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FULL PAPER

A Topological Data Analysis perspective on noncovalent interactions in relativistic calculations

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Abstract

Topological Data Analysis (TDA) is a powerful mathematical theory, largely unexplored in theoretical chemistry. In this work we demonstrate how TDA provides new insights into topological features of electron densities and reduced density gradients, by investigating the effects of relativity on the bonding of the Au₄-S-C₆H₄-S'-Au'₄ molecule. Whereas recent analyses of this species carried out with the Quantum Theory of Atoms-In-Molecules (a previous study) concluded, from the emergence of new topological features in the electron density, that relativistic effects yielded noncovalent interactions between gold and hydrogen atoms, we show from their low persistence values (which decrease with increased basis set size) these features are not significant. Further analysis of the reduced density gradient confirms no relativity-induced noncovalent interactions in Au₄-S-C₆H₄-S'-Au'₄. We argue TDA should be integrated into electronic structure analysis methods, and be considered as a basis for the development of new topology-based approaches.

KEYWORDS

noncovalent interactions, quantum chemistry, relativistic effects, Topological Data Analysis

1 | INTRODUCTION

A well-exploited consequence of the Hohenberg-Kohn theorem^[1] is that the exact, nondegenerate, ground-state electron density (ED) contains complete molecular information and therefore encodes all interactions in molecular systems. In order to retrieve information on interatomic interactions from the ED, this property is subjected to various analysis schemes, which often involve visual exploration. This is, however, both a technical challenge and a test of chemical concepts. The former relates to a nontrivial choice of analysis techniques of scientific data. For instance, *direct* visualization schemes (an example of which is representing 3D scalar fields as isosurfaces), although intuitive and simple in realization, often lead to images that are cluttered and difficult to grasp, especially when aimed to illustrate complex scientific questions. On the other hand, the so-called *feature-based* visualization methods are designed to recapture only the meaningful features of data, however, are more complex to comprehend and therefore not as widely used.^[2,3]

Topological Data Analysis (TDA) is a recent research field, at the interface between mathematics and computer science, which suggests to investigate data based on its structure.^[4] Rooted in sound theoretical settings such as Morse theory,^[5] TDA provides to users a set of tools for the robust extraction of topological features in their data. This includes, for instance, critical points, integral lines, separating surfaces, voids, and so forth. Moreover, Persistent Homology (PH)^[6] brings an appealing framework for measuring the salience of topological features in the data, which is well established both at a theoretical and practical level.^[7] In practice, it allows users to discriminate important features from nonsignificant configurations and provides the necessary basis for multiscale data analysis.^[8] Recent years have witnessed some successful applications of

TDA to chemistry, where it helped to understand hydrogen-bonding networks in ion and molecule aggregates,^[9–11] aqueous solubility of molecules,^[12,13] stability of fullerenes,^[14] molecular transition pathways,^[15] conformational spaces of molecules,^[16] or the bonding patterns in molecular systems.^[17–20]

In this manuscript, we focus on the use of TDA to investigate noncovalent interactions (NCIs) for systems containing heavy elements. The description of NCIs is by itself a challenge for existing chemistry concepts, and has been a driving force for the development of new analysis techniques.^[21,22] For systems containing heavy elements, there is the additional challenge of including the effects of relativity in the electronic structure as these can qualitatively change the properties of a molecular system.^[23–27] The bonding analysis in such systems is especially demanding if the spin-orbit coupling (SOC) is non-negligible.^[28] There is currently intense activity in the development and practical use of new analysis tools and methods that can reliably characterize chemical bonding irrespective of the strength of SOC, and as we hope to show, TDA is a promising approach.

In order to illustrate and further extend the idea of using TDA for interatomic interactions, we have chosen a molecular system studied recently by Anderson et al,^[29] Au₄-S-C₆H₄-S'-Au'₄ (Figure 1), which is interesting in the context of gold-sulfur^[30–34] and gold-hydrogen^[35–39] interactions due to their ubiquity in metalorganic complexes and on interface—a proof of many faces of the fascinating chemistry of gold.^[40–43] A remarkable finding of the study of Anderson et al^[29] is that it characterized the emergence of NCIs between gold and hydrogen centers, using the Quantum Theory of Atoms-In-Molecules (QTAIM).^[44,45]

QTAIM is a very popular analysis tool,^[46] which describes the stationary points of the ED gradient and interprets these points as chemical objects—maxima correspond to nuclei (although some molecules exhibit non-nuclear attractors^[47]), while saddle points are identified as *bond critical points (BCP)*, *ring critical points (RCP)*, or *cage critical points (CCP)*. ED gradient paths emanated from a saddle point, terminating at two maxima and traced along maximum ED, originally named *bond paths*,^[48] are interpreted as interaction lines between atoms and the areas of space delimited by zero-flux surfaces of ED gradient define *atomic basins*. However, recent ambiguous results raised a discussion on the limitations of this method,^[49–58] an important point in this debate being precisely the failure of QTAIM to study NCIs.

While it can be particularly appealing to use QTAIM in this context, especially due to its availability in many standard quantum chemistry codes and supposedly simple interpretation of topological features of ED, it is known that the latter is not a sensitive descriptor of subtle interatomic interactions, due to its exponentially decaying behavior away from the nuclei. This observation has led to the development of other descriptors, better suited for studying NCIs, such as the NCI index^[59–62] based on the reduced density gradient (RDG), $s(\vec{r})$:

$$s(\vec{r}) = \frac{1}{2(3\pi^2)^{1/3}} \frac{|\nabla\rho(\vec{r})|}{\rho(\vec{r})^{4/3}}. \quad (1)$$

At critical points of ED where $\nabla\rho(\vec{r}) = 0$, RDG reaches its minimum value, $s(\vec{r}) = 0$, therefore tracing the minima of $s(\vec{r})$ can be used to locate the interaction sites in molecular systems. The minima of $s(\vec{r})$ may also appear in areas not associated with QTAIM critical points—referred to as “non-AIM critical points” or “interaction critical points (ICPs)”^[63,64]—in which case they indicate very weak interactions.

In practical applications, both ED and RDG are approximate and their quality depends on the quantum mechanical (QM) model adopted in calculations. The approximations introduced with this model,^[65–67] involving the choice of the Hamiltonian (determining whether and which relativistic effects are included), the method (responsible for the description of electron correlation) and the basis set (establishing the quality of one-electron functions), as well as other numerical parameters (such as the numerical thresholds, grids), can be assessed in a systematic way, yet there are relatively few studies illustrating how all of them affect the aforementioned fields and their topological features.^[68–71]

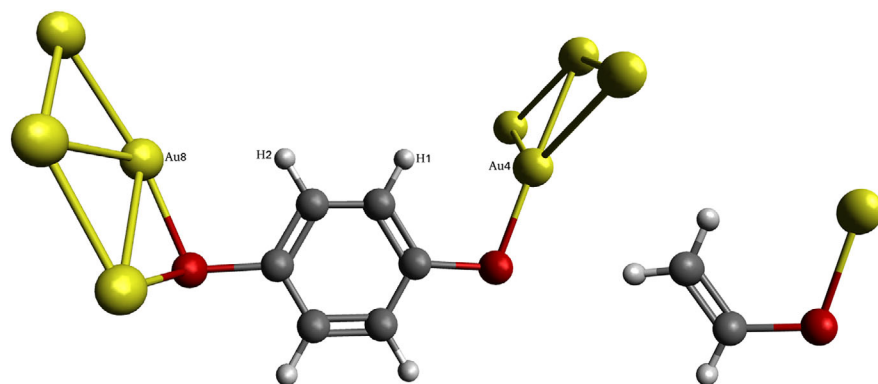


FIGURE 1 Molecular systems under study: Au₄-S-C₆H₄-S'-Au'₄ (left) and Au-S-CH-CH₂ (right). The numbering of selected Au and H centers is shown and is the same as by Anderson et al.^[29] Color coding for atoms: Au—yellow, S—red, C—dark gray, H—light gray. Additional information is available at 10.5281/zenodo.3358788

Inaccuracies of QTAIM results also stem from theoretical and semantic reasons. The former is related to the fact that QTAIM uses concepts from the graph theory, hence inherits its limitations, in particular it can give the “static” or “binary” information about topological features of ED, but not on their evolution, stability or importance (contrary to TDA). The latter reason is a source of continuous debate, related to the misinterpretation of QTAIM concepts (an example of which is a confusion between “bond paths” and “chemical bonds”^[56,72–74]), and to the overuse of the adjective “topological” in chemistry that should not be confused with mathematical topology.^[75,76]

The TDA approach to the interatomic interactions is more general. TDA provides information on the topological features of a scalar field, including their properties, configuration, evolution and significance, while staying oblivious to what the scalar field represents. This detachment of a tool and an interpretation of its output is very helpful to gain a new perspective on the research problem or to redefine existing concepts in a more robust way. In particular, TDA applied to ED, RDG, and a signed electron density ($\tilde{\rho}(\vec{r}) = \text{sign}(\lambda_2(\vec{r}))\rho(\vec{r})$ with λ_2 being the second eigenvalue of the ED Hessian)^[60] enabled to automatically extract and classify interatomic interactions merely based on the *topological persistence* (vide infra) of critical point pairs of these fields.^[18]

2 | COMPUTATIONAL DETAILS

Following Anderson et al,^[29] the geometry of the $\text{Au}_4\text{-S-C}_6\text{H}_4\text{-S'-Au}'_4$ molecule was optimized in the ADF software^[77–79] using three different models: (a) scalar-relativistic Zeroth-Order Regular Approximation (sr-ZORA),^[80–83] PBE functional,^[84] and TZP basis set^[85] at the singlet spin-restricted level (denoted throughout the text as **g1**), (b) sr-ZORA, PBE, TZP at the triplet spin-unrestricted level (**g2**), and (c) sr-ZORA, PBE, QZ4P^[85] at the triplet spin-unrestricted level with tighter SCF convergence criteria (**g3**). The study of a triplet state of this molecule, motivated by its proximity to the singlet ground state, was conducted to explore how the spin state affects the ED topology.

We considered simpler model systems of that molecule to investigate the importance of a local environment to the topological features of interest, Au-S-CH-CH_2 (Figure 1). Its geometry was obtained (without reoptimization) by removing atoms from the **g1** model of $\text{Au}_4\text{-S-C}_6\text{H}_4\text{-S'-Au}'_4$ molecule, so that it contains only one gold atom linked by sulfur to the ethene group that replaced the benzene ring. As the positions of all nuclei in Au-S-CH-CH_2 were preserved as in the original molecule and due to the lack of symmetry in $\text{Au}_4\text{-S-C}_6\text{H}_4\text{-S'-Au}'_4$, this simplification resulted in two geometries of the small model (atom numbers in parenthesis, after Anderson et al^[29])— $\text{Au}(4)\text{-S}(1)\text{-C}(1)\text{H-C}(2)\text{H}(1)\text{H}$ (“a”) and $\text{Au}(8)\text{-S}(2)\text{-C}(4)\text{H-C}(3)\text{H}(2)\text{H}$ (“b”), hence the nomenclature **m-a-g1** and **m-b-g1**, respectively.

For all studied molecules, ED has been evaluated in single-point calculations with the ADF software. These employed the nonrelativistic (NR) Hamiltonian as well as scalar and spin-orbit ZORA Hamiltonians (sr-ZORA and so-ZORA, respectively), DFT with different exchange-correlation (XC) functionals (PBE, B3LYP,^[86] SAOP,^[87,88] M06-2X^[89,90]) and three basis sets (DZ,^[85] TZP, and QZ4P). Additionally, for the small Au model we performed calculations in the locally modified version of the DIRAC software^[91] with the Lévy-Leblond (LL),^[92] spinfree Dirac-Coulomb (SFDC)^[93] and Dirac-Coulomb (DC)^[94] Hamiltonians, two methods involving different levels of electron correlation (Hartree-Fock [HF] and DFT with the PBE functional) and a basis set of triple-zeta quality.^[95–98]

The TDA of the logarithms of ED and RDG ($\log(\rho)$ and $\log(s)$, respectively^[99]) was performed with the TTK software package.^[100] All details can be found at <https://doi.org/10.5281/zenodo.3358788>.

3 | RESULTS AND DISCUSSION

Even though the optimized geometries of the Au complex are slightly different than by Anderson et al,^[29] our analysis confirms the main conclusions of the cited work: additional pairs of critical points of ED (1-saddles and 2-saddles, interpreted as “RCPs” and “BCPs”, respectively) between Au and H nuclei appear for certain combinations of Hamiltonians, methods, basis sets, molecular geometries, and spin states, but they never appear in the nonrelativistic context.

The comparison of topological features of relativistic (so-ZORA) and nonrelativistic EDs of **g1** molecule obtained with the PBE/QZ4P QM model is illustrated in Figure 2, where, in particular, the topological skeleton of nonrelativistic ED (without these additional saddle-saddle pairs) and relativistic ED (with these pairs) are shown. The results from TDA of these densities are described in detail in the caption to Figure 2.

The TDA pattern of critical points and 1-separatrices of ED is principally the same as the one obtained from QTAIM analysis.^[18] However, as we shall show, TDA of ED calculated with various QM models paints a more nuanced picture.

Before continuing on that thought, we shall clarify that the presence of 1-saddles accompanying 2-saddles between Au and H atoms is a result of Morse inequalities^[51] and is in accordance with chemical intuition due to the ring-shaped geometry of the Au-S-C-C-H chains. In PH, the critical points of a scalar function are paired in a nonambiguous way (called the Elder rule^[6]) and in this particular case, the additional 1-saddle and 2-saddle points, if present, form a pair on each side of the aromatic ring. The *lifespan* of this pair is measured by *persistence*, calculated as a difference of scalar field values in corresponding critical points. Accordingly, we define the persistence for the additional “BCP-RCP” pair as the difference of the (logarithm of) densities in the position of relativistic “BCPs” and “RCPs” as²:

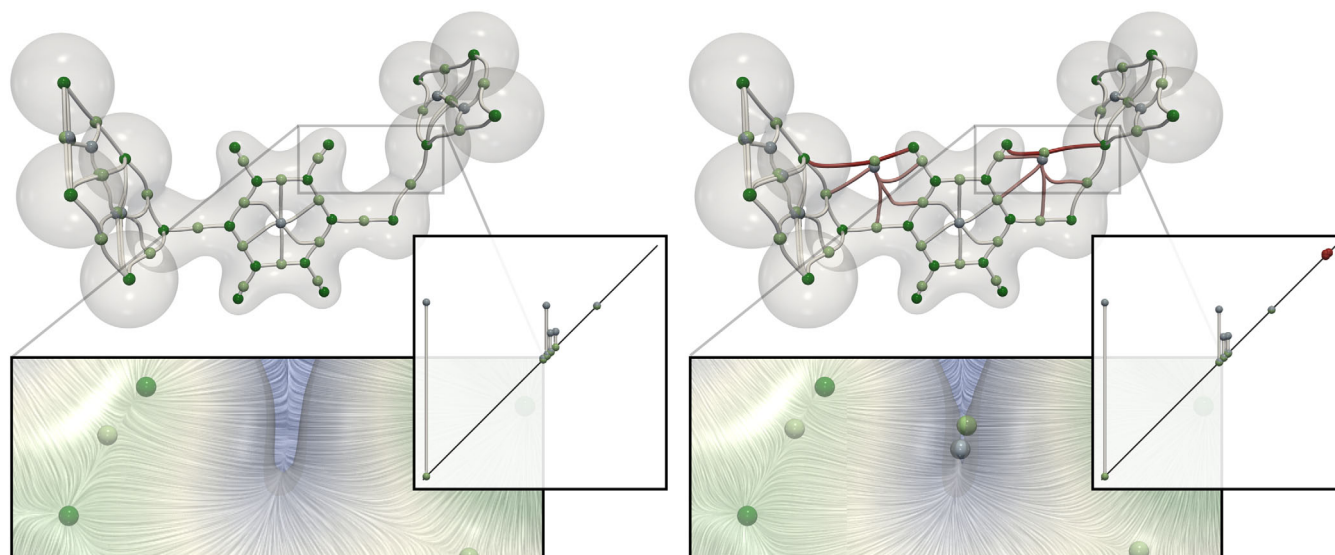


FIGURE 2 Comparison of the topological structure (one-dimensional separatrices and persistence diagram of the one-dimensional persistent homology group) for the logarithm of the electron density, $\log(\rho)$,^[99] of the Au complex without (left) and with (right) relativistic effects. Isosurfaces of $\log(\rho)$ (for the same isovalues) are shown in transparent gray to illustrate the geometry of the molecule. Top: the critical points of $\log(\rho)$ are shown with colored spheres (green: maxima, light green: 2-saddles, light blue: 1-saddles) and integral lines connecting pairs of critical points are shown with white curves. With relativistic effects (right), an additional 1-saddle/2-saddle pair emerges, indicating the possible existence of a hydrogen bond (shown in dark red). Bottom: zoom on the corresponding regions (background: line integral convolution of $\nabla\log(\rho)$, colored with $\log(\rho)$ blue to green). With relativistic effects (right), the isosurface changes its genus (β_1 ^[103]) in the vicinity of the extra saddle-saddle pair (tiny handle in the surface around the pair of light green and light blue spheres). Without relativistic effects (left), the isosurface (for the same isovalue) does not exhibit any handle and $\log(\rho)$ admits no saddle-saddle pair in this region of space. The persistence diagram^[6] (foreground insets) of the one-dimensional persistent homology group of the sublevel sets of $\log(\rho)$ displays each saddle-saddle pair (s_1, s_2) in the 2D plane at coordinates $(\log(\rho)[s_1], \log(\rho)[s_2])$. The height to the diagonal $(\log(\rho)[s_2] - \log(\rho)[s_1])$, white bar) denotes an importance measure called in Topological Data Analysis the *Persistence* of the critical point pair (illustrating the lifespan in the data of the corresponding topological feature). On the right diagram, the extra saddle-saddle pair is displayed in dark red (rightmost spheres). Its extremely low persistence indicates a very weak salience of the corresponding topological feature, questioning its interpretation as a hydrogen bond suggested in Anderson et al^[29]

$$p_{BCP-RCP} = \begin{cases} |\log(\rho_{BCP}) - \log(\rho_{RCP})| & \exists BCP-RCP \\ 0 & \nexists BCP-RCP \end{cases} \quad (2)$$

and use this definition to calculate the persistence values of extra saddle-saddle pairs of $\log(\rho)$ determined with various QM models. In all cases these values are very small, constituting typically less than 0.1% of the full persistence range (represented by red bars on the persistence diagram plotted for relativistic ED in Figure 2). Furthermore, they decrease with an increasing accuracy of the QM models, notably in molecules **g1**, **g2**, and **g3**, these persistence values systematically reduce along the DZ-TZP-QZ4P basis sets order for all XC functionals (Figure 3), a trend that is observed for both spin states and all geometries. The most remarkable change is seen for the combination of QZ4P basis and the meta-GGA M06-2X functional, for which these saddle-saddle pairs disappear (as signified by the zero-persistence values on the plots for all but the Au4-H1 pair in the **g2** system). This functional is also the one for which persistence values are generally lower. Taking at face value the claim of its superior performance for describing NCIs compared to the other functionals used in this work,^[89,101] our findings that this functional gives zero-persistence values would therefore imply the absence of these topological features.

At this point it is also interesting to compare results for **g2** and **g3**, as both represent the triplet state of the $\text{Au}_4\text{-S-C}_6\text{H}_4\text{-S'-Au}'_4$ molecule, only with a larger basis set and tighter SCF criteria used in the geometry optimization and ED calculations of the latter. It turns out that these improvements in the calculations have a tremendous effect on the persistence values of extra saddle-saddle pairs on both sides of the aromatic ring, namely these pairs are absent in **g3** in the most accurate setups (QZ4P basis set) for all selected XC functionals. This confirms the conclusion by Anderson et al^[29] that the topology of ED significantly changes depending on how close the structure is to a minimum of the Born-Oppenheimer (BO) energy surface, but also underscores that this pair is not as significant as the conventional QTAIM analysis would make it to be.

Similar analysis of **m-a-g1** and **m-b-g1** (Figure 3), this time focusing on the comparison of different Hamiltonians and methods, reveals the strong influence of the electron correlation on the persistence values of the additional saddle-saddle pair, seemingly counteracting the relativistic effects. This appears from the DIRAC calculations involving the fully relativistic 4-component DC Hamiltonian with HF and DFT/PBE methods:

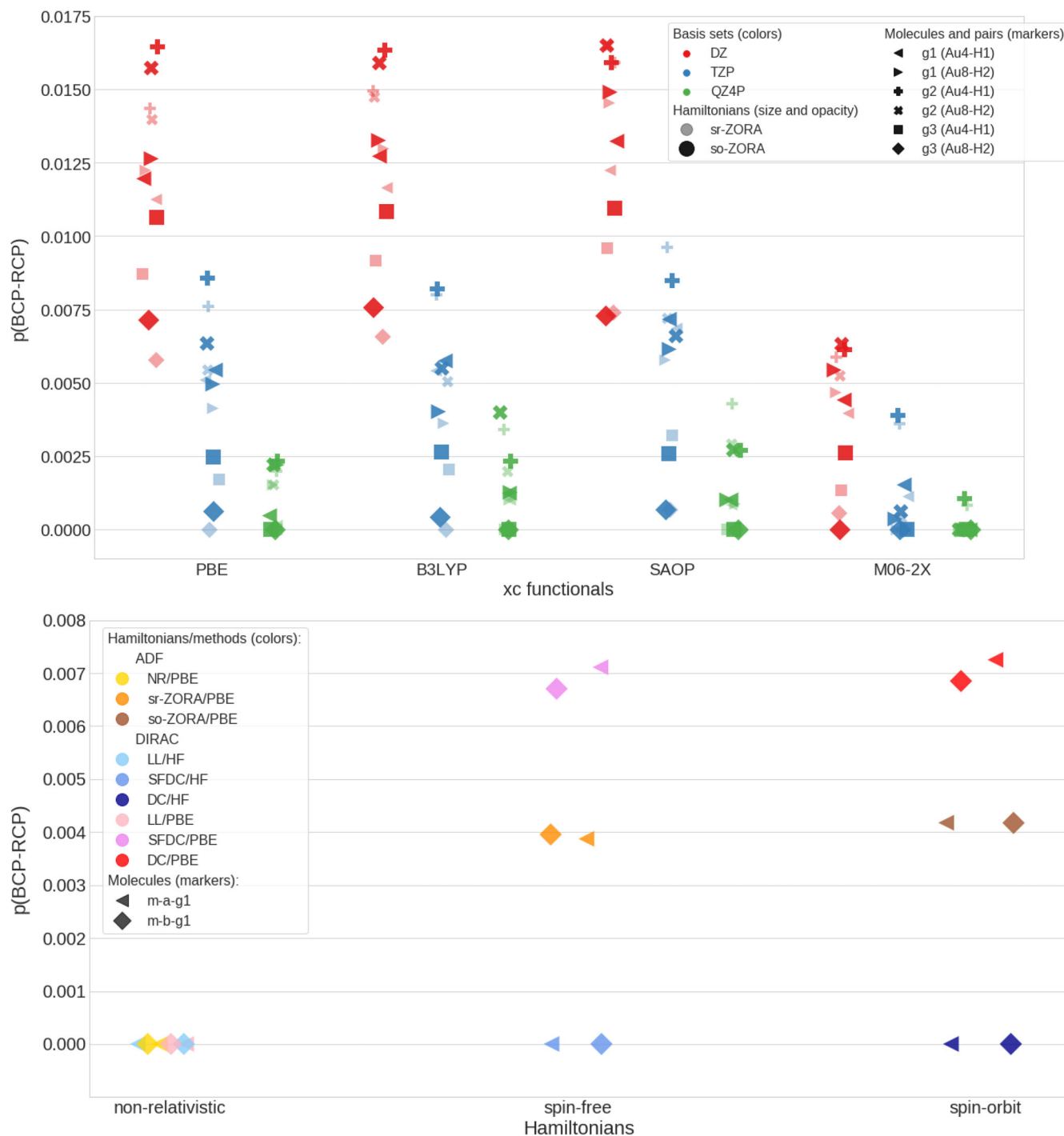


FIGURE 3 Persistence values of the extra saddle-saddle pairs of $\log(\rho)$ between H1 and Au4 and between H2 and Au8 atoms in $\text{Au}_4\text{-S-C}_6\text{H}_4\text{-S'-Au}'_4$ molecule (top) and between Au and H atoms in Au-S-CH-CH_2 molecule (bottom) calculated with different QM models. Top: the plot demonstrates that the persistence values of interesting pairs can be clustered by the basis set used in ED calculations, indicating that overall it is the basis set quality that affects the ED topology to a largest extent, while the choice of the XC functional (within tested ones), the inclusion of the spin-orbit coupling (at the ZORA level) or the chemical environment of the extra saddle-saddle pair do not have that significant effect. Bottom: the plot exposes the role of electron correlation evaluated at the DFT/PBE level in the DIRAC software and shows the differences in results obtained with ADF and DIRAC codes. The persistence values of absent saddle-saddle pairs are set to zero (Figure 2). Numerical values are available at [10.5281/zenodo.3358788](https://doi.org/10.5281/zenodo.3358788)

the ED calculated at HF level does not exhibit the extra saddle-saddle pair, while this pair is present in the DFT calculations and its persistence is (slightly) larger than when an approximate relativistic Hamiltonian (SFDC) is used. This observation, together with the negligible role of the spin-orbit coupling on the persistence values (as evaluated on scalar-relativistic geometries), suggests further studies on **m-a-g1** and **m-b-g1** with wavefunction-based methods at the scalar-relativistic level of theory. Apart from a change in electronic structure method, it may well be that

changes in equilibrium structures take place with the consideration of spin-orbit coupling in the geometry optimization procedure, as evidenced by a recent study on SOC-triggered bond contraction in gold clusters.^[102] We have nevertheless opted in this work to focus on selected geometries such as those previously considered by Anderson et al.^[29] This was motivated, first, by the comparison of the results for these model systems with the original molecular system, which indicates that the chemical surrounding of interacting Au and H atoms does not necessarily affect the topology of ED in these Au–H interatomic areas. And, second, that we considered that this would divert attention to what we believe is the central point of this work, which is to underscore the differences and similarities between TDA and QTAIM. We note, however, that further studies on these gold systems should indeed consider the effect of structural changes brought about by SOC.

In TDA the consensus is that the persistence of a topological feature is associated with its *importance* (in particular in the presence of additive noise), therefore it is frequently used in the topological simplification of complex scalar fields. Features manifested by so small persistence values would typically be deleted, which raises the question of whether the extra “BCPs” and “RCPs” of ED are an important feature of this scalar field. The answer to this question relies on the domain knowledge to provide interpretation of such feature. In the light of late research that cast “serious doubts on the interpretation of BCPs in QTAIM analyses as indicating bonding interactions, however weak”,^[54] confronted with the recently proposed translation of these additional saddle-saddle pairs as indicators of “favorable interaction”,^[29] we are inclined to answer “no.”

There are however more arguments standing for this answer, which do not involve the chemical interpretation of these additional critical points. For instance, the line integral convolution plots of the ED gradient demonstrated in Figure 2 have a very similar pattern in relativistic and nonrelativistic contexts, what is understandable considering that there need to be a border separating the atomic basins of Au and H (regardless of the QM model). In addition, as also exposed in Figure 2, the Euclidean distance between the extra “RCP” and the extra “BCP” in relativistic case is so small, that their presence can be considered as a negligible (*unimportant*) perturbation to the topology of ED in this interatomic area.

In order to test the robustness of the presented analysis with respect to grid sampling, we have analyzed the topology and geometry of ED of **m-a-g1** molecule calculated with a selected QM model (so-ZORA Hamiltonian, PBE functional, and three basis sets, DZ, TZP and QZ4P) on a series of regular grids of size ranging from 100^3 to 320^3 points. The results of this analysis are demonstrated in Figure 4 and they confirm prior observations: The extra saddle-saddle pairs (if present) can indeed be considered as unimportant topological features of ED, if not artificial due to the expected exponential decay of the persistence values of the extra pairs of the ρ scalar field with the grid size (linear dependence of the persistence values of the corresponding pairs exhibited by the $\log(\rho)$ scalar field).

What clearly discerns the relativistic ED exhibiting “BCP-RCP” pairs and nonrelativistic EDs without these pairs (for the same QM setup) is the fact that in the former case $\rho^R(r_{BCP}^R) > \rho^R(r_{RCP}^R)$, what causes the change of genus of the ED surface when sweeping over all values of ED, manifested by the appearance of additional critical points, as demonstrated in Figure 2. In the latter case, the value of ED probed in the location of “relativistic BCP,” r_{BCP}^R , is lower than the value of ED in the position of the “relativistic RCP,” r_{RCP}^R , hence $\rho^{NR}(r_{BCP}^R) < \rho^{NR}(r_{RCP}^R)$. Consequently, the

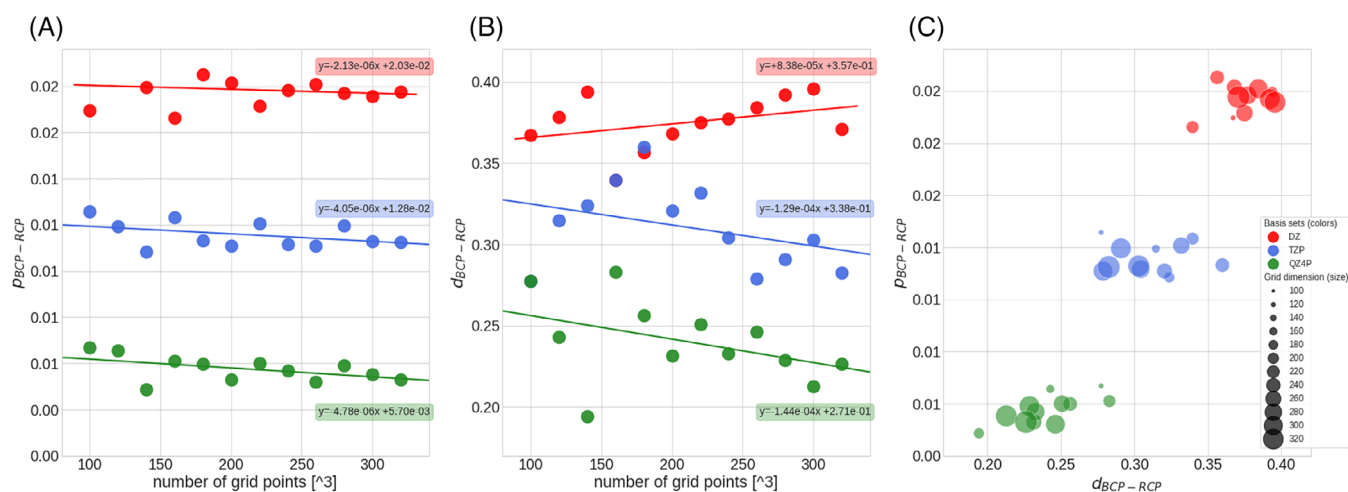


FIGURE 4 Au–S–CH–CH₂ molecule (**m-a-g1** model). A, The dependence of the persistence values of the extra saddle-saddle pair of $\log(\rho)$ between H and Au atoms ($p_{BCP-RCP}$) on the grid size. B, The dependence of the Euclidean distance between these saddle points ($d_{BCP-RCP}$) on the grid size. C, The correlation of $p_{BCP-RCP}$ and $d_{BCP-RCP}$ values, with the dependence on the basis set and the grid size marked on the plot. A and B, The linear regression models were used to estimate a simple relationship between variables. The actual persistence values of the additional saddle-saddle pairs of ρ decay exponentially with the grid size (note the $\log[\rho]$ scalar field under analysis), which suggests that the topological features under study may be artificial. The comparison of slopes of the fitted lines in DZ–TZP–QZ4P cases highlights the faster convergence of $p_{BCP-RCP}$ and $d_{BCP-RCP}$ with the grid size in the most accurate QM setup (QZ4P). C, Clear data clustering with respect to basis set further emphasizes the importance of the basis set in QM-based ED topology analysis. Numerical values are available at 10.5281/zenodo.3358788. Color coding common for all plots: DZ—red, TZP—blue, QZ4P—green markers

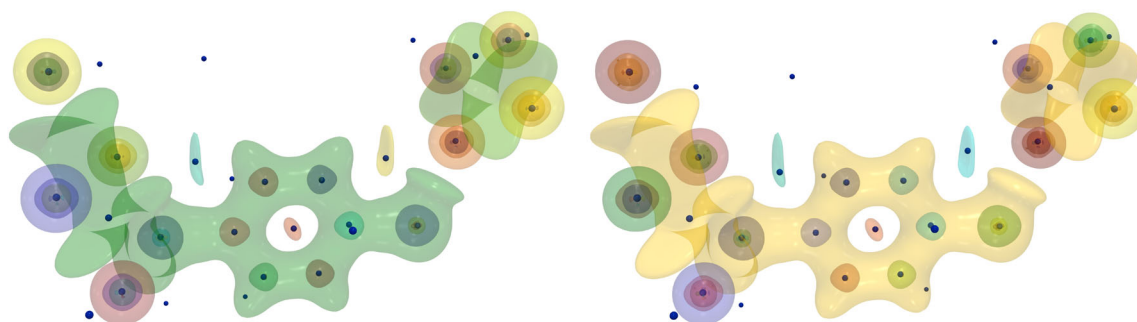


FIGURE 5 Comparison of data segmentations based on isosurfaces of reduced density gradient (same isovalue) without (left) and with (right) relativistic effects for the Au complex. Each connected component of isosurface is displayed with a unique, randomly chosen, color. The number of connected components is equal in the two cases, and their geometry is highly similar. In particular, the regions exhibiting extra saddle-saddle pairs of $\log(\rho)$ (Figure 2) contain isosurface components with very similar geometry in both cases

surface does not change its genus and the critical points do not appear. Yet, the difference of density values in these two locations (r_{RCP}^R and r_{BCP}^R) is very small (Table 13 at 10.5281/zenodo.3358788), which calls for a more accurate analysis involving properties that are based on the higher-order derivatives of the ED, such as the ED gradient or the ED Hessian.^[68] Other QM approximations, such as the ones determining the description of electron correlation or the quality of one-electron functions do not have such a strong influence on ED that could effectively change the total order of density values in these two locations, however, as the final picture is a result of interplay of many effects (eg, counteracting relativistic corrections and electron correlation), none of them should be neglected. Notably, relativistic formalism should be used if the interacting sites involve heavy elements, as already discussed elsewhere.^[29,68,70]

The use of accurate QM setup is essential to assert a good quality of calculated ED; however, it does not change the fact that one should be cautious when drawing conclusions on bonding interactions based on the QTAIM output only. We have therefore continued with the analysis of the RDG in studied systems. As illustrated in Figure 5, there is a striking similarity between the relativistic and nonrelativistic RDGs, in geometrical (the shape of the RDG isosurfaces) and topological (the same number of connected components ($\beta_0^{[103]}$) and the presence and location of minima) senses. In particular, the interaction sites between Au and H atoms are present in both contexts. This is an additional argument for the resemblance of Au–H NCIs with or without relativistic effects included.

4 | CONCLUSIONS

In conclusion, we have used TDA to reinterpret the QTAIM results for gold complexes reported recently,^[29] in which relativistic effects are considered to be behind the emergence of new 1-saddle-2-saddle point pairs between noncovalently bonded Au and H atoms.

The choice of TDA is motivated by its favorable performance (by using combinatorial algorithms instead of numerical grid searches) and mathematical robustness, allowing for an explicit translation between topological features and their chemical interpretation. TDA offers the possibility of visualizing the “topological skeleton” of a scalar field, including for the EDs the same features as QTAIM (critical points and 1-separatrices between 2-saddles and maxima that correspond to “bond paths”) as well as others that do not have their QTAIM correspondence (1-separatrices between 1-saddles and 2-saddles, 2-separatrices, manifolds of the Morse-Smale complex and outputs from other TDA data abstractions).

Our results demonstrate that the relationship between the topology of ED and the existence of NCIs is more complex than what can be deduced from a QTAIM picture based on the presence or absence of “BCPs” and “bond paths” between atoms. While TDA confirms that the additional saddle-saddle pairs between Au and H atoms are indeed the features of the relativistic ED that are absent for a nonrelativistic ED, these feature appear not to be meaningful, due to their low persistence values and, more importantly, to the fact that these values approach zero with increased sophistication of the calculations (larger basis set and denser grids).

Furthermore, our observations are another example of the by now established fact that the ED is not the best descriptor of such weak interactions, for which other quantities such as the RDG are better suited.^[52,63,64,104] Our geometrical and topological analysis of the RDG, however, does not show qualitative differences between the nonrelativistic and relativistic calculations, a fact that further calls into question the importance of relativistic effects to NCIs in such systems.

Taken together, these conclusions point to the need for a deeper analysis of the influence of the relativistic effects on the NCIs. To this end, we have shown here that much smaller model systems retain the same topological features as the original molecules, paving the way for more accurate calculations, such as those based on correlated wavefunction methods.

Finally, we suggest that TDA, and notably the concept of persistence—which can be considered as a process of transforming discrete invariant into a continuous one^[105]—should be considered in the development of real-space analysis techniques of the chemically relevant scalar fields.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

DATA ACCESSIBILITY

The data that support the findings of this study are openly available in Zenodo at DOI:10.5281/zenodo.3358788.

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ENDNOTES

¹ Equivalently it can be deduced from the Poincaré-Hopf theorem, which applies to generic vector fields.

² The mathematical background of persistence can be found elsewhere.^[6]

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