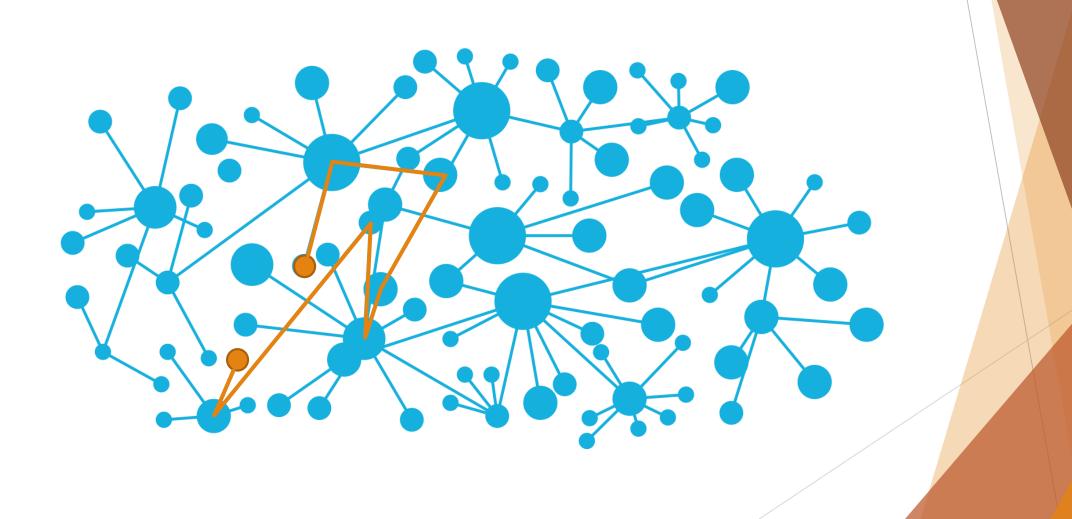
Comparing Oblivious and Robust Routing Approaches

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What is network routing?



What is network routing?

- What path should each packet take throughout the network?
- Most basic solution: shortest path
 - Fewest number of routers crossed
 - ▶ No prior knowledge of the flows that will traverse the network
- Traffic engineering
 - Optimise some metric: network congestion, end-to-end delay, etc.

Why robust/oblivious routing?

- In traffic engineering, a lot of uncertainty!
 - ▶ What is the traffic between a source and a destination?
 - ► How does it evolve over time?
- Robust/oblivious routing deals with this uncertainty!
- Ancillary question: should you invest in higher-bandwidth links?
 - Better routing leads to better hardware utilisation





How to perform traffic engineering

Using mathematical optimisation

Fundamental tool: multicommodity flow (MCF)

- Knowing the traffic the network will face, how to route it?
- For now, no uncertainty!
- How does the model work?
 - Objective function: minimise the maximum link utilisation
 - Usual formulation: path-based (column generation)
 - ► Variables: fraction of each demand in each path

Fundamental tool: multicommodity flow (MCF)

 $\min \mu$

$$\sum_{p \in \mathcal{P}_d} routing_{d,p} = 1 \quad \forall d \in \mathcal{D}$$

$$\sum_{d \in \mathcal{D}} \sum_{\substack{p \in \mathcal{P}_d:\\e \in p}} demand_d \times routing_{d,p} \leq \mu \times capacity_e \quad \forall e \in E$$

- Intuition?
 - \triangleright If μ is 1, at least one edge needs its full capacity, and has a 100% utilisation
 - \blacktriangleright If μ is 0.5, no edge needs more than half its capacity: the worst utilisation is 50%

Oblivious routing



Uncertainty? Where?



- Problem of the MCF formulation?
 - Only one traffic matrix is considered!
- In practice: the traffic is probably far from this matrix
 - ► Even from the average matrix
 - Congestion can be arbitrarily far from that of the optimal routing of the average matrix
- Oblivious routing: consider several traffic matrices
 - Only those that correspond to the worst traffic conditions
 - Only one routing for the network that works reasonably well for any traffic scenario

The new MCF

 \blacktriangleright Main difference? Multiple traffic matrices, taken in Δ (details later)

$$\min \mu$$

$$\sum_{p \in \mathcal{P}_d} routing_{d,p} = 1 \qquad \forall d \in \mathcal{D}, \boxed{\forall \mathcal{D} \in \Delta}$$

$$\sum_{d \in \mathcal{D}} \sum_{\substack{p \in \mathcal{P}_d: \\ e \in p}} demand_d \times routing_{d,p} \leq \mu \times capacity_e \times \boxed{OPT(D)} \qquad \forall e \in E, \boxed{\forall \mathcal{D} \in \Delta}$$

- OPT(D) is the optimum congestion for D
- In the end: μ is the worst ratio between the oblivious routing and optimal routing
- For the first constraint, no real impact, as long as all matrices $\mathcal{D} \in \Delta$ have the same origin-destination pairs

How to solve the new MCF?

- $ightharpoonup \Delta$ is potentially infinite: all traffic matrices that could be routed through the network
 - Only depends on the topology and the allowed origin-destination pairs
 - ▶ No dependency on the usual traffic, for instance
- How can you solve an optimisation problem with infinitely many constraints?
- Iterative algorithm:
 - Compute a routing and its ratio μ
 - Find a new traffic matrix
 - If the congestion is worse than the current μ: add the traffic matrix
 - Otherwise: converged

How to solve the new MCF?

- Focus on the subproblem: how to find a matrix D to consider?
- For each edge: solve a new optimisation program
 - ▶ Objective function: maximise the congestion in that edge
 - ► Variables: demand matrix
 - ► Constraint: current oblivious routing



How bad is oblivious routing?

- ► It is robust against **any** traffic conditions
 - You would expect that this routing can be arbitrarily bad for any scenario
 - ► Theoretical proof: the ratio between the optimal routing for any given network conditions and the behaviour of the oblivious solution to the same conditions is bounded:

 $\mathcal{O}(\log^3 n)$ where n is the number of nodes in the graph

- And in practice? According to simulations on random traffic matrices, the promises seem to hold
- ► Could still do better: not all situations are possible in practice

Robust routing



Maybe oblivious is too extreme

- Oblivious routing keeps everything under control
 - ▶ Whatever the uncertainty, it is dealt with
- In practice, not that useful
 - Operators monitor their networks and know what traffic they usually see
 - ► Future traffic can be estimated with a high accuracy (± 5%)
- \blacktriangleright How about restraining the set of traffic matrices \triangle ?
 - Robust routing



Smaller traffic matrix set \(\Delta \)

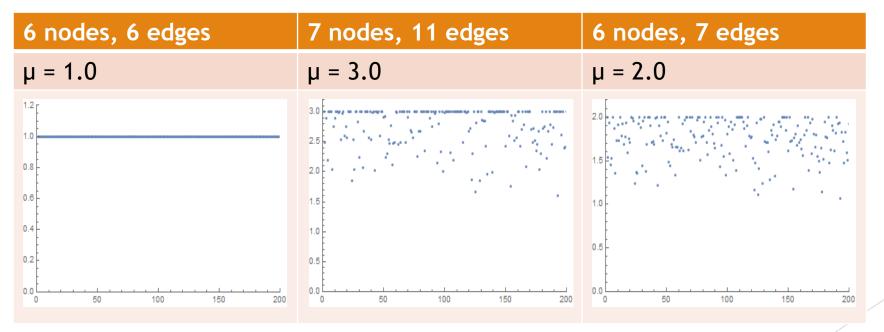
- Only real constraint:
 - ► The subproblem can still be solved "efficiently"
- Usual uncertainty sets:
 - Convex combinations of known traffic matrices
 - Box around some standard traffic (equivalent to intervals for each demand)
- All of these examples keep the subproblem convex!



Numerical results

Main theoretical property: does the bound hold?

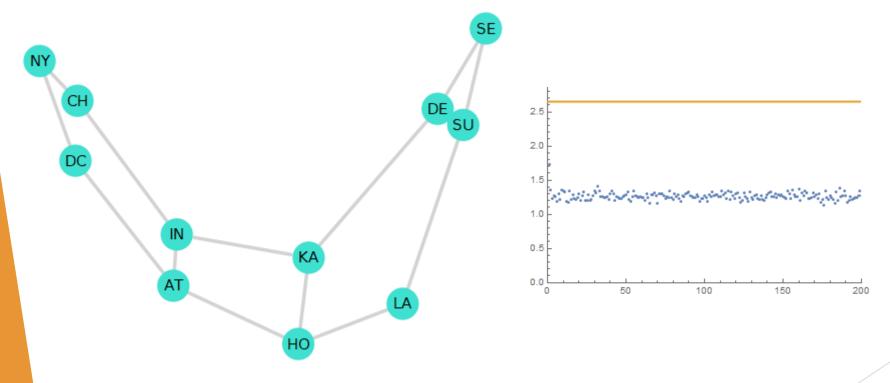
- In theory, solving the oblivious problem gives a **bound** on how bad the congestion can be when compared to optimal routing
 - ► To check whether this is the case: 200 experiments with random traffic (the origin-destination pairs are not random)
 - ► Three "synthetic" graphs



Main theoretical property: does the bound hold?

► And for a more realistic graph? Take Abilene, for instance

$$\mu = 2.65$$



More realistic uncertainty

- Consider a bounded uncertainty set Δ
 - Maximum deviation of 10% around a nominal value
 - ► Traffic can be predicted within 10% with high reliability

6 nodes, 6 edges	7 nodes, 11 edges	6 nodes, 7 edges
Oblivious $\mu = 1.0$	Oblivious $\mu = 3.0$	Oblivious $\mu = 2.0$
Robust $\mu = 1.0$	Robust $\mu = 1.65$	Robust $\mu = 1.1$
D C E	B F F F C D	B E F

Conclusions

Conclusion and future work

- Oblivious routing is a very interesting paradigm
 - ▶ Implemented at Facebook between their datacenters, for instance
 - But not really well studied for now
 - ▶ Able to provide a trade-off between robustness and performance
- ► To tune this trade-off: robust routing
- ► The end of the story? What about failures (link, node)?

References

- Theoretical results about oblivious routing:
 Räcke, Harald. "Minimizing congestion in general networks." In *The Proceedings of the 43rd Annual IEEE Symposium on Foundations of Computer Science*, 2002, pp. 43-52. IEEE, 2002.
 - ► In particular, Theorem 8
- First LP-based algorithm for oblivious routing:
 Azar, Yossi, Edith Cohen, Amos Fiat, Haim Kaplan, and Harald Räcke.
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