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# Exact Algorithms for Fixed Charge Network Design Problem with User-Optimal Flows

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**Abstract** We study the fixed charge network design problem with user-optimal flows, which is modeled as a bi-level programming problem. We propose a new Binary Integer Programming (BIP) formulation. This formulation is used to solve the problem by cutting plane and branch-and-cut algorithms. Numerical experiments are performed on realistic instances as well as on random data sets generated with different criteria to examine the difficulty of the instances. The results also show the efficiency of our proposed approaches.

**Keywords:** *Network design, Bi-level programming, Cutting plane, Branch-and-Cut.*

## 1 Introduction

The Fixed Charge Network Design Problem (FCNDP) consists in selecting a subset of edges from a given network, in such a way that a set of commodities can be transported from its origins to its destinations. The objective is to minimize the sum of fixed costs (depending on the selected edges) and variable costs (depending on the flow of commodities on the edges). There are several variants of FCNDP in the literature, each of which considers a particular objective function or takes into account specific additional constraints.

Herein, we are interested in a specific variant of the FCNDP, namely Fixed Charge Network Design Problem with User-Optimal Flows (FCNDP-UOF) with applications in the transportation network design problem for transportation of hazardous materials. This problem combines the FCNDP with multiple shortest path problems.

## 2 Problem description

We consider a transport network modeled by an undirected graph  $G(V, E)$ , where  $V$  represents the set of facilities and  $E$  represents the connections between them, which are uncapacitated.

There is a set of  $K$  commodities to be delivered in the transport network  $G(V, E)$ , where each edge  $e = \{i, j\}$  is associated with several parameters: a length  $c_{ij}$ , a fixed cost  $f_e$  and a variable cost  $g_{ij}^k$  for each commodity  $k$ . Our objective is to design a subnetwork with minimum total cost to transport  $K$  commodities such that each commodity  $k \in K$  with a flow  $\phi^k$  can be delivered through the shortest path from its origin  $o(k)$  to its destination  $d(k)$  in this subnetwork. The FCNDP-UOF has been first formulated as a bi-level integer programming problem [2], and then it is further reformulated as one-level mixed integer programming models through replacing the second level by its optimality conditions in [2] and [3]. More recently, a cutting plane algorithm based on a set of valid inequalities is proposed in [1].

### 3 Proposed methods

We propose in this work a new Binary Integer Programming model (BIP) for the FCNDP-UOF. To this end, we replace the objective function of the second level by valid inequalities eliminating infeasible solutions. Denoting the set of all paths in the graph by  $\mathcal{P}$ , our BIP formulation is given as follows:

$$\min \sum_{e \in E} f_e y_e + \sum_{k \in K} \sum_{(i,j) \in E} \phi^k g_{ij}^k x_{ij}^k \quad (1a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \delta^+(i)} x_{ij}^k - \sum_{(i,j) \in \delta^-(i)} x_{ji}^k = b_i^k, \quad \forall i \in V, \forall k \in K, \quad (1b)$$

$$x_{ij}^k + x_{ji}^k \leq y_e, \quad \forall e \in E, \forall k \in K, \quad (1c)$$

$$\sum_{(i,j) \in E} x_{ij}^k \leq c(P') + (|P'| - \sum_{e \in P'} y_e)M, \quad \forall P' \in \mathcal{P}, k \in K, \quad (1d)$$

$$x_{ij}^k \in \{0, 1\}, \quad \forall i, j \in E, \forall k \in K, \quad (1e)$$

$$y_e \in \{0, 1\}, \quad \forall e \in E, \forall k \in K. \quad (1f)$$

Where,  $y_e$  is a binary variable takes value 1 if the edge  $e$  is chosen as a part of the solution network, and the variable  $x_{ij}^k$  represents the flow of commodity  $k$  through the arc  $(i, j)$ .

In (1b), we have flow conservation constraint. Constraint (1c) does not allow flow to enter arcs whose corresponding edges are closed, (1d) eliminates all paths with a cost bigger than the cost of an opened path in the network. Finally, (1e) and (1f) impose the variables  $x_{ij}^k$  and  $y_e$  to be binary.

Based on this formulation, we implemented two methods to solve the problem. The first one is a cutting plane algorithm in which we solve the initial problem defined in (2) below so that we can obtain a subnetwork  $y^*$ , and a set of paths  $x^*$  for all origin-destination pairs.

$$\min \left\{ \sum_{e \in E} f_e y_e + \sum_{k \in K} \sum_{(i,j) \in E} \phi^k g_{ij}^k x_{ij}^k, \quad \text{s.t.} \quad (1b), (1c), (1e), (1f) \right\} \quad (2)$$

Then, they check if  $x^*$  is the shortest path in  $y^*$ . If it is the case, then  $(x^*, y^*)$  is optimal for the FCNDP-UOF. Otherwise, a valid inequality (1d) is added to problem (2) for the commodities with infeasible paths. We repeat the procedure until an optimal solution is obtained.

The above approach can be expensive in terms of number of iterations to get the optimum. Thus, our second strategy is to combine the two methods together to form a ‘‘branch-and-cut’’ algorithm, known to be an efficient way to handle such constraints. The latter algorithm

solves the problem (2) by a branch-and-cut procedure that adds the inequalities to each integral node violating the shortest path constraint.

Our second contribution lies in assessing the difficulty of the instances used in the simulations. To this end, we categorize the instances by calculating the angle  $\alpha$  between the objective function vectors of the first and the second levels.

We have coded and compared the performance of six algorithms listed as follows:

**B&B1, B&B2:** One-level formulations from [2] and [3], respectively;

**CP1, B&C1:** The cutting plane and branch-and-cut algorithms proposed in this work;

**CP2, B&C2:** The Cutting plane algorithm proposed by [1] and branch-and-cut algorithm coded using the same valid inequalities.

Numerical tests are performed using both random and real data sets. Tab. 1 compares the number of instances solved to optimality in 3600s by the six methods. The table shows that the two proposed cutting plane approaches have a comparable performance and they solve more instances optimally than the others. Our cutting plane algorithm **CP1** has also the best CPU times for the both data sets.

Table 1: Number of instances solved to optimality

	Total	B&B1	B&B2	CP1	CP2	B&C1	B&C2
$0^\circ \leq \alpha \leq 10^\circ$	135	100	109	110	110	110	109
$40^\circ \leq \alpha \leq 50^\circ$	135	88	89	95	95	94	92
$80^\circ \leq \alpha \leq 90^\circ$	135	88	97	105	105	102	102
Ravenna data	7	6	3	7	7	4	4
Sum	412	283	301	317	317	310	307

## 4 Conclusions and future work

For the FCNDP-UOF, we proposed a new BIP formulation, which is solved by both a cutting plane and a branch-and-cut algorithms. Numerical results demonstrate that the cutting plane algorithm based on the proposed BIP formulation outperforms the other algorithms. The instances with an angle of  $40^\circ \leq \alpha \leq 50^\circ$  are the most difficult. For future work, we intend to work on exact approaches for the case of the FCNDP-UOF, where the capacity constraint on edges is considered.

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