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# Mathematics and music: loves and fights

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**Abstract.** We present different aspects of the special relationship that music has with mathematics, in particular the concepts of rigour and realism in both fields. These directions are illustrated by comments on the personal relationship of the author with Jean-Claude, together with examples taken from his own works, specially the “Duos pour un pianiste”.

**Keywords:** music, mathematics, rigour, philosophy, Jean-Claude Risset.

## Prelude: in memoriam

October 3rd 2016 I attended the premiere of what is probably the last work by Jean-Claude. It was in Rome, with also works and in the presence of John Showning. Unfortunately Jean-Claude couldn't come but I exchanged with him the following emails, alas the last ones.

Cher Jean-Claude,  
je sors du concert au conservatorio à Rome,  
où ton absence nous a surpris et ta presence  
beaucoup manqué. Ton œuvre est très belle.  
(...) Mais tu manquais aux manettes et je  
crois que cela se sentait dans ta piece.  
Très amicalement,  
Thierry.

Cher Thierry,  
Je te remercie vivement pour ton message de  
Rome : j'avais escompté assister au concert  
de Rome, mais j'ai dû rester à Marseille pour  
des examens un peu énigmatiques.  
Ton message amical m'a fait très plaisir.  
Bien à toi,  
Jean-Claude

## 1 Introduction

Jean-Claude Risset used to say: “[for example] *in music, we don’t even have a Heisenberg principle, yet*”.

This complain might seem strange, coming from a composer who used all along his life a lot of physics and mathematics in his action of composing. And on the other side, how not to imagine a bit awkward such a sentence in the mouth of a “pure” researcher other than Jean-Claude ?

This reference to the quantum uncertainty relations - which, by the way, is a (mathematical) theorem - is also very interesting if one thinks that quantum mechanics brought a deep change of paradigm in our way of seeing the world, which could be put in correspondence with the big changes we got from music - between the loss of tonality to nowadays music inseparable from computer sciences - in our way of hearing the sound [5].

But there is more in the relation between math and music, according to me. And much more than the link between the d’Arezzo notation and Descartes analytic geometry (much later), much more than group theory versus inversions, dilations, translations in the art of composing or improvising fugues (much earlier, again). Exactly like there is more than this analogy between wavelets and music scores, an analogy more pleasant to (some) mathematicians than to (any) musician.

For me, one of the deeper links between math and music, which also reveals fundamental differences, is located in the concept of rigour. Both mathematicians and musicians are rigorous in their action. No need to talk about the yoke that constitutes the obligation for the mathematician to prove things. Choosing a tonality, later a series, now a set of patches also consists in fixing some constraints which tie the composer in an a priori quite rigid frame. But this use, sometimes abuse, of rigour - and we claim that this is the same type of rigour which is truly used in both domains, a fact which characterizes them in between other sciences and art - is performed in very different places in music and in mathematics. As an a priori for the composer who feels free, in fine, to “cheat” with the constraints (in the same way with respect to tonality as to dodecaphonism). And, at the contrary, as a final achievement for the mathematician.

Rigour versus reality is another concept I would like to see math and music to share. In fact the famous (and for me a bit “has been”) debate on mathematical Platonism - to put it in a nutshell, do mathematicians invent or discover theorems? - has some resonances in music: after all, any sound belongs (already(?)) to the nature, can one say naively. A comparison between (what one can call) realism in mathematics and music deserve, according to me, to be exhibited.

I will first discuss the position of rigour in both actions of doing mathematics and doing (composing) music: a priori for musicians, in fine for mathematicians. I believe Jean-Claude was a perfect illustration of this, and his works continue

to illustrate this way of handling this quite schizophrenic bridge between math and music.

Then I will briefly discuss the duality rigour/realism, and the tied concept of emergence in music and illustrate it by the “Duos pour un pianiste”, this fantastic work by Jean-Claude where the unplayable rings real piano sounds.

## 2 Rigour in mathematics and in music: confluences and divergences

Music shares with mathematics a particular mandatory use of rigour. This is obviously, intrinsically true in mathematics. But it is also true for music, at least a certain kind of, as choosing tonality, series, a certain use of randomness, numerical patches, are constraint which constitute, according to me, a full tool box of rigour.

Composing, as proving, are actions very rigour consuming. But, at the contrary, the action of proving uses also a lot of other behaviours that escape completely from rigour. This is, this time, obvious for music, as everybody knows the disaster that created, in the past, a too rigid way of composing. But this is also true for mathematicians, who pass most of their time outside of rigour, being wrong.

Let us illustrate briefly these two common points for mathematics and music, namely the rigour and the loss of it, by the famous article “... wie die Zeit vergeht ...” by K. Stockhausen [11] and by looking at examples of writing (and I believe thinking) mathematics in an article by H. Poincaré published in the early twentieth century and a book by É. Goursat in the thirties (see [6] for more details).

In [11], Stockhausen makes the radical bet that one can compose by using in the same manner all the (very) different time scales (from the time of rythm (or even more, of the concert) to the one of timbre). This - in a certain sense quite natural - remark, very embedded in the rigorous serialism philosophy of music in the fifties, constitutes a starting point, a way of starting inside some very rigorous rules. Rules that the musician will be free to abandon during the process of composing.

At the contrary, in 1912, Poincaré [10] “defines” the famous Dirac  $\delta$  function (a function equal to zero except at the origin where it takes an infinite value) in a significant but highly nonrigorous way. Strictly speaking the definition is empty. But it gives the whole flavour of what it should be, using a non defined notion of “infinitesimal small”, a concept that Gourçat defines in a very floppy way<sup>1</sup> [4] twenty years later. In fact, one had to wait twenty years more to have

<sup>1</sup> “On dit qu’un nombre variable  $x$  a pour limite un nombre fixé  $a$ , ou tend vers  $a$ , lorsque la valeur absolue de la différence  $x - a$  finit par devenir et “rester” plus petite que n’importe quel nombre positif donné à l’avance. Lorsque  $a = 0$ , le nombre  $x$  est dit “un infiniment petit”.”, Gourçat [4]

a rigorous definition of  $\delta$ , much after its extensive effective use. One sees that, here, rigour enters the game after nonrigorous considerations, much after.

In fact the initial data are of the same type for the two fields: an original material (a chord, a theme, a patch, an equation, a conjecture, an equality), but the way that have musicians and mathematicians to “honour their material” [2] are very different: limiting the imagination by strong constraints for the musician, fishing ideas outside any rigour for the mathematician. Eventually, the musicians will free themselves from this rigorous straitjacket (sometimes quite quickly [3]) in the name of musicality and the mathematician will put some strict rigorous order in the different arguments necessitated by the constitution of the final proof.

Therefore, if mathematics and music share both the use of rigour and the loss of rigour, we arrive to the conclusion that there is a fundamental difference between them. In the action of doing mathematics and composing music, the musician start with rigour and eventually get rid of it by a typical artistic gesture. In the contrary of the mathematician, who starts outside any rules, fishing ideas, and end up with a perfectly rigorous situation produced by a typical scientific gesture<sup>2</sup>.

In conclusion, musicians and mathematicians both need rigour, but at different times in their process of creation.

### 3 Rigour versus realism, and all that

What is real in mathematics, and in music? After having discussed the fundamental role of rigour in the processes of proving and composing, the aforementioned question concerns more the result of the production both in math and in music. Of course, the answer seems easier in music: what is real is the execution of the work during a concert.

There are no concerts in mathematics. Mathematical results are exchanged mostly through articles in specialized journals and discussions between experts. The diffusion of mathematics to non-mathematicians is something else, something apart, considered as non fundamental for the evolution of the field mathematics. What questions realism in mathematics is generally circumscribed to the notion of mathematical Platonism. Mathematical Platonism [8] states the problem of knowing if the contents of theorems are truly invented by mathematicians, as coming from a “nowhere”, or, at the contrary, if mathematicians “just” discover their production in a “somewhere else” susceptible to contain everything.

There is a temptation [8] to consider that this “somewhere else” should be incarnated in music by the concert, a place of reality for music. But the situation seems to me less simple that it appears. Indeed: which concert? Which interpretation of the piece? And which “realization” in the the case of an open

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<sup>2</sup> The reader interested in the concept of rigour in mathematics, philosophy and music, might consult the proceedings [1] of the conference RIGUEUR held in Paris, July 2 and 3 2019, to be published by Spartacus editions (Paris).

work? Placing realism in music too close to the acoustic event is, according to me, problematic.

I also strongly believe that the Platonistic debate in mathematics is a bit “has been” and that there is no “somewhere else”: mathematics are just the result of their own construction. And I also think that realism in music doesn’t seat in the place of concert: what is more real concerning the “Valses nobles et sentimentales”? The version for orchestra or the version for piano? What is real in “Duos pour un pianiste”?

In fact, putting realism in music inside the execution of a piece is by far too naive. It seems to me that realism emerge in music as a consequence of, among other things but necessary including it, the rigour present in the process of composing. reading a fugue without playing it is a real experience, after all. And conversely, considering that the mathematical realism calls only the rigorous part of mathematics is too simple, too reductive. Let us quote René Thom: “Ce qui limite le vrai, ce n’est pas le faux, c’est l’insignifiant”<sup>3</sup>.

The conclusion of this short section could be expressed by saying that, both in music and in mathematics, the rigour has the role of providing a kind of “emergence of realism”.

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<sup>3</sup> What limits the true is not the false, it is the insignificant

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### **Postlude: solo for a scientist and a musician**

This conclusion will be an homage to Jean-Claude Risset, who all along his too short life, played continuously and successfully a permanent duo between him physicist and him musician [9].

“Duos pour un pianiste”, which, together with this postlude, has a quite surrealistic but rigorous title, illustrate marvellously this duality. First of all, the work addresses the issue of the limits of virtuosity, a very musical one. These limits will be overcome thanks to the use of a computer, a every mathematical object. Not by a computer creating electroacoustic sounds, but by a computer playing, through a precise and rigorous reaction to what the pianist just played, a piano Disklavier. And the only limit of virtuosity for the computer will be the one of the acoustic instrument.

Nobody knows really who, during the execution, is more influenced by the other: the pianist and the computer. Rather than a computer assisted piece of music, it is more of a mind assisted Nancarrow studies style piece.

Only the double-hatted mind of Jean-Claude could achieve such a miracle.