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► **To cite this version:**

Cécile Capdessus, Asoke K. Nandi, N. Bouguerriou. Source extraction algorithm based on cyclic properties. ICA Research Network international Workshop, 2008, Liverpool, United Kingdom. hal-02188508

**HAL Id: hal-02188508**

**<https://hal.science/hal-02188508>**

Submitted on 18 Jul 2019

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# SOURCE EXTRACTION ALGORITHM BASED ON CYCLIC PROPERTIES

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## ABSTRACT

We present an algorithm that aims to extracting from a  $N \times N$  additive mixture a cyclostationary source whose cyclic frequency is known. It consists in minimising an uncomplicated criterion based on the cyclic and stationary temporal moments of the measures. We show that it exhibits an absolute minimum and that this minimum ensures the source extraction. The method can be applied whatever the temporal and spectral structure of the mixed sources are, provided that the source to be extracted exhibits one cyclic frequency that is a priori known and that is not shared with any other source.

**Keywords:** source extraction, cyclostationarity.

## 1 INTRODUCTION

This study arose from an issue specific to predictive monitoring of rotating machinery. Among the various parameters that are collected in order to monitor a machine under operation, some very widely spread are the vibrations. Indeed, each rotating part generates vibrations whose shape reveals its state and can be used to diagnose it. Such an early diagnostics avoids unplanned stopping and costly delays.

Due to their symmetric geometry and periodic movements, rotating machines produce repetitive, i.e. cyclostationary, vibrations [1],[2]. They are usually designed to have unsynchronised movements for reducing wear, so that they exhibit different cyclic frequencies. These vibrations usually spread over a wide frequency band and mingle with each other, so that diagnosing a specific part of the machine from the mixtures recorded on the accelerometric sensors can be difficult.

Since the cyclic frequency of each specific part can

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be computed from its geometry and rotation frequency,

we wish to use this information to extract its vibrations. Our algorithm is based on the only cyclic frequency of the part to be diagnosed, in order to be applicable to complex machinery for which a thorough review of all the contributing parts is not always feasible. It must be simple and allow embedded implementation.

Some cyclostationary source separation methods already exist [3]-[6], that surpass our goal but would not be easily implemented on an embedded calculator. The method presented here is an extension to the  $N \times N$  case of a  $2 \times 2$  algorithm previously presented in [7].

In part 2, we give the hypothesis and the principle of the algorithm and its implementation. In part 3 we prove theoretically that the algorithm leads to extraction. In part 4, some examples are given on artificial mixtures, with either artificial or real vibration signals.

## 2 HYPOTHESES

We suppose that  $N$  sensors receive additive mixtures of  $N$  sources. These sources can be either stationary or cyclostationary. The one to be extracted is cyclostationary and exhibits a cyclic frequency that is a priori known and shared by no other sources. The sources are supposed to be uncorrelated and zero mean.

The source vector will be denoted by  $S = [s_1 s_2 \dots s_N]^T$  and the mixture vector by  $X = [x_1 x_2 \dots x_N]^T$  where  $T$  denotes the transpose operator. These two vectors are related by the  $N \times N$  mixture matrix  $A : X = AS$ . Let us suppose, with no loss of generality, that the cyclostationary source to be retrieved is  $s_1$  and denote its cyclic frequency  $\alpha_1$ .

We are looking for an estimate  $\hat{s}_1$  of  $s_1$ , i.e. for a vector  $B = [b_1 b_2 \dots b_N]$ , such that  $z_1 = BX = \hat{s}_1$ .

### 2.1. Criterion

The criterion that we propose to minimise in order to estimate the source  $s_1$  is

$$C(B) = \frac{|R_{z_1}(0)|}{|R_{z_1}^{\alpha_1}(0)|} \quad (1)$$

where  $R_{z_1}(0)$  and  $R_{z_1}^{\alpha_1}(0)$  are coefficients of the Fourier series decomposition of the autocorrelation function of  $z_1$  respectively for cyclic frequencies 0 and  $\alpha_1$  and zero time lag, that is to say :

$$R_{z_1}(0) = \frac{1}{T_1} \int_0^{T_1} R_{z_1}(t,0) dt \quad (2)$$

$$R_{z_1}^{\alpha_1}(0) = \frac{1}{T_1} \int_0^{T_1} R_{z_1}(t,0) e^{-2\pi j \alpha_1 t} dt \quad (3)$$

with

$$R_{z_1}(t,0) = E[z_1(t)z_1^*(t)] \quad (4)$$

where  $E[\cdot]$  denotes statistical averaging, \* conjugation and  $T_1 = \frac{1}{\alpha_1}$ .

## 2.2. Implementation

The criterion can be computed from the Fourier coefficients of the correlation matrix of the measures for zero time lag and from matrix  $B$ , and minimised over  $B$  coefficients. As shown in the proof section 3, it exhibits an absolute minimum versus  $B$  coefficients that leads to the extraction vector. From (1) and from the definition of  $B$  we derive :

$$C(B) = \frac{BR_X(0)B^T}{BR_X^{\alpha_1}(0)B^T} \quad (5)$$

with

$$\begin{cases} R_X(0) = \frac{1}{T_1} \int_0^{T_1} E[X(t)X^{*T}(t)] dt \\ R_X^{\alpha_1}(0) = \frac{1}{T_1} \int_0^{T_1} E[X(t)X^{*T}(t)] e^{-2\pi j \alpha_1 t} dt \end{cases} \quad (6)$$

## 3 PROOF

The criterion can be theoretically calculated versus

the mixture matrix  $A = \begin{bmatrix} a_{11} & \dots & a_{1N} \\ & \dots & \\ a_{N1} & \dots & a_{NN} \end{bmatrix}$ , the extrac-

tion vector  $B$  and the Fourier coefficients of the correlation matrix of the sources.

$$\begin{cases} R_S(0) = \frac{1}{T_1} \int_0^{T_1} E[S(t)S^{*T}(t)] dt \\ R_S^{\alpha_1}(0) = \frac{1}{T_1} \int_0^{T_1} E[S(t)S^{*T}(t)] e^{-2\pi j \alpha_1 t} dt \end{cases} \quad (7)$$

Due to the hypotheses made in section 2 on the sources,

$$R_S(0) = \begin{bmatrix} \sigma_1^2 & \dots & 0 \\ & \dots & \\ 0 & \dots & \sigma_N^2 \end{bmatrix} \quad (8)$$

is diagonal with  $\sigma_i^2$  the power of source  $s_i$  and

$$R_S^{\alpha_1}(0) = \begin{bmatrix} R_{s_1}^{\alpha_1}(0) & \dots & 0 \\ & \dots & \\ 0 & \dots & 0 \end{bmatrix} \quad (9)$$

exhibits only one non-zero component.

From (1), (8) and (9) we derive :

$$C(B) = \frac{\sigma_1^2}{|R_{s_1}^{\alpha_1}(0)|} + \frac{\sum_{j=2}^N \left( \sum_{i=1}^N b_{li} a_{ij} \right)^2 \sigma_j^2}{|R_{s_1}^{\alpha_1}(0)| \left( \sum_{i=1}^N b_{li} a_{i1} \right)^2} \quad (10)$$

By introducing  $Y_j = \frac{W_j}{W_1}$ , with  $W_j = \sum_{i=1}^N b_{li} a_{ij}$ , equation (10) can be written as

$$C(B) = \frac{\sigma_1^2}{|R_{s_1}^{\alpha_1}(0)|} + \frac{1}{|R_{s_1}^{\alpha_1}(0)|} \sum_{j=2}^N Y_j^2 \sigma_j^2 \quad (11)$$

If the set of  $\{Y_j\}$  can take any value over  $R^N$ , then the criterion exhibits an absolute minimum for  $\{Y_2, \dots, Y_N\} = \{0, \dots, 0\}$ . Let us investigate what are the conditions for this requirement to be fulfilled.

The question can be expressed in another way: for any  $(N-1)$ -uplet  $\{Y_2, \dots, Y_N\}$  does there exist a  $B$  vector that leads to that  $(N-1)$ -uplet? Now, this question can be re-expressed: does equation (13) always have a solution?

$$\sum_{i=1}^N (a_{ij} - Y_j a_{i1}) b_{1i} = 0 \quad (12)$$

This depends on the determinant of a G matrix defined by

$$G = \begin{bmatrix} a_{11} & & a_{N1} \\ (a_{12} - Y_2 a_{11}) & (a_{22} - Y_2 a_{21}) & (a_{N2} - Y_2 a_{N1}) \\ (a_{1N} - Y_N a_{11}) & (a_{2N} - Y_N a_{21}) & (a_{NN} - Y_N a_{N1}) \end{bmatrix} \quad (13)$$

Due to the fact that G is a combination of A matrix and the first row of A, and based on the properties of matrix determinants, it can easily be shown that  $\det(G) = \det(A)$ . Thus, unless  $\det(A) = 0$ , the proposed criterion exhibits a unique minimum that is reached for  $W_2 = W_3 = \dots = W_N = 0$ .

Then,  $z_1 = W_1 s_1$  is proportional to the source  $s_1$ , which means that the extraction is achieved. We thus demonstrated that provided that the matrix A is not ill-conditioned, minimising the proposed criterion leads to extraction.

## 4 SIMULATIONS

All the simulations are performed over artificial mixtures and in a 3 sources \* 3 measures case. The first simulation is applied to a mixture of three synthetic signals: two cyclostationary ones at different cyclic frequencies and a stationary one. The second simulation is applied to one real vibration and two artificial sources (one cyclostationary and the other stationary).

The mixture matrix is a rotation matrix, which ensures that it is well conditioned. Its coefficients are :

$$A = \begin{bmatrix} 0.4330 & 0.5537 & 0.7113 \\ -0.7500 & 0.6590 & -0.0564 \\ -0.5000 & 0.3000 & 1.288 \end{bmatrix} \quad (14)$$

The statistics of the measures are estimated over 60 realisations, which corresponds to different numbers of samples depending on the chosen cyclic frequency. Indeed, one realisation corresponds to the period of cyclostationarity.

For the first simulation, the sampling frequency is  $f_e = 40kHz$  and 60 realisations correspond to 18001 samples. For the second one,  $f_e = 100kHz$  and 60 realisations correspond to 15383 samples.

The coefficients of matrix B are optimised by being made vary with a step equal to 0.1 between -1 and 1. It is not a problem to restrict the research to this interval,

since any solution B could lead to a solution within that interval by being divided by its biggest coefficient.

In order to evaluate the extraction method efficiency, we compute the mean square error between normalised versions of the source  $s_1$  and its estimate. This error is given in dB. We also compute the product  $B \cdot A$  that should exhibit two zero coefficients in case of perfect extraction.

### 4.1 Artificial sources

The sources  $s_1$  and  $s_2$  are both white random noises modulated by cosine functions at different frequencies, i.e. typical second order cyclostationary signals. The third one is a white random noise. The three of them spread over the whole spectrum so that no classical filtering method would extract the source  $s_1$ . The criterion is computed with the cyclic frequency of the source  $s_1$  in order to extract it.

Fig 1 shows the temporal representations of the three sources. The two cyclostationary ones have close cyclic frequencies (62 Hz and 66 Hz) so that they look like each other.

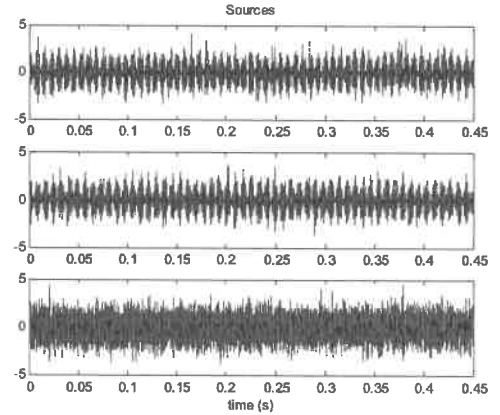


Figure 1. Temporal representations of the three sources.

The mean square error between the source  $s_1$  and its estimate is -35 dB and the matrix product is  $BA = [-0.8031 \quad -0.0040 \quad -0.0051]$ , which means that the two other sources are rather well suppressed.

The algorithm was applied to 100 different mixture rotation matrices and the figure 2 shows the error obtained for these 100 tests. In 99% of the tests the error does not exceed -25 dB, in 68% of the tests it is smaller than -30 dB and in two of these cases it is even smaller than -40 dB.

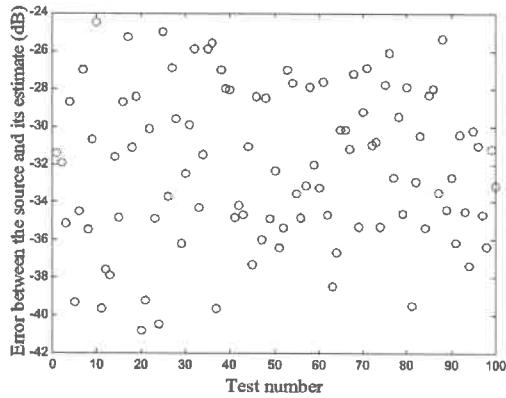


Figure 2. Error between the source and its estimate for 100 different mixture matrices.

#### 4.2. Real vibrations

The source to be retrieved  $s_1$  is now a damaged roller bearing vibration, whose vibration spectrum is shown on fig. 3. The vibration spreads over the whole frequency range of the sensor and is amplified by some resonances of the structure.

It exhibits no peculiar spectral line but is a typical cyclostationary signal whose cyclic frequency can be computed from its geometric characteristics and the rotation speed of the shaft. This cyclic frequency is equal to 390.05 Hz and is used to extract the vibration from the mixture.

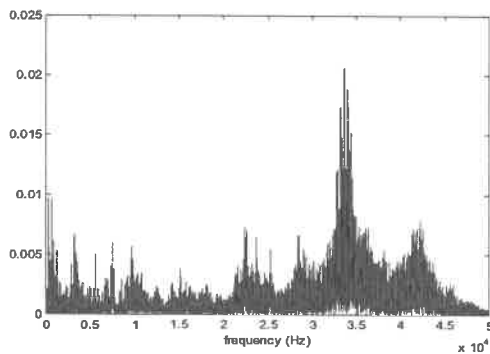


Figure 3. Spectrum of the roller bearing vibration.

The mean square error between the source  $s_1$  and its estimate is  $-42$  dB and the matrix product is  $BA = [-0.8031 \quad -0.0040 \quad -0.0051]$ . Extraction has been properly achieved.

## 5 CONCLUSION

The source extraction method presented here allows retrieving a cyclostationary source from set of additive

mixtures. The cyclic frequency of that source is supposed to be known and shared with no other source. The additional assumptions that were made are the second order decorrelation of the sources and having as many sensors as sources.

The method was shown to be efficient on any number of sources from a theoretical point of view and some simulations were given in the  $3 \times 3$  case, both for simulated signals and real vibrations.

In order for this method to be applied to real mixtures, the hypothesis must be enlarged to a more realistic scheme. In particular, the additive mixture assertion does not fit to these real mixtures features. An extension of the algorithm to the convolutive case is in progress.

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