



**HAL**  
open science

# Perturbations of a rotating black hole in DHOST theories

Christos Charmousis, Marco Crisostomi, David Langlois, Karim Noui

► **To cite this version:**

Christos Charmousis, Marco Crisostomi, David Langlois, Karim Noui. Perturbations of a rotating black hole in DHOST theories. *Classical and Quantum Gravity*, 2019, 36 (23), pp.235008. 10.1088/1361-6382/ab4fb1 . hal-02188141

**HAL Id: hal-02188141**

**<https://hal.science/hal-02188141>**

Submitted on 20 Mar 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Perturbations of a rotating black hole in DHOST theories

Christos Charmousis,<sup>1</sup> Marco Crisostomi,<sup>1,2,3</sup> David Langlois,<sup>4</sup> and Karim Noui<sup>5,4</sup>

<sup>1</sup>Laboratoire de Physique Théorique, CNRS, Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France

<sup>2</sup>Institut de physique théorique, Univ. Paris Saclay, CEA, CNRS, 91191 Gif-sur-Yvette, France

<sup>3</sup>AIM, CEA, CNRS, Univ. Paris-Saclay, Univ. Paris Diderot, Sorbonne Paris Cité, F-91191 Gif-sur-Yvette, France

<sup>4</sup>AstroParticule et Cosmologie, UMR CNRS 7164, Université Paris-Diderot,

10 rue Alice Domon et Léonie Duquet, 75205 Paris, France

<sup>5</sup>Institut Denis Poisson, UMR CNRS 7013, Université de Tours,

Université d'Orléans, Parc de Grandmont, 37200 Tours, France

(Dated: July 8, 2019)

We study linear perturbations of a rotating black hole solution that has been recently discovered in degenerate higher-order scalar-tensor (DHOST) theories. We find a parametrization which permits the explicit resolution of the scalar perturbation while the tensor perturbation is obtained as a Teukolsky equation supplemented by an effective source term. The effective source term is related to the black hole hair and can be computed exactly for any value of the black hole spin. We discuss how the perturbations of the geometry and thus the emitted gravitational waves could be modified in comparison with general relativity.

## I. INTRODUCTION

The ability to detect gravitational waves (GW) has opened an extraordinary new window for cosmology and astrophysics. It also offers the possibility of testing General Relativity (GR) directly and in the presence of very strong gravitational fields. From the latter perspective, the ringdown phase of a black hole merger is particularly interesting. Indeed, with the next generation of GW interferometers (such as the Einstein Telescope and LISA) it will be possible to measure multiple quasi-normal modes (QNM) emitted by the newly formed black hole and perform what is called *black hole spectroscopy* [1]. This will permit to investigate potential deviations from the well-known QNM in GR [2–5] (see [6–8] for reviews) and will provide a crucial test for gravity.

To fully exploit this new possibility, it is useful to explore what type of modifications could be expected from black holes in various models of modified gravity. Several no-hair theorems [9, 10] state that, under certain hypotheses, the only modification that can be obtained is at the level of perturbations [11], while the black hole solution is indistinguishable from GR. However, if some of the hypotheses are relaxed, differences can enter already at the background level: the black hole geometry can differ from Kerr [12], or a Kerr metric can be dressed with some non-trivial field – or “hair”, in which case we are in presence of the so called “stealth” solutions.

Several works have analysed perturbations and QNM of spherically symmetric black holes (with and without hair) in alternative theories of gravity, e.g. [13–18]. However, only few articles<sup>1</sup> deal with rotating black holes, one of the main reasons being the mere paucity of non-trivial rotating solutions in theories of modified gravity.

In the present work we consider a stealth rotating black hole solution obtained very recently [23] in the context of Degenerate Higher Order Scalar Tensor (DHOST) theories [24–26],

which represent the most general class of covariant scalar-tensor theories with a single scalar degree of freedom (see [27] for a review). This black hole solution is characterised by an exact Kerr (or, more generally, Kerr-de Sitter) geometry and a non-trivial scalar field which can be identified with families of geodesics of the spacetime itself<sup>2</sup>. For simplicity, in this paper we restrict ourselves to the solution of [23] with no effective cosmological constant.

In the present work, we study the linear perturbations about this hairy black hole solution and obtain the tensor and scalar perturbation equation in a relatively simple form, so that one can disentangle the scalar and tensor perturbations. Remarkably, the tensor equation differs from the Teukolsky equation in GR only by the presence of an effective source term that depends on the “hairy” scalar perturbation, which can be explicitly solved.

## II. PERTURBATIONS OF KERR BLACK HOLES IN GR

Before computing the relevant equations in our theory, it is useful to recall the main steps for the computation of gravitational perturbations of a Kerr black hole in general relativity. This problem is far more involved than the case of a spherically symmetric black hole, due to the complexity of the geometry. In Boyer-Lindquist coordinates, the Kerr metric reads

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a \sin^2 \theta d\phi)^2 + \rho^2 \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{\sin^2 \theta}{\rho^2} (adt - (r^2 + a^2)d\phi)^2, \quad (1)$$

where

$$\Delta \equiv r^2 + a^2 - 2Mr, \quad \rho^2 \equiv r^2 + a^2 \cos^2 \theta. \quad (2)$$

<sup>1</sup> See [19] for Einstein-Maxwell theory, [20] for recent results in  $f(R)$  theory and [21, 22] for slowly rotating black holes without hair in Horndeski theories

<sup>2</sup> The spherically symmetric versions of [23] were initially found for Horndeski theory in [28], whilst their disformed DHOST version and stability was investigated in [29] and [30]. A recent comprehensive review of hairy black hole solutions in scalar tensor gravity can be found in [31].

$M$  is the mass and  $a$  the angular momentum parameter ( $a \leq M$ ) of the black hole. Kerr's solution has an inner ( $r = r_I$ ) and an outer ( $r = r_H$ ) event horizon, corresponding to the roots of  $\Delta = 0$ .

Taking into account the fact that the background solution is stationary and axisymmetric, one can decompose the perturbations into modes of the form

$$\psi(r, \theta) e^{-i\omega t + im\phi}, \quad (3)$$

where  $m$  is an integer and  $\omega$  is the frequency which turns out to be complex because of the damping of the modes due to gravitational radiation;  $\psi$  is a function (parametrized by  $\omega$  and  $m$ ) of the radial and angular coordinates.

In the case of spherical symmetry, the perturbation equations, after decomposition into spherical harmonics, reduce to ordinary differential equations along the radial direction. By contrast, in the Kerr case one ends up with partial differential equations that depend on both  $r$  and  $\theta$ . Remarkably, as shown by Teukosky [2] upon using the Newman-Penrose formalism [32], the perturbation equations can be expressed in a separable form. Indeed, they are of the form

$$\mathcal{O}(\omega, m)\psi = 0, \quad (4)$$

where the second order differential operator  $\mathcal{O}(\omega, m)$  can be written as

$$\mathcal{O}(\omega, m) = \mathcal{O}_r(\omega, m) + \mathcal{O}_\theta(\omega, m), \quad (5)$$

where  $\mathcal{O}_r$  and  $\mathcal{O}_\theta$  are second order differential operators involving respectively the variables  $r$  and  $\theta$  only. As a consequence  $\psi$  can be written as a (sum of) products  $\psi(r, \theta) = R(r)S(\theta)$  where  $R$  and  $S$  satisfy separately ordinary differential equations. The details and the explicit equations can be found in the original Teukosky paper [2].

The separability property renders the calculation of quasinormal modes much more tractable. In this paper, we show that the equations for the tensor perturbations in DHOST theories that are known to admit a hairy Kerr solution remain separable. More precisely, these equations are exactly given by the classical equation (4) where now an effective ‘‘source’’ term appears and depends on the scalar perturbation, thus on the hair of the black hole solution. Remarkably, we find a parametrization of the modified scalar-tensor perturbations which allow us to compute the source term.

### III. DHOST THEORIES AND STEALTH KERR SOLUTION

Let us now consider DHOST theories, which represent the most general family of covariant scalar-tensor theories propagating a single scalar degree of freedom. We restrict our discussion to a subclass of DHOST theories which are shift and reflection symmetric ( $\phi \rightarrow \phi + c$ ,  $\phi \rightarrow -\phi$ ) and whose tensor perturbations propagate at the speed of light [33–36]. Their Lagrangian can be written in the form

$$\mathcal{L} = K(X) + G(X)R + A_3(X)L_3 + A_4(X)L_4 + A_5(X)L_5, \quad (6)$$

where  $G$ ,  $K$  and  $A_I$  are functions of  $X \equiv \phi_\mu \phi^\mu$ , and the Lagrangians  $L_I$  are defined by

$$L_3 \equiv \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi, \quad L_4 \equiv \phi^\mu \phi_{\mu\rho} \phi_\nu \phi^{\nu\rho}, \quad L_5 \equiv (\phi^\mu \phi^\nu \phi_{\mu\nu})^2.$$

In our simplified notation, upper or lower indices on  $\phi$  correspond to (covariant) derivatives, e.g.  $\phi^\mu = \nabla^\mu \phi$  and  $\phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi$ . Due to the degeneracy conditions, which guarantee the presence of a single scalar degree of freedom,  $A_4$  and  $A_5$  are not free but depend on  $G$  and  $A_3$  as follows [24]

$$A_4 = -A_3 + \frac{1}{8G} (48G_X^2 + 8A_3 G_X X - A_3^2 X^2), \quad (7)$$

$$A_5 = \frac{A_3}{2G} (4G_X + A_3 X). \quad (8)$$

Furthermore, without loss of generality, since for our purposes we can consider the theory in vacuum, we can set  $G = 1$  by means of an  $X$ -dependent conformal transformation [25, 37].

In this subclass of theories, it is possible to construct stealth rotating black hole solutions, where the geometry is exactly Kerr (or Kerr-de Sitter if one adds an effective cosmological constant) and the scalar field  $\phi$  is non-trivial [23]. These solutions are characterised by a constant value of the kinetic term<sup>3</sup>  $X = X_0 = -\mu^2$ , and require that the following conditions are satisfied for the two independent functions  $A_3$  and  $K$  in the Lagrangian (6) (while  $G = 1$ ):

$$A_3(X_0) = 0, \quad K(X_0) = 0, \quad K_X(X_0) = 0. \quad (9)$$

Note that this implies, according to the degeneracy conditions (7-8), that  $A_4(X_0) = 0$  and  $A_5(X_0) = 0$  as well.

The scalar field  $\phi$  can be obtained by exploiting the analogy with families of Kerr geodesics [38] and reads

$$\phi(t, r) = -\mu t + \varepsilon \mu \int \frac{\sqrt{R(r)}}{\Delta(r)} dr, \quad (10)$$

where

$$R(r) \equiv 2Mr(r^2 + a^2), \quad (11)$$

and  $\varepsilon$  can take the values  $\pm 1$ , corresponding to the two branches of the square root<sup>4</sup> in (10).

### IV. EQUATIONS FOR THE PERTURBATIONS

We now expand the equations of motion for the scalar field and the metric to first order in perturbations, using

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad \phi = \bar{\phi} + \delta\phi, \quad (12)$$

<sup>3</sup> Here we take  $\phi^\mu$  to be timelike, should a spacelike  $\phi^\mu$  be required, substitute  $\mu^2 \rightarrow -\mu^2$  in what follows.

<sup>4</sup> Notice that, in absence of an effective cosmological constant (i.e. Kerr and not Kerr-dS metric), it is impossible to realise the merging of branches that characterises the solutions in [23] and that provides a finite scalar field at both the event and cosmological horizons. We leave the study of perturbations around such solutions for future work.

where barred quantities refer to the background whereas  $\delta g_{\mu\nu}$  and  $\delta\phi$  are perturbations. Interestingly, the leading order terms in the expansion of the equations of motion simplify drastically, and one obtains, after straightforward calculations, the equations

$$\bar{\nabla}_\mu \left( \Xi \bar{\phi}^\mu \delta X \right) = 0, \quad (13)$$

$$\delta G_{\mu\nu} = \delta T_{\mu\nu} \equiv \frac{1}{2} \Xi \bar{\phi}_\mu \bar{\phi}_\nu \delta X, \quad (14)$$

where  $\delta G_{\mu\nu}$  is the linearised Einstein tensor,  $\delta T_{\mu\nu}$  is the effective source term associated with the scalar field and  $\delta X$  is the first order perturbation of the kinetic term. We have also introduced the notation

$$\Xi \equiv A_{3X}(X_0) \mathcal{E}_3 - 2K_{XX}(X_0), \quad (15)$$

with

$$\mathcal{E}_3 \equiv (\bar{\square}\bar{\phi})^2 - (\bar{\phi}_{\mu\nu})^2. \quad (16)$$

When one replaces the background metric by the Kerr solution (1) and the background scalar field by the expression (10), the function  $\Xi$  becomes

$$\Xi = 2M\mu^2 a^2 \frac{3\cos^2\theta + 1}{\rho^6} A_{3X}(X_0) - 2K_{XX}(X_0). \quad (17)$$

At this stage, let us make two comments. First, the equations for the perturbations do not involve the Lagrangians  $L_4$  and  $L_5$  in (6). Indeed, since the functions  $A_3, A_4$  and  $A_5$  vanish on the background, the quadratic expansion of the last three terms in the Lagrangian (6) is given by

$$\delta X [A_{3X}(X_0)\delta L_3 + A_{4X}(X_0)\delta L_4 + A_{5X}(X_0)\delta L_5], \quad (18)$$

where  $\delta L_I$  is the first order perturbation of Lagrangian  $L_I$ . The latter can be easily rewritten as

$$L_3 = \frac{1}{2} \square\phi\phi^\mu\partial_\mu X, \quad L_4 = \frac{1}{4} \partial_\mu X \partial^\mu X, \quad L_5 = \frac{1}{4} (\phi\phi^\mu\partial_\mu X)^2,$$

hence, we see immediately that  $\delta L_4 = \delta L_5 = 0$  at linear order. As a consequence, the quadratic Lagrangian (18) reduces to

$$\frac{1}{4} A_{3X}(X_0) \bar{\square}\bar{\phi}\bar{\phi}^\mu\partial_\mu(\delta X^2). \quad (19)$$

Second, we see that using the variable  $\delta X$ , which is related to the original perturbations (12) through the equation

$$\delta X = 2\bar{\phi}^\mu\partial_\mu(\delta\phi) - \bar{\phi}^\mu\bar{\phi}^\nu\delta g_{\mu\nu}, \quad (20)$$

considerably simplifies the dynamics. Indeed,  $\delta X$  is totally decoupled from the Einstein tensor perturbations and, as we are going to see in the next section, its equation can be solved explicitly. As a consequence, the equations for the tensor perturbations reduce to the linearized Einstein equations supplemented with a source term which depends on  $\delta X$ . Therefore, the Teukolsky equations (4) for the Newman-Penrose coefficients  $\psi$  is exactly the same as in general relativity with a source term that can be explicitly computed.

## V. SOLUTION FOR THE SCALAR KINETIC DENSITY PERTURBATION $\delta X$

We now solve equation (13) for  $\delta X$ . This equation is first order in  $\delta X$  and therefore can be easily integrated.

First, we write it as follows

$$\partial_\mu (\sqrt{-g} \Xi \phi^\mu \delta X) = 0, \quad (21)$$

where  $\Xi$  is given by (17), and the determinant of the metric  $g$  and the non-vanishing components of  $\phi^\mu$  are given by

$$\sqrt{-g} = \rho^2 \sin\theta, \quad (22)$$

$$\phi^t = \frac{\mu}{\Delta\rho^2} [(r^2 + a^2)^2 - a^2\Delta\sin^2\theta], \quad (23)$$

$$\phi^r = \varepsilon\mu \frac{\sqrt{R}}{\rho^2}, \quad \phi^\varphi = 2Ma\mu \frac{r}{\Delta\rho^2}. \quad (24)$$

As  $\phi$  does not depend on  $\theta$  (in the Kerr geometry), the component  $\phi^\theta = g^{\theta\theta}\phi_\theta$  vanishes. Substituting these expressions into (21) leads to the very simple form

$$\varepsilon\Delta\partial_r(\sqrt{R}\chi) + [(r^2 + a^2)^2 - a^2\Delta\sin^2\theta] \partial_t\chi + 2Mar\partial_\varphi\chi = 0,$$

where we have introduced the variable  $\chi \equiv \Xi\delta X$  for simplicity. We decompose the solution into modes

$$\chi = \sum_m \int d\omega \chi_{m,\omega}(r, \theta) e^{-i\omega t + im\varphi}, \quad (25)$$

and we easily obtain the general solution for each mode

$$\chi_{m,\omega}(r, \theta) = \frac{C_{m,\omega}(\theta)}{\sqrt{R(r)}} \exp [i\varepsilon (-\omega I(r) - \omega \sin^2\theta J(r) + mK(r))], \quad (26)$$

where  $C_{m,\omega}$  is, at this stage, an arbitrary function of  $\theta$  (parametrized by  $\omega$  and  $m$ ) and

$$I(r) \equiv - \int dr \frac{(r^2 + a^2)^2}{\Delta(r)\sqrt{R(r)}}, \quad (27)$$

$$J(r) \equiv \int dr \frac{a^2}{\sqrt{R(r)}}, \quad (28)$$

$$K(r) \equiv - \int dr \frac{2Mar}{\Delta(r)\sqrt{R(r)}}. \quad (29)$$

Thus, the components  $\delta X_{m,\omega}$  of the perturbation  $\delta X$  are immediately given by  $\delta X_{m,\omega} = \chi_{m,\omega}/\Xi$ .

## VI. BEHAVIOUR OF THE SOLUTION AT THE BOUNDARIES

Now, let us study the regularity of this solution. We start analyzing the behavior of the modes (26) when  $r$  approaches the horizon  $r_H$ , where  $\Delta$  vanishes. In this limit, one finds a divergence in the integrals  $I$  and  $K$ . However, the Boyer-Lindquist coordinates also become singular at the horizon and one must use, instead, well-behaved coordinates such as the ingoing

Eddington-Finkelstein-like coordinates for Kerr, which we denote  $\{v, r, \theta, \bar{\varphi}\}$ . The coordinate transformation is given by

$$v \equiv t + \int dr \frac{r^2 + a^2}{\Delta(r)}, \quad \bar{\varphi} \equiv \varphi + a \int \frac{dr}{\Delta(r)}. \quad (30)$$

As a consequence, the combination  $(t + \varepsilon I)$  which appears in (25) can be rewritten as

$$t + \varepsilon I = v - \int dr \frac{\rho_0^3 (\varepsilon \rho_0 + \sqrt{2Mr})}{\sqrt{2Mr}(\rho_0 - \sqrt{2Mr})(\rho_0 + \sqrt{2Mr})} \quad (31)$$

with  $\rho_0(r) \equiv (r^2 + a^2)^{1/2}$ , which shows that the singularity disappears for the branch  $\varepsilon = -1$ . Similarly, we have

$$\varphi + \varepsilon K = \bar{\varphi} - a \int dr \frac{\sqrt{2Mr} (\varepsilon \sqrt{2Mr} + \rho_0)}{(\rho_0 - \sqrt{2Mr})(\rho_0 + \sqrt{2Mr})} \quad (32)$$

and the singularity also disappears when  $\varepsilon = -1$ . This is in complete accord with the background solution as  $\varepsilon = -1$  corresponds to the branch where the scalar field is regular at the event horizon.

One can also examine the behavior of the modes (26) at large distances. Clearly, the functions  $J$  and  $K$  converge when  $r$  tends to infinity, whereas  $I$  diverges according to  $I(r) \sim -r^{3/2}$ . Taking into account that the frequency  $\omega$  contains an imaginary part, which depends on the typical damping time,  $\text{Im}(\omega) \equiv -1/\tau < 0$ , one finds that the mode is suppressed at large distances in the branch  $\varepsilon = -1$  (whereas it diverges for  $\varepsilon = 1$ ).

In summary, the explicit solution for  $\delta X$  that we have obtained appear well-behaved both at the horizon and at spatial infinity in the branch  $\varepsilon = -1$ . Note that the solution for each mode depends on an arbitrary function  $C_{m,\omega}$  of the angular variable  $\theta$ , which is in principle determined by the initial conditions.

## VII. DISCUSSION

In this work, we have studied the perturbations of a stealth rotating black hole solution in a subclass of DHOST theories. The background solution is characterized by a constant value of the scalar field kinetic term  $X$  and its perturbation  $\delta X$  fully describes how the tensor modes are modified in this model. Indeed we have found that the equations of motion for the perturbations of the geometry can be reformulated as linearized Einstein's equation with a source term proportional to  $\delta X \bar{\phi}_\mu \bar{\phi}_\nu$ , in contrast with the GR result. In parallel, the quantity  $\delta X$  obeys an equation that does not involve the tensor modes and can thus be solved independently. We have written the general solution of this equation as a superposition of modes and studied their behaviour at the black hole horizon and at spatial infinity, finding that they are well-behaved only for one branch of the background solutions.

Given a solution for  $\delta X$ , one can then compute the source term in the Teukolsky equation for the Newman-Penrose variables  $\psi$ . Contrary to general relativity, the equation for  $\psi$  is no longer homogeneous. The situation thus appears analogous to the simpler case of a vibrating string obeying a dissipative wave equation with a source term, i.e. of the form

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial t^2} - \frac{2}{\tau} \frac{\partial \psi}{\partial t} = S(x, t). \quad (33)$$

Assuming the boundary conditions  $\psi(0, t) = \psi(L, t) = 0$  (and for the source as well),  $\psi$  can be decomposed as

$$\psi(x, t) = \sum_n A_n \sin(n\pi x/L) e^{-i\omega_n t}, \quad (34)$$

where the complex frequencies  $\omega_n$ , are given by

$$\omega_n \tau \equiv -i \pm \sqrt{n^2 \frac{\pi^2 \tau^2}{L^2} - 1}. \quad (35)$$

These discrete  $\omega_n$ 's are analogous to the black hole QNM. In the absence of source, the full solution is given by (34) where the coefficients  $A_n$  are fixed by the initial conditions at  $t = 0$ .

When a source term is present, it is convenient to rewrite it in the form

$$S(x, t) = \sum_n \int d\omega \sin(n\pi x/L) e^{-i\omega t} \hat{S}(n, \omega), \quad (36)$$

then, the solution of the wave equation is formally given by

$$\psi(x, t) = - \sum_n \int d\omega \frac{\hat{S}(n, \omega) \sin(n\pi x/L) e^{-i\omega t}}{n^2 \pi^2 / L^2 - \omega^2 - 2i\omega/\tau}. \quad (37)$$

Such an analysis for the QNMs of a GR Kerr black hole in presence of a source term due to some matter around the black hole, can be found in [39].

In the specific case of the stealth black hole, one could proceed along the same lines. This would however require a more precise description of the source term, i.e. identify which of the solutions for  $\delta X$  are physically relevant and how they can be generated in some physical process, analysis which goes beyond the scope of the present work.

To conclude, our analysis is a first direct attempt to tackle perturbations of non trivial hairy rotating black holes. It is still not clear whether this model can be seen as a viable alternative to GR or needs to be further restricted. There could also be some issues concerning the validity of hairy stealth solutions from an effective field theory point of view (see e.g. discussions in [30] and very recently [40]). These aspects should be explored further in the future (see for example [41]).

## ACKNOWLEDGEMENTS

We thank Stas Babak, Eugeny Babichev, Enrico Barausse, Brando Bellazzini, Pedro Ferreira, Shinji Mukohyama, Francesco Nitti, George Pappas and Nick Stergioulas for useful discussions. CC thanks the Laboratory of Astronomy of AUTH in Thessaloniki for hospitality. CC and KN acknowledge support from the CNRS project 80PRIME. MC is supported by the Labex P2IO.



- 
- [1] E. Berti, A. Sesana, E. Barausse, V. Cardoso and K. Belczynski, *Spectroscopy of Kerr black holes with Earth- and space-based interferometers*, *Phys. Rev. Lett.* **117** (2016) 101102, [1605.09286].
- [2] S. A. Teukolsky, *Perturbations of a rotating black hole. I. Fundamental equations for gravitational electromagnetic and neutrino field perturbations*, *Astrophys. J.* **185** (1973) 635–647.
- [3] W. H. Press and S. A. Teukolsky, *Perturbations of a Rotating Black Hole. II. Dynamical Stability of the Kerr Metric*, *Astrophys. J.* **185** (1973) 649–674.
- [4] S. Chandrasekhar, *The mathematical theory of black holes*, in *Oxford, UK: Clarendon (1992) 646 p.*, Oxford, UK: Clarendon (1985) 646 P., 1985.
- [5] E. W. Leaver, *An Analytic representation for the quasi normal modes of Kerr black holes*, *Proc. Roy. Soc. Lond.* **A402** (1985) 285–298.
- [6] H.-P. Nollert, *Topical Review: Quasinormal modes: the characteristic ‘sound’ of black holes and neutron stars*, *Class. Quant. Grav.* **16** (1999) R159–R216.
- [7] K. D. Kokkotas and B. G. Schmidt, *Quasinormal modes of stars and black holes*, *Living Rev. Rel.* **2** (1999) 2, [gr-qc/9909058].
- [8] E. Berti, V. Cardoso and A. O. Starinets, *Quasinormal modes of black holes and black branes*, *Class. Quant. Grav.* **26** (2009) 163001, [0905.2975].
- [9] J. D. Bekenstein, *Novel “no-scalar-hair” theorem for black holes*, *Phys. Rev. D* **51** (1995) R6608.
- [10] L. Hui and A. Nicolis, *No-Hair Theorem for the Galileon*, *Phys. Rev. Lett.* **110** (2013) 241104, [1202.1296].
- [11] E. Barausse and T. P. Sotiriou, *Perturbed Kerr Black Holes can probe deviations from General Relativity*, *Phys. Rev. Lett.* **101** (2008) 099001, [0803.3433].
- [12] C. A. R. Herdeiro and E. Radu, *Kerr black holes with scalar hair*, *Phys. Rev. Lett.* **112** (2014) 221101, [1403.2757].
- [13] T. Kobayashi, H. Motohashi and T. Suyama, *Black hole perturbation in the most general scalar-tensor theory with second-order field equations I: the odd-parity sector*, *Phys. Rev. D* **85** (2012) 084025, [1202.4893].
- [14] T. Kobayashi, H. Motohashi and T. Suyama, *Black hole perturbation in the most general scalar-tensor theory with second-order field equations II: the even-parity sector*, *Phys. Rev. D* **89** (2014) 084042, [1402.6740].
- [15] O. J. Tattersall, P. G. Ferreira and M. Lagos, *General theories of linear gravitational perturbations to a Schwarzschild Black Hole*, *Phys. Rev. D* **97** (2018) 044021, [1711.01992].
- [16] O. J. Tattersall and P. G. Ferreira, *Quasinormal modes of black holes in Horndeski gravity*, *Phys. Rev. D* **97** (2018) 104047, [1804.08950].
- [17] G. Franciolini, L. Hui, R. Penco, L. Santoni and E. Trincherini, *Effective Field Theory of Black Hole Quasinormal Modes in Scalar-Tensor Theories*, *JHEP* **02** (2019) 127, [1810.07706].
- [18] K. Takahashi, H. Motohashi and M. Minamitsuji, *Linear stability analysis of hairy black holes in quadratic degenerate higher-order scalar-tensor theories: Odd-parity perturbations*, [1904.03554].
- [19] P. Pani, E. Berti and L. Gualtieri, *Gravitoelectromagnetic Perturbations of Kerr-Newman Black Holes: Stability and Isospectrality in the Slow-Rotation Limit*, *Phys. Rev. Lett.* **110** (2013) 241103, [1304.1160].
- [20] A. G. Suvorov, *Gravitational perturbations of a Kerr black hole in  $f(R)$  gravity*, [1905.02021].
- [21] O. J. Tattersall, *Kerr-(anti-)de Sitter black holes: Perturbations and quasinormal modes in the slow rotation limit*, *Phys. Rev. D* **98** (2018) 104013, [1808.10758].
- [22] O. J. Tattersall and P. G. Ferreira, *Forecasts for Low Spin Black Hole Spectroscopy in Horndeski Gravity*, *Phys. Rev. D* **99** (2019) 104082, [1904.05112].
- [23] C. Charmousis, M. Crisostomi, R. Gregory and N. Stergioulas, *Rotating Black Holes in Higher Order Gravity*, [1903.05519].
- [24] D. Langlois and K. Noui, *Degenerate higher derivative theories beyond Horndeski: evading the Ostrogradski instability*, *JCAP* **1602** (2016) 034, [1510.06930].
- [25] M. Crisostomi, K. Koyama and G. Tasinato, *Extended Scalar-Tensor Theories of Gravity*, *JCAP* **1604** (2016) 044, [1602.03119].
- [26] J. Ben Achour, M. Crisostomi, K. Koyama, D. Langlois, K. Noui and G. Tasinato, *Degenerate higher order scalar-tensor theories beyond Horndeski up to cubic order*, *JHEP* **12** (2016) 100, [1608.08135].
- [27] D. Langlois, *Dark energy and modified gravity in degenerate higher-order scalar-tensor (DHOST) theories: A review*, *Int. J. Mod. Phys. D* **28** (2019) 1942006, [1811.06271].
- [28] E. Babichev and C. Charmousis, *Dressing a black hole with a time-dependent Galileon*, *JHEP* **08** (2014) 106, [1312.3204].
- [29] E. Babichev, C. Charmousis, G. Esposito-Farèse and A. Lehbel, *Stability of Black Holes and the Speed of Gravitational Waves within Self-Tuning Cosmological Models*, *Phys. Rev. Lett.* **120** (2018) 241101, [1712.04398].
- [30] E. Babichev, C. Charmousis, G. Esposito-Farèse and A. Lehbel, *Hamiltonian unboundedness vs stability with an application to Horndeski theory*, *Phys. Rev. D* **98** (2018) 104050, [1803.11444].
- [31] A. Lehbel, *Compact astrophysical objects in modified gravity*. PhD thesis, Orsay, 2018. 1810.04434.
- [32] E. Newman and R. Penrose, *An Approach to gravitational radiation by a method of spin coefficients*, *J. Math. Phys.* **3** (1962) 566–578.
- [33] J. M. Ezquiaga and M. Zumalacregui, *Dark Energy After GW170817: Dead Ends and the Road Ahead*, *Phys. Rev. Lett.* **119** (2017) 251304, [1710.05901].
- [34] P. Creminelli and F. Vernizzi, *Dark Energy after GW170817 and GRB170817A*, *Phys. Rev. Lett.* **119** (2017) 251302, [1710.05877].
- [35] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller and I. Sawicki, *Strong constraints on cosmological gravity from GW170817 and GRB 170817A*, *Phys. Rev. Lett.* **119** (2017) 251301, [1710.06394].
- [36] J. Sakstein and B. Jain, *Implications of the Neutron Star Merger GW170817 for Cosmological Scalar-Tensor Theories*, *Phys. Rev. Lett.* **119** (2017) 251303, [1710.05893].
- [37] J. Ben Achour, D. Langlois and K. Noui, *Degenerate higher order scalar-tensor theories beyond Horndeski and disformal transformations*, *Phys. Rev. D* **93** (2016) 124005, [1602.08398].
- [38] B. Carter, *Global structure of the Kerr family of gravitational fields*, *Phys. Rev.* **174** (1968) 1559–1571.
- [39] S. L. Detweiler, *Resonant oscillations of a rapidly rotating black hole*, *Proc. Roy. Soc. Lond.* **A352** (1977) 381–395.

- [40] C. de Rham and J. Zhang, *Perturbations of Stealth Black Holes in DHOST Theories*, [1907.00699].
- [41] S. Endlich, A. Nicolis, R. Rattazzi and J. Wang, *The Quantum mechanics of perfect fluids*, *JHEP* **04** (2011) 102, [1011.6396].