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► **To cite this version:**

Mostafa Safdari Shadloo, Sushank Sharma, A. Hadjadj. Turbulent flow topology in supersonic boundary layer with wall heat transfer. *International Journal of Heat and Fluid Flow*, 2019, 78, pp.108430. 10.1016/j.ijheatfluidflow.2019.108430 . hal-02189352

**HAL Id: hal-02189352**

**<https://hal.science/hal-02189352>**

Submitted on 25 Oct 2021

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# Turbulent Flow Topology in Supersonic Boundary Layer with Wall Heat Transfer

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## Abstract

Direct numerical simulations (DNS) are performed for the supersonic boundary layers (SBLs) with a free-stream Mach number  $M_\infty = 2.2$ . Different cases including the adiabatic and the isothermal (cooled and heated) walls are investigated. The laminar boundary layer is excited by means of a blowing and suction strip with single-frequency and multiple spanwise wave-numbers. The incoming laminar flow is strongly perturbed with a perturbation intensity of 2.4% of the free-stream velocity to obtain the turbulent boundary layer. In the fully developed turbulent regions, the joint probability density function (JPDF) distribution and the covariance integrands' analyses of different parameters are performed to find out the contribution of various physical mechanisms towards different transfer processes. The results reveal that behavior of the turbulent shear stress is similar to its incompressible counterpart and the wall-temperature impacts are dominant in the buffer layer region (at  $y^+ = 10$ ). The inclination angles of coherent structures show variations arising from the wall-temperature in both the buffer-layer and the

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log region. The covariance integrands' analyses of different components of the heat flux reveal the dominance of a different transfer process in case of the cooled wall, and as a result of this difference, the cooled wall acts as a heat sink.

*Keywords:* Supersonic boundary layer, Turbulent flow, Direct numerical simulation (DNS), Joint Probability Distribution Function (JPDF) distribution, Covariance integrand analysis

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## 1. Introduction

An increasing focus towards the improvement in the designs of the supersonic aircraft, calls for a better understanding of the high-speed flows. Various other applications such as the flow through a supersonic propulsive nozzle [1], demand for better characterization of the supersonic turbulent boundary layers. Due to the complexity posed by the compressibility effects in case of the high-speed flows, it therefore becomes necessary to explore the implications of different physical parameters such as the surface temperature on the flow itself [2].

The scientific community is trying to characterize the turbulent flows from a very long period of time. The study performed by Theodorsen [3] brings out the importance of the coherent structures in case of the incompressible turbulent wall-bounded flows. Their results shed light on the fact that these structures are responsible for low-momentum fluid transport and Reynolds shear-stress production. The morphology of these structures were experimentally verified by Head and Bandyopadhyay [4]. The investigation presented in [5] suggests that in the turbulent boundary layer, the asymmetric

18 one-legged hairpin vortex is the most-probable shape of the coherent struc-  
19 tures. Later on, the numerical study performed by Wu and Moin [6] stated  
20 that the forests of hairpin vortices dominate the turbulent boundary layer.  
21 Experimentally, the events of ejections and sweeps which are responsible  
22 for Reynolds shear-stress production were visualized by Corino and Brod-  
23 key [7]. Wallace et al. [8] quantified the turbulent processes and provided  
24 further insight about Reynolds stress production in the near-wall region for  
25 the incompressible turbulent channel flows. Their results revealed that ejec-  
26 tions and sweeps together contribute more than 100% to the Reynolds stress,  
27 and the additional stress was countered by other contributing factors named  
28 interactions.

29 For the incompressible turbulent channel flows, Wallace and Brodkey [9]  
30 performed the joint probability density distribution function (JPDF) and the  
31 covariance integrands' analyses for the streamwise and wall-normal velocity  
32 fluctuations in order to find out the contribution of different transport pro-  
33 cesses towards the Reynolds shear-stress. Their results suggest that when  
34 moving from the near wall-region i.e.  $y^+ = 5$  to the end of the log region,  
35 different physical phenomena dominate the transfer processes. They also  
36 showed that the most-probable velocity pairs did not have the largest contri-  
37 bution towards the shear-stress. Major contribution of ejections towards the  
38 Reynolds shear stress was also reported by the experimental investigation  
39 of Willmarth and Lu [10]. The results presented by Ong and Wallace [11]  
40 highlighted the ability of the JPDF and covariance analyses in determining  
41 the topology of the turbulent flows. The results of this study helped in de-  
42 termining the most probable angles of inclination of the vorticity filaments

43 using the covariance integrands' analyses. The events of vortex stretching  
44 and compression were also discussed in detail. It was found that the average  
45 stretching of the filaments was greater than compression at all of the con-  
46 sidered locations [11]. Direct numerical simulations (DNS) of Le et al. [12]  
47 investigated the changes in three-dimensional turbulent boundary layer by  
48 employing a combination of different statistical and visualization methods.  
49 Their results uncovered that mean three-dimensionality was responsible for  
50 breaking up the symmetry and alignment of the near-wall coherent structures  
51 disrupting their self-sustaining mechanisms, and resulting in the reduction  
52 of the turbulent kinetic energy.

53 Fewer investigations have been performed so-far for the compressible tur-  
54 bulent boundary layers. For low Mach number turbulent boundary layers,  
55 the DNS results of Bechlars and Sandberg [13] found the potential backscat-  
56 ter mechanism for the transfer of the kinetic energy from smaller scales to  
57 the larger scales. The effects on the first three invariants of the velocity  
58 gradient tensor with wall-normal distance for weakly compressible flow are  
59 studied by [14]. The experimental database available for the compressible  
60 problems is scarce due to the difficulty in measurements. The experimental  
61 investigation of Spina et al. [15] revealed that the compressibility has little  
62 impact on the statistical properties of the flow. One of the first investigations  
63 reported by Morkovin [16] suggests that the effects of compressibility on tur-  
64 bulence are due to the variations of the thermodynamic properties across the  
65 boundary layer. The experimental data also confirms that the supersonic  
66 boundary layers bear close similarities to the incompressible ones [17, 18]. Li  
67 and Xi-Yun [18] have reported that the angles of inclination of the vortical

68 structures with the streamwise direction increases from sub-layer to buffer  
69 layer and then decreases from the buffer layer to the wake region. Maeder  
70 et al. [19], Pirozzoli et al. [20] have investigated the structural characteris-  
71 tics of the supersonic turbulence and found the presence of the organized  
72 motions in the outer layer. The study presented in [21] tried to quantita-  
73 tively characterize the statistical features of the coherent structures for the  
74 case of turbulent supersonic boundary layer and found that the inner layer  
75 was mostly populated by the quasi-streamwise vortices while the outer layer  
76 (including the log and the wake regions) was populated by different types of  
77 structures such as the hairpin vortices and the hairpin packets.

78 The careful examination of the existing scientific literature reveals that the  
79 studies pertaining to the effects of wall heat-transfer on turbulent flow topol-  
80 ogy for the compressible supersonic boundary layer are scarce (almost none  
81 for the heated wall). The study investigating the supersonic cooled turbulent  
82 channel flows in [22] deals with the effects of compressibility on the pressure-  
83 strain correlation and the dissipation rate tensors in the Reynolds stress  
84 budgets. The results of this study revealed that the fluctuations conditioned  
85 on ejections and sweeps in the wall-layer were instructive, and showed that  
86 the positive temperature fluctuations were mainly due to sweeps in case of  
87 the cooled wall. Moreover, the comparison with the incompressible flow data  
88 underlined that the compressibility effects persisted in the wall-layer only.  
89 Relevant statistical properties of the compressible turbulent flows (including  
90 the heated wall) are assessed in [23]. This study found that the Morkovin's  
91 hypothesis was neither valid for the heated walls nor for the cooled walls. The  
92 analysis of the turbulent kinetic energy budget showed that the dilatational

93 to solenoidal dissipation ratio increases/decreases with heating/cooling of the  
94 wall. Later on, Trettel and Larsson [24] proposed the transformations of the  
95 velocity and the wall-coordinate simultaneously for the supersonic isother-  
96 mal turbulent channel flows and the turbulent boundary layers, relating the  
97 compressible mean velocity profile at any given Mach number. For low-Mach  
98 number heated channel flows, Patel et al. [25] found that the van Driest trans-  
99 formed mean temperature profiles of variable property cases collapsed with  
100 the constant property cases if the semilocal Reynolds number and the local  
101 Prandtl number distributions are constant across the channel. Chu et al.  
102 [26] studied the effects of wall temperature on the orientation of the vortical  
103 structures and other statistical properties like Morkovin’s scaling. It was  
104 found that with increasing wall-temperature, the spanwise distance between  
105 the legs of the hairpin vortex increased, the mean swirling strength and the  
106 angle of the vorticity filament with the wall also increased in the inner layer.  
107 However, the statistical properties of the vortical structures were nearly in-  
108 sensitive to the wall temperature in the outer layer. Moreover, they also put  
109 forward a new criteria for better characterizing the angles of inclination of  
110 the vortical structures. Other works characterized the factors influencing the  
111 transition scenarios for the compressible supersonic flows [27, 28, 29].

112 For the supersonic boundary layers, it is important to address the impacts  
113 of wall-heating and cooling on the arrangement and the orientation of the  
114 vortical structures, and the heat-transfer mechanisms, which are the funda-  
115 mental and still open questions for the community. In this study, the JPDF  
116 and the covariance integrands’ analyses are utilized to unravel the physical  
117 mechanisms responsible for the heat-transfer in the streamwise and the wall-

118 normal directions. Various quadrant analyses have been put forward to find  
 119 out the most-significant and contributing transfer process responsible for the  
 120 turbulent shear stress, the vortical structures' orientation and the turbulent  
 121 heat-flux.

122 This paper is structured as follows: the governing equations and details  
 123 of the computational setup including the boundary conditions are given in  
 124 §2, followed by the description of the turbulent boundary layer in §3. Then  
 125 a detailed discussion about the turbulent shear stress, topology of the co-  
 126 herent structures and different components of the turbulent heat-transfer is  
 127 presented in §4.1, §4.2 and §4.3, respectively. The conclusions of the paper  
 128 are presented in §5.

## 129 **2. Description of the numerical setup**

### 130 *2.1. Governing equations*

131 The motion of a Newtonian fluid is governed by the set of equations  
 132 known as the *Navier – Stokes equations* (NSE) comprising of the equa-  
 133 tions of conservation of mass, momentum and total energy. The NSE are  
 134 non-dimensionalized using the free-stream quantities and the boundary layer  
 135 thickness at the inlet  $\delta_{in}^*$  as the reference length:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0, \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (2)$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho E + p) u_i}{\partial x_i} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial u_i \tau_{ij}}{\partial x_j}, \quad (3)$$



136 where, density  $\rho = \rho^*/\rho_\infty^*$ , velocity  $u = u^*/u_\infty^*$ , time  $t = t^* \times u_\infty^*/\delta_{in}^*$ , pres-  
 137 sure  $p = p^*/(\rho_\infty^* u_\infty^{*2})$  and energy  $E = E^*/u_\infty^{*2}$ . Throughout this paper, the  
 138 free-stream quantities are marked by the subscript  $\infty$  and the dimensional  
 139 quantities are marked by the asterisk superscript (\*).

140  $\tau$  being the symmetric viscous stress tensor, which is given by:

$$\tau_{ij} = \frac{\mu}{Re} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right). \quad (4)$$

141 where, viscosity  $\mu = \mu^*/\mu_\infty^*$ , Reynolds number  $Re = \rho_\infty^* u_\infty^* \delta_{in}^*/\mu_\infty^*$  and  $\delta_{ij}$  is  
 142 the Kronecker delta. The pressure and the heat-flux are computed using the  
 143 equation of state and the Fourier law of heat conduction respectively:

$$p = (\gamma - 1) \left( \rho E - \frac{1}{2} \rho u_i u_i \right) = \frac{1}{\gamma M_\infty^2} \rho T, \quad (5)$$

144 and

$$q = \frac{-\mu}{(\gamma - 1) M_\infty^2 Re Pr} \frac{\partial T}{\partial x_j}. \quad (6)$$

145 with temperature  $T = T^*/T_\infty^*$ , constant specific heat ration  $\gamma = 1.4$  and  
 146 Mach number  $M_\infty = u_\infty^*/\sqrt{\gamma R^* T_\infty^*}$  with gas constant  $R^* = 287 J/K kg^{-1}$  and  
 147 Prandtl number  $Pr = 0.72$ .

148 The Sutherland's law has been used to calculate the dynamic viscosity:

$$\mu^*(T^*) = \frac{C_1^* T^{*3/2}}{T^* + S^*}, \quad (7)$$

149 where,  $S^* = 110.4$  K is Sutherland's temperature for air and  $C_1^*$  is a constant,  
 150  $1.458 \times 10^{-6} kg/ms \sqrt{K}$  which can be written as:

$$C_1^* = \frac{\mu_r^*}{T_r^{*3/2}} (T_r^* + S^*), \quad (8)$$

151 where  $\mu_r^*$  is the reference dynamic viscosity of the air,  $1.716 \times 10^{-5}$  kg/ms  
152 at the reference temperature,  $T_r^*$  of 273.15 K. The subscript  $r$  refers to the  
153 reference values.

## 154 2.2. Numerical solver

155 We have utilized a well validated DNS - LES numerical solver named  
156 CHOC-WAVES to solve the three-dimensional, compressible, unsteady NSE  
157 for perfect gases. This solver discretizes the convective fluxes by a hybrid  
158 conservative sixth-order central scheme with fifth-order Weighted Essential  
159 Non-Oscillatory (WENO) scheme [30, 31]. Convective terms are splitted in  
160 a skew-symmetric form to minimize the aliasing error and to enforce discrete  
161 conservation of the kinetic energy which results in better numerical stability.  
162 Approximation of the diffusive terms is done with the fourth or the sixth  
163 order formulas, and they are expanded in the Laplacian form. The time  
164 integration is performed using the third-order Runge-Kutta (RK-3) scheme.  
165 More details on validation can be found in ([32, 33, 34]).

## 166 2.3. Problem setup

167 This study utilizes the supersonic flow over a flat plate with free-stream  
168 Mach number  $M_\infty = 2.2$ , temperature  $T_\infty^* = 177$  K, pressure  $p_\infty^* = 23796$   
169 Pa and viscosity  $\nu_\infty^* = 2.55 \times 10^{-5}$  m<sup>2</sup>/s. The choice of the Mach number  
170 is based on the fact that at higher Mach numbers, the second mode insta-  
171 bilities or the Mack modes [35] dominate the flow and hence the effects of  
172 wall heat-transfer could not be distinguished prominently [27, 28, 29]. The  
173 computational domain is free of the shocks generated at the leading edge of  
174 the flat-plate because the inlet is placed downstream of the leading edge at

175  $x_{in}^* = 0.1016$  m with inlet Reynolds number  $Re_{x_{in}} = 2.33 \times 10^6$  and unit  
 176 Reynolds number  $Re_{unit}^* = 2.293 \times 10^7/m$ . The length and the height of the  
 177 domain are  $L_x^* = 0.15$  m and  $L_y^* = 0.0127$  m respectively. The height of  
 178 the computational domain is chosen such that the boundary layer thickness  
 179 towards the end of the domain is approximately one-third of the height of the  
 180 domain. The spanwise width of the domain is set equal to the fundamental  
 181 wavelength of the excited mode i.e.  $L_z^* = \lambda_z^* = 0.00605$  m corresponding  
 182 to the most-unstable mode predicted by the Linear stability theory (LST)  
 183 [27, 28]. Two-point correlations in the spanwise direction are plotted (not  
 184 shown here) which assure that the periodicity does not affect the generated  
 185 turbulence. Uniform mesh spacing is used in both the streamwise and the  
 186 spanwise directions with  $N_x = 4096$  and  $N_z = 280$  being the number of points  
 187 in the given directions. However, in the wall-normal direction ( $N_y = 150$ ),  
 188 points are more concentrated close to the wall in order to resolve the bound-  
 189 ary layer. The stretching function in the wall-normal direction is given by:

$$y^* = L_y^* \frac{1 + \tanh(\kappa_o y^*)}{\tanh(\kappa_o)}, \quad (9)$$

190 with,  $\kappa_o \approx 3$  being the stretch parameter.

191 Details about various DNS cases under investigation are enlisted in table  
 192 1. In this table, A, C and H stand for the adiabatic, cooled and heated walls  
 193 respectively. As seen in table 1, constant excitation frequency i.e.  $\omega^* = 150$   
 194 krad/s (or 23.87 kHz) is chosen for the blowing and suction strip for all the  
 195 cases which corresponds to the most-unstable frequency according to the LST  
 196 [27, 28]. Moreover, the perturbation intensity is kept high i.e. 2.4% of the  
 197 free-stream velocity in order to strongly excite the boundary layer, so that

Table 1: Computational parameters for various test cases. A, C and H stand for the adiabatic, the cooled and the heated walls, respectively. Subscript *min* denotes the wall-normal spacing. Superscript + denotes the quantities in wall-units. *I* is the disturbance amplitude of blowing and suction ( $I = v_{wall,max}^*/u_\infty^*$ ).

| Cases  | $T_w^*/T_{aw}^*$ | $I(\%)$ | $\omega^*$ (krad/s) | $\Delta x^+$ | $\Delta y_{min}^+$ | $\Delta z^+$ |
|--------|------------------|---------|---------------------|--------------|--------------------|--------------|
| A0[27] | 1.00             | 2.4     | 150                 | 5.52         | 0.34               | 2.85         |
| A      | 1.00             | 2.4     | 150                 | 5.52         | 0.34               | 3.26         |
| C      | 0.75             | 2.4     | 150                 | 8.11         | 0.50               | 4.78         |
| H      | 1.50             | 2.4     | 150                 | 3.30         | 0.20               | 1.95         |

198 the turbulent boundary layer exists in the majority of the computational  
 199 domain ( $> 50\%$ ).

### 200 2.3.1. Boundary conditions

201 Figure 1 represents a schematic the computational domain and the bound-  
 202 ary conditions. At the inlet of the domain, the streamwise and the wall-  
 203 normal velocities, as well as the density profile are set to the laminar Bla-  
 204 sius profile, without any disturbance. These profiles are calculated using a  
 205 dedicated solver to obtain similarity solutions for adiabatic and isothermal  
 206 compressible laminar boundary layers which utilizes the Illingworth transfor-  
 207 mation [36, 37]. The boundary layer thickness at the inlet of the domain  $\delta_{in}^*$   
 208 for A, C and H cases are  $4.44 \times 10^{-4}$  m,  $3.91 \times 10^{-4}$  m and  $4.86 \times 10^{-4}$  m  
 209 respectively. For the adiabatic case, the wall temperature  $T_w^* = T_{aw}^*$  (adiabatic  
 210 wall temperature), while for cooled and heated walls the temperature is set  
 211 as  $T_w^* = 0.75 T_{aw}^*$  and  $T_w^* = 1.5 T_{aw}^*$  respectively, where  $T_{aw}^* \approx 1.82 T_\infty^*$ .  $T_{aw}^*$

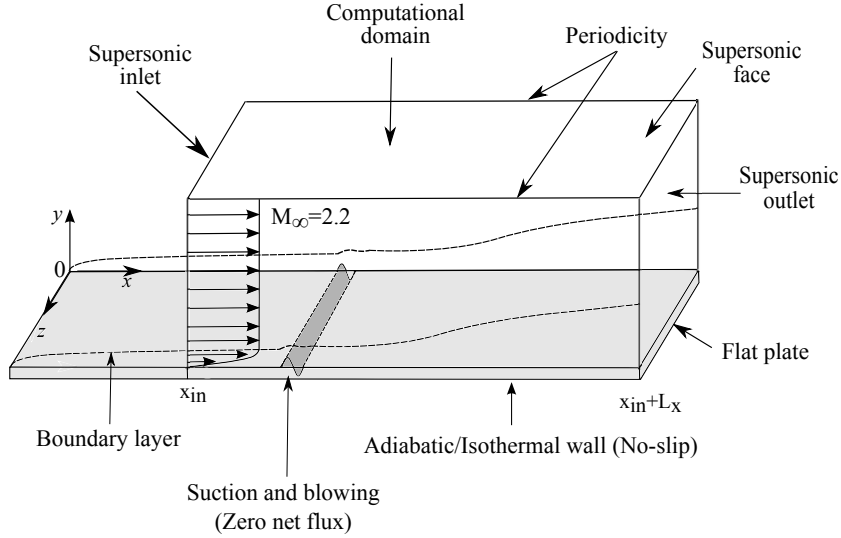


Figure 1: Computational domain and boundary conditions.

212 is calculated using the recovery factor approximation  $Pr^{1/3}$ , [36]:

$$T_{aw}^* = T_{\infty}^* \left( 1 + Pr^{1/3} \times \frac{\gamma - 1}{2} \times M_{\infty}^2 \right) \quad (10)$$

213 Supersonic inflow and outflow boundary conditions are imposed at the inlet  
 214 ( $x_{in}^*$ ) and at the outlet ( $x_{out}^* = x_{in}^* + L_x^*$ ) respectively. The side-walls of the  
 215 domain are periodic and for the upper face of the domain, zero boundary-  
 216 normal gradient is imposed. No-slip and no-penetration condition is used for  
 217 at the surface of the wall ( $y = 0$ ), except for the narrow strip of blowing  
 218 and suction existing between  $x_a^* = x_{in}^* + 0.0127$  m to  $x_b^* = x_{in}^* + 0.0254$   
 219 m. The wall-normal component of the velocity in the blowing and suction  
 220 strip is prescribed by the single-frequency and multiple-spanwise wavenumber  
 221 boundary condition given as:

$$v^*(x, y = 0, z, t) = Iu_{\infty}^* f(x) \left[ \frac{g(z)}{\max(g(z))} \right] \left[ \frac{h(t)}{\max(h(t))} \right], \quad (11)$$

222

223 where  $I$  is the disturbance amplitude,  $f(x)$ ,  $g(z)$  and  $h(t)$  are the streamwise,  
224 spanwise and time-dependent variations respectively, defined as:

$$f(x) = 4 \sin \theta (1 - \cos \theta) \sqrt{27}, \quad (12)$$

$$g(z) = \sum_{l=1}^{l_{max}} Z_l \sin(2\pi l(z^*/L_z^* + \phi_l)), \quad (13)$$

$$h(t) = \sum_{m=1}^{m_{max}} T_m \sin(\omega^* t^* + \phi_m). \quad (14)$$

225

226 Here,  $\omega^*$  is the fundamental frequency of the induced disturbance,  $\theta =$   
227  $2\pi(x^* - x_a^*)/(x_b^* - x_a^*)$ , and  $\phi_l$  and  $\phi_m$  are the random numbers between 0  
228 and 1. The random numbers are generated using the FORTRAN subroutines  
229 of RANDOM\_NUMBER and RANDOM\_SEED which generate the pseudo-  
230 random numbers with uniform distribution between 0 and 1.  $\sum_{l=1}^{l_{max}} Z_l = 1$ ,  
231  $Z_l = 1.25Z_{l=1}$ , with  $l_{max} = 20$  and  $\sum_{m=1}^{m_{max}} T_m = 1$ ,  $T_m = 1.25T_{m=1}$ , with  
232  $m_{max} = 20$ . The above mentioned methodology for generating fully devel-  
233 oped turbulent boundary layer is a modified version of the method used by  
234 Pirozzoli et al. [20]. This methodology has been used by Shadloo et al. [27]  
235 and Shadloo and Hadjadj [28], and their results present good agreement with  
236 the turbulent boundary layer results of Shadloo et al. [23] (cf. figures 4 and  
237 8 in [28]).

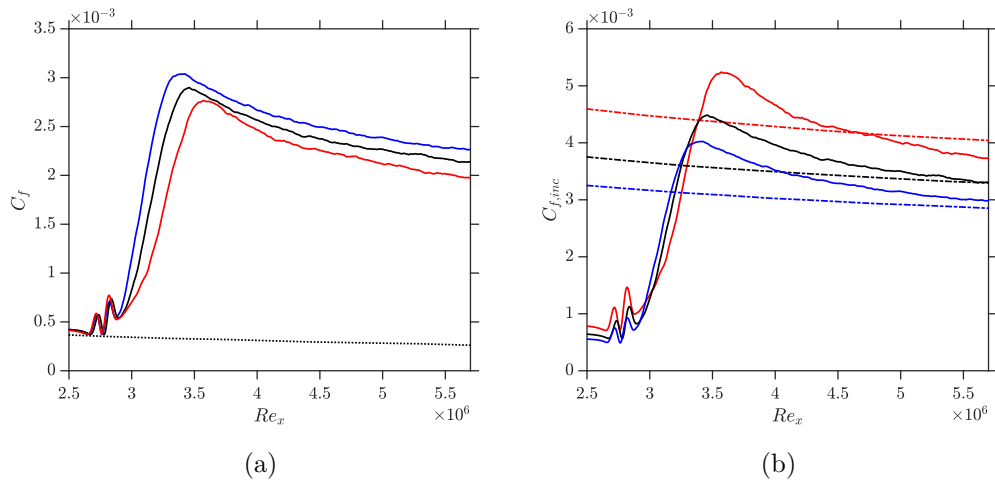


Figure 2: Streamwise evolution of (a) compressible, and (b) incompressible skin-friction coefficients as a function of  $Re_x$  for cooled (—), adiabatic (—) and heated (—) cases. Here, (····) is  $(C_{f,lam} = 0.664 \times \frac{\sqrt{\rho_w^* \mu_w^* / \rho_\infty^* \mu_\infty^*}}{\sqrt{Re_x}})$  and (— · —) is  $(C_{f,inc(turb)} = 0.074 \times \sqrt{\frac{\rho_{aw}^*}{\rho_w^*}} \times Re_x^{-0.2})$  lines represent the theoretical curves for the laminar and the turbulent regimes respectively.

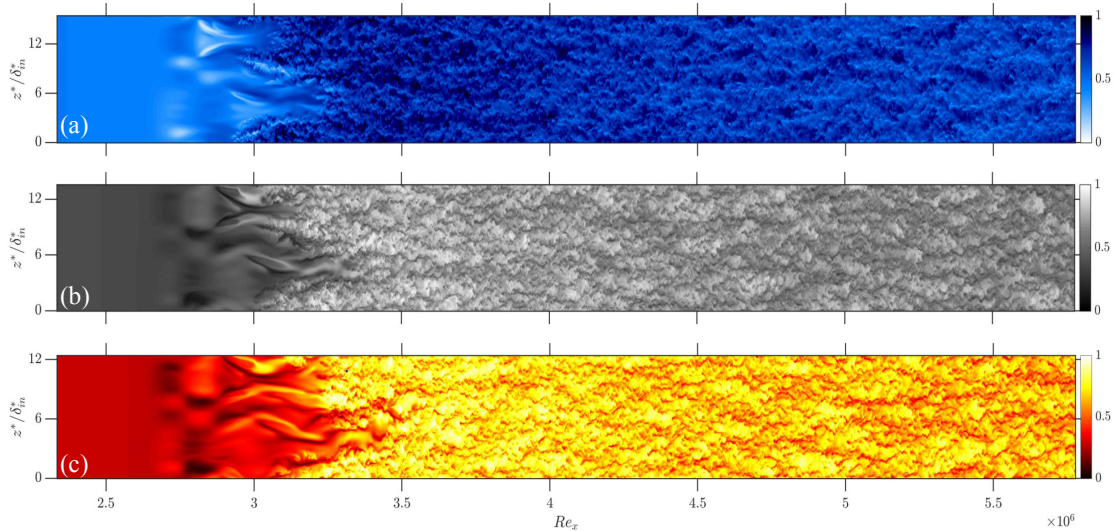


Figure 3: Instantaneous flow fields for (a) cooled, (b) adiabatic and (c) heated walls: contours of  $u^*/u_\infty^*$ , shown at  $y^*/\delta_{in}^* = 0.29$ .

### 238 3. State of the turbulent boundary layer

239 Figure 2 represents the evolution of the compressible and the incompressible  
 240 skin-friction coefficients  $C_f$  and  $C_{f,inc}$  (averaged in time and the spanwise  
 241 direction) in the domain for all the cases under consideration. In this study,  
 242 we would regard the maximum value of  $C_f$  to mark the beginning of the fully  
 243 developed turbulent region.  $C_f$  can be defined as:

$$C_f = \frac{\tau_w^*}{\frac{1}{2}\rho_\infty^* u_\infty^{*2}} \quad (15)$$

244 where,  $\tau_w^*$  is the shear stress at the wall.

245 It can be seen from figure 2a that the boundary layer begins the transition to  
 246 turbulence towards the end of the blowing/suction strip because of the high  
 247 intensity of perturbation, which sets-in the by-pass transition scenario and no



248 secondary instability region (usually marked by the formation of the streaks)  
 249 is formed. The effect of disturbance is visible in the plot due to the high  
 250 intensity of perturbation. Moreover, the levels of skin-friction coefficients  
 251 rise consistently with decreasing wall-temperature in the transitional and the  
 252 turbulent parts of the domain, because of the increasing local density close  
 253 to the wall. More details regarding the effects of various physical parameters  
 254 on the onset of transition can be found in [38, 39]. However, a contrasting  
 255 trend is observed in case of the incompressible  $C_{f,inc}$  (see figure 2b). These  
 256 trends agree well with the findings reported by Shadloo et al. [27], Shadloo  
 257 and Hadjadj [28]. Emperically, the compressible skin-friction coefficient for  
 258 the laminar regime (marked by ..... in figure 2a) is given by [36]:

$$C_{f,lam} = 0.664 \times \frac{\sqrt{\rho_w^* \mu_w^* / \rho_\infty^* \mu_\infty^*}}{\sqrt{Re_x}} \quad (16)$$

259 while the analytical relation for the incompressible skin-friction coefficient  
 260 for the fully developed turbulent region can be given as [27]:

$$C_{f,inc(turb)} = 0.074 \times \sqrt{\frac{\rho_{aw}^*}{\rho_w^*}} \times Re_x^{-0.2} \quad (17)$$

261 Hence, it can be seen in figure 2a that the fully developed turbulent region  
 262 starts from  $Re_x = 3.42 \times 10^6$ ,  $Re_x = 3.46 \times 10^6$  and  $Re_x = 3.57 \times 10^6$   
 263 for cooled, adiabatic and heated walls, respectively. Figure 3 displays the  
 264 instantaneous flow visualizations of different cases showing the existence of  
 265 the turbulent region in the majority of the domain.

266 Frequency spectra for the adiabatic case in the middle of the domain i.e.  
 267 at  $z^*/\delta_{in}^* = 7$  and  $y^+ = 90$  at different streamwise locations;  $Re_x = 3.02 \times 10^6$ ,  
 268  $3.94 \times 10^6$  and  $5.40 \times 10^6$  along with the -5/3rd slope of turbulence decay are  
 269 plotted in figure 4a-c. It can be clearly seen in these spectra plots that as we

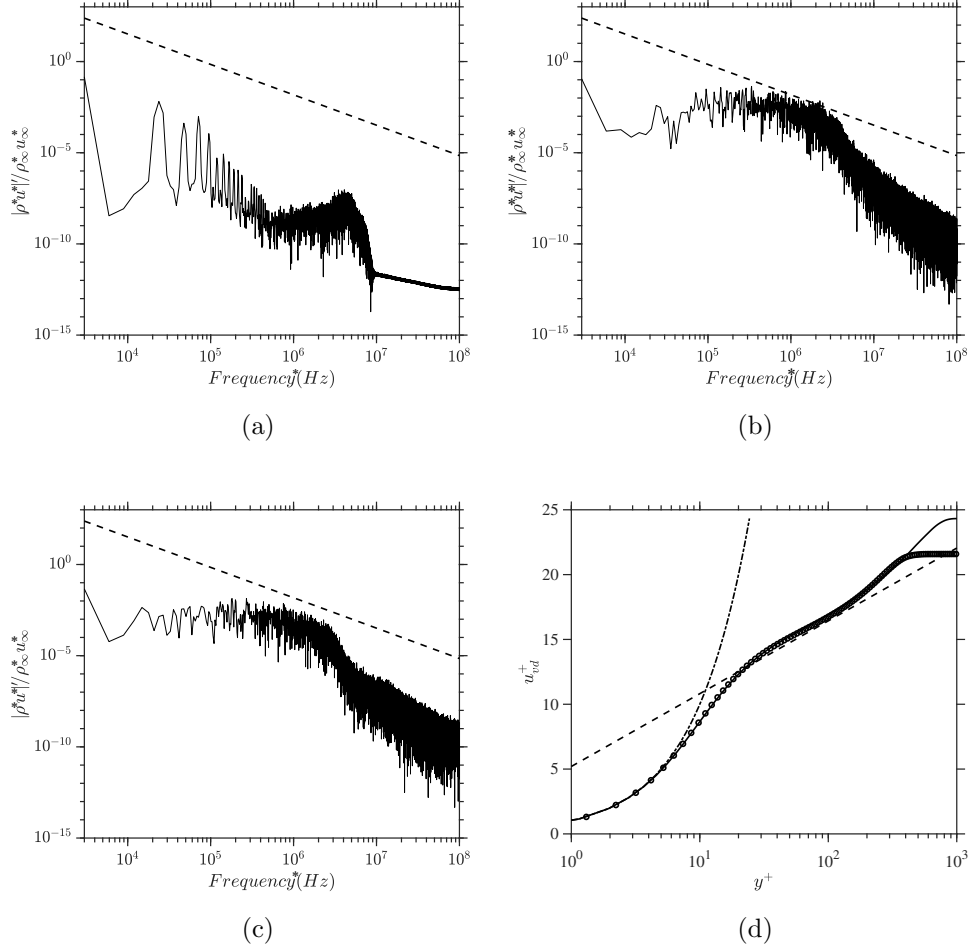


Figure 4: Frequency spectra of  $|\rho^* u^{*'}| / \rho_\infty^* u_\infty^*$  for the adiabatic wall at (a)  $Re_x = 3.02 \times 10^6$ , (b)  $Re_x = 3.94 \times 10^6$ , (c)  $Re_x = 5.40 \times 10^6$ ; where (---) represents the  $(-5/3)$  law of turbulence decay, and (d) Van-Driest transformed mean velocity profile for the adiabatic case compared with [23] (symbols) at  $Re_x = 5.40 \times 10^6$ , where (---) is  $(1/0.41 \log y^+ + 5.2)$  and (-·-) is  $u_{vd}^+ = y^+$ .

270 move from the transition region to the fully developed turbulent regime, the  
271 excitation frequency (23.87 kHz) does no longer remain prominently visible  
272 in the frequency spectrum. Therefore, it can be stated that the resulting  
273 turbulent statistics are not affected by the forcing frequency of the blowing  
274 and suction strip. Figure 4d shows the comparison of the Van-Driest trans-  
275 formed velocity profiles at  $Re_x = 5.40 \times 10^6$  for the adiabatic case vs. Shadloo  
276 et al. [23] revealing the existence of fully developed turbulent flow because  
277 the viscous sublayer and the logarithmic regions are distinctly visible.

#### 278 **4. Joint probability density function and covariance integrands** 279 **analyses**

280 In this section, we will extensively employ the JPDF distribution and the  
281 covariance integrand analyses for investigating different properties affecting  
282 the flow topology of the supersonic turbulent boundary layers. These anal-  
283 yses would be used to describe various structural and dynamical aspects of  
284 the vortical structures and also to get a deeper insight about the physical  
285 mechanisms contributing the most to the turbulent shear-stress and the tur-  
286 bulent heat-flux. Both the JPDFs and the covariance integrands are plotted  
287 using the same bin size. It is to be noted that the results are reported for  
288 a fixed streamwise location in the fully turbulent part of the domain i.e. at  
289  $Re_x = 5.40 \times 10^6$ . A grayscale color palette is used to represent the lev-  
290 els of the contours ranging from white (the **minimum** value) to black (the  
291 maximum value).

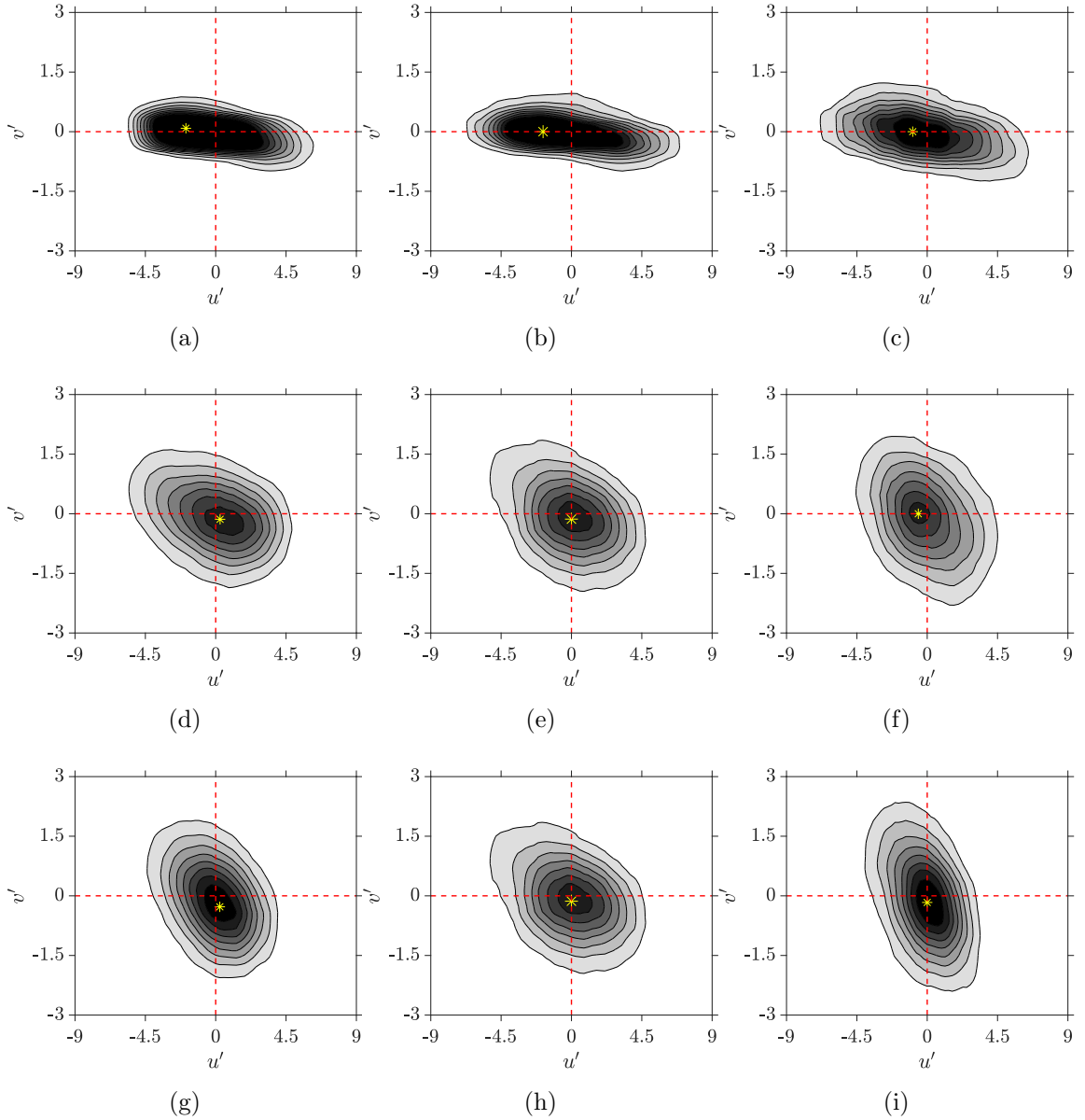


Figure 5: Contours of the joint probability density function (JPDF) distribution of the  $u'$  and  $v'$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The colors of the contour levels vary from 1 to 8 (light to dark). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

Table 2: Locations of the peaks for different cases shown in figure 5. The coordinates are marked as the  $(u', v')$  tuple.

| $y^+$ | C               | A               | H               |
|-------|-----------------|-----------------|-----------------|
| 10    | (-1.906, 0.086) | (-1.815, 0.000) | (-0.926, 0.000) |
| 35    | (0.280, -0.139) | (0.000, -0.135) | (-0.571, 0.000) |
| 90    | (0.264, -0.276) | (0.000, -0.142) | (0.000, -0.166) |

292 *4.1. Velocity fluctuations*

293 Figure 5 depicts the contour plots of the JPDF distribution of the fluc-  
294 tuations of the streamwise ( $u'^*$ ) and wall-normal ( $v'^*$ ) velocity components  
295 scaled by the local friction velocity  $u_\tau^* = \sqrt{\tau_w^*/\rho_w^*}$  at various  $y^+$  locations. In  
296 this figure,  $u' = u'^*/u_\tau^*$  and  $v' = v'^*/u_\tau^*$ . Table 2 enlists the peak locations  
297 for the cases mentioned in the figure 5 (marked by the yellow \*). The JPDF  
298 contours point out that the distribution of the  $v'$  is confined to a very small  
299 area in the buffer layer i.e.  $y^+ = 10$ , hence, the distribution is quite flat  
300 (figures 5a-c). **A comparison of the figures 5a-c shows that, for the near-wall**  
301 **region, the peak tends to move towards zero (see first row of table 2) with**  
302 **increasing wall-temperature confirming the presence of the accelerated flow**  
303 **due to the increase in the momentum transfer process. Moreover, the peak**  
304 **locations marked in the first row of table 2 show negligible effects of the wall-**  
305 **temperature on the  $v'$  which is due to the strong viscous forces close to the**  
306 **wall.** Similar behavior has been reported by Wallace and Brodkey [9] in case  
307 of the incompressible turbulent boundary layer. The comparison of the first  
308 and the second rows of the figure 5, reveals that on moving from the buffer  
309 layer to the log-region i.e  $y^+ = 10$  to 35, the peak moves in the direction of

310 the fourth quadrant which means that an increasing amount of fast moving  
 311 flow going towards the wall (more details on the quadrant analysis will fol-  
 312 low subsequently). However in the log-region, at  $y^+ = 90$  (figures 5g-i), the  
 313 positions of the peaks do not show a prominent variation (also see the third  
 314 row of table 2) with respect to the wall-temperature. This means that the  
 315 effects of the wall-temperature are confined to the near-wall region only. On  
 316 comparing figures 5d to 5f with figures 5g to 5i, it can be observed that the  
 317 distribution of  $v'$  grows more rapidly and dramatically with increase in  $y^+$  as  
 318 the wall temperature increases. This trend of growth mechanism shows an  
 319 increased amount of wall-normal fluctuations in the log region with increasing  
 320 wall-temperature, which generates higher levels of the turbulent shear-stress  
 321 in the heated case. It should be noted here that the JPDF distribution tends  
 322 to align its major axis with the corresponding dominant quadrants. From  
 323 our discussion of this set of figures, it can be said that impact of wall heat  
 324 transfer on the distribution of  $u'$  and  $v'$  can be seen predominantly in the  
 325 buffer layer region.

326 In order to have a better insight about the flow topology, we would now  
 327 use the quadrant analysis previously reported in [8, 9, 10]. The quadrant  
 328 analysis of the contour plots of the covariance integrands provides a better  
 329 understanding of various physical phenomena related to the corresponding  
 330 quantities. Each quadrant represents a particular transfer process and the  
 331 dominant quadrant represents the most influential of these. The turbulent  
 332 shear-stress covariance,  $\overline{u'v'}$  can be written as

$$\overline{u'v'} = \int \int_{-\infty}^{\infty} u'v'P(u', v')du'dv', \quad (18)$$

333 where,  $P(u', v')$  is the joint probability density function of the  $u'$  and  $v'$  over

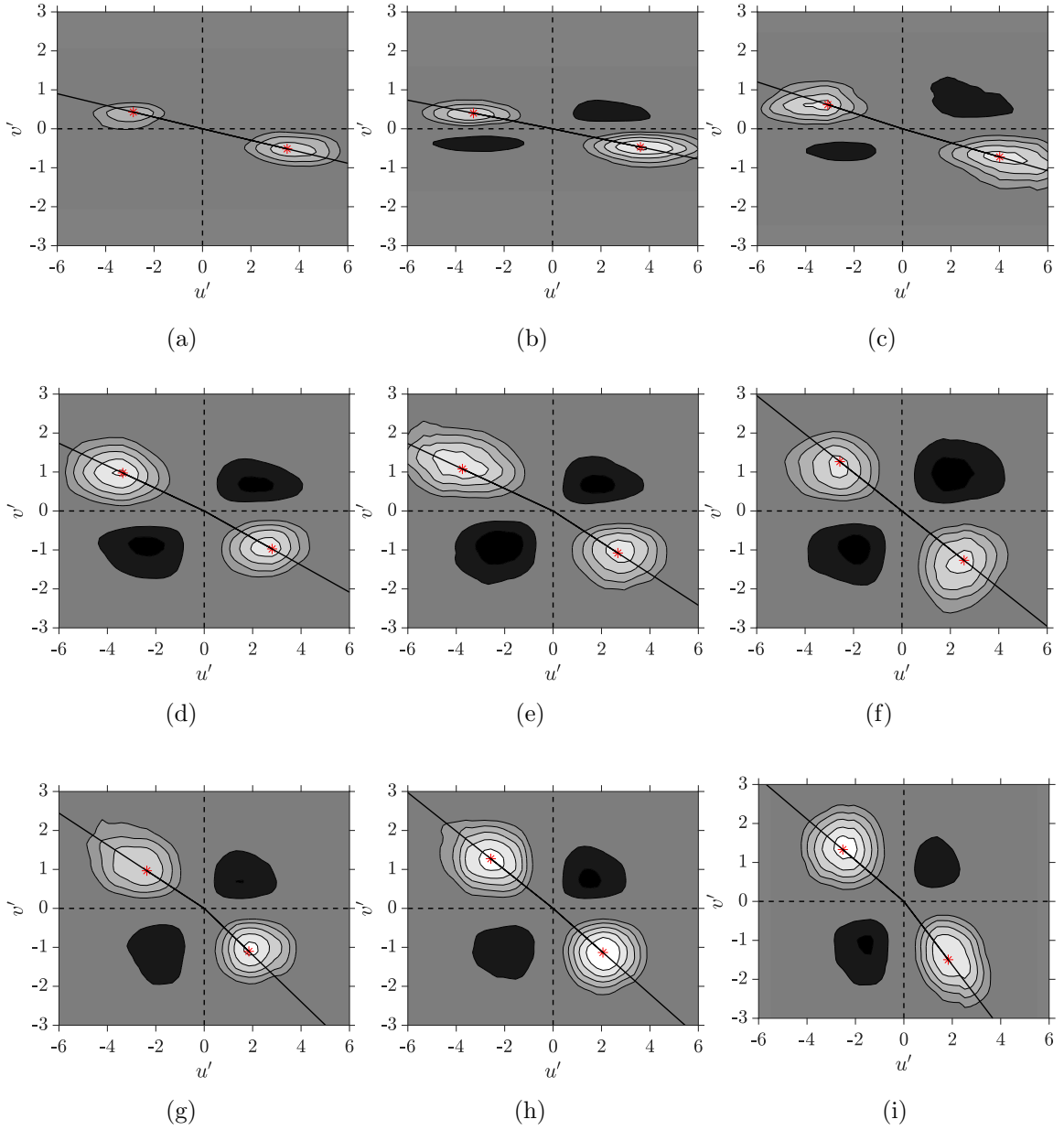


Figure 6: Contours of the covariance integrands of the  $u'$  and  $v'$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The colors of the contour levels vary from -8 to 2, excluding the zero level (light to dark). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

334 a bin area of  $du'dv'$  and the quantity  $u'v'P(u'v')$  is known as the covariance  
335 integrand.

336 The contour plots of the covariance integrands of  $u'v'$  represent the con-  
337 tribution of signs and magnitude of a given component of velocity i.e.  $u'$  or  $v'$   
338 towards the turbulent shear-stress covariance,  $\overline{u'v'}$ . The first quadrant ( $Q_1$ ),  
339 where  $u' > 0$  and  $v' > 0$ , represents the outward interactions. The second  
340 quadrant ( $Q_2$ ), where  $u' < 0$  and  $v' > 0$ , represents the events of ejections.  
341 The third quadrant ( $Q_3$ ), where  $u' < 0$  and  $v' < 0$ , represents the inward  
342 interactions, and the fourth one ( $Q_4$ ), with  $u' > 0$  and  $v' < 0$ , represents the  
343 sweeps [9, 10, 18].

344 Figure 6 represents the contour plots of the covariance integrands of the  $u'$   
345 and  $v'$  for the corresponding  $y^+$  positions mentioned in figure 5. In this figure,  
346 dark contours signify the positive contour levels i.e. levels 1 and 2 while  
347 the light ones are the negative contour levels (levels -8 to -1, see figure 5).  
348 The solid black lines in each plot serve as a visual reference for determining  
349 the angles of inclination with respect to the positive  $u'$  axis. These lines  
350 originate from the origin and pass through the peaks of the most dominant  
351 quadrants (marked by the red \*). The changes in the angles of inclination  
352 quantify the shift in the observed physical phenomenon for the corresponding  
353 quadrant. At a glance of figure 6, it can be said that for all of the cases  
354  $Q_2$  and  $Q_4$  are the dominant quadrants which means that the ejection and  
355 sweep mechanisms contribute the most to the turbulent shear stress. For  
356 incompressible channel flows, as per the findings of Wallace et al. [8], the  
357 ejections and sweeps contribute more than 100% towards the shear-stress,  
358 and additional stress generated is countered by the positive and negative



359 interactions represented by the events of  $Q_1$  and  $Q_3$  respectively. Hence, the  
360 results shown in figure 6 are in good agreement with their incompressible  
361 counterparts. A closer look to this figure reveals that in the buffer region,  
362 at  $y^+ = 10$ , the sweeps are more important in comparison to the ejections  
363 in case of the cooled and to some extent for the adiabatic wall (figures 6a  
364 and b) because there exists an additional lower contour level. This means  
365 that the high-speed fluid moving towards the wall is the major contributor  
366 to the turbulent shear-stress. However, figure 6c suggests that for  $y^+ = 10$ ,  
367 the ejections and sweeps become comparable for the heated wall. Therefore,  
368 it can be said that in the buffer-region, decrease in wall-temperature favors  
369 the sweep events. At  $y^+ = 35$  (figures 6d-f), it can be seen that the ejection  
370 events are dominant for the cooled and the adiabatic walls, whereas for the  
371 heated wall, both the ejections and sweeps become comparable, which means  
372 that the decreasing wall-temperature favors the ejections. Towards the end  
373 of the log-region, at  $y^+ = 90$ , the ejections and the sweeps are comparable for  
374 the cases A and H while for the case C sweeps contribute the most towards  
375 the shear-stress covariance.

#### 376 4.2. Vorticity fluctuations

377 In this section we would talk about the topology and the physical ori-  
378 entation of the coherent structures in the flow field. The JPDF and the  
379 covariance integrand contours of the vorticity components shed some light  
380 on the flow topology. The alignment of these contours indicates the most-  
381 likely orientation of the projections of the coherent structures at that given  
382 position in different planes. It should be noted here that this representation  
383 is global in nature and reveals the most-likely inclinations of the projections

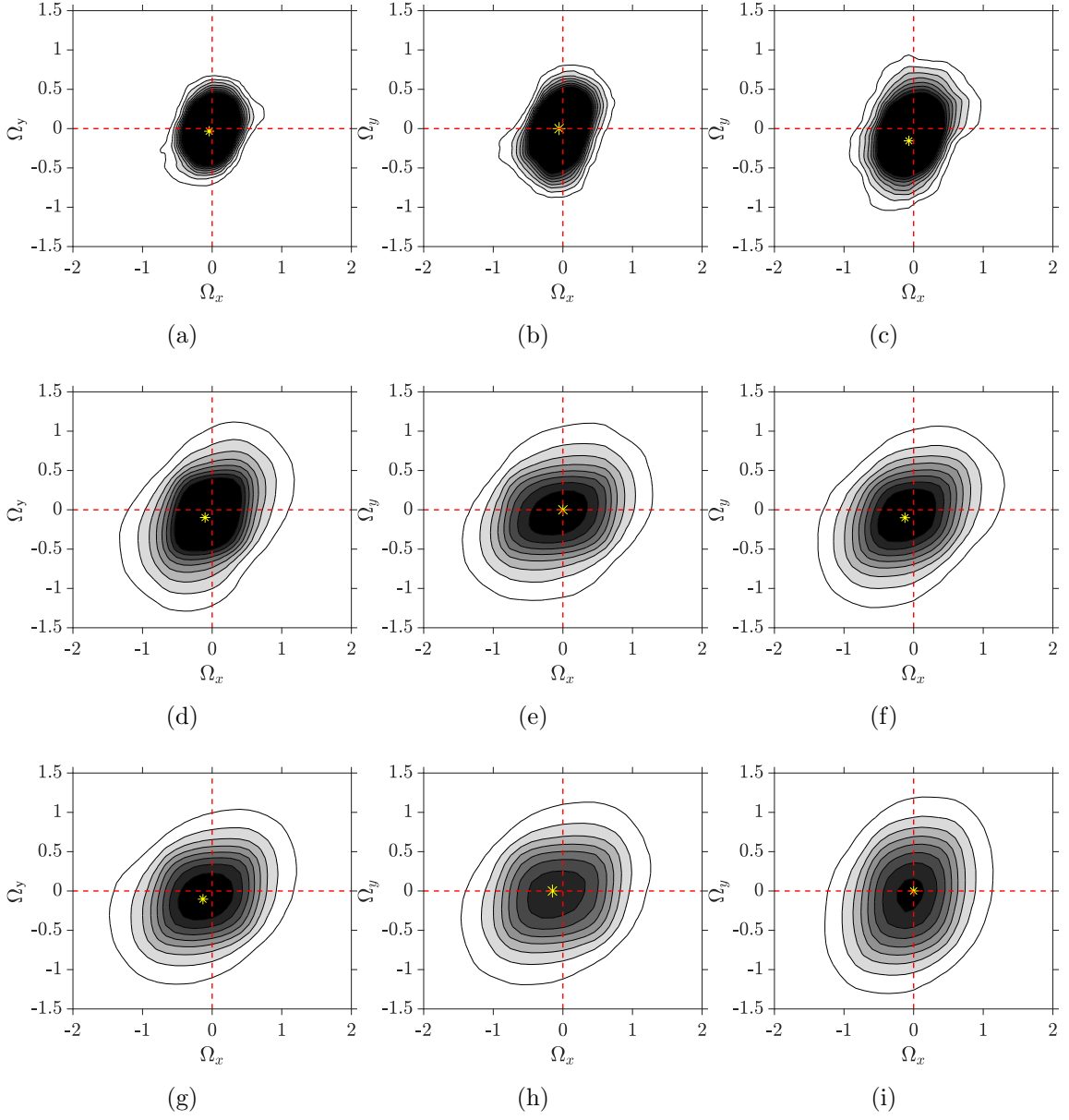


Figure 7: Contours of the JPDF distribution of the  $\Omega_x$  and  $\Omega_y$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The colors of the contour levels vary from 5 to 40 (light to dark). Here, the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

384 of the coherent structures in three-dimensional sense. The structures present  
385 in the turbulent field can have any sense of rotation or they may exist in  
386 pairs of counter-rotating vortices which in three-dimensional sense represent  
387 a hairpin (or lambda) structure. Other possibilities of flow configuration also  
388 exist such as the hairpin forests [6], or the cane structures (or asymmetrical  
389 hairpins) [18] among others.

390 Figure 7 displays the contours of the JPDF distribution of the streamwise  
391 and wall-normal vorticity components;  $\Omega_x^*$  and  $\Omega_y^*$  respectively which are  
392 normalized by the local vorticity magnitude i.e.  $\Omega_x = \Omega_x^*/\Omega^*$  and  $\Omega_y =$   
393  $\Omega_y^*/\Omega^*$ , where  $\langle\langle\Omega^*\rangle\rangle = \sqrt{\Omega_x^{*2} + \Omega_y^{*2} + \Omega_z^{*2}}$ ,  $\langle\langle\rangle\rangle$  represents local time-averaged  
394 quantity, and  $\Omega_x^* = \frac{\partial w^*}{\partial y^*} - \frac{\partial v^*}{\partial z^*}$ ,  $\Omega_y^* = \frac{\partial u^*}{\partial z^*} - \frac{\partial w^*}{\partial x^*}$  and  $\Omega_z^* = \frac{\partial v^*}{\partial x^*} - \frac{\partial u^*}{\partial y^*}$ . These  
395 contours represent the most probable projections of the coherent structures  
396 in the streamwise-wall-normal plane (x-y plane) at some angle to the wall  
397 that can vary along the length of the vortical structure [11]. The orientation  
398 of these contours shows the positive correlation between the two quantities  
399 which is obvious due to the high-speed of the flow in the streamwise direction.  
400 On comparing the first, second and third rows of figure 7 it can be stated that  
401 the contours' orientation **increases** in the streamwise direction which is due  
402 to the increment in the streamwise velocity with increasing  $y^+$ . Moreover,  
403 it can be seen from these figures that the wall-temperature does not has  
404 significant impact on the inclination of the contours.

405 The contours of the covariance integrands of  $\Omega_x$  and  $\Omega_y$  i.e.  $\Omega_x\Omega_yP(\Omega_x, \Omega_y)$   
406 are shown in figure 8. It should be noted that, for this quadrant analysis,  
407 the physical significance of each quadrant is not the same as stated before  
408 for figure 6. In this case, the dominant quadrants highlight the most-likely

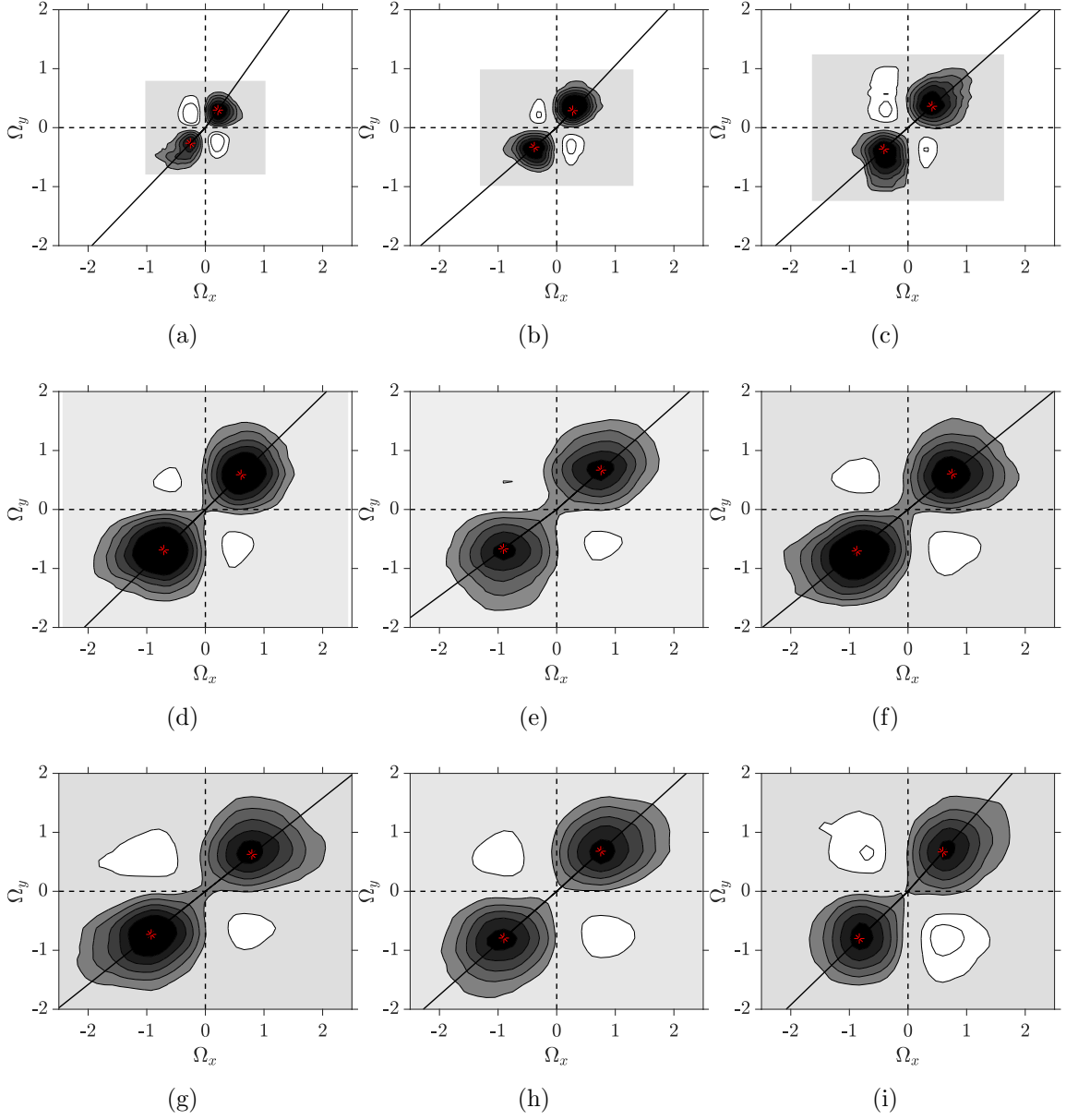


Figure 8: Contours of the covariance integrands of the  $\Omega_x$  and  $\Omega_y$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The colors of the contour levels vary from -0.5 to 3, excluding the zero level (light to dark). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

409 orientation of the vorticity filaments (their projections) at the given loca-  
 410 tions which contribute to the covariance  $\overline{\Omega_x \Omega_y}$ . It can be clearly seen from  
 411 these plots that as a result of the positive correlation between the two quan-  
 412 tities,  $Q_1$  and  $Q_3$  are the dominant quadrants here. The orientation of the  
 413 projections of the vorticity filaments in the (x-y) plane can be given as

$$\alpha = \tan^{-1} \left( \frac{\Omega_y}{\Omega_x} \right) \quad (19)$$

414  $\alpha$  is the angle made by the peaks of the dominant quadrants ( $Q_1$  and  $Q_3$   
 415 here) with respect to the positive  $\Omega_x$ -axis. The solid black lines are marked  
 416 to serve as a visual aid to estimate the angles of inclination. In figures 8a-  
 417 c, the white regions surrounding the gray rectangles are the areas with no  
 418 data. On comparing the inclinations of the vorticity filaments at different  $y^+$   
 419 positions, it can be observed that  $\alpha$  is  $54^\circ$ ,  $44^\circ$  and  $38^\circ$  in  $Q_1$  and  $-134^\circ$ ,  $-136^\circ$   
 420 and  $-142^\circ$  in  $Q_3$  for  $y^+ = 10, 35$  and  $90$ , respectively for the cooled wall (i.e.  
 421 figures 8a, d and g). This trend suggests that on moving from the buffer layer  
 422 ( $y^+ = 10$ ) to the log region ( $y^+ = 35$ ), the filaments tend to rotate in the  
 423 streamwise direction. The same trend is observed for all the three cases when  
 424 moving from  $y^+ = 10$  to  $35$  (see first two rows of table 4). However, for the  
 425 adiabatic wall, the  $\alpha$  is about  $46^\circ$ ,  $41^\circ$  and  $42^\circ$  in  $Q_1$  and  $-139^\circ$ ,  $-144^\circ$  and  
 426  $-139^\circ$  in  $Q_3$  respectively. Hence, the adiabatic wall shows subtle variation  
 427 in the log-region (angles in  $Q_1$ ), this trend is similar to the one reported  
 428 by Ong and Wallace [11] for the incompressible turbulent flows (keeping in  
 429 mind the  $\pm 5^\circ$  error for the bin size used by them). The visual inspection of  
 430 the orientation and dominance of the quadrants with the literature reveals  
 431 that the results of both the cooled as well as the adiabatic walls bear close  
 432 similarities with the results of the compressible flows [26] (due to the different

Table 3: Locations of the peaks for different cases shown in figure 9. The coordinates are marked as the  $(\Omega_x, \Omega_z)$  tuple.

| $y^+$ | C                | A                | H                |
|-------|------------------|------------------|------------------|
| 10    | (0.000, -0.983)  | (0.000, -0.884)  | (-0.068, -0.889) |
| 35    | (-0.102, -0.497) | (0.000, -0.398)  | (-0.125, -0.456) |
| 90    | (-0.132, -0.304) | (-0.150, -0.286) | (-0.119, -0.348) |

433  $y^+$  locations reported, a direct comparison is not possible). However, for the  
434 heated wall, the inclination angles decrease in the early log region followed  
435 by a strong increase i.e.  $42^\circ$ ,  $39^\circ$  and  $48^\circ$  in  $Q_1$  for  $y^+ = 10, 35$  and  $90$ ,  
436 respectively. This variation in the trend of inclination means that in the log  
437 region the wall-normal vorticity component is still on the rise due to increased  
438 heat-transfer from the wall. A comparison between the angles of inclination  
439 of the second and the third rows of figure 8 (see table 4) reveals that on  
440 moving from  $y^+ = 35$  to  $90$ , the vortical filaments tend to orient themselves  
441 lesser in the streamwise direction (as evident from the increasing values of  $\alpha$ )  
442 with increasing wall-temperature. Therefore, from our discussion of about  
443 this set of figures, it can be stated that the wall temperature affects the usual  
444 orientation of the vorticity filaments in the buffer-layer region as well as in  
445 the log-region due to the strong heat transfer.

446 Now, we consider the orientation of the projections of the coherent struc-  
447 tures in the x-z plane (streamwise-spanwise plane). Figure 9 shows the JPDF  
448 distribution,  $P(\Omega_x, \Omega_z)$  of the streamwise and the wall-normal vorticity com-  
449 ponents,  $\Omega_x^*$  and  $\Omega_z^*$ , respectively which are normalized by the time-averaged  
450 local vorticity magnitude  $\langle \Omega^* \rangle$  at different  $y^+$  positions for all the three cases

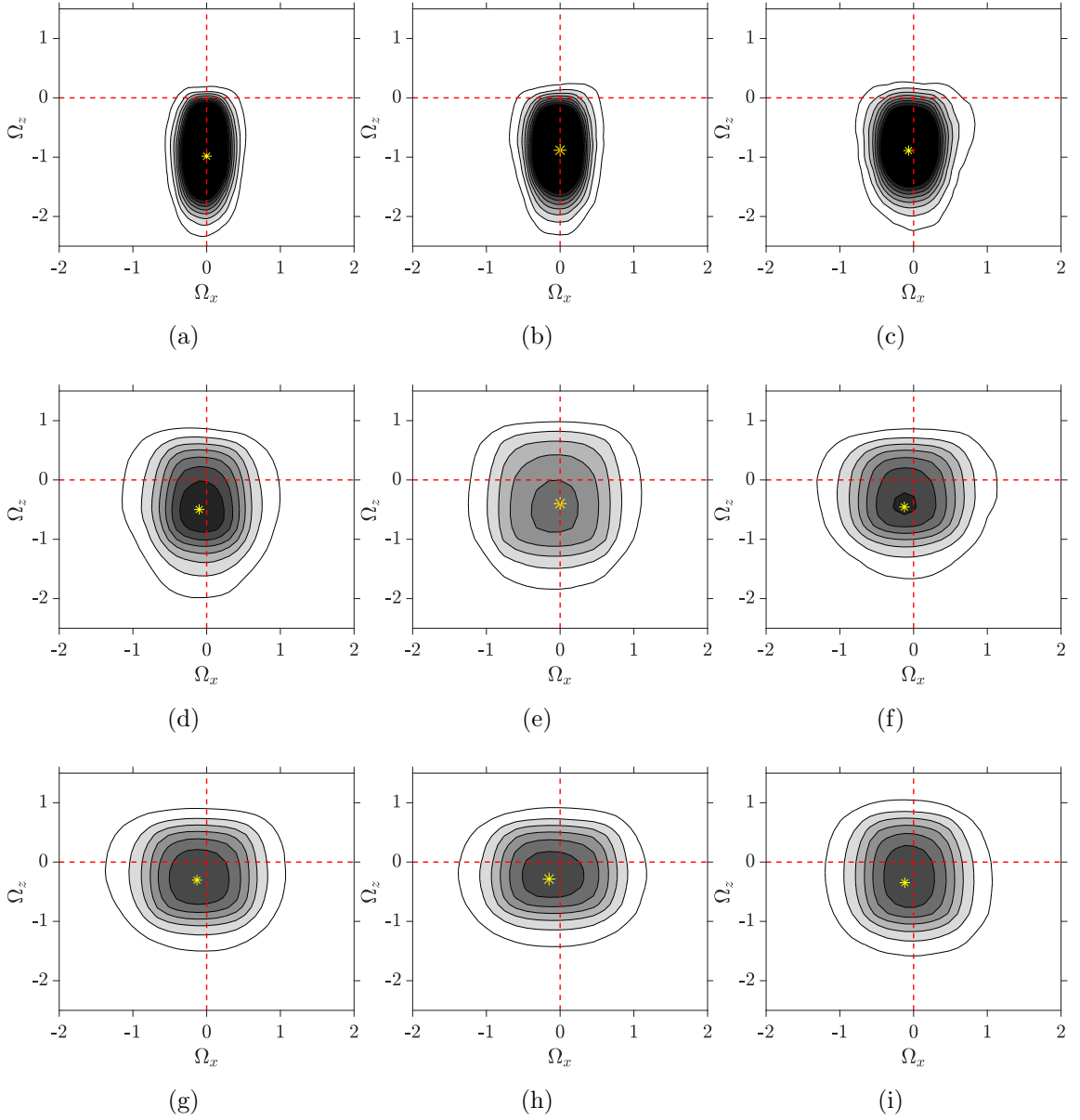


Figure 9: Contours of the JPDF distribution of the  $\Omega_x$  and  $\Omega_z$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The color of the contour levels vary from 5 to 40 (light to dark). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

451 (A, C and H), and table 3 enlists the peak locations for the same cases. It  
 452 can be seen in these figures that in the buffer layer,  $y^+ = 10$  (figures 9a to  
 453 9c) the JPDF contours are somewhat triangular in shape. This triangular  
 454 shape becomes predominant with increasing wall-temperature which indi-  
 455 cates a net increase in the magnitude of  $\Omega_x$  as the wall-temperature rises. As  
 456 we progress farther from the buffer-layer region ( $y^+ = 10$ ) to the log-region  
 457 ( $y^+ = 35$ ), the contours start to get more dilated in the  $\Omega_x$  direction irre-  
 458 spective of the wall-temperature, however, the peak remains in the vicinity  
 459 of zero for the  $\Omega_x$ -axis (see table 3). The dilatation of the contours gets in-  
 460 creased with increasing wall-temperature (figures 9a to f) which means that  
 461 increasing wall-temperature also increases the net magnitude of  $\Omega_x$ . These  
 462 triangular shapes of JPDF contours imply that the vorticity filaments do not  
 463 show much inclination in the streamwise direction and the spanwise vorticity  
 464 component ( $\Omega_z$ ) largely remains negative, meaning that it has the same sign  
 465 as that of the mean shear. It can be clearly seen from table 3 that there  
 466 exists a considerable shift in the location of the peak towards the positive  
 467  $\Omega_z$ -axis ( $\approx 50\%$ ) irrespective of the wall-temperature as one moves to the  
 468 higher  $y^+$  locations which implies a sudden and dramatic increase of the  
 469 wall-normal vorticity component. This comparison of the peak location also  
 470 reveals that the shift of the peak locations is less significant with increasing  
 471 wall-temperature. From figure 9, it can be concluded that  $\Omega_z$  is dominant in  
 472 comparison to  $\Omega_x$  because the location of the peak never changed consider-  
 473 ably in the  $\Omega_x$ -axis.

474 Figure 10 shows the plots of the contours of the covariance integrand of  
 475 the  $\Omega_x$  and  $\Omega_z$  at different  $y^+$  locations. **The white regions around the dark**



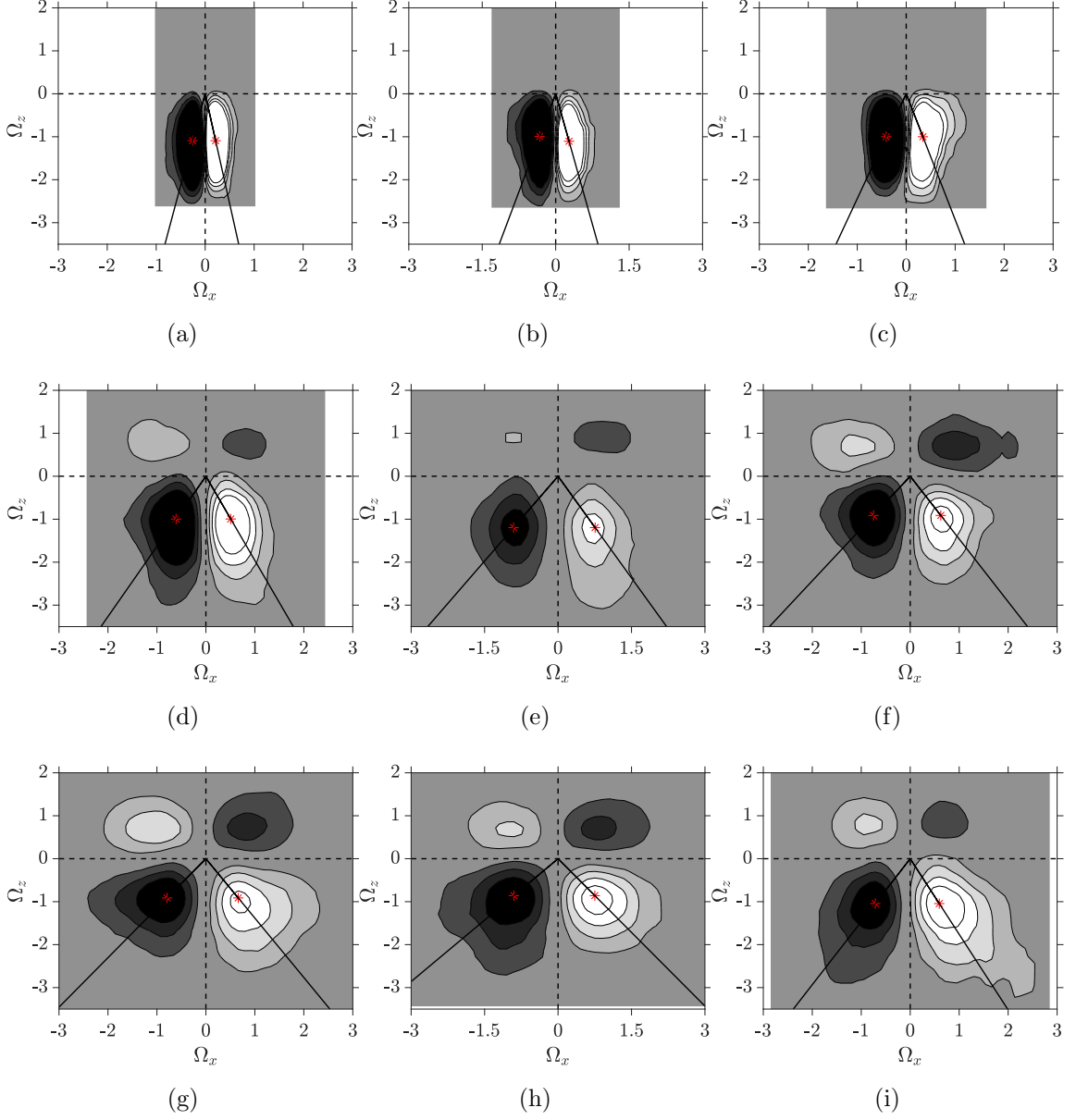


Figure 10: Contours of the covariance integrands of the  $\Omega_x$  and  $\Omega_z$  at  $y^+ = 10$  ((a)-(c)), 35 ((d)-(f)) and 90 ((g)-(i)). The color of the contour levels vary from -2 to 1.5, excluding the zero level (light to dark). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively.

476 rectangles in figures 10a-d and i are the regions without any data. In the  
 477 buffer-layer ( $y^+ = 10$ , figures 10a-c), no contour levels exist in the  $Q_1$  nor  
 478 in the  $Q_2$  which is consistent with figures 9a-c which show that the JPDF  
 479 distribution of  $\Omega_x$  and  $\Omega_z$  exists only in the  $Q_3$  and  $Q_4$ . Therefore, figure  
 480 10 clearly shows that the  $Q_3$  and  $Q_4$  are the dominant quadrants. The  
 481 angles of inclination of projections of the vorticity filaments in the x-z plane  
 482 (represented by the solid black lines) can be defined as

$$\beta = \tan^{-1} \left( \frac{\Omega_x}{\Omega_z} \right). \quad (20)$$

483 These inclinations are mentioned here with respect to the negative  $\Omega_z$ -axis.  
 484 Like before, the counter-clockwise sense is considered as the positive sense of  
 485 rotation while the clockwise sense is the negative one. On moving from the  
 486 buffer-layer to the log-region; from  $y^+ = 10$  to 35, the inclination angles in-  
 487 crease for all the cases (see table 4). Moreover, for these locations, the angles  
 488 of inclination increase with increasing wall-temperature. For case C, in the  
 489 buffer layer region, i.e.  $y^+ = 10$  (figure 10a), the vortical filament projections  
 490 are inclined at  $11^\circ$  and  $-13^\circ$  in the  $Q_4$  and  $Q_3$  quadrants, respectively. These  
 491 low values imply that in the x-z plane, the coherent structures are primarily  
 492 oriented in the negative spanwise direction. However, with an increment in  
 493 the distance from the wall, the filaments tend to rotate in the streamwise  
 494 direction resulting in the augmentation of the angles of inclination to  $27^\circ$   
 495 and  $-31^\circ$  for the  $Q_4$  and  $Q_3$ , respectively at  $y^+ = 35$ , and to  $36^\circ$  and  $-41^\circ$   
 496 at  $y^+ = 90$ . The same trend is observed for the adiabatic wall as well (see  
 497 table 4). Significant differences exist regarding the angles of inclination for  
 498 the heated wall at different  $y^+$  locations because of the increased transfer  
 499 processes due to higher wall-temperature. Notably, at  $y^+ = 90$ , the values

Table 4: Angles of inclination of the projections of the vorticity filaments for different cases at various  $y^+$  positions.

|       | Cooled wall |       |         |       | Adiabatic wall |       |         |       | Heated wall |       |         |       |
|-------|-------------|-------|---------|-------|----------------|-------|---------|-------|-------------|-------|---------|-------|
|       | $\alpha$    |       | $\beta$ |       | $\alpha$       |       | $\beta$ |       | $\alpha$    |       | $\beta$ |       |
| $y^+$ | $Q_1$       | $Q_3$ | $Q_3$   | $Q_4$ | $Q_1$          | $Q_3$ | $Q_3$   | $Q_4$ | $Q_1$       | $Q_3$ | $Q_3$   | $Q_4$ |
| 10    | 54          | -134  | -13     | 11    | 46             | -139  | -18     | 14    | 42          | -138  | -22     | 19    |
| 35    | 44          | -136  | -31     | 27    | 41             | -144  | -37     | 32    | 39          | -141  | -39     | 34    |
| 90    | 38          | -142  | -41     | 36    | 42             | -139  | -40     | 35    | 48          | -136  | -34     | 30    |

500 of  $\beta$  decrease for the heated wall (see table 4). This can be explained by the  
501 JPDF distribution shown in figure 9i, which shows a dramatic increase in the  
502 distribution of the  $\Omega_z$  as a consequence of the increased wall-temperature.  
503 Hence, the distribution of  $\Omega_x$  slightly shrinks. Therefore, it can be said that  
504 the wall-temperature affects the topology of the vortical elements in the x-z  
505 plane also in both the buffer-layer and the log-region.

506 Table 4 enlists all the values of  $\alpha$  and  $\beta$  for all the cases at different  $y^+$   
507 positions, and also brings out the impacts of wall temperature on these angles.  
508 From this table, it can be clearly interpreted that the wall-temperature affects  
509 the turbulent flow topology in both the buffer-layer as well as the log-region  
510 for the SBLs.

### 511 4.3. Streamwise velocity component and temperature fluctuations

512 So far, we have discussed the kinetic aspects of the flow in detail which  
513 dealt with the velocity fluctuations and the vorticity components. For this  
514 study, we are utilizing different wall temperatures, hence, it becomes vital to  
515 investigate the implications of the wall-temperature on the heat-flux. More-

Table 5: Locations of the peaks for different cases shown in figure 11. The coordinates are marked as the  $(u', T')$  tuple.

| $y^+$ | C                | A               | H               |
|-------|------------------|-----------------|-----------------|
| 5     | (-1.145, -0.004) | (-1.391, 0.013) | (-1.221, 0.011) |
| 10    | (-1.906, -0.000) | (-1.815, 0.026) | (-0.926, 0.026) |
| 35    | (0.280, -0.006)  | (-0.268, 0.000) | (-0.570, 0.025) |
| 90    | (-0.264, -0.006) | (0.000, -0.009) | (0.000, -0.029) |

516 over, in the supersonic regime, the wall-temperature is one of the important  
 517 factors to be looked into because it is impacted by different physical quanti-  
 518 ties like the local density and viscosity which themselves are affected by the  
 519 compressibility.

520 Figure 11 shows the JPDF distribution of the streamwise velocity fluc-  
 521 tuations ( $u'^*$ ) and the temperature fluctuations ( $T'^*$ ) for all the cases at  
 522 different  $y^+$  locations, and table 5 enlists the locations of the peaks for the  
 523 corresponding cases. In this figure, an additional location in the viscous  
 524 sub-layer ( $y^+ = 5$ ) is also shown, in order to explore the near-wall region  
 525 in greater detail. Here, the temperature axis is scaled by the time-averaged  
 526 local temperature,  $\langle T^* \rangle$  and the velocity by the local friction velocity ( $u_\tau^*$ ).  
 527 The relation between the  $u'$  and the  $T'$  represents the turbulent heat-flux in  
 528 the streamwise direction. For the rest of the manuscript, the correlation and  
 529 the anti-correlation between the mentioned quantities refers to the existence  
 530 of the positive, and the negative slopes respectively of the JPDF contours  
 531 with the axis of abscissae. A comparison of the figures 11a to 11c high-  
 532 lights the effects of the wall-temperature in the viscous sub-layer. It can be

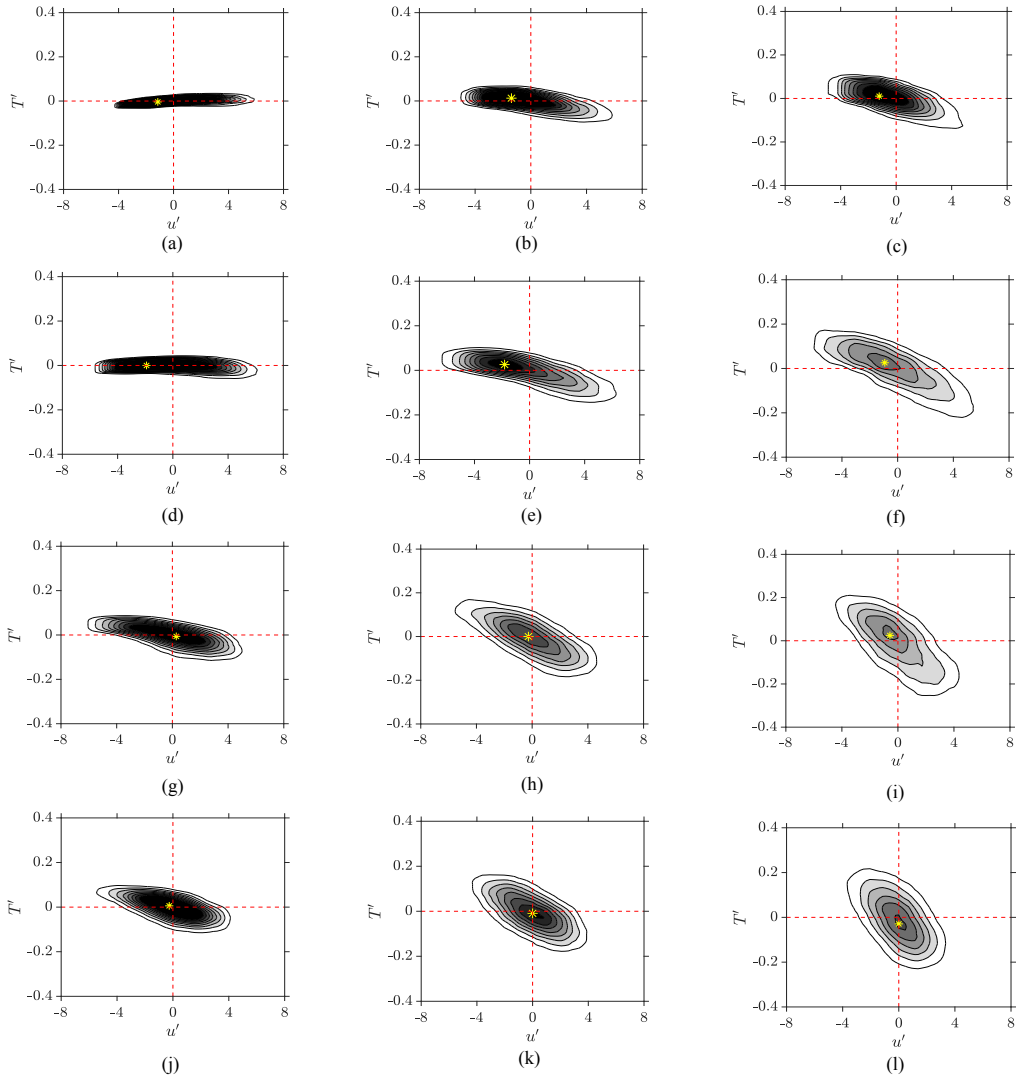


Figure 11: Contours of the JPDF distribution of the  $u'$  and  $T'$  at  $y^+ = 5$  ((a)-(c)), 10 ((d)-(f)), 35 ((g)-(i)) and 90((j)-(k)). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively. the color of the contour levels vary from 15 to 120 (light to dark).

533 seen that in case of the cooled wall, in the viscous sub-layer, the  $u'$  and  $T'$   
 534 are directly correlated, while a clear anti-correlation can be seen in case of  
 535 the adiabatic and the heated walls. This trend has been reported by Duan  
 536 et al. [2], Lechner et al. [22], Shadloo et al. [23] for the supersonic turbu-  
 537 lent flat plates and the channel flows. This set of figures clearly reveals a  
 538 trend of change in the orientation of the major axis of the JPDF contours  
 539 with increment in the  $y^+$  with respect to the  $u'$  axis. For case C, in the  
 540 buffer layer region (at  $y^+ = 10$ ), the JPDF contours become parallel to the  
 541  $u'$  axis representing a flat distribution. It can be seen from figure 11d that  
 542 the peak of  $u'$  is shifted to the negative side (see first column of table 5)  
 543 which implies the existence of the retarded flow in the streamwise direction.  
 544 **On the other hand, the flow is comparatively less retarded for the adiabatic**  
 545 **and the heated walls.** This clearly indicates that in the buffer-layer region  
 546  $y^+ = 10$ , the increasing wall-temperature tends to decrease the deceleration  
 547 of the flow in the streamwise direction. For the higher  $y^+$ , the  $u'$  and the  $T'$   
 548 become anti-correlated for the cooled wall. However, the adiabatic and the  
 549 heated cases remain anti-correlated from the viscous sub-layer itself. It can  
 550 be clearly seen from the figures 11g-l that in the log-region, for all the cases,  
 551 the peak location remains very close to zero (see table 5) which indicates  
 552 the existence of the homogenous turbulence. This set of figures also clarifies  
 553 that the increasing wall-temperature favors the anti-correlation between the  
 554  $u'$  and the  $T'$ .

555 The contours of the covariance integrands of the  $u'$  and  $T'$ , i.e.  $u'T'P(u', T')$   
 556 are shown in figure 12 which represent the contributions of the  $u'$  and the  
 557  $T'$  towards the streamwise component of the heat-flux,  $\overline{u'T'}$ . In order to

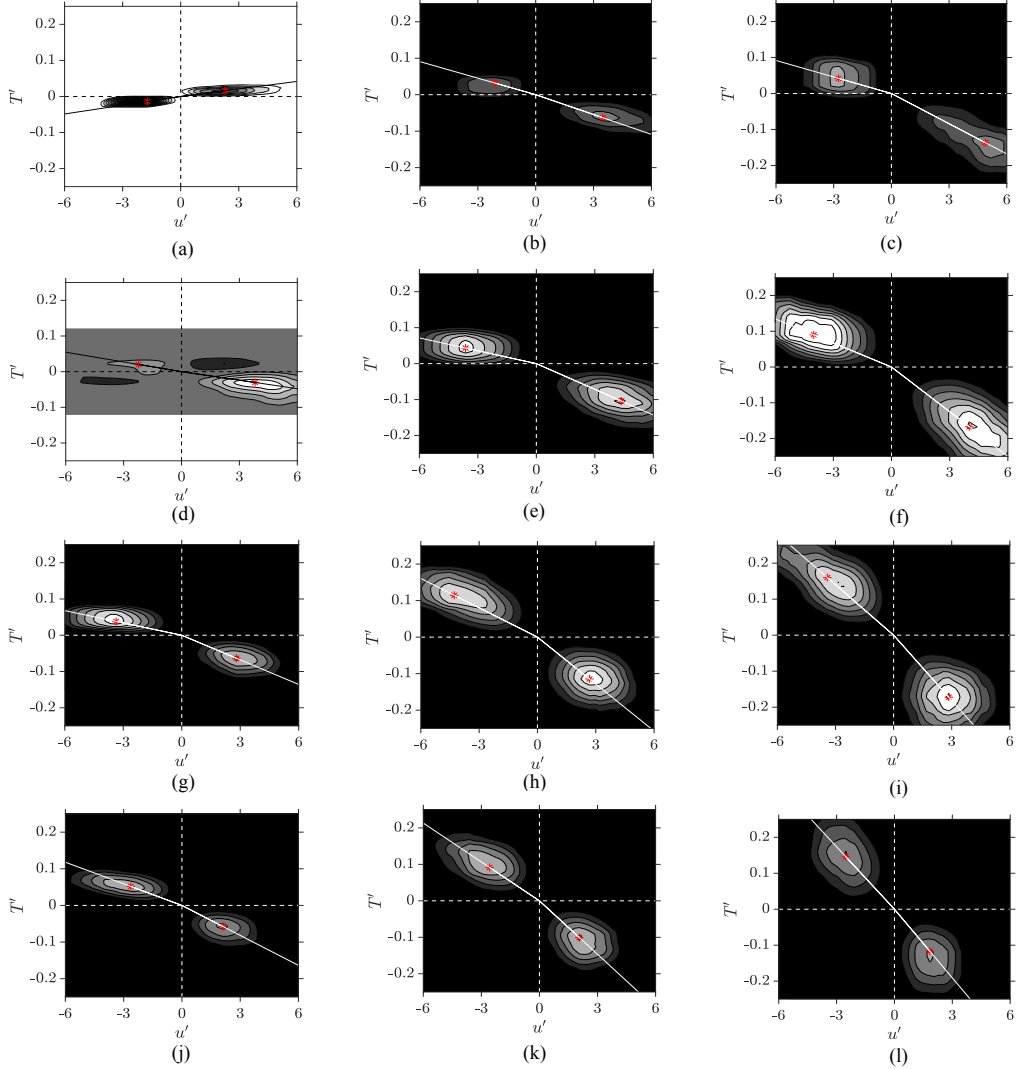


Figure 12: Contours of the covariance integrands of the  $u'$  and  $T'$  for  $y^+ = 5$  ((a)-(c)), 10 ((d)-(f)), 35 ((g)-(i)) and 90((j)-(l)). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively. The colors of the contour levels vary from 2 to 9 (light to dark) for (a), from -5 to 2, excluding the zero level for (d) and from -16 to -4 for the rest.

558 understand the physical phenomena responsible for this transfer process, we  
 559 present the following quadrant analysis. The first quadrant ( $Q_1$ ), where  
 560  $u' > 0$  and  $T' > 0$ , indicates fast moving heated fluid. The second quadrant  
 561 ( $Q_2$ ), with  $u' < 0$  and  $T' > 0$ , indicates slow moving heated fluid. The third  
 562 quadrant ( $Q_3$ ), having  $u' < 0$  and  $T' < 0$ , means that the cooled fluid is  
 563 moving slower while the fourth quadrant ( $Q_4$ ), where  $u' > 0$  and  $T' < 0$ ,  
 564 denotes the events of fast moving cooled fluid. As can be seen, in all the fig-  
 565 ures except figure 12a where clear dominance of the  $Q_1$  and  $Q_3$  can be seen,  
 566 the  $Q_2$  and  $Q_4$  are the dominant quadrants owing to the anti-correlation  
 567 between the  $u'$  and  $T'$ . For the cooled wall, at  $y^+ = 5$  (figure 12a),  $Q_3$  is  
 568 dominant in comparison to  $Q_1$  which means that the slow moving cold fluid  
 569 contributes more to the turbulent heat-flux in the streamwise direction than  
 570 the fast moving hot fluid. In the buffer-layer, at  $y^+ = 10$  (figure 12d), where  
 571 the change in the inclination of the contours is registered, it can be seen that  
 572  $Q_4$  is more dominant in comparison to  $Q_2$ , highlighting the fact that fast  
 573 moving cooled fluid has more contribution towards the streamwise turbulent  
 574 wall heat-transfer. In case of the adiabatic and the heated walls, it can be  
 575 seen that the major contribution comes from  $Q_2$  than  $Q_4$ , extending from  
 576 the viscous sub-layer to the log region. The comparable dominance of the  $Q_2$   
 577 and  $Q_4$  implies the existence of homogeneous turbulence towards the outer  
 578 layer. It can also be observed from these figures that on moving away from  
 579 the surface of the wall i.e. towards the higher  $y^+$ , the angles of inclination  
 580 of the contours also increase because of the increased perturbations. From  
 581 this comparison we can state that for the cooled wall, the effects of wall tem-  
 582 perature on the streamwise turbulent heat-flux can be seen from the viscous



583 sub-layer to the buffer-layer. And, in the log-region, the wall temperature  
584 does not has significant effects due to the turbulent mixing happening in the  
585 outer layers.

#### 586 4.4. Wall-normal velocity component and temperature fluctuations

587 In order to have a complete overview of the heat-transfer phenomenon,  
588 we would now focus on the wall-normal component of the turbulent heat-flux  
589 which is given by the covariance of the wall-normal velocity and temperature  
590 fluctuations. The contours plots of the JPDF distribution of the wall-normal  
591 velocity fluctuations ( $v'^*$ ) and temperature fluctuations ( $T'^*$ ) are shown in  
592 figure 13. Here, the velocity and the temperature fluctuations are scaled by  
593 the local friction velocity ( $u_\tau^*$ ) and the time-averaged local temperature  $\langle T^* \rangle$ ,  
594 respectively. Likewise in figure 11, a comparison between the figures 13a to  
595 13c highlights a different trend in case of the cooled wall, as in the viscous sub-  
596 layer the quantities  $v'$  and  $T'$  are **weakly anti-correlated** while they exhibit a  
597 **subtle correlation** for the other two cases (A and H). For case C, on moving  
598 towards higher  $y^+$ , we see that at  $y^+ = 10$ , the major axis of the JPDF  
599 contours becomes parallel to the  $v'$  axis and the distribution becomes flat,  
600 and the peak of the contours remains close to the origin. However, in the log  
601 region, figures 13g and 13j, the major axis of the contours aligns itself in the  
602  $Q_1$  and  $Q_3$  in an anti-clockwise sense which means that more amount of fluid  
603 is going towards the wall. This set of figures shows that for the adiabatic and  
604 the heated walls, the quantities  $v'$  and  $T'$  show **correlation** from the viscous  
605 sublayer itself. This means that the effects of temperature on the turbulent  
606 boundary layers can be seen from the viscous sublayer to the buffer-layer  
607 region. Afterwards, for  $y^+ \geq 35$ , as a result of the turbulent mixing, the

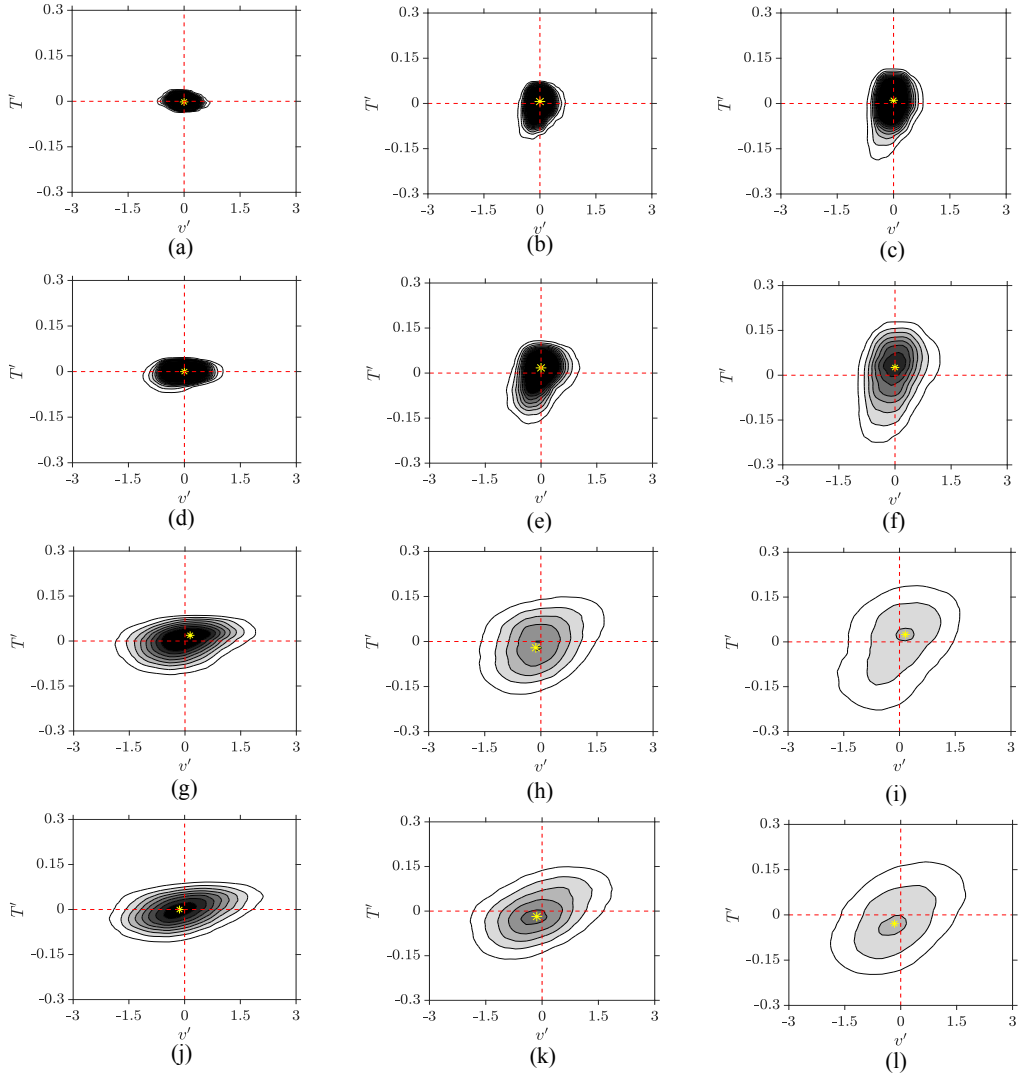


Figure 13: Contours of the JPDF distribution of the  $v'$  and  $T'$  for  $y^+ = 5$  ((a)-(c)), 10 ((d)-(f)), 35 ((g)-(i)) and 90 ((j)-(l)). Here the first, second and third columns represent the cooled, adiabatic and heated walls, respectively. The color of the contour levels vary from 40 to 320 (light to dark).

608 differences arising from the wall-temperature are not so significant in terms  
609 of the peak position, but the maximum value of the JPDF contours does  
610 decrease with increasing wall-temperature.

611 We would now discuss about the contour plots of the covariance integrand  
612 of the  $v'$  and  $T'$  i.e.  $v'T'P(v',T')$  in order to reveal the contributions of  
613 different physical phenomena towards the turbulent wall-normal heat-flux  
614 covariance,  $\overline{v'T'}$ . Following quadrant analysis is presented for the  $v'T'$  plane.  
615 The first quadrant ( $Q_1$ ), with  $v' > 0$  and  $T' > 0$ , indicates that the heated  
616 fluid is moving away from the wall. The second quadrant ( $Q_2$ ), where  $v' < 0$   
617 and  $T' > 0$ , means that the heated fluid moving down towards the wall. The  
618 third quadrant ( $Q_3$ ), with  $v' < 0$  and  $T' < 0$ , highlights the events where  
619 cooled fluid moves towards the wall while the fourth quadrant ( $Q_4$ ), with  
620  $v' > 0$  and  $T' < 0$ , indicates the events of the cooled fluid moving away from  
621 the wall. The contours of the covariance integrands of the  $v'$  and  $T'$  are shown  
622 in figure 14 for all the cases at different  $y^+$  locations. Like before, the white  
623 region surrounding the gray rectangles represents the region with no data. It  
624 should be noted here that in order to have a clear representation, figure 14a,  
625 and figures 14b and c are zoomed-in by four and two times respectively for  
626 both the axes with reference to the planes' dimensions used for figures 14d  
627 to l. It can be seen in figure 14 that for all the cases the  $Q_1$  and  $Q_3$  are the  
628 dominant quadrants except for figure 14a where the  $Q_2$  and  $Q_4$  are dominant.  
629 In the viscous sub-layer, at  $y^+ = 5$ , it can be seen that for the cooled wall, the  
630  $Q_2$  is more dominant in comparison to the  $Q_4$  which means that the heated  
631 fluid is going towards the wall has more contribution towards the turbulent  
632 wall-normal heat-flux which is the reason why the wall is behaving as the

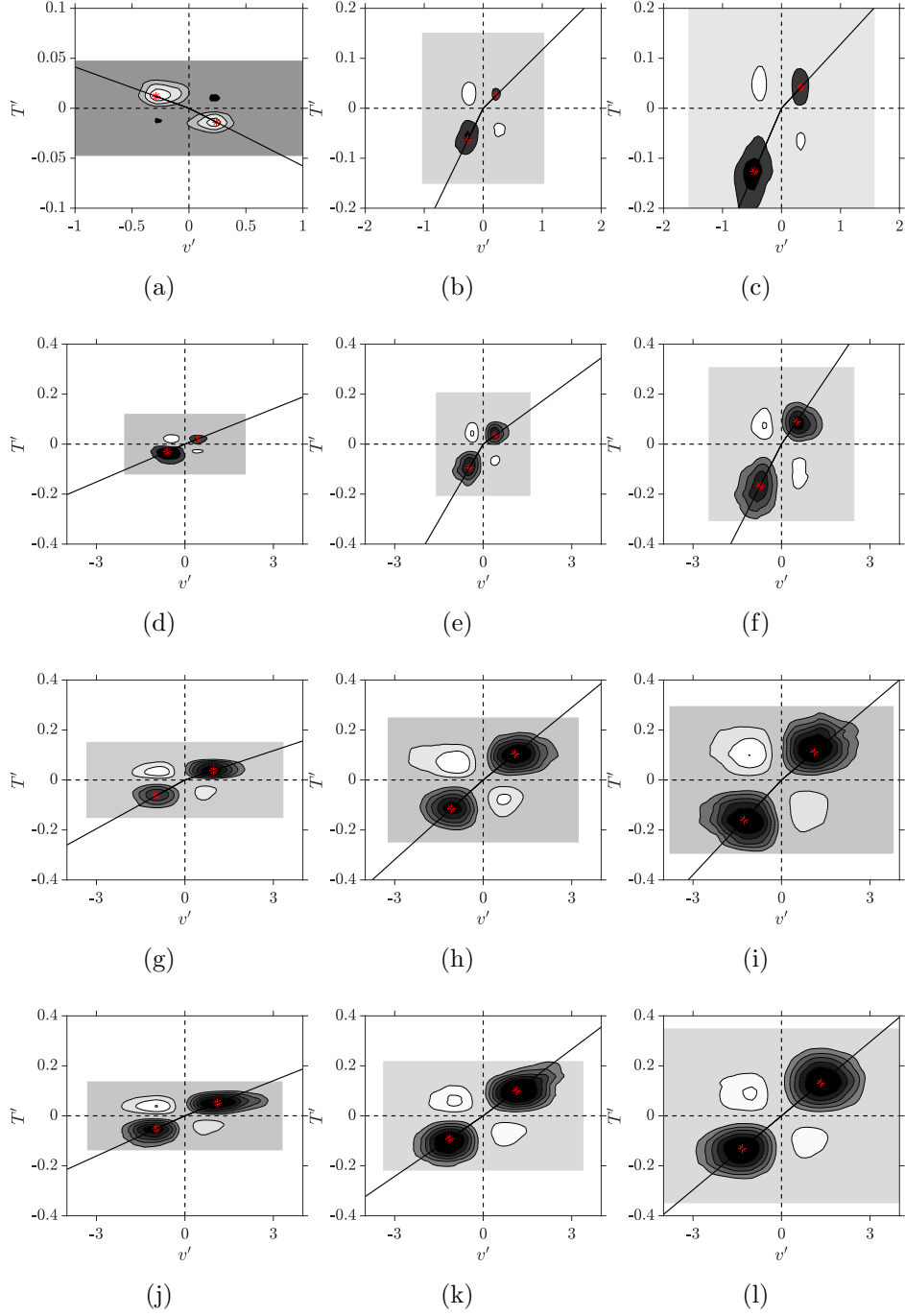


Figure 14: Contours of the covariance integrands of the  $v'$  and  $T'$  at  $y^+ = 5$  ((a)-(c)), 10 ((d)-(f)), 35 ((g)-(i)) and 90 ((j)-(l)). Here, the first, second and third columns represent the cooled, adiabatic and heated walls, respectively. The colors of the contour levels vary from -2.5 to 1, excluding the zero level (light to dark) for (a), from -1 to 1, excluding the zero level for (d) and from -3 to 6 for the rest.

633 heat sink here, as pointed out by Lechner et al. [22]. On the other hand,  
 634 for the adiabatic and the heated cases, at the same  $y^+$  location (figures 14b  
 635 and 14c), the  $Q_3$  is more dominant in comparison to the  $Q_1$  which means  
 636 that the cooled fluid moving towards the wall is the major contributor to the  
 637 turbulent wall-normal heat-flux. Therefore, in these cases, the wall is acting  
 638 as a heat source. In the buffer layer,  $y^+ = 10$ , the  $Q_3$  is more dominant than  
 639 the  $Q_1$  irrespective of the wall temperature. From figures 14g-l, it can be seen  
 640 that in the log region from  $y^+ = 35$  to 90, the  $Q_1$  is more dominant than the  
 641  $Q_3$  for all of the cases highlighting that the principal contribution is coming  
 642 from the events of the  $Q_1$  than the events of the  $Q_3$ . From the observations  
 643 drawn from this set of figures, it can be concluded that the wall-cooling  
 644 has significant effect on the heat-transfer mechanisms for the compressible  
 645 turbulent boundary layer which is clearly highlighted in the near-wall region  
 646 (up to  $y^+ = 5$ ). In the higher  $y^+$  regions, the physical mechanisms responsible  
 647 for the heat-transfer do not change significantly with the wall-temperature,  
 648 but slight variations in their amplitudes are registered. This explains the  
 649 difference in the levels of the wall-normal Reynolds heat-flux observed for  
 650 the heated and the cooled walls found by Sharma et al. [29].

## 651 5. Conclusion

652 Direct numerical simulations (DNS) for the supersonic boundary layers  
 653 (SBLs) with free-stream Mach number of  $M_\infty = 2.2$  were carried out. Three  
 654 DNS test cases were investigated in order to unravel the effects of the wall-  
 655 temperature on the turbulent flow topology for the SBLs. **The implications**  
 656 **on the important physical parameters like the turbulent shear-stress, the**

657 orientation of the projections of the coherent structures in different planes,  
658 and different components of the turbulent heat-flux were analyzed using the  
659 joint probability density function (JPDF) distribution and the covariance  
660 integrands' analyses.

661 The results showed that the sweeps were the dominant physical phe-  
662 nomenon majorly contributing to the turbulent shear-stress in the buffer  
663 layer. But, for the heated wall, both the ejections and the sweeps became  
664 comparable transfer processes. In the log-region, ejections had the domi-  
665 nant contribution to the shear-stress irrespective of the wall-temperature.  
666 These trends showed similarities with the findings reported by Wallace et al.  
667 [8], Ong and Wallace [11] regarding the adiabatic incompressible boundary  
668 layers. The results presented also highlighted different trends for the angles  
669 of inclination ( $\alpha$  and  $\beta$ ) of the projections of the coherent structures in case  
670 of the heated wall, as a result of the increased heat transfer from the sur-  
671 face of the wall. The trends of  $\alpha$  and  $\beta$  showed good agreement with the  
672 compressible and the incompressible counterparts reported in the literature.

673 The plots of the covariance integrands of the  $u'$  and  $T'$  showed that for the  
674 adiabatic and the heated walls, the  $Q_2$  and  $Q_4$  were the dominant quadrants  
675 implying the principal contribution of the fast moving cooled fluid towards  
676 the streamwise turbulent wall heat-transfer extending from the viscous sub-  
677 layer to the log-region. Whereas for the cooled wall, the  $Q_1$  and  $Q_3$  were  
678 found to be the major contributors in the viscous sub-layer. A similar con-  
679 trasting trend was observed for the cooled wall again, for the wall-normal  
680 component of the turbulent heat-flux in the viscous sub-layer where the  $Q_2$   
681 had dominance in comparison to the  $Q_4$ , meaning that heated fluid going

682 towards the wall had more contribution towards the turbulent wall-normal  
683 heat-flux. For the rest, the  $Q_1$  and  $Q_3$  were the dominant quadrants.

## 684 **6. Acknowledgements**

685 This work was granted access to HPC resources of IDRIS under the allo-  
686 cation 2017-100752 made by GENCI (Grand Equipement National de Calcul  
687 Intensif- A0022A10103). The authors acknowledge the access to HPC re-  
688 sources of French regional computing center of Normandy named CRIANN  
689 (Centre Régional Informatique et d'Applications Numériques de Normandie)  
690 under the allocations 1998022 and 2017002. The funding resources provided  
691 from European projects entitled FEDER and NEPTUNE 1 are gratefully  
692 acknowledged. Authors would also like to extend their gratitude to both the  
693 reviewers whose critical comments helped us to substantially **improve** the  
694 quality of the paper.

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