



**HAL**  
open science

# Exact Algorithms for Fixed Charge Network Design Problem with User Optimal Flow

Ikram Bouras, Rosa Figueiredo, Michael Poss, Fen Zhou

► **To cite this version:**

Ikram Bouras, Rosa Figueiredo, Michael Poss, Fen Zhou. Exact Algorithms for Fixed Charge Network Design Problem with User Optimal Flow. ROADEF 2018 - 19e Congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision, Feb 2018, Lorient, France. hal-02192562

**HAL Id: hal-02192562**

**<https://hal.science/hal-02192562>**

Submitted on 24 Jul 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Exact Algorithms for Fixed Charge Network Design Problem with User Optimal Flow

Ikram Bouras<sup>2</sup>, Rosa Figueiredo<sup>1</sup>, Michael Poss<sup>2</sup>, Fen Zhou<sup>1</sup>

<sup>1</sup> University of Avignon, LIA, 84911 Avignon, France {firstname.lastname}@univ-avignon.fr

<sup>2</sup> University of Montpellier, Lirimm, 34090 Montpellier, France {firstname.lastname}@lirimm.fr

**Keywords** : *Network design, Bi-level programming, Cutting plane, Branch-and-Cut.*

## 1 Introduction

The Fixed Charge Network Design Problem (FCNDP) consists in selecting a subset of edges from a given network, in such a way that a set of commodities can be transported from its origins to its destinations. The objective is to minimize the sum of fixed costs (depending on the selected edges) and variable costs (depending on the flow of commodities on the edges). There are several variants of FCNDP in the literature, each of which considers a particular objective function or takes into account specific additional constraints.

Herein we are interested in a specific variant of the FCNDP, namely Fixed Charge Network Design Problem with User Optimal Flow (FCNDP-UOF). This problem combines the FCNDP with multiple shortest path problems.

## 2 Problem description

We consider a transport network modeled by an undirected graph  $G(V, E)$ , where  $V$  represents the set of facilities and  $E$  represents the connections between them, which are uncapacitated. There is a set of  $K$  commodities to be delivered in the transport network  $G(V, E)$ , where each edge  $e = (i, j)$  is associated with several parameters : a length  $c_{ij}$ , a fixed cost  $f_e$  and a variable cost  $g_{ij}^k$  for each commodity  $k$ . Our objective is to design a subnetwork with minimum total cost to transport  $K$  commodities such that each commodity  $k \in K$  with a flow  $\phi^k$  can be delivered through the shortest path from its origin  $o(k)$  to its destination  $d(k)$  in this subnetwork. The FCNDP-UOF has been formulated as a bi-level integer programming problem [2], and as an ILP formulations through replacing the second level by its optimality conditions in [2] and [3]. A cutting plane algorithm based on a set of valid inequalities is proposed in [1].

## 3 Proposed methods

We propose in this work a new Binary Integer Programming model (BIP) for the FCNDP-UOF. To this end, we replace the objective function of the second level by a valid inequality (1c) eliminating infeasible solutions. Our BIP formulation is given as follows :

$$\min \sum_{e \in E} f_e y_e + \sum_{k \in K} \sum_{(i,j) \in E} \phi^k g_{ij}^k x_{ij}^k \quad (1a)$$

$$\text{s.t.} \quad \sum_{j \in \delta^+(i)} x_{ij}^k - \sum_{j \in \delta^-(i)} x_{ji}^k = b_i^k \quad \forall i \in V, \forall k \in K \quad (1b)$$

$$\sum_{(i,j) \in E} x_{ij}^k \leq \sum_{(i,j) \in P'} c_{ij} + (|P'| - \sum_{e \in P'} y_e) M \quad \forall P' : \text{path for } k \in K \quad (1c)$$

$$x_{ij}^k + x_{ji}^k \leq y_e, x_{ij}^k \in \{0, 1\}, y_e \in \{0, 1\} \quad \forall e \in E, \forall k \in K \quad (1d)$$

Where (1b) represents the flow conservation constraint, and  $M$  is a large number. Our first contribution is to implement several cutting plane algorithms. These algorithms solve the initial problem defined by Eq. (2) so that we can obtain a subnetwork  $y^*$ , and a set of paths  $x^*$  for all origin-destination pairs.

$$\min \left\{ \sum_{e \in E} f_e y_e + \sum_{k \in K} \sum_{(i,j) \in E} \phi^k g_{ij}^k x_{ij}^k, \quad s.t. \quad (1b), (1d) \right\} \quad (2)$$

Then, they check if  $x^*$  is the shortest path in  $y^*$ . If it is the case, then  $(x^*, y^*)$  is optimal for the FCNDP-UOF. Otherwise, a valid inequality (1c) is added to problem (2) for the commodities with infeasible paths. We repeat the procedure until an optimal solution is obtained.

The above approach can be expensive in terms of number of iterations to get the optimum. Thus, our second strategy is to combine the two methods together to form a “branch-and-cut” algorithm, known to be an efficient way to handle such constraints. The latter algorithm solves the problem (2) by a branch-and-bound procedure, and adds the inequalities to each integral node violating the shortest path constraint. Our second contribution lies in assessing the difficulty of the instances used in the simulations. To this end, we categorize the instances by calculating the angle  $\alpha$  between the objective function vectors of the first and the second levels. We have coded and compared the performance of six algorithms listed as follows :

**B&B1, B&B2** : one-level formulations from [2] and [3], respectively ;

**CP1, B&C1** : The proposed cutting plane and branch-and-cut algorithms ;

**CP2, B&C2** : Cutting plane and branch-and-cut algorithms using valid inequalities in [1].

TAB. 1 – Number of instances solved to optimality

	Total	B&B1	B&B2	CP1	CP2	B&C1	B&C2
$0^\circ \leq \alpha \leq 10^\circ$	135	100	109	110	110	110	109
$40^\circ \leq \alpha \leq 50^\circ$	135	88	89	95	95	94	92
$80^\circ \leq \alpha \leq 90^\circ$	135	88	97	105	105	102	102
Ravenna data	7	6	3	7	7	4	4
Sum	412	283	301	317	317	310	307

Numerical tests are performed using random and real data sets. Tab. 1 compares the number of instances solved to optimality in 3600s by the six methods. The table shows that **CP1** and **CP2** have a comparable performance and they solve more instances optimally than the others.

## 4 Conclusions and future work

For the FCNDP-UOF, we proposed a new BIP formulation, a cutting plane and a branch-and-cut algorithms to solve it. Numerical results demonstrate that the cutting plane algorithms outperform the other algorithms. The instances with an angle of  $40^\circ \leq \alpha \leq 50^\circ$  are the most difficult. For future work, we intend to work on exact approaches for the general case of the FCNDP-UOF, where the capacity constraint on edges is considered.

## Références

- [1] Fatma Gzara. A cutting plane approach for bilevel hazardous material transport network design. *Operations Research Letters*, 41(1) :40–46, 2013.
- [2] Bahar Y. Kara and Vedat Verter. Designing a road network for hazardous materials transportation. *Transportation Science*, 38(2) :188–196, 2004.
- [3] A. Mauttone, M. Labbé, and R. M. V. Figueiredo. A tabu search approach to solve a network design problem with user-optimal flows. In *VI ALIO/EURO Workshop on Applied Combinatorial Optimization*, 2008.