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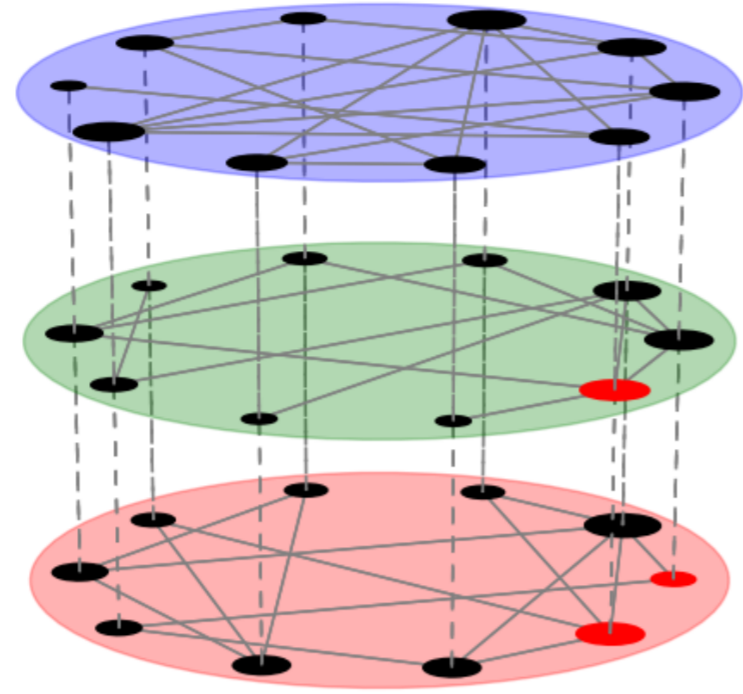
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Learning of Weighted Dynamic Multi-layer Networks via Latent Gaussian Processes.

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Motivation

In a number of natural, social and economic systems the dynamic interaction between agents across time can be recorded as longitudinal network-valued data, with directed and weighted edges (eg. financial transactions, trading between countries, migration between regions, etc). These interactions often occur in multiple types of connectivity, thus generating **Multilayer networks**, which should be jointly modelled for an adequate understanding of the system under study.



The motivation for our model lies in the context of *Financial Networks*, specifically the analysis of transactions between commercial banks and its implications for systemic risk. We aim to characterise the underlying probabilistic structure that dictates interactions between banks and across various types of transactions.

There have been a recent surge of statistical models for dynamic networks using latent factors (see [3]), but is focused on undirected, binary and single-layer Networks.

Dynamic Network Mixture Model via Latent GP Factors

Consider a dynamic system with V agents. Their activity generates relational data recorded as a Multiplex Network with K levels. We denote as $Y_{ij}^{(k)}(t)$ the directed interaction from agent i to agent j ($i, j = 1, \dots, V$), for the layer $k = 1, \dots, K$, recorded at time $t = t_1, \dots, t_T$.

The data of interest has a semi-continuous nature, $Y_{ij}^{(k)}(t) \geq 0$, with a probability mass at zero for pairs with no interaction, and a continuous distribution over the value of positive interactions between agents.

Observational equations

The observational model consist of a mixture between a Gaussian distribution and a probability mass at zero.

$$y_{ij}^{(k)}(t) \sim \lambda_{ij}^{(k)}(t) \mathcal{N}(\mu_{ij}^{(k)}(t), \sigma_{(k)}^2) + (1 - \lambda_{ij}^{(k)}(t)) \delta_{y_{ij}^{(k)}(t)}(0). \quad (1)$$

Here $\lambda_{ij}^{(k)}(t)$ is the **probability of incidence** of an interaction from agent i to agent j in layer k at time t . The Gaussian mean $\mu_{ij}^{(k)}(t)$ is the **expected strength** of such interaction -if it existed-. Finally, $\sigma_{(k)}^2$ accounts for the **variance of interactions**, which is shared by all pairs within layer k .

Evolving Latent Space

The complexity of the system and the dynamics of the relation between agents is captured by **latent effects** for both the incidence probability $\lambda_{ij}^{(k)}(t)$, and the expected strength $\mu_{ij}^{(k)}(t)$.

The latent structure for the **expected strength** is given by

$$\begin{aligned} \mu_{ij}^{(k)}(t) = & \theta^{(k)}(t) + \\ & u_i(t)'v_j(t) + u_i^{(k)}(t)'v_j^{(k)}(t) + \\ & s_{\mu,i}(t) + p_{\mu,j}(t) \end{aligned} \quad (2)$$

The *baseline processes*, $\theta^{(k)}(t) \in \mathbb{R}$, captures the **average intensity of interactions between all agents in layer k** .

The *multiplicative effects*, $u_i(t)'v_j(t)$ and $u_i^{(k)}(t)'v_j^{(k)}(t)$, are interpreted as the **distance between agents within the latent social space**, where $u_i(t) = \{u_{i,1}(t), \dots, u_{i,H}(t)\}' \in \mathbb{R}^H$ is the vector of latent coordinates of node i at time t within the *global sender space*; similarly, $v_j(t)$ is the location of agent j in the *global receiver space*. $u_i^{(k)}(t)$ and $v_j^{(k)}(t)$ denote the corresponding coordinates within the *layer-specific spaces*.

The *additive effects*, $s_{\mu,i}(t)$ and $p_{\mu,j}(t)$, represent agent i **sociability** (out-degree) and agent j **popularity** (in-degree), respectively.

The *systemic structure of dependency* that governs all layers will be captured by the latent elements that affect all layers, i.e. the global social spaces and the agent-specific additive effects.

The evolution of the network through time will be produced by dynamic latent components. We adopt **smooth trajectories** in our formulation by using *Gaussian Processes* with squared exponential correlation, $C(t, t') = \exp(-\frac{|t-t'|}{\delta})^2$, as the priors for the coordinates,

$$\begin{aligned} \theta^{(k)}(\cdot) & \sim GP(\bar{\theta}^{(k)}, C_\mu), \\ u_{i,h}(\cdot) & \sim GP(\bar{u}_{i,h}, C_\mu), \quad v_{i,h}(\cdot) \sim GP(\bar{v}_{i,h}, C_\mu), \\ u_{i,h}^{(k)}(\cdot) & \sim GP(\bar{u}_{i,h}^{(k)}, C_\mu), \quad v_{i,h}^{(k)}(\cdot) \sim GP(\bar{v}_{i,h}^{(k)}, C_\mu), \\ s_{\mu,i}(\cdot) & \sim GP(\bar{s}_{\mu,i}, C_\mu), \quad p_{\mu,i}(\cdot) \sim GP(\bar{p}_{\mu,i}, C_\mu), \end{aligned} \quad (3)$$

The latent structure of the **probability of incidence** $\lambda_{ij}^{(k)}(t)$ is analogous. We use a separate set of latent components and map the measure of similarity into the unit interval using a logistic link.

$$\begin{aligned} \text{logit}(\lambda_{ij}^{(k)}(t)) = & \eta^{(k)}(t) + \\ & a_i(t)'b_j(t) + a_i^{(k)}(t)'b_j^{(k)}(t) + \\ & s_{\lambda,i}(t) + p_{\lambda,j}(t) \end{aligned} \quad (4)$$

The dynamics of the latent components in $\lambda_{ij}^{(k)}(t)$ also follow Gaussian Processes priors.

Posterior Computation

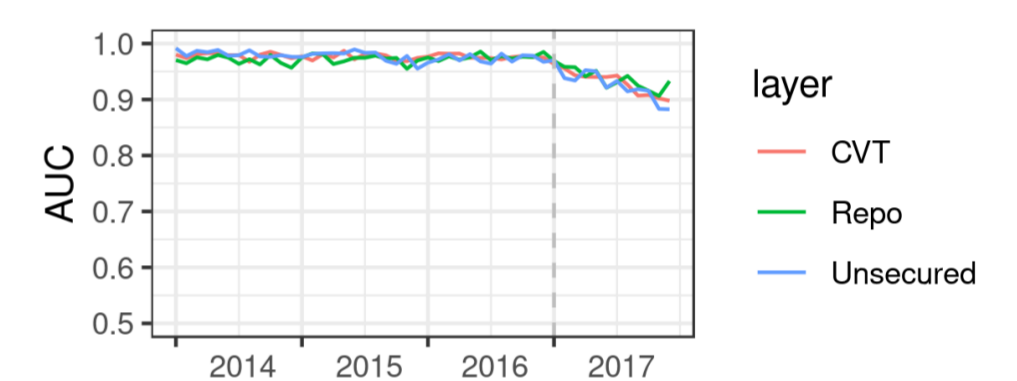
We perform MCMC using a Gibbs sampler. The method is based on [1, 2], but expanded to consider the additional latent elements in our model. **The implementation is available in the R package DynMultiNet.**

Interbank transactions in the Mexican Financial System

We applied our model to financial transactions between $V=46$ banks in Mexico, aggregated monthly between Jan-2014 and Dec-2017. The layers correspond to three types of transactions: 1) Unsecured lending, 2) Repo agreements, and 3) Securities trading (CVT).

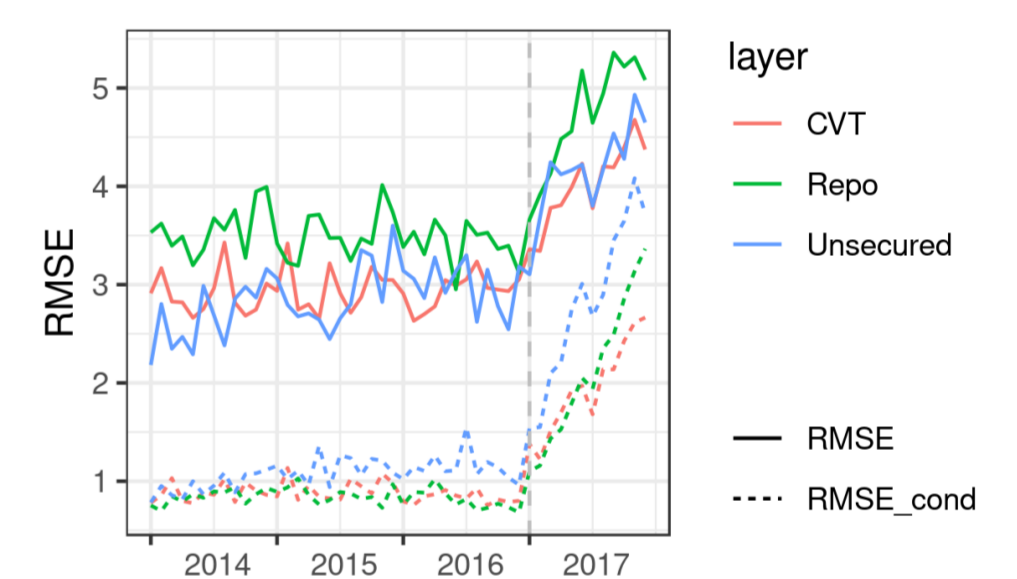
To assess the performance of the model we consider criteria for both classification (**AUC**) and regression (**RMSE**), due to the semi-continuous nature of the data. The plots below illustrate an exercise training the model with 36 months (2014-2016) and testing on the next 12 months (2017). We calculate the criteria for in- and out-of-sample periods.

We evaluate the prediction of edge incidence by computing the **AUC** (Area Under the ROC curve). The performance is outstanding, reaching almost 1 for in-sample data, and values above 0.85 for out-of-sample periods in the three layers.



To assess the accuracy of the predicted weight, we calculate two versions of the **RMSE**: one using predictions for all observations, and one only using connected pairs ($Y > 0$).

As a reference, the average weights for each layer (CVT, Repo, Unsecured) are 13.4, 15.2 and 13.8, respectively, with standard deviations of 2.1, 1.8, and 2.3.

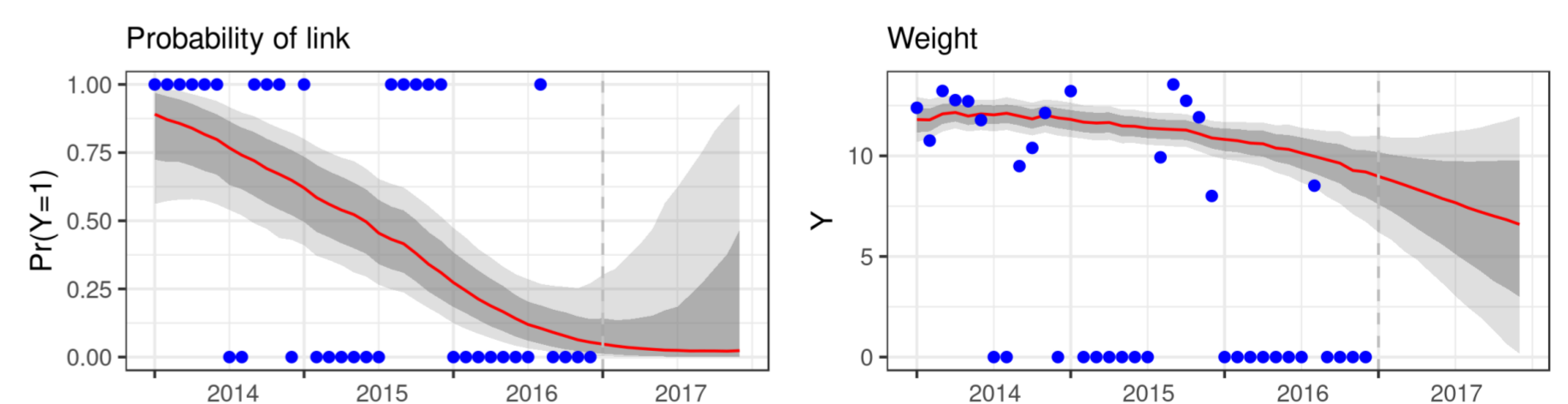


The probabilistic structure captured by the model provides information that can be used for insightful visualization of the system under study.

For example, combining posterior expected weights with force-directed network plots, it is possible to visualize the core-periphery structure of the financial system and identify important actors.



It is possible to visualise changes in the relationship between any pair of agents across time. The plots below show the evolution of the probability of connection and expected weight for two banks (blue points indicate observed values).



Conclusions

This work provides a flexible model that efficiently captures the underlying systemic dependency in weighted multilayer networks, showing an exceptional prediction performance and good interpretability.

References

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