Type Synthesis of Kinematically Redundant 3T1R Parallel Manipulators

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TYPE SYNTHESIS OF KINEMATICALLY REDUNDANT 3T1R PARALLEL MANIPULATORS

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ABSTRACT

A kinematically redundant parallel manipulator (PM) is a PM whose degrees-of-freedom (DOF) are greater than the DOF of the moving platform. It has been revealed in the literature that a kinematically redundant PM has fewer Type II kinematic singular configurations (also called forward kinematic singular configurations, static singular configurations or parallel singular configurations) and/or constraint singular configurations than its non-redundant counterparts. However, kinematically redundant PMs have not been fully explored, and the type synthesis of kinematically redundant PMs is one of the open issues. This paper deals with the type synthesis of kinematically redundant 3T1R PMs (also called SCARA PMs or Schoenflies motion generators), in which the moving platform has four DOF with respect to the base. At first, the virtual-chain approach to the type synthesis of kinematically redundant parallel mechanisms is recalled. Using this approach, kinematically redundant 3T1R PMs are constructed using several compositional units with very few mathematical derivations. The type synthesis of 5-DOF 3T1R PMs composed of only revolute joints is then dealt with systematically. This work provides a solid foundation for further research on kinematically redundant 3T1R PMs.

KEY WORDS Parallel manipulator, Kinematically redundant parallel manipulator, Type synthesis, Virtual-chain approach, Screw theory

NOMENCLATURE

3T1R Three-DOF translation and one-DOF rotation.
$\mathcal{C}$ Order of the twist system (also the connectivity) of the moving platform of a parallel manipulator.
c Order of the wrench system of a parallel manipulator.
c^i Order of the wrench system of the i-th leg.
f^i Sum of DOF of all the joints in leg i.
$\mathcal{F}$ DOF of a parallel manipulator.
P Prismatic joint.
PKC Parallel kinematic chain.
PM Parallel manipulator.
PPPR= PPPR virtual-chain equivalent.
R Revolute joint.
Ŕ R joints within a 3T1R PM whose axes are parallel to the axes of rotation of the 3T1R motion.
 R̀ R joints except for the Ŕ joints within a leg that have parallel axes.
$\underline{\text{R}}$ Actuated R joint.
R Redundant DOF of a parallel manipulator.
R^i Redundant DOF of leg i.
V= Virtual-chain equivalent.
$\mathcal{W}$ Wrench system of a parallel manipulator.
$\mathcal{W}^i$ Leg wrench system of leg i.
ζ_0 A wrench of zero-pitch (or constraint force).
ζ_∞ A wrench of ∞-pitch (or constraint couple).

1 INTRODUCTION

During the past decade, great advances have been made on the type synthesis of parallel mechanisms (PMs). In addition to a large number of PMs (see [1–8] for example), several systematic approaches have been proposed for the type synthesis of PMs, such as the method based on the displacement group [2, 9–15], the method based on screw theory [16–19], the single-opened-chain approach [20], the virtual-chain approach [21, 22], the linear transformation approach [23] and other approaches such as [24].

While most works have been focusing on non-redundant PMs, several works on redundant PMs, including redundantly actuated PMs and kinematically redundant PMs, have also been published [4, 25–32]. In a kinematically redundant PM, the degrees-of-freedom (DOF) of the PM are greater than the DOF of the moving platform (also the connectivity of the moving platform with respect to the base). It has been revealed that a kinematically redundant PM has fewer Type II kinematic singular configurations than its non-redundant counterpart [29, 30]. Recently, a systematic study on the type synthesis of kinematically redundant translational PMs has been published in [31], where the characteristics of kinematically redundant translational PMs have been identified as compared with their associated non-redundant counterparts from the perspective of constraint singularity and Type II kinematic singularity. Kinematically redundant PMs have been classified into four categories: (a) PMs that have fewer constraint singularities and Type II kinematic singularities than the associated non-redundant PMs, (b) PMs that have fewer constraint singularities than the associated non-redundant PMs, (c) PMs that have fewer Type II kinematic singularities than the associated non-redundant PMs and (d) PMs that have the same constraint singularities and Type II kinematic singularities as the associated non-redundant PMs. Considering the needs of 3T1R PMs (also PPPR= PMs [22], SCARA PMs or Schoenflies motion generators) and the fact that a 3T1R PM may suffer from constraint singularities and Type II singularities [33], this paper will focus on the type synthesis of kinematically redundant 3T1R PMs, which is still an open issue.

In this paper, the type synthesis of kinematically redundant 3T1R PMs will be investigated. The virtual-chain approach to the type synthesis of kinematically redundant PMs [21, 22, 31] will be recalled in Section 2. In Sections 3–6, the type synthesis of 5-DOF 3T1R PMs (Fig. 1) will be dealt with in detail. Here and throughout this paper, ζ_∞ (ζ_0) denotes a wrench of ∞-pitch (0-pitch) representing a constraint couple (force). ξ_∞ (ξ_0) denotes a twist of ∞-pitch (0-pitch) representing a translation (rotation). Finally, conclusions will be drawn.

Due to space limitation, we confine ourselves to the type synthesis of 5-DOF 3T1R PMs involving only R (revolute) joints. For simplicity reasons, here and throughout this paper, $\underline{\text{R}}$ denotes actuated R joints. R̋ denotes R joints within the same 3T1R PM whose axes are parallel to the axes of rotation of the 3T1R motion. Ŕ or R̀ denotes R joints with parallel axes within a single-loop kinematic chain or a leg of PMs but are not parallel to the axes of the R̋ joints.

2 THE VIRTUAL-CHAIN APPROACH

In [21, 22, 31], a virtual-chain approach has been proposed to the type synthesis of non-redundant and kinematically redundant PMs. Families of PMs have been constructed using several classes of compositional units. One of the characteristics of the virtual-chain approach is that few derivations are needed. In [34, 35], the virtual-chain approach has been extended to the type synthesis of PMs with multiple operation modes. This section will recall the virtual-chain approach to the type synthesis of kinematically redundant PMs.

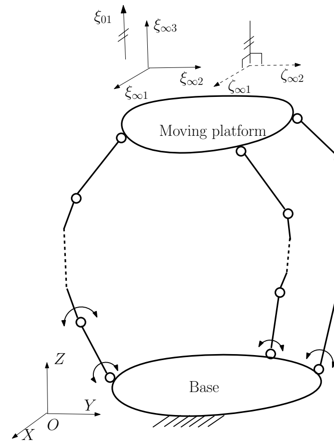


Fig. 1 A 5-DOF PPPR= PMs.

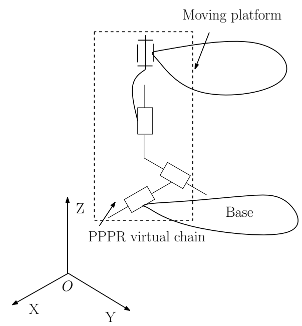


Fig. 2 PPPR virtual chain.

2.1 Virtual chain

Different applications may have different motion patterns. Virtual chains were introduced in [22] to represent the motion patterns of PMs. For example, 3T1R motion, one of the commonly used motion patterns, can be represented with a PPPR virtual chain (Fig. 2). For clarity, a virtual chain is encircled by a rectangle drawn with dashed lines. The wrench system of the PPPR virtual chain is a 2-ζ_∞-system.

2.2 Instantaneous mobility analysis of kinematic chains

The instantaneous mobility analysis of kinematic chains is the starting point in the type synthesis of PMs if the virtual-chain approach is applied.

During the past few years, several formulas (see [22, 23, 36] for example) have been proposed to calculate the DOF of PMs. The instantaneous mobility criteria proposed in [22] based on screw theory is recalled below.

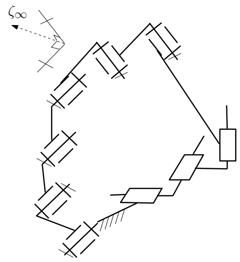


Fig. 3 R̀R̀R̀R̀ŔŔPPP single-loop kinematic chain.

2.2.1 Single-loop kinematic chain The instantaneous mobility, $\mathcal{F}$, of a single-loop kinematic chain is

$$\mathcal{F} = f - 6 + c. \tag{1}$$

where f denotes the sum of DOF of all the joints, and c denotes the order of the common wrench system, which is the reciprocal system of the twist system composed of all the joint twists, of the single-loop kinematic chain.

Let us take the R̀R̀R̀R̀ŔŔPPP single-loop kinematic chain (Fig. 3) as an example. For this kinematic chain, the common wrench system is a 1-ζ_∞-system. The basis wrench of the 1-ζ_∞-system is the ζ_∞ whose direction is perpendicular to the axes of all the R joints. One then has $c = 1$. In addition, one has $f = 9$. Using Eq.(1), one obtains the DOF of the kinematic chain as

$$\mathcal{F} = f - 6 + c = 9 - 6 + 1 = 4.$$

2.2.2 Parallel kinematic chains Consider an m-legged parallel kinematic chain (PKC) (Fig. 4). The wrench system of a PKC [Fig. 4(a)] is equal to the sum of the leg-wrench systems [Fig. 4(b)], i.e.,

$$\mathcal{W} = \sum_{i=1}^{m} \mathcal{W}^i \tag{2}$$

Let $\mathcal{C}$ denote the order of the twist system of the PKC (also the connectivity). R^i and R denote the redundant DOF of leg i and the PKC respectively. c and $\mathcal{F}$ denote the order of the wrench system, $\mathcal{W}$, and DOF of the PKC. c^i and f^i denote the order of the leg wrench system, $\mathcal{W}^i$, and the sum of DOF of all the joints in leg i. One has

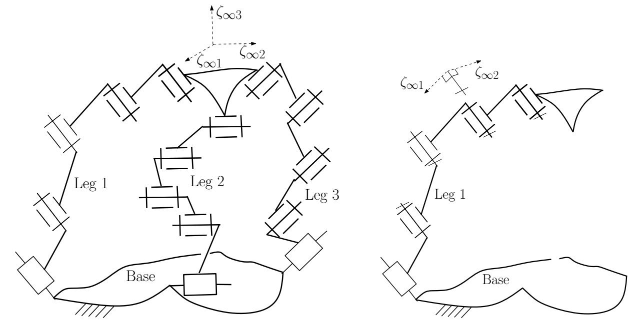
$$\mathcal{C} = 6 - c \tag{3}$$

$$R = \sum_{i=1}^{m} R^i \tag{4}$$

$$R^i = f^i - (6 - c^i) = f^i - 6 + c^i \tag{5}$$

The DOF of the PKC is

$$\mathcal{F} = \mathcal{C} + \mathcal{R} = \mathcal{C} + \sum_{i=1}^{m} R^i = 6 - c + \sum_{i=1}^{m} R^i \tag{6}$$



(a) The PKC. (b) Leg 1.

Fig. 4 3-PŔŔŔŔ PKC.

The DOF obtained using Eq. (6) is usually instantaneous. If c, c^i and R^i are the same in different general configurations, the DOF is full-cycle.

In addition to the mobility, another important index, the number of overconstraints or redundant constraints Δ, of a PKC is defined as

$$\Delta = \sum_{i=1}^{m} c^i - c \tag{7}$$

Consider the 3-PŔŔŔŔ PKC [32] shown in Fig. 4. In this PKC, all the axes of the R joints within a same leg are parallel. The direction of a P joint is not perpendicular to the axes of the R joints within the same leg. Not all the axes of the R joints on the moving platform are parallel. The wrench system of each leg is a 2-$\boldsymbol{\zeta}_\infty$-system [Fig. 4(b)]. The wrench system of the PKC is the 2-$\boldsymbol{\zeta}_\infty$-system [Fig. 4(a)]. One has $c^i = 2$, $c = 3$, $R^i = 5 - (6 - 2) = 1$ and $m = 3$. Then one obtains

$$\mathcal{C} = 6 - c = 3$$

$$\mathcal{F} = \mathcal{C} + \sum_{i=1}^{3} R^i = 6$$

and

$$\Delta = \sum_{i=1}^{3} c^i - c = 6 - 3 = 3.$$

To facilitate the type synthesis of PMs, we substitute Eq. (7) into Eq. (6) and then obtain

$$\sum_{i=1}^{m} c^i = 6 - \mathcal{C} + \Delta = 6 - \mathcal{F} + R + \Delta \tag{8}$$

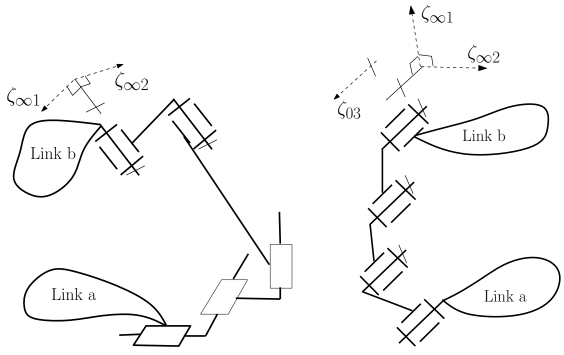
Equations (4), (5) and (2) can then be respectively rewritten as

$$\sum_{i=1}^{m} R^i = R \tag{9}$$

$$f^i = 6 + R^i - c^i \tag{10}$$

$$\sum_{i=1}^{m} \mathcal{W}^i = \mathcal{W}. \tag{11}$$

Equations (8)–(11) will be used in the type synthesis of PMs.



(a) $(\text{PPPRR})_A$ (b) $(\text{RRRR})_E$

Fig. 5 Example compositional units.

2.3 Compositional units

If one uses only the instantaneous mobility criteria to perform the type synthesis of PMs, there is no guarantee that the PMs can undergo the desired finite motion. Compositional units are therefore identified in [22] using which PMs with full-cycle mobility can be constructed directly.

A compositional unit is a serial kinematic chain with the following kinematic characteristic: *In any configuration of a compositional unit, its wrench system always includes a specified number of independent wrenches of zero-pitch or infinity-pitch.* The compositional units will be used in the construction of single-loop kinematic chains and PKCs. Only two classes of compositional units (Fig. 5) that will be used in the type synthesis of PPPR= PMs are recalled below.

- Parallelaxis compositional units. A parallelaxis compositional unit is a serial kinematic chain composed of at least one R joint and at least one P joint in which the axes of all the R joints are parallel and not all the directions of the P joints are perpendicular to the axes of the R joints. The characteristic of a compositional unit of this class is that the axes of all the R joints are always parallel. The wrench system of this compositional unit always includes a 2-$\boldsymbol{\zeta}_\infty$-system. The 2-$\boldsymbol{\zeta}_\infty$-system is composed of all the $\boldsymbol{\zeta}_\infty$ whose directions are perpendicular to all the axes of the R joints [Fig. 5(a)]. A parallelaxis compositional unit is denoted by $()_A$.

- Planar compositional units. A planar compositional unit is a serial kinematic chain composed of at least two R and/or P joints which include at least one R joint and in which all the links are moving along parallel planes. In a compositional unit of this class, the axes of all the R joints are parallel, and the direction of each P joint is perpendicular to all the axes of the R joints. The wrench system of this compositional unit always includes a 2-$\boldsymbol{\zeta}_\infty$-1-$\boldsymbol{\zeta}_0$-system. The 2-$\boldsymbol{\zeta}_\infty$-1-$\boldsymbol{\zeta}_0$-system is composed of all the $\boldsymbol{\zeta}_0$ whose axes are parallel to the axes of the R joints as well as all the $\boldsymbol{\zeta}_\infty$ whose directions are perpendicular to the axes of all the R joints [Fig. 5(b)]. A planar compositional unit is denoted by $()_E$.

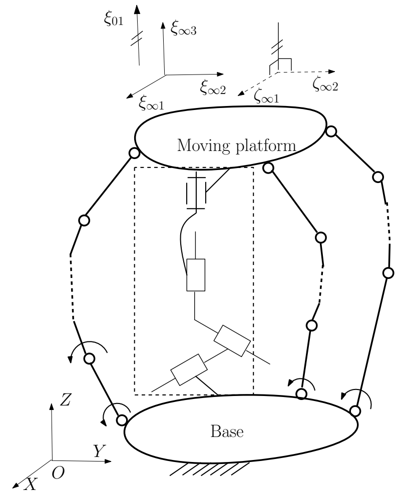


Fig. 6 A 5-DOF PPPR= parallel mechanisms with a PPPR virtual chain added.

2.4 Conditions for a PKC to be an $\mathcal{F}$-DOF V= PKC

Let us consider an $\mathcal{F}$-DOF V= PKC (Fig. 1). When one connects the base and the moving platform of a PKC by an appropriate virtual chain (Fig. 2) matching the motion pattern of the PKC, the function of the PKC is not affected (Fig. 6). It is apparent that a PKC is an $\mathcal{F}$-DOF V= PKC if it satisfies the following three conditions:

(1) The redundant DOF, R^i, of the legs satisfy

$$\sum_{i=1}^{m} R^i = \mathcal{F} - \mathcal{C} = \mathcal{F} + c - 6 \tag{12}$$

(2) Each leg of the PKC and the same virtual chain constitute a $(\mathcal{C} + R^i)$-DOF single-loop kinematic chain.

(3) The wrench system of the PKC is the same as that of the virtual chain in any general configuration.

Conditions (1) and (2) for V= PKCs guarantee that the moving platform can undergo the V= motion and the DOF of the PM is at least $\mathcal{F}$, while Condition (3) for V= PKCs further guarantees that the DOF of the moving platform is the same as that of the virtual chain and the DOF of the PKC is $\mathcal{F}$.

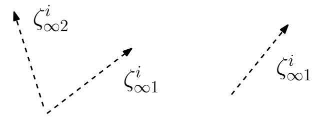
2.5 Systematic type synthesis of V= parallel mechanisms

Based on the above conditions, a general systematic procedure can be proposed for the type synthesis of $\mathcal{F}$-DOF V= PMs. The proposed procedure can be divided into four steps, namely:

Step 1 Determination of the combinations of leg structural parameters. Step 1 can be performed using Eqs. (11), (8) and (9).

Step 2 Type synthesis of legs for V= PKCs.

According to Condition (2) of V= PKCs, each leg, together with a virtual chain, forms a $(\mathcal{C}+R^i)$-DOF single-loop kinematic chain. For a set of leg structural parameter c^i and R^i



(a) 2-ζ^i_∞ system (b) 1-ζ^i_∞ system

Fig. 7 Leg-wrench systems ($c^i > 0$) of PPPR= PKCs.

obtained in Step 1, the types of legs can be obtained by constructing $(\mathcal{C} + R^i)$-DOF single-loop kinematic chains using the compositional units and then removing the virtual chain from these single-loop kinematic chains.

Step 3 Assembly of legs for V= PKCs.

The m-legged V= PKCs can be generated by assembling m legs for V= PKCs that are obtained in Step 2 according to the combinations of leg structural parameters obtained in Step 1, such that the sum of the leg-wrench systems is the same as that of the virtual chain (Condition (3) for V= PKCs). The conditions, if any, can be easily satisfied by inspection.

Step 4 Selection of actuated joints.

V= PMs can be generated by selecting the actuated joints for each V= PKC obtained in Step 3.

3 STEP 1: DETERMINATION OF THE COMBINATIONS OF LEG STRUCTURAL PARAMETERS

For the PPPR= PKC and a specified number of overconstraints Δ, the combinations of leg structural parameters of a PKC can be determined using Eqs. (11), (8) and (9).

In any general configuration, the wrench system of a PPPR= PKC is the same as that of its PPPR virtual chain, i.e., a 2-$\boldsymbol{\zeta}_\infty$-system (Eq. (11)). It then follows that any leg-wrench system with order $c^i > 0$ of a PPPR= PKC is the 2-$\boldsymbol{\zeta}_\infty$-system or 1-$\boldsymbol{\zeta}_\infty$-system (Fig. 7). Since all the wrenches in the wrench systems, $\mathcal{W}$, and the leg-wrench systems, $\mathcal{W}^i$, are of the same pitch, the combinations of leg-wrench systems can be simply represented by the combinations of the orders, c^i, of leg-wrench systems.

The combinations of the orders, c^i, of leg-wrench systems can be determined by solving Eq. (8). Using Eq. (9) , one can further determine the combination of redundant DOF of legs. Table 1 shows all the combinations of leg structural parameters c^i and R^i for $m(2 \le m \le \mathcal{C})$-legged 5-DOF PPPR= PKCs. The combinations of leg structural parameters corresponding to all possible values of Δ have been listed for completeness.

From Table 1, one obtains that there are six sets of structural parameters, c^i and R^i, of legs for 5-DOF PPPR= PKCs (Table 2).

4 STEP 2: TYPE SYNTHESIS OF LEGS

Once the structural parameters of legs have been obtained, the types of legs with $c^i > 0$ can be constructed using the compositional units as follows[1]. The type synthesis of legs with structural

[1]Types of legs with $c^i = 0$ have been well documented in the literature and are therefore not discussed in this paper. In addition, the types of legs with inac-

Table 1 Combinations of c^i and R^i for m-legged 5-DOF PPPR= PKCs (Case $2 \le m \le 3$)

m	c	Δ	Leg 1		Leg 2		Leg 3			
			c^1	R^1	c^2	R^2	c^3	R^3	c^4	R^4
2	2	2	2	1	2	0				
		1	2	1	1	0				
				0		1				
		0	2	1	1	0				
				0		1				
			1	1	1	0				
3	2	4	2	1	2	0	2	0		
		3	2	1	2	0	1	0		
				0		0		1		
		2	2	1	2	0	0	0		
				0		0		1		
			2	1	1	0	1	0		
				0		1		0		
		1	2	1	1	0	0	0		
				0		1		0		
				0		0		1		
			1	1	1	0	1	0		
		0	2	1	0	0	0	0		
				0		1		0		
			1	1	1	0	0	0		
				0		0		1		
4	2	6	Omitted							
		5	Omitted							
		4	Omitted							
		3	Omitted							
		2	2	1	2	0	0	0	0	0
				0		0		1	0	0
			2	1	1	0	1	0	0	0
				0		1		0		0
				0		0		0		1
			1	1	1	0	1	0	1	0
		1	2	1	1	0	0	0	0	0
				0		1		0		0
				0		0		1		0
			1	1	1	0	1	0	0	0
				0		0		0		1
		0	2	1	0	0	0	0	0	0
				0		0		0		1
			1	1	1	0	0	0	0	0
				0		0		1		

Table 2 Structural parameters, c^i and R^i, of legs for 5-DOF PPPR= PKCs.

R^i	0			1		
c^i	2	1	0	2	1	0
f^i	4	5	6	5	6	7

parameters $c^i = 1$ and $R^i = 1$ will be used as an example to illustrate the synthesis procedure.

4.1 Step 2a: Calculate the number of joints in a leg

The number of joints in a leg with structural parameters c^i and R^i can be obtained using Eq. (10). For example, for legs with structural parameters $c^i = 1$ and $R^i = 1$, one has

$$f^i = 6 + R^i - c^i = 6 + 1 - 1 = 6.$$

All the numbers of joints of legs for 5-DOF PPPR= PKCs obtained are shown in the third row of Table 2.

4.2 Step 2b: Type synthesis of $(\mathcal{C} + R^i)$-DOF single-loop kinematic chains

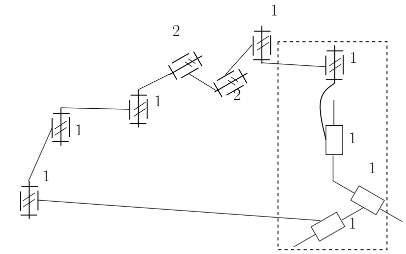
A $(\mathcal{C} + R^i)$-DOF single-loop kinematic chain that contains the PPPR= virtual chain and has $\mathcal{W}^i$ as its common wrench system can be constructed using the compositional units. According to [22], a 5-DOF single-loop kinematic chain with a 1-$\boldsymbol{\zeta}_\infty$-system as its common wrench system is composed of two spatial Parallelaxis and/or planar compositional units. Figure 8 shows two of the 5-DOF single-loop kinematic chains. For example, the R̋R̋R̋R̀R̀R̋V single-loop kinematic chain shown in Fig. 8(a) is composed of one spatial Parallelaxis compositional unit, R̋R̋R̋R̀R̀R̋V denoted by 1, and one planar compositional unit, R̀R̀ denoted by 2. The basis wrench of the 1-$\boldsymbol{\zeta}_\infty$-system of the single-loop kinematic chain is the $\boldsymbol{\zeta}_\infty$ whose direction is perpendicular to the axes of all the R joints.

4.3 Step 2c: Discard single-loop kinematic chains in which the twists of all the joints but the PPPR= virtual chain are reciprocal to a ζ that does not belong to the wrench system of the virtual chain.

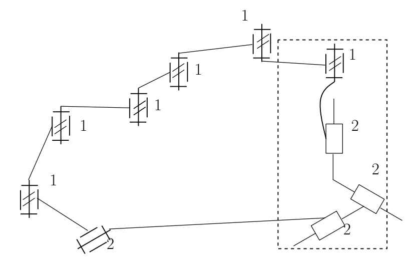
In a $(\mathcal{C} + R^i)$-DOF single-loop kinematic chain that contains the PPPR= virtual chain and has $\mathcal{W}^i$ as its common wrench system obtained in Step 2b, the wrench system of the serial kinematic chain composed of all the joints except the PPPR= virtual chain contains $\mathcal{W}^i$. In order to ensure the wrench system of the serial kinematic chain composed of all the joints except the PPPR= virtual chain is $\mathcal{W}^i$, one needs to discard the single-loop kinematic chains in which the twists of all the joints but the PPPR= virtual chain are reciprocal to a $\boldsymbol{\zeta}$ that does not belong to the wrench system of the PPPR= virtual chain.

For example, the R̀R̋R̋R̋R̋R̋V kinematic chain [Fig. 8(b)] should be discarded since the twists of the six R-joints are reciprocal to any $\boldsymbol{\zeta}_0$ with its axis parallel to the axes of the R̋ joints and intersects the axis of the R̀ joint. Such a $\boldsymbol{\zeta}_0$ does not belong to the wrench system of the PPPR virtual chain.

tive joints [18,22] will not be discussed in this paper due to space limitation.



(a) R̋R̋R̋R̀R̀R̋V.



(b) R̀R̋R̋R̋R̋R̋.

Fig. 8 Two 5-DOF single-loop kinematic chains.

4.4 Step 2d: Generation of types of legs

In each of the $(\mathcal{C} + R^i)$-DOF single-loop kinematic chains obtained in step 2c, the wrench system of the serial kinematic chain composed of all the joints except the PPPR= virtual chain is $\mathcal{W}^i$. Therefore, the types of legs for PPPR= PMs can be readily obtained by removing the virtual chains from the $(\mathcal{C} + R^i)$-DOF single-loop kinematic chains.

In removing the virtual chain, the specific geometric conditions that the joint axes or directions satisfy before removing the virtual chain need to be satisfied by the leg. Such conditions can be easily indicated by the notations introduced before.

For example, by removing the PPPR virtual chain from the 5-DOF R̋R̋R̋R̀R̀R̋V single-loop kinematic chain [Fig. 8(a)], one obtains an R̋R̋R̋R̀R̀R̋ leg for PPPR= PKCs (Fig. 9(a)). The wrench system of this leg is a 1-$\boldsymbol{\zeta}_\infty$-system, the basis wrench of which is the $\boldsymbol{\zeta}_\infty$ whose direction is perpendicular to the axes of all the R joints.

A number of legs for 5-DOF PPPR= PMs have been obtained. The geometric conditions that the legs satisfy are summarized in Table 3. One can easily list the types of legs according to these geometric conditions. For example, in legs for PPPR= PMs with structural parameters $c^i = 1$ and $R^i = 1$, one can obtain the following 11 types of legs composed of six R joints.

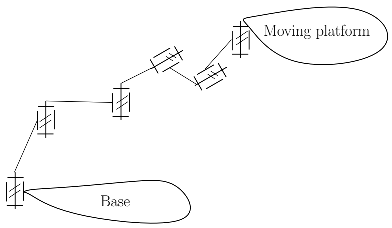
R̋R̋R̋R̋R̀R̀, R̋R̋R̋R̀R̀R̋, R̋R̋R̀R̀R̋R̋, R̋R̀R̀R̋R̋R̋, R̀R̀R̋R̋R̋R̋, R̋R̋R̋R̀R̀R̀, R̋R̋R̀R̀R̀R̋, R̋R̀R̀R̀R̋R̋, R̀R̀R̀R̋R̋R̋, R̋R̋R̀R̀R̀R̀ and R̋R̀R̀R̀R̀R̋.

5 STEP 3: ASSEMBLY OF LEGS

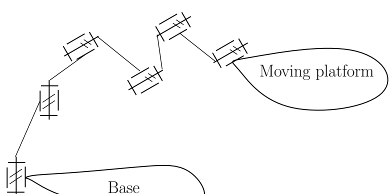
The type synthesis of PPPR= PKCs consists in obtaining the types of PPPR= PKCs by assembling the legs obtained in Step 2 according to the combinations of leg structural parameters obtained in Step 1. In the assembly of legs, conditions that guarantee that the sum of all the leg-wrench systems constitutes a 2-ζ_∞-system (Conditions (3) for V= PKCs) should be revealed, if any [22].

Table 3 Legs for PPPR= PKCs (Cases R^i=0 or 1)

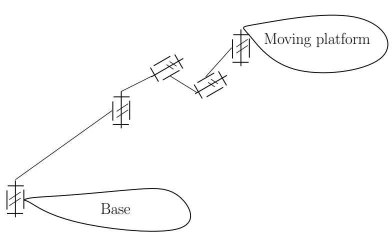
R^i	Class	No.	Type	Description	Leg-wrench system
0	5R	1	R̋R̋R̋R̀R̀	The axes of two or three successive R̀ joints within a leg are parallel, while the axes of the remaining joints are R̋ joints whose axes are parallel to the axes of rotation in the 3T1R PM.	1-ζ_∞-system
		2	R̋R̋R̀R̀R̋		
		3	R̋R̀R̀R̋R̋		
		4	R̀R̀R̋R̋R̋		
		5	R̋R̋R̀R̀R̀		
		6	R̋R̀R̀R̀R̋		
		7	R̀R̀R̀R̋R̋		
1	6R	8	R̋R̋R̋R̋R̀R̀		
		9	R̋R̋R̋R̀R̀R̋		
		10	R̋R̋R̀R̀R̋R̋		
		11	R̋R̀R̀R̋R̋R̋		
		12	R̀R̀R̋R̋R̋R̋		
		13	R̋R̋R̋R̀R̀R̀		
		14	R̋R̋R̀R̀R̀R̋		
		15	R̋R̀R̀R̀R̋R̋		
		16	R̀R̀R̀R̋R̋R̋		
		17	R̋R̋R̀R̀R̀R̀		
		18	R̋R̀R̀R̀R̀R̋		



(a) R̋R̋R̋R̀R̀R̋ ($R^i = 1$).



(b) R̋R̋R̀R̀R̀R̀ ($R^i = 1$).



(c) R̋R̋R̀R̀R̋ ($R^i = 0$).

Fig. 9 Several legs for PPPR= PMs with a 1-ζ_∞-system.

For example, one combination of leg structural parameters for the 5-DOF PPPR= PMs are $c^1 = 1$, $R^1 = 1$, $c^2 = 1$, $R^2 = 0$, $c^3 = 1$, $R^3 = 0$, $c^4 = 1$, and $R^4 = 0$ (see the 10^{th} row from the last in Table 1). According to Step 2, one type of the legs

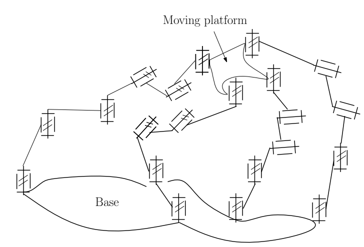


Fig. 10 R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PKC.

with structural parameter $c^1 = 1$ and $R^1 = 1$ is an R̋R̋R̋R̀R̀R̋ leg (Fig. 9(a)). One type of the legs with structural parameter $c^1 = 1$ and $R^1 = 0$ is an R̋R̋R̀R̀R̋ leg (Fig. 9(c)). By assembling oneR̋R̋R̋R̀R̀R̋ leg and three R̋R̋R̀R̀R̋ legs, one obtains the R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PKC shown in Fig. 10. The leg wrench system of leg i is a 1-ζ_∞-system the basis wrench of which is the ζ^i_∞ whose direction is perpendicular to the axes of all the R joints. The direction of ζ^i_∞ may vary with the position of the moving platform. In order to guarantee that the sum of ζ^1_∞ and ζ^2_∞ constitutes the 2-ζ_∞-system in a general configuration, the following case should be avoided: The axes of all the R joints on the moving platform (base) are parallel, and the axes of all the R joints on the base (moving platform) are parallel but are not parallel to the axes of R joints on the moving platform (base).

6 STEP 4: SELECTION OF ACTUATED JOINTS

The selection of actuated joints for PPPR= PMs involves finding all the possible PPPR= PMs for a given PPPR= PKC and removing the cases for which the set of actuated joints is invalid.

Since finding all the candidate PPPR= PMs for a PPPR= PKC is trivial, only the validity condition that the actuated joints of PMs need to satisfy is given below:

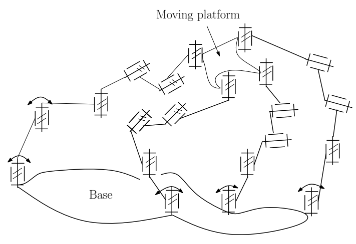


Fig. 11 R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PM.

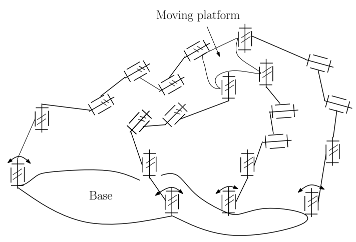


Fig. 12 R̋R̋R̀R̀R̀R̀-3-R̋R̋R̀R̀R̋ 5-DOF PPPR= PM with an idle DOF.

A set of $\mathcal{F}$ actuated joints for an $\mathcal{F}$-DOF PPPR= PM is valid if and only if, in a general configuration, all the basis wrenches of the actuation wrench systems $\mathcal{W}^i_{\not D}$ ($i = 1, 2, \cdots, m$), of all the legs, together with the basis wrenches, $\zeta_{\infty 1}$ and $\zeta_{\infty 2}$, of the wrench system of the PPPR= PKC constitute a basis of the 6-system.

As defined in [31], the actuated wrench system, $\mathcal{W}^i_{\not D}$, of leg i is composed of all the wrenches that are reciprocal to the twists of all the joints in leg i except the actuated joints and do not belong to the wrench system, $\mathcal{W}^i$, of the leg i.

The R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PM (Fig. 11) is one of the potential 5-DOF PPPR= PMs obtained from the R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PKC. The leg actuation wrench system of each leg is a 1-ζ_0-system the basis wrench of which is the $\zeta^i_{0\not D}$ whose axis intersects the axes of all the unactuated R joints in the same leg. The axes of $\zeta^i_{0\not D}$ may vary with the pose of the moving platform. It can be verified that $\zeta^1_{0\not D}$, $\zeta^2_{0\not D}$, $\zeta^3_{0\not D}$ and $\zeta^4_{0\not D}$, together with ζ^1_∞ and ζ^2_∞, constitute a basis of the 6-wrench system in a general configuration. Therefore, the R̋R̋R̋R̀R̀R̋-3-R̋R̋R̀R̀R̋ PPPR= PM is valid.

A large number of 5-DOF PPPR= PMs can be obtained following the above procedure. It is noted that there may exist PMs with idle DOF, which refer to $\mathcal{F}$-DOF PMs in which less than $\mathcal{F}$ actuated joints can be used to fully control the motion of the moving platform. For example, the R̋R̋R̀R̀R̀R̀-3-R̋R̋R̀R̀R̋ 5-DOF PPPR= PM (Fig. 12), a set of four R joints can be used to fully control the motion of the moving platform [32]. One of the Ŕ joints in the R̋R̋R̀R̀R̀R̀ leg has an idle DOF.

7 CONCLUSIONS

The type synthesis of 5-DOF PPPR= PMs has been studied and several types of 5-DOF PPPR= PMs have been obtained. The class of PPPR= PMs that have fewer constraint singularities and Type II kinematic singularities than their associated non-redundant PMs, such as the R̋R̋R̋R̀R̀R̋ -3-R̋R̋R̋R̀R̀R̋ PPPR= PM, deserve further investigation.

This work, together with [31], lays the foundation to the optimal type synthesis of kinematically redundant PMs and may contribute to the development of energy-efficient reconfigurable PMs.

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