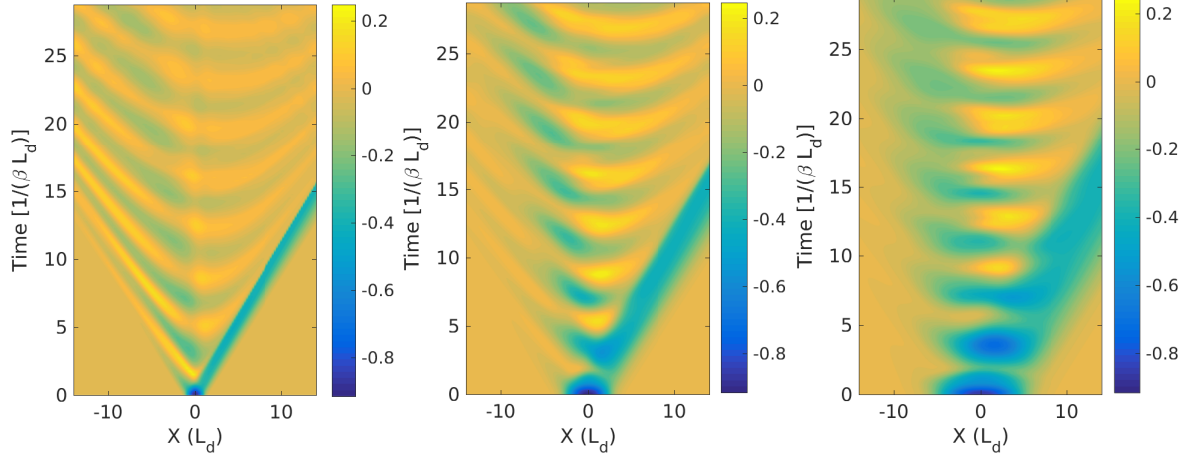


FIG. 2. Hovmöller diagrams of pressure signal along $y = 1[L_d]$ corresponding, respectively, to the left and the right upper panels of Fig. 1 (*left and middle panels*), and to the adjustment of the pressure anomaly with large aspect ratio $a = 5$ (*right panel*).

The main result of the numerical simulations of the MC adjustment is a discovery of new scenarios with generation of steady eastward-moving twin cyclones, or cyclones moving, first, eastward, then westward, and detaching from the Equator. Smaller-scale intense vortices moving off the Equator also appear. In MC case we used exactly the same initial pressure perturbations as in the dry case. We found that, although both circular and zonally elongated *weak* initial anomalies produce a Gill - like scenario, cf. Fig. 3, the MC adjustment of the circular anomaly in the westward sector clearly generates WIGW. What is most important, when the amplitude of the anomaly increases at the same aspect ratio, the scenario of MC adjustment drastically changes, as follows from Fig. 4. While the initial perturbation still produces a Kelvin wave in the eastern sector, and inertia-gravity waves in the western sector, a new structure emerges: an intense twin dipole. The dipole arises from the vorticity pattern appearing at the early stages (cf. upper panel of Fig. 5), which becomes a pair of cyclones starting eastward and ejecting vorticity filaments, as follows from the top panels in Fig. 5. The fate of this dipole depends on the amplitude of the initial perturbation. For circular perturbations with $0.15 < \max|\Delta H/H| < 0.175$ the dipole stops moving eastward after some time, and engages in a reverse motion. Its northern and southern components start moving off the Equator in North-West and South-West directions, respectively, merging on their way with the smaller vortices, previously formed by the vortex filaments, and thus intensifying. For circular perturbations with $\max|\Delta H/H| > 0.175$, the dipole ejects the filaments, which

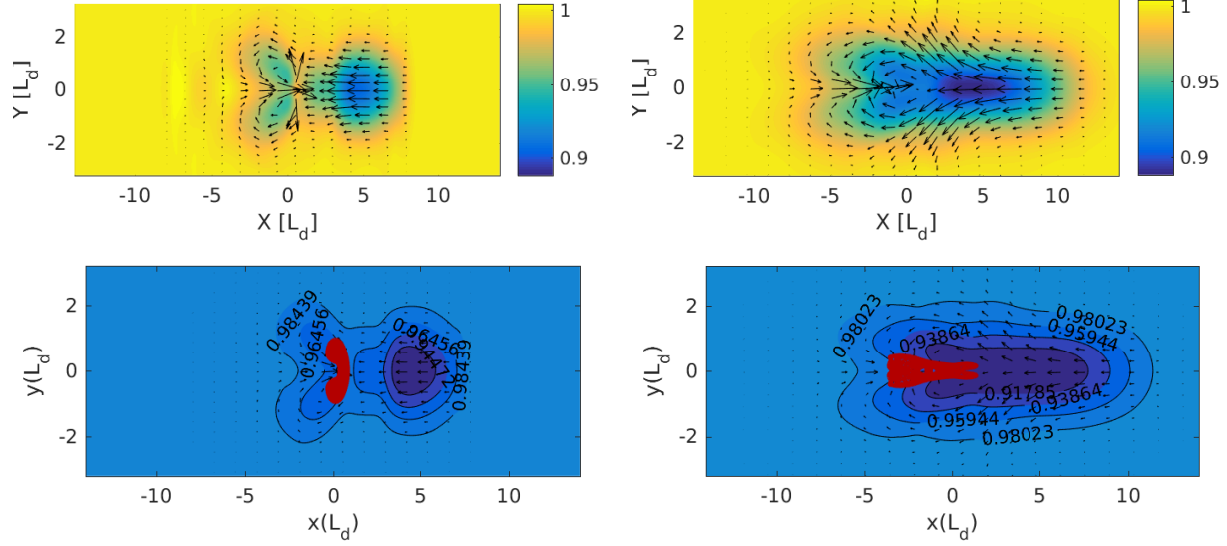


FIG. 3. From top to bottom: Snapshots at $t = 7 [1/\beta L_d]$ of pressure (colours) and velocity (arrows), and pressure (contours) and condensation (red), during adjustment of negative pressure anomalies with $\max |\Delta H/H| = 0.1$, and $a = 1$ (left column) and $a = 5$ (right column).

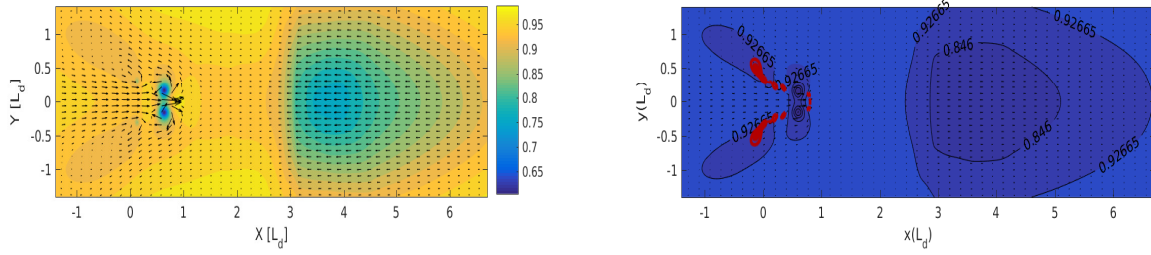


FIG. 4. Snapshots at $t = 7 [1/\beta L_d]$ of pressure (colours) and velocity (arrows) (left panel), and pressure (contours) and condensation (red) (right panel) during MC adjustment of a circular negative pressure anomaly with $\max |\Delta H/H| = 0.15$.

form secondary small-scale intense vortices left behind and moving westward and off the Equator, and continue a steady eastward motion. Both scenarios are presented in Fig. 5. The cyclonic dipole emerging from sufficiently strong anomalies keeps moving slowly eastwards, and is accompanied by a persistent condensation pattern (not shown). The structure of such dipoles strongly resembles that of the moist-convective eastward-moving equatorial modons recently described in [14].

Recently the equatorial modons were proposed as a possible dynamical explanation of the MJO in the dry shallow-water model in spherical geometry [15] and in mcRSW on the equatorial

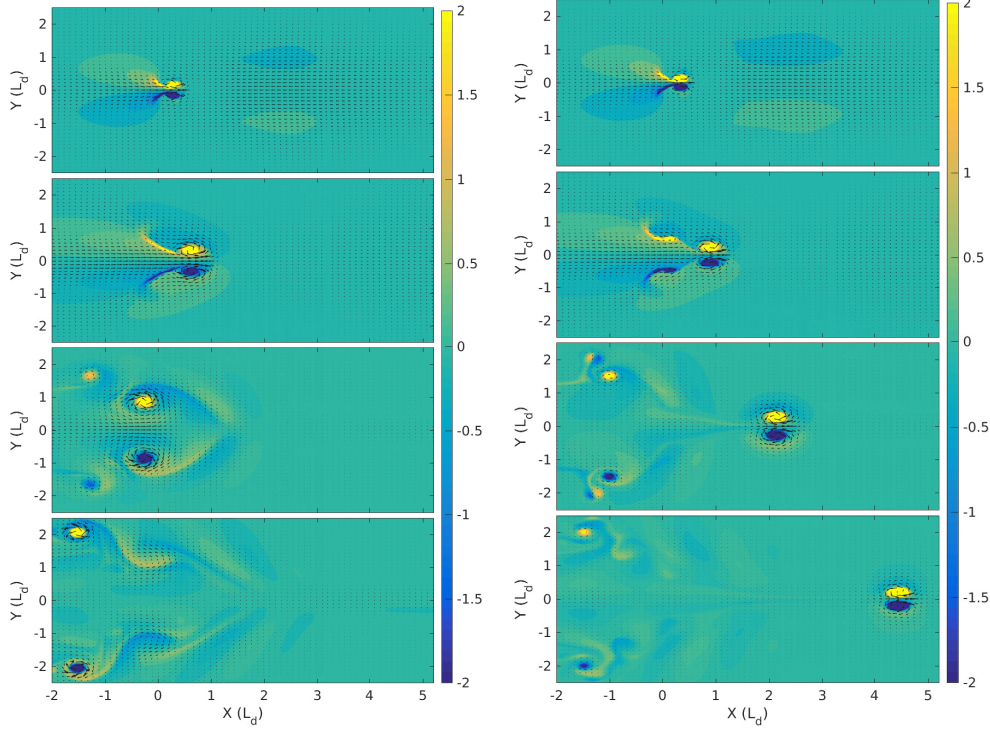


FIG. 5. Evolution at later times $t = 5, 30, 45, 75 [1/\beta L_d]$ of relative vorticity during the MC adjustment of circular negative pressure anomalies with $\max|\Delta H/H| = 0.15$ (left panel), and $\max|\Delta H/H| = 0.18$ (right panel)

beta-plane [14]. We hereby exhibit a mechanism of generation of such structures from pressure anomalies. We should stress that without the effects of moisture even strong depressions, e.g. with $\Delta H/H = 0.25$, do not produce modons. The difference between initial stages of dry and moist-convective adjustments with $\Delta H/H = 0.1$, and the role of convergence due to moist convection, are clearly seen in Fig. 6. At the very beginning, an almost isotropic inflow starts toward the center of the depression. Gradually the direction of the winds deviate from the center because of the Coriolis force. From this step on “dry” and MC adjustment follow two different evolution scenarios. While the “moon like” vorticity distributions become intensified, localized, and compactified because of moist-convection, it is not the case in “dry” adjustment. Once the emerging dipole acquires a vorticity large enough, it propagates eastward as an equatorial modon, otherwise other aforementioned scenarios are realized. The transient moist-convective scenario, when the eastward-moving dipole transforms into a pair of cyclones moving off the Equator can be of interest in the context of the genesis of tropical cyclones, and their possible link to Madden-Julian

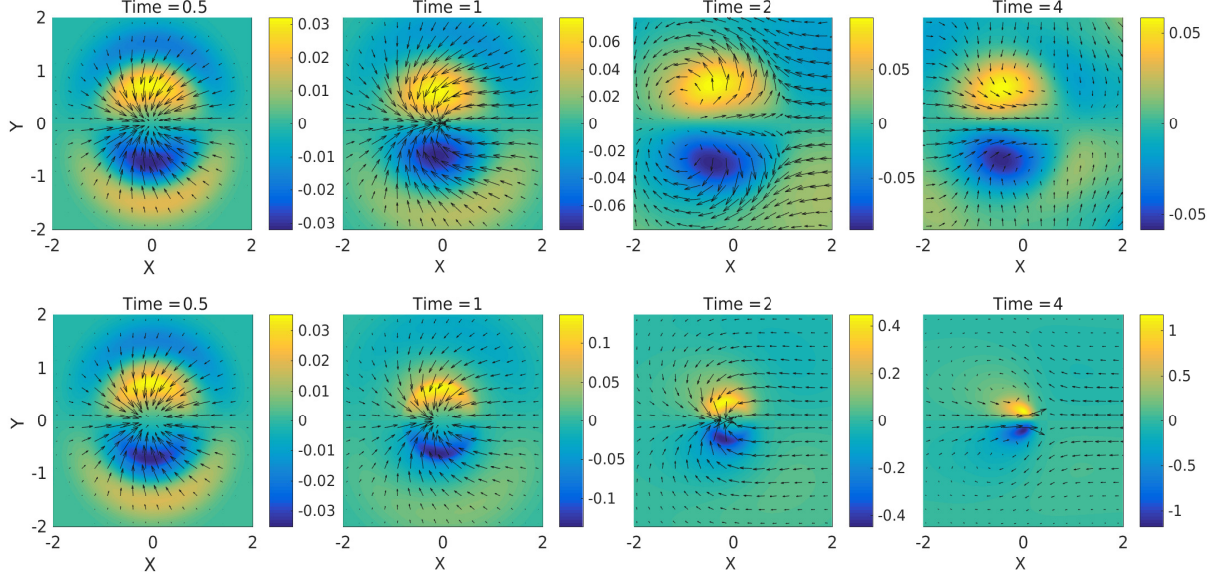


FIG. 6. Snapshots of vorticity (colors) and velocity (arrows) of the initial stages of “dry” (*top*) and MC (*bottom*) adjustments of a circular negative pressure anomaly with $\max |\Delta H/H| = 0.1$. Notice a pronounced convergence at $t = 2[1/\beta L_d]$ due to condensation in the bottom panel.

events. In the study [3] this link was investigated using the “dry” RSW model on the sphere where the MJO was modeled as a stationary, or slowly eastward-moving, meridionally elongated mass sink acting for several days. A scenario close to our results for initial anomaly of intermediate intensity, cf. Fig. 5, left panel, was observed with a stationary sink of finite duration, while eastward-moving sink was producing a scenario resembling that of the right panel of Fig. 5. We displayed results for negative initial pressure perturbations symmetric with respect to the equator. The positive ones do not produce eastward-moving dipoles in the MC case, while in the dry case the results are similar. Relaxing the symmetry with respect to Equator would lead to appearance of Yanai waves in the response of the system to initial perturbations.

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