



HAL
open science

Cyclostationarity: a new signal processing concept for vibration analysis and diagnosis

Cécile Capdessus, Jean-Louis Lacoume, Ménad Sidahmed

► **To cite this version:**

Cécile Capdessus, Jean-Louis Lacoume, Ménad Sidahmed. Cyclostationarity: a new signal processing concept for vibration analysis and diagnosis. 1994 International Gear Conference, 1994, Newcastle, United Kingdom. hal-02188462

HAL Id: hal-02188462

<https://hal.science/hal-02188462>

Submitted on 18 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

SYNOPSIS Toothed gearing vibrations are shown to be cyclostationary. An application of this property to the early diagnosis of a system is presented.

NOTATION

f_E = sampling rate.

f_e = meshing frequency.

f_{ri} ($i=1,2$) = rotation frequencies.

$R_x(t,\tau)$ = autocorrelation function.

$S_x(f)$ = power spectrum.

$S_x^\alpha(f)$ = spectral correlation function.

1 INTRODUCTION

The keeping up of an equipment is both a problem of safety and of saving money. An early diagnosis of damages in the machinery can avoid some accidents, some further damages of the system, or can prevent from stopping the machinery for long repairs. The object of this paper is to present some new parameters, based on recent signal processing methods, which are well-adapted to the characterization of the condition of toothed-gearings. We first propose a model of the vibrations, and we then give some details about cyclostationarity and the way it can be characterized by the spectral correlation function. By applying this function to the model of the vibrations we show that they are cyclostationary and determine their cycle frequencies. At last, we present some practical results that we obtain on an experiment realised on a toothed-gearing and we discuss about the parameters which seem to best characterize the condition of this toothed-gearing and the future prospects it opens about vibration analysis, possible industrial applications and automatic diagnosis.

2 TOOTHED-GEARING VIBRATIONS.

2-1 Model of the vibrations.

Many papers have been written on the subject of modeling of toothed-gearing vibrations, but one of them, published by W. Mark [1] in 1978, is very comprehensive. It can be used to establish a model from which we can predict what kind of an effect the emergence of a damage on one of the wheels is going to have on the aspect of the vibrations.

Mark first defines the different parameters which contribute to the model. The source of the vibrations is the transmission error. Three main frequencies have to be defined : both the two rotation frequencies f_{r1} and f_{r2} of the

wheels, and the meshing frequency which is the one of the shocks between the teeth, and is equal to $f_e = N_1.f_{r1} = N_2.f_{r2}$, where N_i is the number of teeth of the wheel number i .

W. Mark shows that the static transmission error, which generates the vibrations, can be expressed in the following way :

$$x(t) = \frac{W}{v(t)} + \frac{u(t)}{v(t)}$$

where $v(t)$ is periodical at the frequency f_e and $u(t)$ is the sum of three periodical terms, whose frequencies are f_e for the first one and respectively f_{r1} and f_{r2} for the other ones, that are tied to the shape of each of the wheels. W is the total load transmitted by the system, and we suppose it is constant.

We can reexpress this model so as to use it for signal processing applications. It then becomes :

$$x(t) = s_e(t) \cdot (W + s_m(t) + s_{r1}(t) + s_{r2}(t)),$$

where $s_e(t)$ and $s_m(t)$ are deterministic periodical signals with frequency f_e and $s_{r1}(t)$ and $s_{r2}(t)$ are periodical with respective frequencies f_{r1} and f_{r2} , and with random, stationary, centered amplitudes. Since each of these terms is tied respectively to the shape of one wheel, and this shape has no reason to be tied from one wheel to the other, we can suppose that their random amplitudes are independent random variables. We will write this more precisely when necessary.

We can try to forecast what is going to happen if a damage affects the gearing. In the particular case that we have studied, the defect is a flaking off that appears on one of the wheels, which means that what will be changed in the expression of the vibrations is the term related to the shape of the teeth of this wheel, say $s_{ri}(t)$, where i is the number of this wheel, since the defect is a new component of the deviation of the tooth profile from the ideal profile.

2-2 Presentation of the experiment.

The experiment was realised at the C.E.T.I.M.*. It is composed of two toothed-gearings of respectively 20/21 and 40/42 teeth. The sensors are accelerometers which were placed so as to record the vibrations of the 20/21 toothed-gearing.

* Centre Technique des Industries Mécaniques.

The experiment lasted thirteen days, and the vibrations were recorded every day. At the beginning, the gearing under study was in a good condition, and it gradually became damaged. The sampling rate f_E is 20 000 Hz. The meshing frequency f_c is 340 Hz. The rotation frequencies f_{r1} and f_{r2} are about the same (since the two wheels have almost the same number of teeth), and they are equal to 16.7 Hz. Each of the records lasted about 3 s, which means 50 rotations of the wheels.

One of the problems that we have to cope with is the closeness of the rotation frequencies. The two modulations, due respectively to the shape of the two wheels, will appear on the temporal representation under the shape of a very low-frequency beat, which is hardly seen on the temporal representation of the signal. In the following paragraph, we will show what kind of a structure these modulations generate on the spectral representation.

2-3 Temporal and spectral representation. Modulations and sidebands.

Fig. 1 shows both a temporal representation and spectrum of the vibration. We can notice a fast phenomenon which could be the meshing one, and a slower modulating one that can be attributed to the rotation of the wheels. It is not possible on this temporal representation to separate the parts of the vibrations owing respectively to each of the wheels. On the last day record, a dramatic change in the shape of the vibrations appears at the rate of the damaged wheel, but this happens at a time when the toothed-gearing is already seriously damaged, and the goal of this study is to investigate the field of signal processing methods that could help us for an early diagnosis of the system.

The hypothesis of a periodically modulated vibration is confirmed when observing the spectral representation of the signal. Indeed, the spectrum presents one series of spectral lines whose fundamental frequency is the meshing frequency, and a second series of spectral lines which appears several times and is located around the gearing harmonics. This phenomenon is called sidebands. We can forecast, by referring to the model, that two different series of sidebands are going to appear on the spectral representation, since there are two different modulating terms, owing to the rotation of the two different wheels. Actually, on the spectrum, we can observe only one series of sidebands, since the spectral resolution does not allow us to separate the two rotation frequencies.

In the following paragraph, we are going to define cyclostationarity, and give the main properties of cyclostationary signals and the ways they can be characterized.

3 CYCLOSTATIONARITY.

3-1 Autocorrelation function. Stationarity, time dependence [2], [3].

One can compute for any signal the different moments of this signal, at any order, either in the time domain or in the frequency domain. These moments are a statistically averaged value of a lag product of the signal. Statistical average means that we dispose of a set of different realizations of the same function or signal and that each parameter that we compute from one or several samples of this function can be averaged

on the whole set of realizations. A lag product is a product of the values obtained for one function at different times.

For instance, the first and second order moments of a function $x(t)$ both in time and frequency are given by :

Time	Frequency
First order (mean) : $E [x(t)]$	First order : $E [X(f)]$
Second order (autocorrelation function) : $E [x(t).x^*(t-\tau)]$	Second order (spectrum) : $E [X(f).X^*(f-\alpha)]$

where $x(t)$ is the temporal representation of the signal and $X(f)$ its frequential representation. The (*) symbolises the conjugate of the value and $E[...]$ symbolises statistical averaging.

The time-domain moments and frequency-domain moments are a pair of Fourier transform (Wiener-Kintchine), at any order. For instance, second-order moments are related by a two-dimensional Fourier transform :

$$E [X(f).X^*(f-\alpha)] = TF/t,\tau \{ E [x(t).x^*(t-\tau)] \}.$$

In this paper, we are interested only in second order moments, which are autocorrelation function and spectral correlation function. We can see from the previous definitions that both of them depend on two different variables and should be represented on a two dimensional diagram. Whereas, when we usually compute autocorrelation function or spectrum, it only depends on one variable and can be represented on a lineal diagram. This comes from the following reason :

We usually suppose that the signal is second order stationary, which means that its second order temporal moment does not depend on time, that is to say :

$$E [x(t).x^*(t-\tau)] = R_x(\tau) \text{ does not depend on the variable } t.$$

Then, the frequency domain second order moment is equal to :

$$E [X(f).X^*(f-\alpha)] = TF/t,\tau \{ R_x(\tau) \} \cdot \delta(\alpha),$$

where $\delta(\alpha)$ symbolises a Dirac impulsions.

As a consequence, the frequential second order moment $E [X(f).X^*(f-\alpha)]$ is non-zero only for the zero value of the variable α , so that we compute it for the only value $\alpha=0$ and obtain a one-dimensional result $S_x(f)$, depending only on the variable f and called power spectrum.

Nevertheless, it can be more realistic in some cases to suppose that the signal is non-stationary and to compute the frequential second-order moments for more than one value of the variable α which is then called cycle frequency. Actually, this new way of seeing the signal finds expression in the way we compute the moments : we will not be able to simply replace the statistical averaging by a temporal averaging, as we are used to doing in the case of stationary signals, since we cannot suppose any more that the signal is ergodic. This is going to be shown in the following section.

3-2 Statistical averaging, time-averaging, Ergodism, cycloergodism.

Let us define a signal $x(t)$, stationary and ergodic, and come back to the problem of statistical averaging. It is obvious that in most of the cases, we don't actually dispose of a set of different realizations of the studied signal. So, to palliate this

problem, we are used to replacing the statistical averaging by time averaging. This procedure finds its justification in the following remark : for one given value of t , the lag-product can be computed and averaged on all of the realizations of the signal (Fig. 2). We then suppose that the signal is ergodic, which means that the different values taken by the lag-product $\{x(t).x^*(t-\tau)\}_i$ for one definite value of i and t varying describe the same set as the values taken by this lag-product for a definite value of t with i varying. This allows us to replace the statistical mean $E[\dots]$ by a time averaging $\int_{-\infty}^{\infty} \dots dt$ on one of the realizations.

In the case of a non-stationary signal, if we just replace the statistical averaging by an integration on time, we loose information about possible variations of the second-order moment as a function of time t . Let us now consider a signal $x(t)$ whose temporal second-order moment is not any more a constant as a function of t , but depends on the time t periodically with frequency α_0 . This means that the statistical mean of the lag-product of the signal is a periodical function of frequency α_0 . Fig. 3 illustrates this possibility.

In this case, the statistical averaging is not any more replaced by a simple time averaging, but by a synchronized averaging, at a rate equal to the frequency of the moment. It is to say that $E[x(t).x^*(t-\tau)]$ will be replaced by

$$\sum_{n \in \mathbb{N}} x\left(t + \frac{n}{\alpha_0}\right).x^*\left(t - \tau + \frac{n}{\alpha_0}\right). \text{ Such a signal is called}$$

cyclostationary signal of frequency α_0 , and if the statistical averaging can be replaced by synchronized averaging, it is said to be cycloergodic. Let us notice that in this case, the frequency-domain moment, say the spectral correlation function, given by

$$S_x^\alpha(f) = E[X(f).X^*(f-\alpha)] = TF_{/t,\tau} \{E[x(t).x^*(t-\tau)]\},$$

then presents spectral lines as a function of cycle frequency α , since the temporal moment $E[x(t).x^*(t-\tau)]$ is periodical as a function of t .

The problem with synchronized averaging is that in order to perform it on some data, they must have been recorded with a sampling rate synchronized on the frequency of cyclostationarity of the signal. Therefore, we are not supposed to know this frequency before we acquire the data. This is why we actually do not compute the autocorrelation function of the data, but two other parameters called cyclic autocorrelation function and spectral correlation function, that we will define further.

This was a practical approach of cyclostationarity and cycloergodism. We can now give a better definition for cyclostationary signals and study which of the second-order representations of the signal best characterize second-order cyclostationarity.

Stationarity :

A signal is said to be stationary at the n^{th} order if its time-domain n^{th} order moment does not depend on time t .

Cyclostationarity :

A signal is said to exhibit cyclostationarity at the n^{th} order if its time-domain n^{th} order moment is a periodical function of time t .

In this paper, we will focus on second-order moments, which are the ones that we have applied to the real vibrations. These are the autocorrelation function and its Fourier transform, both in t and τ (see Fig. 4). Both "cyclic autocorrelation function" and "spectral correlation function" have a very peculiar shape when applied to a cyclostationary signal. These two functions then present spectral lines as functions of cycle frequency α . Fig. 5 shows a schematic representation of the spectral correlation function for a stationary or cyclostationary signal.

3-3 Estimation of spectral correlation function.

We have chosen, for this study, to use the spectral correlation function, which shows both the spectral lines in cycle and spectral frequencies.

We estimate the spectral correlation function, $E[X(f).X^*(f-\alpha)]$, by a frequency-smoothed periodogram. This method consists in computing the lag-product $X(f).X^*(f-\alpha)$ on the whole set of data and smoothing it in spectral frequency f with an adequate smoothing window so as to improve the estimation.

One important remark about the estimation is that by choosing the width of the smoothing window, we define the bandwidth of the spectral components whose correlation to each other we are going to study. It has to be well-adapted to the characteristics of the signal that we want to point out. Indeed, the efficiency of spectral correlation function comes from the fact that it can be well-estimated by smoothing in spectral frequency, without losing resolution in cycle frequency. The resolution in cycle frequency is given by the

$$\text{total length of the studied signal, that is to say } \Delta\alpha = \frac{1}{T} .$$

where T is the total duration of the recorded signal. Whereas the resolution in spectral frequency depends on both the length of the signal and the width of the smoothing window that we choose so as to estimate the spectral correlation function. If the chosen smoothing window has a width N (which means N samples), then the resolution in spectral frequency is equal to

$$\Delta f = \frac{N}{T} .$$

The spectral correlation functions, given by $E[X(f).X^*(f-\alpha)]$, measures the correlation between the components of the signal whose respective frequencies are f and $f+\alpha$. These components have a bandwidth which is given by the physical nature of the signal and which must be taken into account when choosing the width N of the smoothing window. The width N must be about equal to this bandwidth, so that the correlation be well-estimated and be not merged with information concerning other physical components of the signal.

2-4 Normalisation. Spectral coherence.

In the case where the signal comprises spectral components of various level of energy, it is interesting to normalise spectral correlation by the energy of the components whose correlation we measure, so as to be able to detect the correlation between components of very weak energy. This is why we have chosen to compute spectral coherence, which is spectral correlation normalised by the energy of the components in the spectrum.

The value of spectral coherence for the spectral and cycle frequencies (f,α) is given by (its symmetrical form, that we use):

$$C_x^\alpha(f) = \frac{S_x^\alpha(f)}{\sqrt{\left|S_x(f+\frac{\alpha}{2})\right| \cdot \left|S_x(f-\frac{\alpha}{2})\right|}}$$

Then if two weak energy components $X(f+\frac{\alpha}{2})$ and $X(f-\frac{\alpha}{2})$ of the signal are strongly correlated, this correlation will be better pointed out by spectral coherence than by spectral correlation.

4 CYCLOSTATIONARITY OF THE TOOTHED-GEARING VIBRATIONS.

4-1 Frequential components of the vibrations.

Let us come back to the model of the vibrations and show that they are cyclostationary. This model is given by :

$$x(t) = s_e(t) \cdot (W + s_m(t) + s_{r1}(t) + s_{r2}(t)),$$

where we have seen that W is a constant, $s_e(t)$ and $s_m(t)$ are both definite periodical functions of frequency f_e , (the meshing frequency), and $s_{r1}(t)$ and $s_{r2}(t)$ are periodical functions, whose frequencies are the rotation frequencies of the wheels, modulated by a random signal. These two last terms can be written in the following way :

$$s_{ri}(t) = a_{ri}(t) \cdot \sum_{n=-N_i}^{N_i} e^{2\pi j n f_{ri} t + \Phi_n}$$

where i ($=1$ or 2) characterizes one of the two wheels, N_i is the number of harmonics of the corresponding periodical modulating signal, and $a_{ri}(t)$ is the real, random amplitude of this periodical component.

The autocorrelation function, which is given by $R_x(t, \tau) = E[x(t) \cdot x^*(t-\tau)]$ comprises two kinds of terms : those that come from the deterministic components of the vibration, in which we are not interested, and the ones produced by the random components of the signal, that are worth being detailed.

Among these terms, some are equal to zero, due to the fact that the amplitudes $a_{ri}(t)$ are zero mean and $a_{ri}(t)$ and $a_{rj}(t)$ are independent when $i \neq j$. Two of them are non-zero and are given by :

$$s_e(t) \cdot s_e^*(t-\tau) \cdot E[a_{ri}(t) \cdot a_{ri}^*(t-\tau)] \cdot$$

$$\sum_{n_i=-N_i}^{N_i} \sum_{m_i=-N_i}^{N_i} e^{2\pi j (n_i - m_i) f_{ri} t + m_i f_{ri} \tau + (\Phi_{n_i} - \Phi_{m_i})},$$

for $i=1,2$.

Since the signals $a_{ri}(t)$ are supposed to be random and stationary, their autocorrelation function $R_{a_{ri}}(\tau) = E[a_{ri}(t) \cdot a_{ri}^*(t-\tau)]$ depends only on τ , so that the expression given by (4) is periodical as a function of t with frequency $(n_i - m_i) \cdot f_{ri}$, and does not present any peculiar periodicity as a function of τ , since its shape, in this last case, depends on the shape of $R_{a_{ri}}(\tau)$. The multiplicative term $s_e(t) \cdot s_e^*(t-\tau)$ just introduces a series of translations, both in spectral and cycle frequencies, since it is periodical with frequency f_e as a

function of each of these. Anyway, due to the proportionality between this frequency and the two frequencies of cyclostationarity, the spectral lines will still appear at positions harmonic of f_{r1} and f_{r2} .

We can see from the previous remark that several spectral lines are going to appear in the spectral correlation function in cycle frequency. Their fundamental frequencies are respectively the rotation frequencies of the two wheels. These two series of spectral lines characterize the existence of two cyclostationary components in the vibrations. We are going to see how these two frequencies of cyclostationarity can be used so as to separate the modulations due to the wheels and to diagnose the toothed-gearing.

The fig. 6 illustrates these results by showing a schematic representation of the spectral correlation function of the vibrations. This function can take any shape as a function of spectral frequency f but is discrete as a function of cycle frequency α .

4-2 Application of spectral correlation to the vibrations.

Here it is necessary to give some details about the figures that are going to be shown. We would like to be able to separate the two modulation phenomena, which means that we need the cycle frequency resolution $\Delta\alpha$ to be lower than the difference between the two rotation frequencies, that is to say $\Delta\alpha < 16.7 \cdot (1-20/21) = 0.79$ Hz. Since the sampling rate is equal to 20 kHz, we need to use an amount of 25150 samples at least to be able to separate these frequencies. We decide to use the whole set of samples (60000 samples).

In order to reduce the time of computation, we can use our knowledge of the vibrations. Let us remember that the two rotation frequencies appear in the spectrum as sidebands, which means they appear around each of the harmonics of meshing frequency. If a defect appears on one of the wheels, for instance on one or two teeth, then it is going to make the shape of the corresponding modulation less smooth, which will produce on the spectrum of the modulation an enlargement of the bandwidth. This means that what we need to detect is the coming out of high order harmonics of the modulation corresponding to the damaged wheel, that is to say, harmonics of the corresponding sideband distant from the center meshing harmonic.

So, before applying spectral correlation to the vibrations, we first filter them around one harmonic of the meshing frequency and undersample it as much as we can. We have chosen to perform this filtering around the second meshing harmonic because it is the tallest one and the associated sidebands are of great energy. We can by this way reduce the amount of samples to 4096, which is much easier to process than the first 60000 samples. The practical results of this study are presented in the following section.

3-4 Practical results. Diagnosis.

Fig. 7 shows the spectrum and spectral coherence computed on the second day recording. On fig. 6, lines are drawn at the places where we expect to see some lines in cycle frequency. Indeed, we can notice that at these places on fig. 7 appear some discrete phenomena whose amplitude is close to one, as expected. Some artefacts appear at other places, that

we cannot really explain, but they are of lower amplitude. They would probably disappear if we could chose a wider smoothing window and improve the estimation, but the width of this smoothing window is limited by the fact that we don't want to merge the information concernig two successive harmonics of a same family of sidebands, so that this width must be lower than 16.7 Hertz.

Having performed a detailed study of the shape and amplitude of these spectral lines for each day of the experiment, we have decided to concentrate our attention on the second harmonic of both the two rotation frequencies, for these are the ones which appear with the best stability on each computed spectral coherence. Each of the figures that we are going to show from now will be a cut at one of these two cycle frequencies $\alpha=2.f_{r1}$ and $\alpha=2.f_{r2}$.

Spectrum (Fig. 8)

The greatest peak that appears on the spectrum is the meshing frequency around which the signal has been filtered. On each side of this central harmonic appears a series of sidebands. Actually, two series of sidebands are known to be present in the spectrum but the spectral resolution in spectral frequency being lowered by the estimation smoothing, these two series of sidebands are merged.

Spectral correlation (Fig. 8)

The peaks which appear on the two other curves, obtained for non-zero values of the cycle frequency, correspond for each of the curves to one of the wheels only. Let us explain how to interpretate them. For instance, on the curve corresponding to the twenty one toothed wheel, the central peak measures the degree of correlation between the first harmonics (on both left and right side of the meshing harmonic) of the corresponding sidebands, that is to say between the components at

frequencies f_e+f_{r21} and f_e-f_{r21} . Indeed, $f+\frac{\alpha}{2} = f_e+f_{r21}$, $f-\frac{\alpha}{2} = f_e-f_{r21}$, $f = f_e$, and $\alpha = 2.f_{r21}$.

Diagnosis

At last, fig. 9 shows the value of spectral coherence for each of the chosen values of cycle frequency ($\alpha_{21}=2.f_{r21}$ and $\alpha_{20}=2.f_{r20}$), for each day of the experiment. We can notice that, for the twenty toothed wheel, spectral coherence detects the coming out of higher order harmonics of the corresponding sidebands, (far from the center meshing harmonic), after the 9th day of the experiment, which reveals the coming out of a damage on the same wheel, according to our model.

5 CONCLUSION

We have pointed out that toothed gearing vibrations are second order cyclostationary. Let us point out that the widely used synchronous averaging is a first order cyclostationary study of the signal. This second order cyclostationary study allows to separate from each other the phenomena tied respectively to the rotation of the two wheels, and we show that this can be used for an early diagnosis of the toothed-gearing. We show how spectral correlation function performs this separation by keeping the best resolution in cycle frequency while improving the estimation in spectral frequency. The main improvement of the second order study, compared to synchronous averaging, is that it does not require a synchronous acquisition of the signal. It is temptating to compare this method to some other second order methods, such as time and frequency representations. The main

difference between these methods and the cyclostationary study is that time and frequency representations are mainly used on deterministic signals, to point out local non stationarity properties, whereas spectral correlation function is used to detect statistical properties which are spread on the whole duration of the signal.

Computing the spectral correlation function for all possible cycle frequencies takes much time but once determined the frequencies of cyclostationarity of the gearing, and once elaborated a strategy of comparison to previously stocked values, it has to be computed for only a set of discrete values of the cycle frequency. So, the use of spectral correlation so as to perform an automatic diagnosis of such a device can be envisaged.

REFERENCES

- [1] MARK "Analysis of the Vibratory Excitation of Gear Systems : Basic Theory" J. Acoust. Soc. Am., Mai 1978, Vol. 63(5), pp. 1409-1430.
- [2] GARDNER "Statistical Spectral Analysis. A Non Probabilistic Theory" 1988 by Prentice-Hall, Inc.
- [3] CAPDESSUS "Aide au diagnostic des machines tournantes par traitement du signal" Thèse de l'Institut National Polytechnique de Grenoble, 1992.

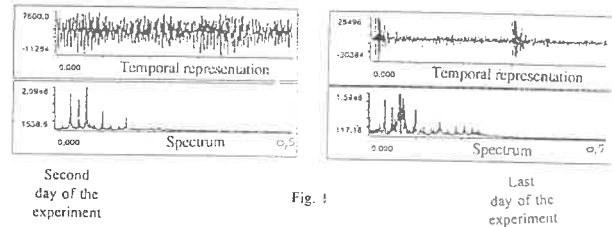


Fig. 1

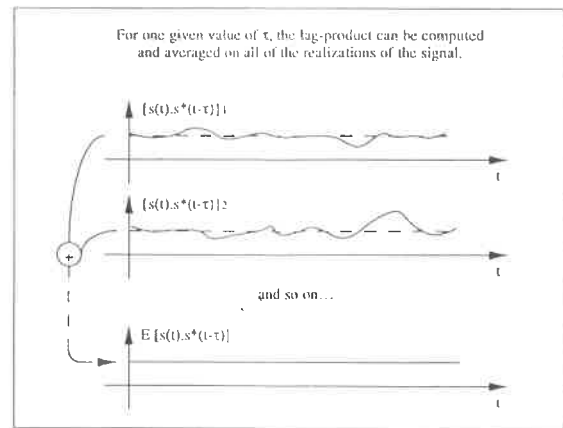


Fig. 2

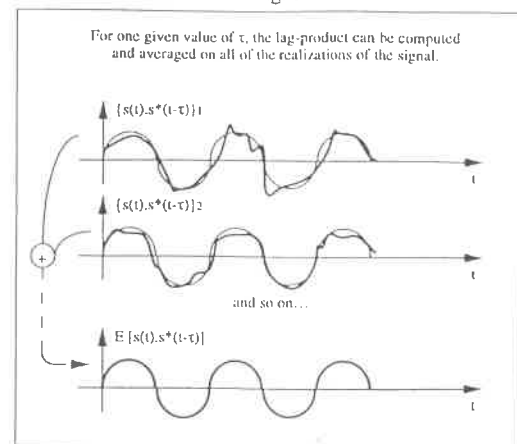
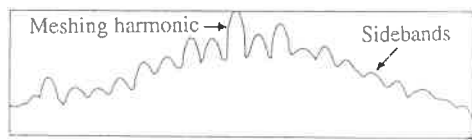
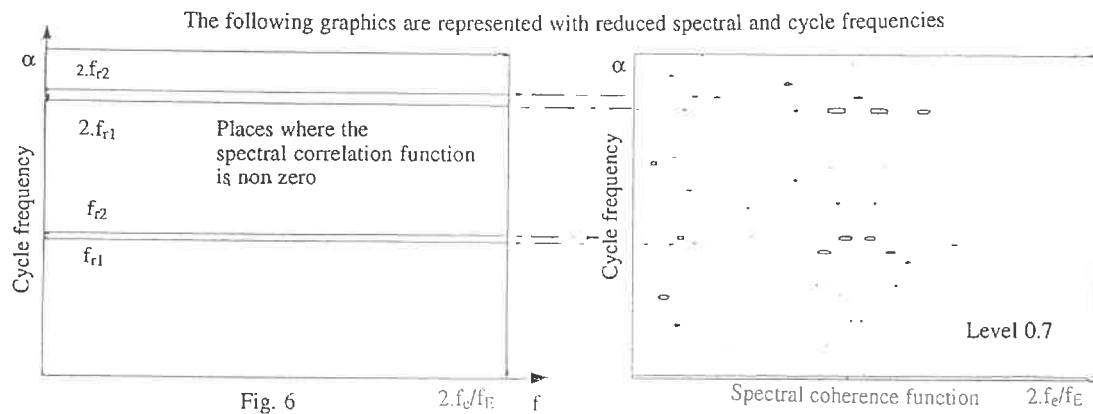
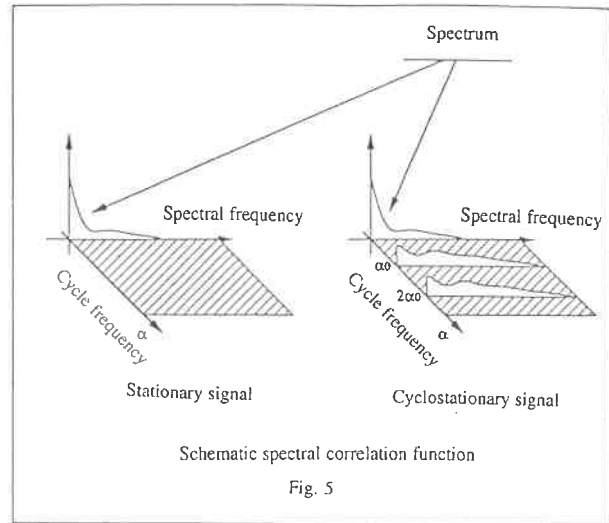
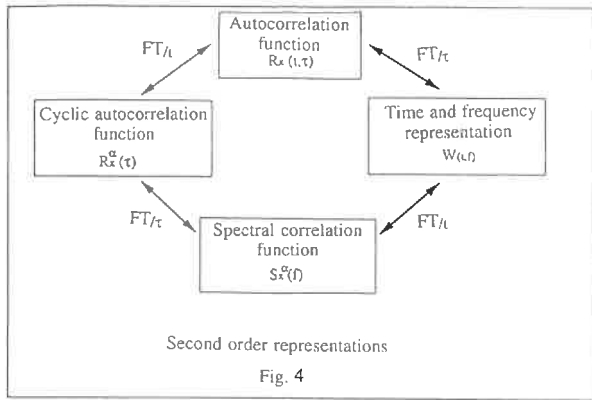


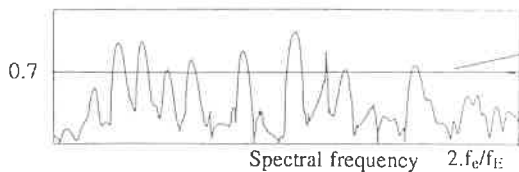
Fig. 3



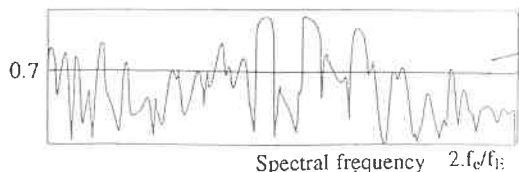
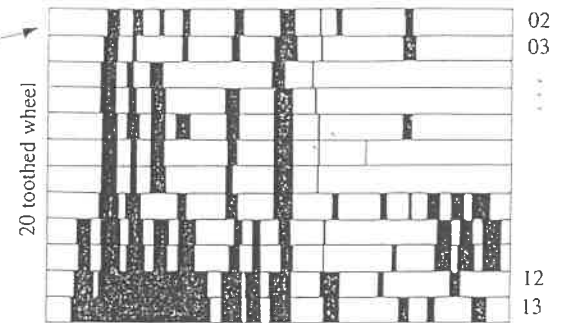
Logarithmic spectrum cycle frequency $\alpha = 0$
Second day of the experiment



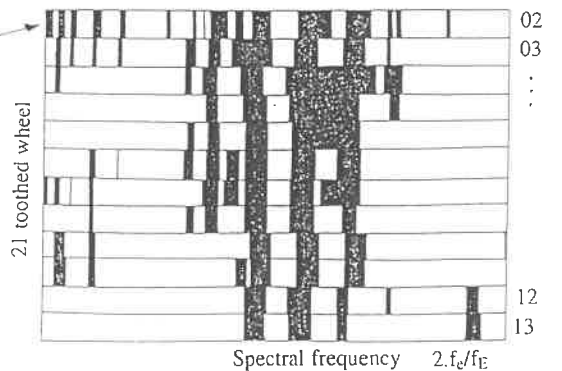
Logarithmic spectrum
Fig. 7



Value of the spectral coherence function for the cycle frequency $\alpha = 2f_{r20}$
Second day of the experiment



Value of the spectral coherence function for the cycle frequency $\alpha = 2f_{r21}$
Second day of the experiment



Spectral frequency $2f_e/f_{r1}$
Fig. 9