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Theory of cyclostationary processes applied to gears diagnostic and analysis.

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SUMMARY

This paper introduces a new vibration analysis method based on spectral correlation function. This method is shown to be an extension of the well-known synchronised averaging method. Each of the two methods is a cyclostationary analysis of the signal, respectively at first and second order. The advantages of spectral correlation function are discussed.

INTRODUCTION

The study presented in this paper takes place in the wider frame of predictive monitoring. Predictive monitoring can be decomposed into two parts. First, some parameters have to be chosen, which are significant of the state of the machinery : for instance lubricant composition and temperature, level of the vibrations, The set of the values that are taken by these parameters when the machinery is safe is called the signature of the machinery. These parameters are then regularly measured and compared with the signature, and any significant difference may be supposed to be caused by a failure. In order to decide whether there is a failure or not, a decision can be taken from these values by data analysis methods [1].

The analysis of machinery vibrations has been used for many years to characterise their state, but the parameters that are usually used (vibrations level, amplitude of some of the harmonics of the power spectrum...), have been chosen intuitively. In this study it is shown that the concept of cyclostationary processes, that has been developed within the frame of signal processing, can be used for the diagnosis of rotating machinery. We point out that the synchronised averaging method, which is classically used in vibration analysis, is in fact a cyclostationary study of the vibrations. We then propose a new parameter, called spectral coherence function, and show that it is an extension to the second order of synchronised averaging. Our study has been performed on the vibrations of a toothed-gearing, but the results that we obtain are general, and could be extended to any kind of rotating machinery.

1 CYCLOSTATIONARY PROCESSES.

Non-stationary study is widely used for signals comprising strongly varying frequency components : for instance, the speed variations of a machinery, the resonance frequencies of a varying structure such as a diesel engine... The vibrations of a toothed-gearing comprising only constant-frequency components, they are usually considered as a stationary signal. But the family of non-stationary signals is actually wider than the one of the signals whose frequency components evolve. Indeed, stationarity is a statistical property which is not always satisfied by a signal whose frequency components are not time-varying. It can be shown that if a signal is stationary, its frequency components cannot be correlated to each other. And toothed-gearing vibrations, as we are going to see from the analytical model, comprise several different families of harmonic components generated by the same physical phenomenon, that can be the shock between the teeth or the rotation phenomenon. We cannot suppose that harmonics from the same family are completely uncorrelated to each other. Since the vibrations are periodical, they can be supposed to be cyclostationary, which is a particular class of non-stationary signals.

1-1 Cyclostationarity, definition [2,3].

Stationarity.

A signal is said to be stationary at orders n if its temporal moments at order smaller or equal to n are not time-varying.

Cyclostationarity.

A signal is said to be cyclostationary at orders n if its temporal moments at order smaller or equal to n are periodical as functions of the time t . The frequency α of this periodicity is called cyclostationarity frequency of the signal.

Cyclostationarity at order 1.

The statistical mean of a random signal $x(t)$ is defined by : $m(t) = E[x(t)]$. The symbol $E[...]$ represents the statistical mean, that is to say the mean over all the values that can be taken by the function $x(t)$ at a given moment t , weighed by the probability associated to these values, or over all of the possible realisations of the signal. This mean usually depends on time t . In the case of a stationary signal, this mean function does not depend on time, and for a cyclostationary signal, it is periodic.

Cyclostationarity at order 2.

The autocorrelation function of $x(t)$ is defined by $R_x(t, \tau) = E[x(t+\tau/2).x^*(t-\tau/2)]$. This function usually depends of the mean date t and of the temporal distance τ between the two observation moments. The signal is second order stationary, cyclostationary, or non-stationary, depending on the properties of the autocorrelation as a function of time t : constant, periodic, or time-varying.

1-2 Characterisation of these properties.

The first order properties of the signal are characterised by the properties of its first order temporal moment, that is to say the mean function. The second order properties of the signal can be characterised by four different functions. Indeed, since the second order moment of the signal, that is to say the autocorrelation function, depends on two variables which are the time t and the temporal distance τ , we can Fourier transform it over each of these two variables. When Fourier transformed over the variable τ , this moment exhibits spectral lines corresponding to the periodicities of the signal, whereas when Fourier transformed over the time t , it exhibits spectral lines corresponding to underlying periodicities of the signal, which reveal its cyclostationary properties. The four possible second order representations of the signal are illustrated by figure 1. The frequency f obtained by Fourier transforming the function over τ is called spectral frequency, and the one which is obtained by Fourier transforming it over time t is noted α and called cycle frequency.

The two of these representations which are best adapted to cyclostationary signals are the cyclic autocorrelation function and the spectral correlation function, which exhibit spectral lines at the cyclostationarity frequencies of the signal. The signal that we are going to study exhibits spectral lines both in spectral and cycle frequencies, this is why we have chosen to apply to them spectral correlation function, which reveals these two types of periodicities.

Furthermore, spectral correlation function, whose expression is given by $S_x^{\alpha}(f) = E[X(f+\alpha/2).X^*(f-\alpha/2)]$, where $X(f)$ is the Fourier transform of $x(t)$, is a measure of the degree of correlation between the different frequency components of the signal, which can be physically interpreted for toothed gearing vibrations.

1-3 Estimation of these functions.

Estimation of mean function.

Since the mean function of a cyclostationary signal is periodic, it cannot be estimated by simply time averaging, as it can be done for stationary and ergodic signals. The cyclostationary signal is not supposed to be ergodic, but cycloergodic. That is to say that the statistical mean is actually replaced by periodical temporal averaging, with frequency equal to the cyclostationarity frequency of the signal :

$$\hat{m}(t) = \frac{1}{2 \cdot N} \cdot \sum_{n=-N}^N x(t - n/\alpha_0), \text{ where } \alpha_0 \text{ is the cyclostationarity frequency of the signal.}$$

Estimation of spectral correlation function.

Spectral correlation function is actually a cross-spectrum between two frequency-shifted versions of the signal. Therefore, it can be estimated by the classical cross-spectra estimation methods, which are the frequency smoothed periodogram, and time averaged periodogram. We have chosen to work with the frequency smoothing method for reasons of calculation speed and complexity.

The principle of this method is illustrated by figure 2. Two different frequency shiftings of the signal are performed (by multiplying the signal by a complex sine wave), of respective frequencies $-\alpha/2$ and $+\alpha/2$. The signals $X(f+\alpha/2)$ and $X(f-\alpha/2)$ are then obtained by Fourier transform. Finally, the cross spectrum of these two signals is obtained by product and frequency smoothing. The properties of this estimator, that is to say the bias and variance, can be deduced from the classical properties of the frequency smoothed periodogram.

1-4 Normalisation. Resolution.

It is interesting to normalise the spectral correlation function by the energy of the components of the power spectrum, so as to emphasise correlations between components of weak energy relatively to those of great energy. The normalised function is called spectral coherence function and is given by :

$$C_x^\alpha(f) = \frac{S_x^\alpha(f)}{\sqrt{S_x(f + \frac{\alpha}{2})} \sqrt{S_x(f - \frac{\alpha}{2})}}$$

where $S_x(f)$ is the power spectrum of the signal, and $S_x^\alpha(f)$ is its spectral correlation function.

Spectral coherence function is always comprised between zero and one, and gives a measure of the degree of correlation between the components at frequencies $+\alpha/2$ and $-\alpha/2$. It is equal to zero for completely uncorrelated components and equal to one for completely correlated components, whatever the energy level of the components is. For the cycle frequency $\alpha=0$, spectral correlation is equal to power spectrum, and spectral coherence is equal to one (since each component is completely correlated to himself).

In the case of our study, the main interest of this function is the resolution it offers in cycle frequency. Indeed, the frequential smoothing that improves estimation is performed over spectral frequency, and it does not worsen the resolution in cycle frequency, which depends only on the duration of the signal. If T is the total duration of the signal, and L the width of the smoothing window (in number of samples), the resolutions in spectral and cycle frequencies are respectively given by :

$$\Delta f = L/T, \text{ and } \Delta \alpha = 1/T.$$

The performances of the estimator can thus be improved by enlarging the width of the smoothing window, without worsening the cycle frequency resolution.

2 MODEL OF THE VIBRATIONS. CYCLOSTATIONARY STUDY.

2 1 Model of the vibrations.

The vibrations of a gear are mainly produced by the shock between the teeth of the two wheels it is composed of. This vibration, called meshing signal, is filtered by the structure of the machinery between the source of the vibrations and the measure. We will suppose that this filtration is linear, and will take the model of the source signal as a model of our measured

signal. Many models have been proposed for toothed gearing vibrations, but most of them derive from a former study performed by W.D. Mark that we have used to derive our own model.

According to Mark [4], the meshing signal is given by the deviation of the meshing point position relatively to the ideal motion of a perfect gear with no load. This deviation is called transmission error, and its components come both from the deflection of the teeth due to the load and deviations of the shape of the teeth from their ideal shape. We are not going to reproduce the calculations performed by Mark, and will just admit that the expression of the transmission error is given by :

$$s(x) = s_e(x) \cdot (W + s_m(x) + s_1(x) + s_2(x)) \quad (12)$$

The terms $s_m(x)$ and $s_e(x)$ are obtained by integrating along the contact lines of the gear, and the number of pairs of teeth being in contact varies periodically, with frequency f_e , (frequency of the shock between the teeth), called meshing frequency, so that these terms are periodical with frequency f_e .

The terms $s_1(x)$ and $s_2(x)$ represent the phenomena produced by the deviation of the teeth shape from their ideal shape, respectively for the first and the second wheels. Since these deviations are different from one tooth to another, each term $s_i(t)$ is periodical with a frequency equal to the rotation frequency f_i of the corresponding wheel.

The different frequencies appearing in our model are tied by the relation : $f_e = N_1 \cdot f_1 = N_2 \cdot f_2$. In the signal thus appear three different periodical terms given by $s_e(x) \cdot (W + s_m(x))$, $s_e(x) \cdot s_1(x)$ and $s_e(x) \cdot s_2(x)$, of respective frequencies f_e , f_1 and f_2 . Each of these terms can be decomposed into a Fourier series, and be seen as a sum of pure sine waves.

The variable x is not time, it is a variable directly tied to the angular position of the wheels. At perfectly constant rotation speed, this variable is a linear function of time, but actually, slight random fluctuations affect this function, and produce a phase randomisation of the pure sine waves.

When a defect appears on the i th wheel, we can say from our model that it will affect the term $s_e(x) \cdot s_i(x)$, so that we are going to focus on the spectral lines corresponding to this periodical term, that is to say on any change in their shape or amplitude.

Studying the first and second order cyclostationarity frequencies of the signal is the same as studying the periodicities of its mean and autocorrelation functions. In this study, we will not be interested in the terms at meshing frequency and its harmonics, since it has been shown in a previous work [5] that they cannot be used to detect the defect.

2-2 First order cyclostationarity.

The two components families that we are interested in can be written :

$$s_e(x) \cdot s_i(x) = \sum_{n=-\infty}^{+\infty} a_{n,i} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n + j \cdot \delta\phi_i) \quad (13)$$

where $\delta\phi_i$ is a zero mean random variable modelling the fluctuations of the angular position of the wheels. This random variable thus appears when replacing the variable x by the time t .

These random fluctuations can be supposed to be weak compared to one rotation of the wheel, that is to say : $\delta\phi_i < 2\pi$,

$$\text{so that } \exp(j \cdot \delta\phi_i) \approx 1 + j \cdot \delta\phi_i,$$

$$\text{and } \exp(2\pi j n f_i t + j \cdot \phi_n + j \cdot \delta\phi_i) \approx \exp(2\pi j n f_i t + j \cdot \phi_n) \cdot (1 + j \cdot \delta\phi_i).$$

So, each sine wave comprises one deterministic term given by $a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n)$, and one random term given by $a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n) \cdot j \cdot \delta\phi_i$, whose frequency is the same and is equal to $n \cdot f_i$.

First order study, mean of the signal.

Since each variable $\delta\Phi_i$ is zero mean, the mean of each sinusoidal term is equal to $a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n)$, so that the signal is first order cyclostationary with frequencies f_1 and f_2 . Since it exhibits two cyclostationarity frequencies, it is said to be wide sense cyclostationary. We can notice that when performing a first order study, we lose all the information about the random components of the signal.

2-3 Second order study, autocorrelation function.

In the autocorrelation function appear cross-terms whose shape is given by the following formula :

$$E \left[a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n + j \cdot \delta\phi_i) \cdot (a_{m_j} \cdot \exp(2\pi j m f_j (t - \tau) + j \cdot \phi_m + j \cdot \delta\phi_j))^* \right]. \quad \text{These}$$

terms are the cross correlations of two components of the signal. When $i = j$, it is the cross correlation between two components from the same family, and if $i \neq j$, between two components from different families. We can approximate the complex exponential functions as we have already done, which gives :

$$E \left[a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n + j \cdot \delta\phi_i) \cdot (a_{m_j} \cdot \exp(2\pi j m f_j (t - \tau) + j \cdot \phi_m + j \cdot \delta\phi_j))^* \right] \approx$$

$$E \left[a_{n_j} \cdot \exp(2\pi j n f_i t + j \cdot \phi_n) \cdot (1 + j \cdot \delta\phi_i) \cdot (a_{m_j} \cdot \exp(2\pi j m f_j (t - \tau) + j \cdot \phi_m))^* \cdot (1 - j \cdot \delta\phi_j) \right] =$$

$$a_{n_j} \cdot a_{m_j}^* \cdot \exp(2\pi j (n f_i - m f_j) t + 2\pi j m f_j \tau + j \cdot (\phi_n - \phi_m)) \quad (1)$$

$$+ a_{n_j} \cdot a_{m_j}^* \cdot \exp(2\pi j (n f_i - m f_j) t + 2\pi j m f_j \tau + j \cdot (\phi_n - \phi_m)) \cdot j \cdot E[\delta\phi_i] \quad (2) \leftarrow$$

$$+ a_{n_j} \cdot a_{m_j}^* \cdot \exp(2\pi j (n f_i - m f_j) t + 2\pi j m f_j \tau + j \cdot (\phi_n - \phi_m)) \cdot j \cdot E[-\delta\phi_j] \quad (3)$$

$$+ a_{n_j} \cdot a_{m_j}^* \cdot \exp(2\pi j (n f_i - m f_j) t + 2\pi j m f_j \tau + j \cdot (\phi_n - \phi_m)) \cdot E[\delta\phi_i \cdot \delta\phi_j] \quad (4) \leftarrow$$

The term (1) is always non-zero. The terms (2) and (3) are equal to zero, since $\delta\Phi_i$ and $\delta\Phi_j$ are zero mean ^{variables} terms. The random variables $\delta\Phi_i$ and $\delta\Phi_j$ can be supposed to be independent when $i \neq j$, since they are not produced by the same wheel. The term (3) is thus non zero only when $i = j$, that is to say when the two components are harmonics from the same family (produced by the same wheel).

annexe 1

Since

The terms (1) and (4) are periodical with frequency $n \cdot f_i - m \cdot f_j$. When $i=j=1$ (resp. 2), the spectral coherence function exhibits a series of spectral lines, whose fundamental frequency is f_1 (resp. f_2). When $i \neq j$, the corresponding term measures the degree of correlation between two components which are not from the same family, so that the spectral lines it produces (at

frequencies $(n.f_i - m.f_j)$ are of weaker energy, since they are produced only by the deterministic components.

So, according to our model of the vibrations, they are wide sense second order cyclostationary, with frequencies f_1 and f_2 .

Remark: *handwritten*

It can be interesting to compute a second order cumulant, instead of a second order moment, that is to say the second order moment minus the contribution of the first order cyclostationarity (the periodic mean). In order to compute this cumulant, we must first extract from the signal the deterministic components $a_{n,i} \cdot \exp(2\pi j n f_i t + j \cdot \Phi_n)$ and keep only the random components $a_{n,i} \cdot \exp(2\pi j n f_i t + j \cdot \Phi_n) \cdot j \cdot \delta\Phi_i$, and then compute the autocorrelation function. The only terms that are left in this function are the ones relative to the random components, given by

$a_{n,i} \cdot a_{m,j}^* \cdot \exp(2\pi j (n f_i - m f_j) t + 2\pi j m f_j \tau + j \cdot (\Phi_n - \Phi_m)) \cdot E[\delta\Phi_i \cdot \delta\Phi_j]$, which are the strictly second order cyclostationary terms.

Such a study has been presented in [] (Lacourte GRETSI)

The improvement that gives a second order study, relatively to a first order study, is that it keeps the information about the random components of the signal.

3 REAL VIBRATIONS

The vibrations we have studied come from an experiment performed at the C.E.T.I.M. The characteristics of these vibrations are the following ones :

The experiment comprises one engine, and two toothed gearings, with respectively 40 and 42 teeth, and 20 and 21 teeth for the one we study. The rotation speed is equal to 1000 tours per minute, (or 16,7 Hz). The vibrations were recorded every day during 13 days. The experiment was running continuously during these 13 days. The sampling rate was 20 kHz. Each record comprises 60 160 samples. At the beginning of the experiment the gear is in a good state, and at the end, the twenty-toothed wheel is damaged.

3-1 Real vibrations. Checking of the model.

Figure 3 show the temporal representation of the vibrations for each day of the experiment. Figure 4 shows the Fourier transform of each record performed over 16384 samples, that is to say a duration of 0,8 s. We can notice on this representation that the emergence of a damage can be detected only on the 12th day of the experiment. We are thus interested in any method allowing to diagnose the gear earlier.

We can also notice that the aspect of the spectrum evolves much from one day to another. For instance, the biggest meshing harmonic can be either the second or the fourth one, and this difference does not seem to be related to the presence of the defect. Since the spectrum keeps changing in a way that cannot be explained, it can hardly be taken as a signature of the gear state. We must find some parameters being more characteristic of the presence of the damage, and less varying.

In order to determine such parameters and to check our model, we need to observe the vibrations in a more detailed way. The spectrum exhibits the three spectral lines family that we expect according to our model. The harmonics of greatest energy are the meshing ones. The fundamental harmonic of this family is the meshing frequency $f_e = 330$ Hz. We notice the presence of two other families of harmonics tied to the shape of the wheels. Figure 5 shows a part of the spectrum comprised between two of the meshing harmonics, with logarithmic scale. This part of the spectrum is shared respectively into 20 and 21 equal smaller parts, by the two families. Some lines have been drawn at the expected positions of these harmonics. Due to the number of teeth of the wheels, the two different families are best separated in the middle of the studied interval, but in this part of the spectrum, the harmonics are of very weak amplitude. This

confirms the hypothesis we have done for the meshing phenomenon to be modulated by the rotation ones.

Figure 6 is a zoom on a smaller part of the spectrum. We can notice on this zoom that the harmonics of the sidebands have a non zero bandwidth, approximately comprised between 5 and 10 Hz. This bandwidth can be explained from our model by a phase modulation due to the fact that the variable x is not exactly a linear function of time t .

3-2 First order cyclostationary study.

In order to perform synchronised averaging in the best way on a discrete signal, this signal must have been sampled with a sampling rate equal to a multiple of the frequency used for the averaging, so that each sample corresponds to an exact angular position. But the signal we work on has been sampled with a constant sampling rate, that does not depend on the rotation frequency of the system. So, when performing synchronised averaging on it, we commit an error. Anyway, this error is always smaller than the sampling period, and can be considered as a zero mean random variable, which is cancelled by the averaging. Since the signal exhibits two different cyclostationarity frequency, when the averaging is synchronised with one of them, the component of the signal which is periodical at the other frequency is cancelled by the averaging. Indeed, if the averaging is synchronised on the term at frequency f_1 , all the terms at frequencies $n.f_2$ can be seen as sine waves with random phase equi-distributed over $[0, 2\pi]$. Then, we have to perform the synchronised averaging twice, once with each of the cyclostationarity frequency.

Figures 7-a and 7-b show the results obtained for each of the two wheels. The averaging is performed over as many slices as possible, with a given length of signal. The results are given for some days of the experiment. We notice that synchronised averaging actually allows to separate the respective contributions of the two wheels. But the defect does not appear clearly before the 12th day, when it also appears clearly on the temporal representation. This lack of efficiency can be explained by several different phenomena : first, the error that is made when averaging, and second, the fact that in this first order study, we lose the information about random terms. Part of the change produced by the damage on the vibrations may appear in these random terms, which are killed by the averaging.

3-3 Second order cyclostationary study.

Spectral coherence function is estimated for each day of the experiment in the following way :

The vibrations are filtered around the second meshing harmonic, which is in most of the case the biggest one, with a bandwidth equal to the meshing frequency. The filtered signal is demodulated so that its central frequency is the meshing frequency, and is under-sampled by a factor 16. These filtration and under-sampling are performed in order to reduced the time it takes to compute the function, the size of the result and the quantity of information to be interpreted.

Spectral correlation function is estimated by frequency smoothing, with a window whose width is of 50 samples, which is about equal to the bandwidth of the harmonics. Spectral coherence function is then estimated only for the components whose amplitude is bigger than a minimum value. Indeed, since we divide the spectral correlation function by the amplitude of the power spectrum, for components of very weak amplitude, which are only noise, we may obtain very big values in the spectral coherence function. Due to this selection of the components, the non zero values of spectral coherence function are located in a triangle corresponding to the components not to be neglected.

Since spectral coherence function presents various symmetry axis [2], we will represent only the part of it which corresponds to both positive spectral and cycle frequencies. The only coherences to be represented are the ones bigger than 0.7, the smaller ones being considered as non significant ones.

Figure 8 shows the spectral coherence function of the vibrations, as well as a schematic spectral coherence function where the cyclostationarity frequencies deduced from the model are given. Some spectral lines do appear at the predicted cycle frequencies, and confirm the cyclostationary character of the vibrations. Some other lines, whose cycle frequencies do not have any common denominator, appear too. This can be explained by the deterministic components, which produce lines at the frequencies $n.f_1 - m.f_2$, as previously seen. When the gear gets damaged, the random terms grow bigger relatively to the deterministic ones, so that the coherences between components which are not of the same family get smaller.

The information given under the shape of a surface is difficult to interpret, so that it is interesting to reduce it by taking particular slices, or by projecting the surface over one of the axis. Figure 9 shows the projection of spectral coherence function over the axis of cycle frequency, for the second and 12th days of the experiment. We can observe that some high energy lines do appear at the frequencies which we expect from our model to be the cyclostationarity frequencies of the vibrations, that is to say the harmonics of the two rotation frequencies. On this figure is also shown the evolution of the amplitude of these lines. The parameter which is represented on this scheme is obtained by computing the ratio between the amplitudes of the lines respectively at frequencies $n.f_1$ and $n.f_2$. This ratio is drawn as a function of time, which means for the twelve days of the experiment. It is represented for two different pairs of harmonics located at the frequencies given by $n=2$ and $n=4$ (that is to say a cycle frequency equal to twice and four times the rotation frequencies). We observe on this curve that a sudden change appears on the ninth day, for both of the cases chosen. The lines generated by the twenty-one toothed wheel are bigger until the eighth day, and during the ninth day, the other family grows bigger and goes on growing until the last day, when the only twenty-toothed wheel family still appears. And we know that this very wheel presents a defect at the end of the experiment. So, it seems that the defect produces on spectral correlation function an increase of the correlation between the harmonics produced by the damaged wheel, or the emergence of new harmonics in the correspondent family.

This hypothesis is confirmed when observing figure 10. This figure shows the spectral coherence function of the vibrations for the two cycle frequencies $\alpha_1=2.f_1$ and $\alpha_2=2.f_2$, for each day of the experiment. New correlation peaks do appear for the damaged wheel. Furthermore, let us recall that the studied signal has been filtered around one of the meshing harmonics, with a bandwidth equal to the meshing frequency. This means that the new harmonics are high order harmonics of the modulation at the frequency of the twenty-toothed wheel, which means that the shape of the temporal modulation presents faster variations. This evolution is confirmed by what can be observed on the temporal representation.

CONCLUSION.

In this paper we showed that synchronised averaging can be considered as a first order cyclostationary study of the signal, and we extended this cyclostationary study at the second order. The parameter chosen to perform this study on the vibrations is spectral coherence function. We explained how spectral coherence function could be used to measure the contribution to the signal of a random component that is lost when performing a first order study. By studying the evolution of this new parameter, we can detect the damage four days before it may be detected on the temporal representation. The efficiency of spectral coherence function comes from two peculiarities : it allows to separate in cycle frequency phenomena that are mixed in spectral frequency, and it gives a normalised information, comprised between 0 and 1, on which can easily be placed a critical limit. Let us notice that since we know the rotation frequencies of the system, we know the cyclostationarity frequencies of the vibrations, and may compute the spectral coherence function for the only particular frequencies we are interested in. This means that the computation complexity could be reduced a lot, relatively to this first study. Since this study has been performed only over one toothed gearing, it should be confirmed by some other examples, before we generalise the results that have been obtained. Anyway, we can already think that this parameter will be advantageously added to the ones which are already used in diagnosis problems.

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Figures

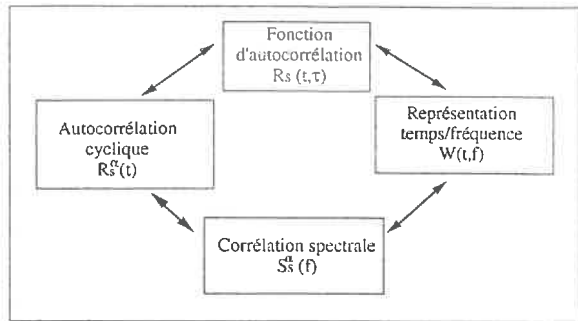


Fig. 1
Représentations du signal au second ordre

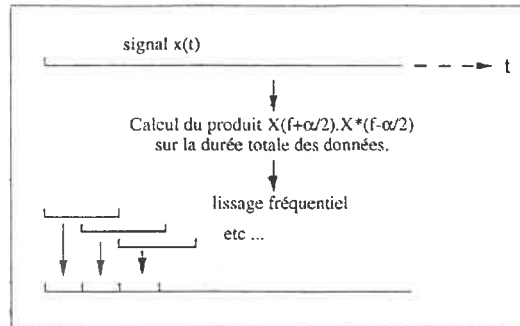


Fig. 2
Estimation de la corrélation spectrale par périodogramme lissé

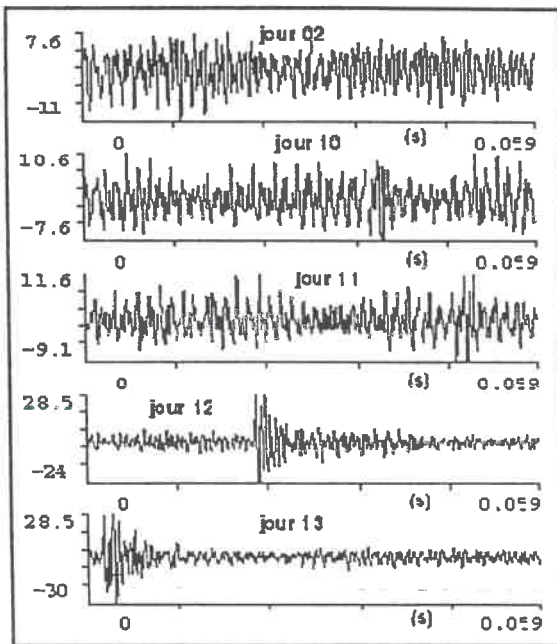


Fig. 3
Représentation temporelle sur une période

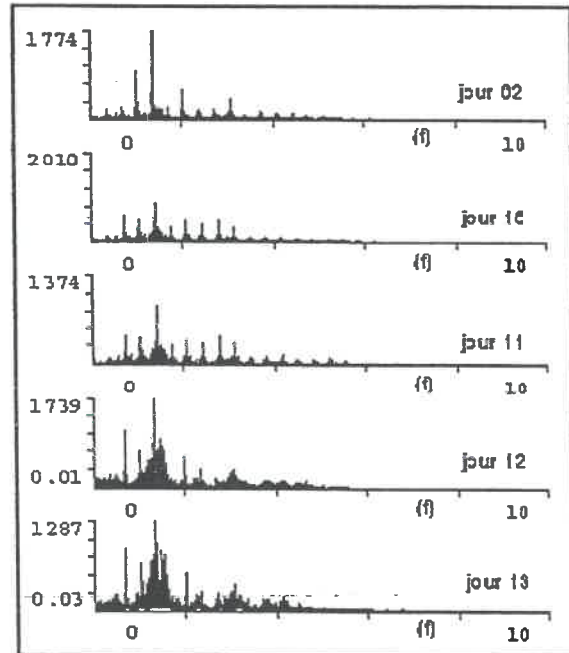


Fig. 4
TF des vibrations sur 16384 échantillons

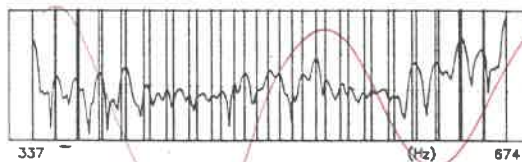


Fig. 5
Spectre du signal entre deux harmoniques d'engrènement

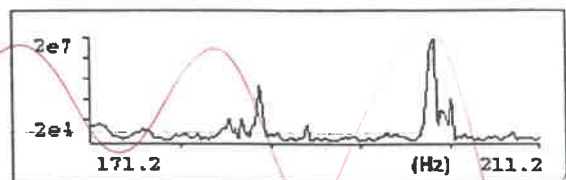


Fig. 6
Loupe sur une partie du spectre

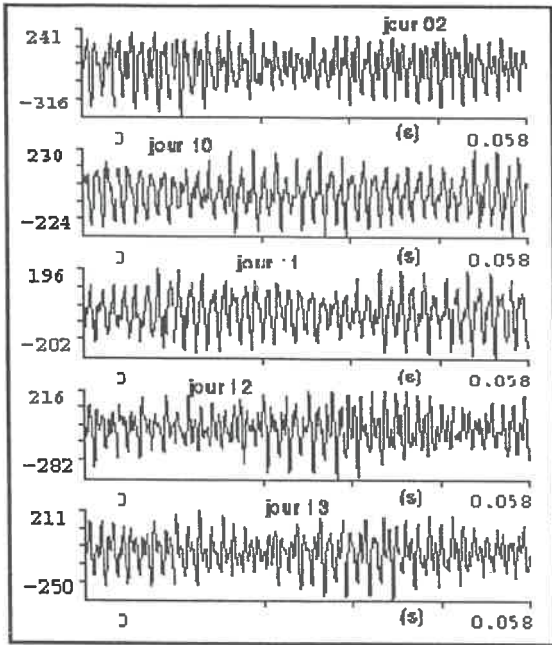
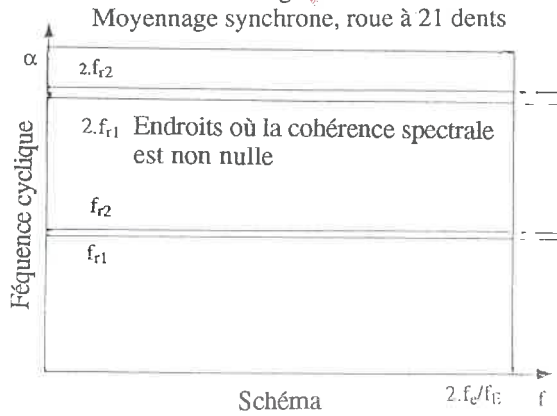


Fig. 5-a



Schéma

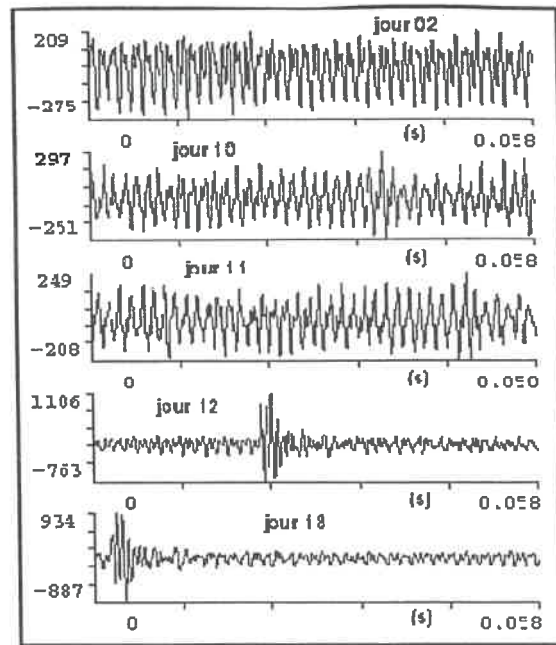
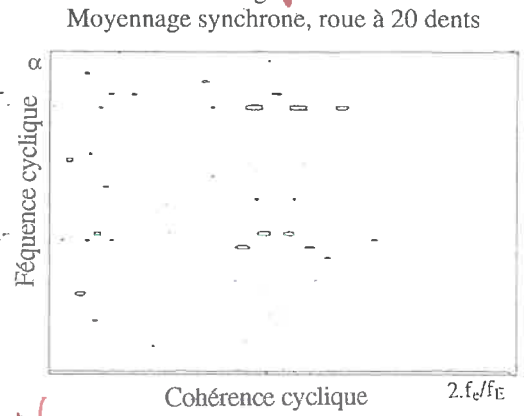


Fig. 5-b



Cohérence cyclique

Fig. 6 Cohérence spectrale, niveau 0.7

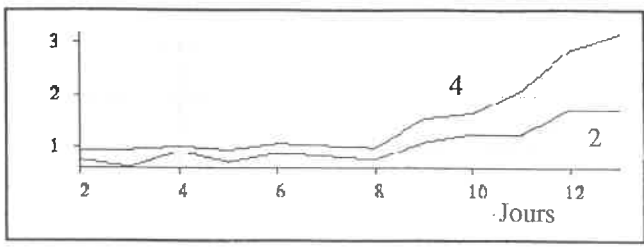
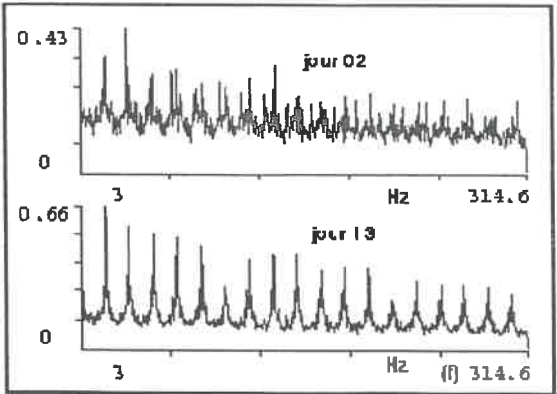


Fig. 7 Projection sur les fréquences cycliques Evolution relative pour les harmoniques 2 et 4

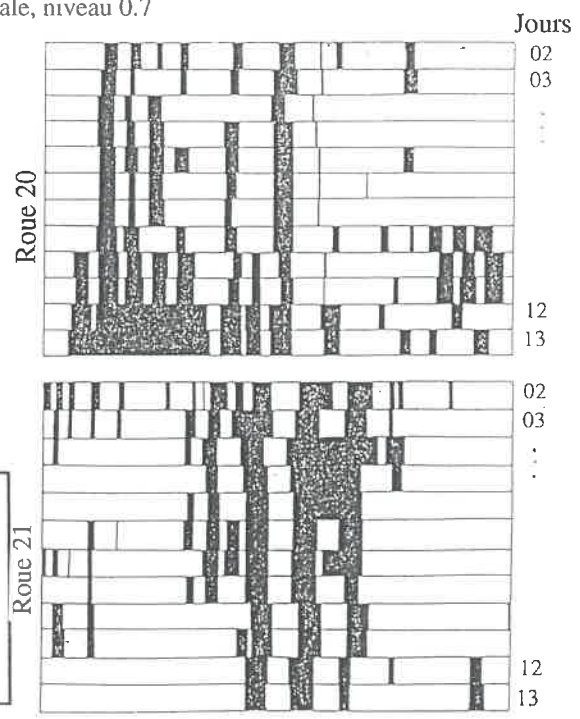


Fig. 8 Evolution de la cohérence spectrale au cours des 12 jours