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# Comparing fractal indices of electric networks to roads and buildings: the case of Grenoble (France)

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## Abstract

We compare the fractal indices of the power grid of the city of Grenoble (France) to those computed on the spatial organization of the buildings and the road infrastructure. We assess the fractal dimensions and the curves of scaling behavior and conduct a concordance analysis. We study different districts within the city and compare the power and road networks with the built-up patterns. We show the importance of using carefully data sets from different sources and how it could sometimes make major differences ahead of further use. We conclude on the optimality of the spatial coverage of the distribution network in Grenoble

*Keywords:* fractals, power grids, data cleaning, Grenoble

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## 1. Introduction

Urban infrastructure analysis has shifted recently from the original considerations about housing and urban segregation to a more systemic view of new facilities and utilities taking into consideration urban metabolism and vulnerabilities, smart cities, communication networks and urban networks (water, roads,..)[1]. Complex relationships between the urban structure and daily mobility were investigated and scrutinized in the literature [2-5].

Power systems are a key infrastructure of smart cities. They are supposed to become in the future more and more scaling because they are made of recursive assembly of active devices, smart buildings, micro-grids, district grids ... Studying the relationships of the power grid with related networks within this urban structure is getting more attention as part of planning more sustainable, energy efficient future cities[6-8].

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Classical approaches used to investigate power systems are mono-scale; hence they do not allow to comprehend complex systems with structural elements often belonging to different scales. Understanding this complexity helps design flexible and resilient architectures for the optimization of smart grids operations. This is a major challenge to increase efficiency and to avoid or better manage random breakdowns.

Moreover, at an urban level, power networks provide energy access to buildings. Their spatial development should thus be correlated to built-up patterns. We may as well expect that power networks go through existing corridors, which means here the street networks. Therefore, it seems interesting to explore to what extent the current power grid fits the existing built-up spaces and road network. This will lead to a better perception of how the current power grid spatial coverage is with regard to the road network. These results would ultimately be used to propose a reconfiguration of the existing urban structures but and also a new architecture for future planning of urban districts.

Traditional models such as complex network theory [9], stochastic geometry or random graph do not consider geometrical, functional and dynamical aspects of a city and its associated networks at the same time [10]. Hence, we carry out a fractal-based approach to analyze the properties of power systems and understand their organization across scales. To show the usefulness of our approach, results are shown for Grenoble's Medium Voltage network. We will focus on the structural concordance between the power grid, the road network and the buildings.

We use in this paper geographic information system files from both the local distribution system operator in Grenoble (power system data) and the National Institute of Geographic and Forest Information (Roads and buildings). However, databases refer often to non-geographical context, e.g. parallel power lines running in the same road space are spatially separated by an arbitrary distance which has no actual meaning. Hence, to have a non-biased analysis, we look first into the data sets and clean the mis-referenced data out. Then we identify the consequences of this correction on the fractal analysis of the networks. The analysis is first performed at a city level and afterwards, we proceed with a local investigation of the fractal behavior of three different areas in the city. The analysis results are used to carry out different concordance analysis: power grid vs roads, power grids vs buildings and roads vs buildings and to conclude on the optimality of the spatial coverage of the distribution network in Grenoble.

## **2. Theoretical background**

### *2.1. Introduction to distribution power systems*

The power system has three levels starting from the main transmission system and interconnectors, which carry large quantities of electricity at 400 kV or 225 kV over long distances with a low levels of losses (sometimes called “energy highways”). The energy is delivered afterwards to regional sub-transmission networks, which distribute energy within regions and supply electricity to the public distribution networks and large

industrial consumers. The voltage range goes from 225 to 63 kV. 20 kV and 400 V distribution networks come last and supply electricity to end consumers at medium voltage (small businesses) or low voltage (households, tertiary sector, light industry) [11].

The French electricity network is managed by 8 operators: RTE for long-distance electricity transmission, and seven electricity distribution system operators (DSO) which serve over 100,000 customers. The main DSO is Enedis, which serves 95% of continental metropolitan France. Besides Enedis, there are six local operators: SER (City of Strasbourg); URM (City of Metz); Gérédis (French department of Deux Sèvres); SRD (City of Vienne); GEG (City of Grenoble); EDF's insular power systems department (EDF SEI), which serves Corsica and most of France's overseas territories.

The coherence between the distribution grid and urban territory is still to be investigated and has been raising more interest due to the needs for optimal solutions to deliver power to urban areas [12]. The coverage of the urban space by the power grid results from the objective of delivering power at any point of the city while lowering the cost and maximizing the robustness. The indices of the concordance between the power grids and urban systems will give us a metric to assess how optimal the system is.

## 2.2. Why using fractality to analyze the power grid?

The fractal concept was introduced in the 1960s when B. B. Mandelbrot published "How Long Is the Coast of Britain? Statistical Self-similarity and Fractional Dimension" and defined fractals as self-similar patterns where the fractal object replicates itself the same way at different scales.

Fractal geometry has been widely and rather successfully used for over twenty years in disciplines like meteorology, biology, physics, thermodynamics, art, history, philosophy of seismology but also in geography [13]. While considering urban fabrics, fractal analysis turned out to be a powerful instrument for exploring their spatial organization [14-20]. Public transportation networks were considered as well [21, 22] and showed a connection between both built-up spaces and street networks [23].

The fractal approach is geometrical, which makes it possible to study spatial phenomena either by using reference models or morphometric fractal measurements. By using fractal measurements, we can verify the existence of hierarchical scaling laws in spatial distribution. Being able to study a phenomenon throughout different scales provides the possibility of discovering thresholds or breaks within spatial organization. A morphological classification of networks becomes possible as this approach highlights the internal organization which does not appear using other approaches.

Urban fabrics and related networks are usually not issued from any coherent planning process and show no obvious specific organization. However, they are deeply multiscale, reaching the metropolitan scale to that of buildings. Hence, using fractals seems to be an interesting way to characterize these forms and unravel the complexity of underlying layers, which is a step further than classical Euclidian approaches.

The main function of networks is to connect the urban fabrics to a service as optimally as possible. Their topology must ensure to get the service to as much of the urban fabric as possible, which could mean lengthening of paths. But in the meantime,

to minimize the overall length of the network, they are spatially organized according to a hierarchical system of ramifications. This reminds the basic properties of fractals [24].

Beyond current methods used in fractal analysis, we use a specific method introduced in [23] : the concordance analysis allowing a direct comparison across scales of different networks or of a network and a built-up space. This allows exploring how spatial organization follows the same logic across scales of both explored systems.

### 2.3. Fractal analysis

Let us remind the definition of the fractal dimension. Fractals are self-similar. By zooming in or out from the structure, we always find the same shape. This means, that while considering an arbitrarily chosen zone of size  $l$  of the structure, the number  $M$  of elements (e.g. points or network length lying within this zone) scales according to the power law

$$M(l) = l^{D_0} = M_0 \quad (1)$$

The fractal dimension  $D_0$  is defined as the exponent of this law. It characterizes the scaling property of the object over scales and is hence constant as far as self-similarity holds. To verify to what extent empirical structures follow fractal behavior, different analysis methods have been developed[25].

The box counting method is one of the most commonly used methods to measure the dimension of fractal objects. Boxes of variable size  $\varepsilon$  are used to cover the objects. For each value  $\varepsilon$  we count the number  $N(\varepsilon)$  of non empty boxes of side  $\varepsilon$  needed to cover the object of length  $L$ . For fractal structures the relation holds:  $N(\varepsilon) = a \varepsilon^{-D_0}$  where  $\varepsilon \rightarrow 0$ . Taking the logarithm of the relation yields  $\log N = \log a - D_0 \log \varepsilon$ ,  $a$  being the fractal measure of the object, which is often called the ‘shape prefactor’[20, 24]. Hence, by logarithmic transformation, we obtain a linear relationship where the slope value is the fractal dimension  $D_0$  [26, 27]. When assuming a discrete series of values  $\varepsilon_i$ , we obtain:

$$D_0 = \frac{\log N(\varepsilon_{i+1}) - \log N(\varepsilon_i)}{\log \varepsilon_{i+1} - \log \varepsilon_i} \quad (2)$$

The analysis is carried out using the software FracGis, developed in the Franche-Comté University (Théba research laboratory), which provides the fractal analysis using the box counting method as described above.

This method allows the minimum coverage of a structure by covering each occupied point of a square of size  $\varepsilon$ . However, if two points are located at a distance less than  $\varepsilon$ , only one box is kept. This method corresponds exactly to the claim of the minimum coverage. This is the main reason why it was preferred in this study. However, the reliability of this method could be questioned as shown in Fig 1. The coverage in the figure at the left shows 7/9 filled boxes while the minimum coverage is 2/9 filled boxes. This issue is considered in the software FracGis, as we can use a gliding coefficient to adjust the grid to the built-up areas and networks.

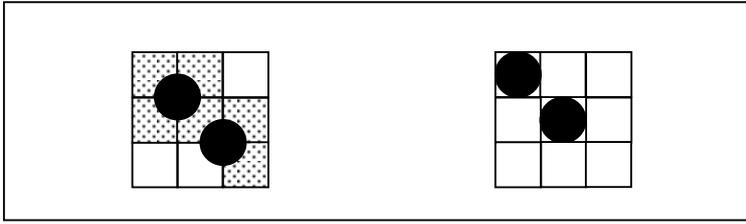


Fig 1: Illustration of the box counting method as used in FracGis (left: box counting method coverage / right: minimum coverage)

## 2.4. Scaling behavior

For a theoretical fractal, the fractal dimension  $D_0$  is strictly the same all over the scales. However, we can suppose that real world objects do not follow strictly a fractal law. To explore if deviations from the fractal law exist, we use the curve of scaling behavior, which turned out to be efficient [19, 26]. For this purpose, we compute for each value  $\epsilon_i$  the slope values in the double logarithmic representation, according to (2), and obtain then a local scaling exponent  $\alpha(\epsilon_i)$ . Obviously, it is constant and equal to  $D_0$  for theoretical fractals, what does not necessarily hold for empirical structures. Hence, by representing the sequence of these slopes as a function of the size  $\epsilon_i$ , we obtain the scaling behavior  $\alpha(\epsilon_i)$ , which variations informs us about changes in the fractal behavior of the analyzed structure across scales.

In order to establish a link between the analyzed objects, we compute the ratio of their scaling behavior (Eq 2). E.g. linking build-up space (*build*) to a network (*net*) yields:

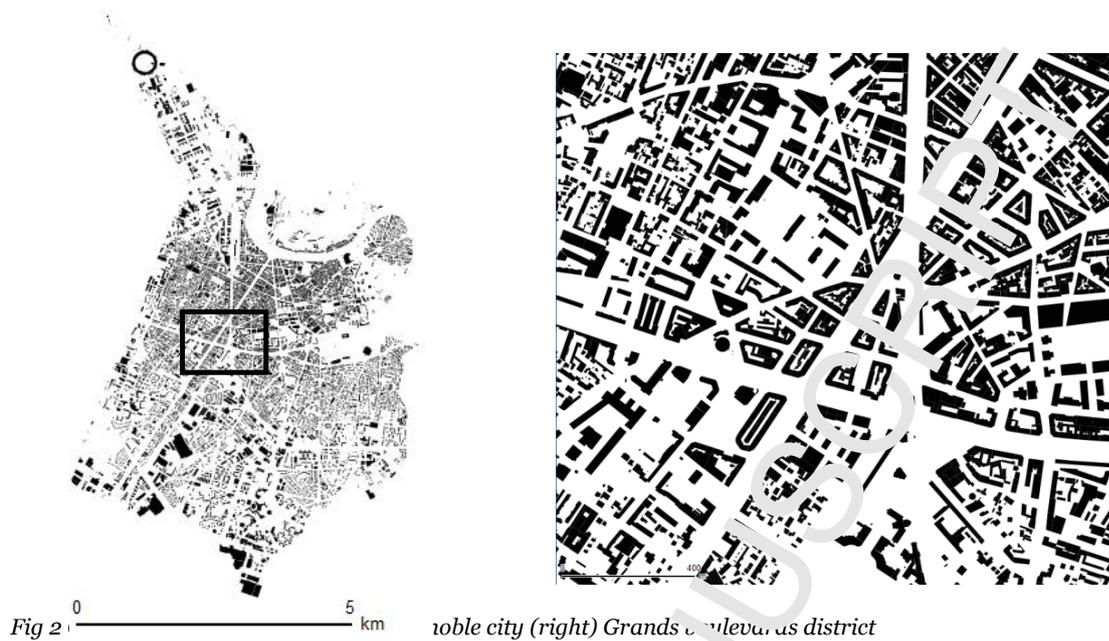
$$\beta_{build/net}(\alpha_i) = \frac{\alpha_{build}(\epsilon_i)}{\alpha_{net}(\epsilon_i)} = \frac{\log N_{build}(\epsilon_{i+1}) - \log N_{build}(\epsilon_i)}{\log N_{net}(\epsilon_{i+1}) - \log N_{net}(\epsilon_i)} \quad (3)$$

We call this type of representation the concordance analysis [23]. It shows to what extent both the objects follow the same logic of covering space at each scale. The  $\beta$ -values becomes equal to one if the logics are the same.

## 3. Data processing

### 3.1. Buildings

The buildings map of Grenoble is available in the Institut géographique national (IGN) open data. It includes all the buildings of the city (residential, industrial, educational. . .). This data set seems to be coherent with reality and no abhorrent elements were identified. No processing was involved while using the building data.



The fractal behavior observed in Fig 3 is typical for medium size European cities [23]: the decrease for small distances is due to the modeling of the buildings. Every building is represented by a surface element whose size is proportional to its footprint. Therefore, at very local scales, the box-counting method provides a dimension closed to 2. These scales refer to distances inferior to that of the size of buildings and the number of occupied sites is thus dominated by build-up footprints.

When the size of boxes increases slightly, the dimension decreases abruptly due to the appearance of the empty spaces around the building footprints, but other buildings do not yet come into play. For higher sizes of the boxes, the relative position of one building to the other buildings is taken into account and the dimension increases. This means exploring the urban pattern configuration. Curves tend to become more regular. Between 100 m and 400 m, the fractal dimension settles around 1.7, which indicates a scale invariant coverage of the buildings.

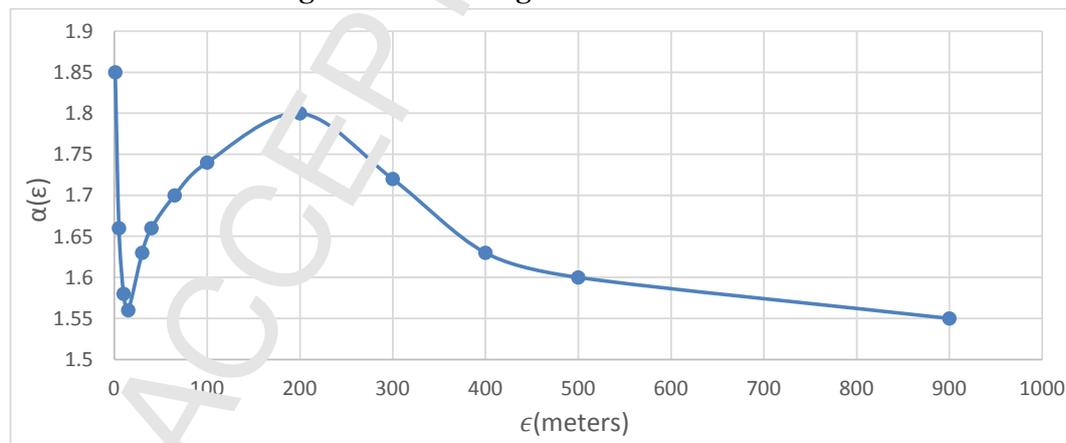


Fig 3: Fractal behavior of the built-up areas

### 3.2. Power grid

The Distribution System Operator (DSO) of the city of Grenoble in France provided us with the data we need for the fractal analysis of the power network and concordance analysis with roads and built-up areas. The data set at hand describes the medium voltage distribution system meaning that we know the shape and the consumption data at a street level. However, as shown in Fig 4, some power lines are roughly drawn. In some cases, a bundle of parallel cables is taking more space than it does in reality. As we are focusing on studying the shape of the network through its scaling behavior, parallel cables become irrelevant and are better represented using one single line.

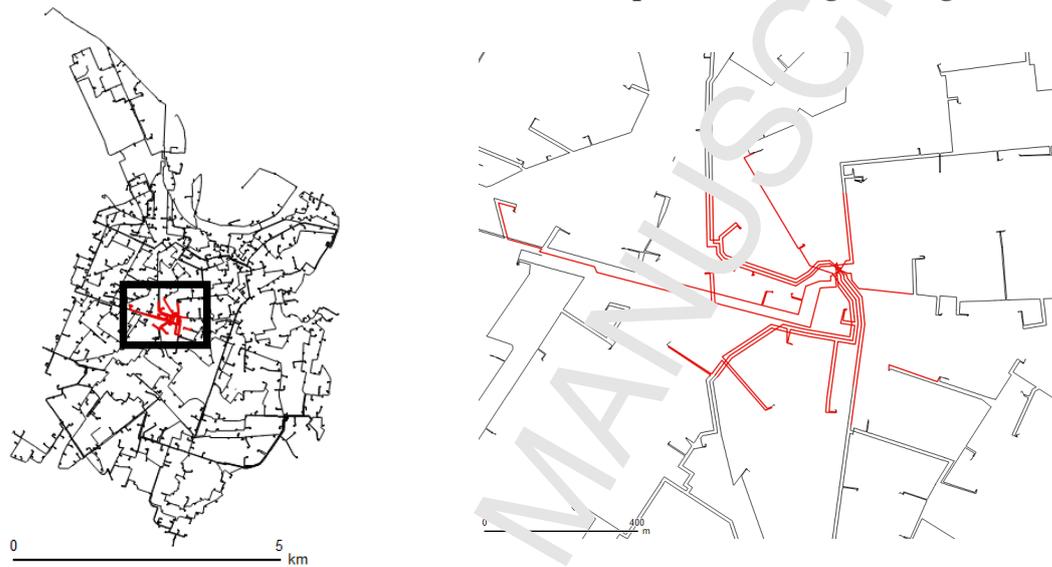


Fig 4: Grenoble's medium voltage power grid: (left) Grenoble city (right) Grands boulevards district

The process to correct the “measurement errors” introduced by parallel cables consisted in merging all the parallel power lines into one single line. As the data set of the city is rather small, the process was executed manually to be able to appreciate the more appropriate way to reshape the drawing.

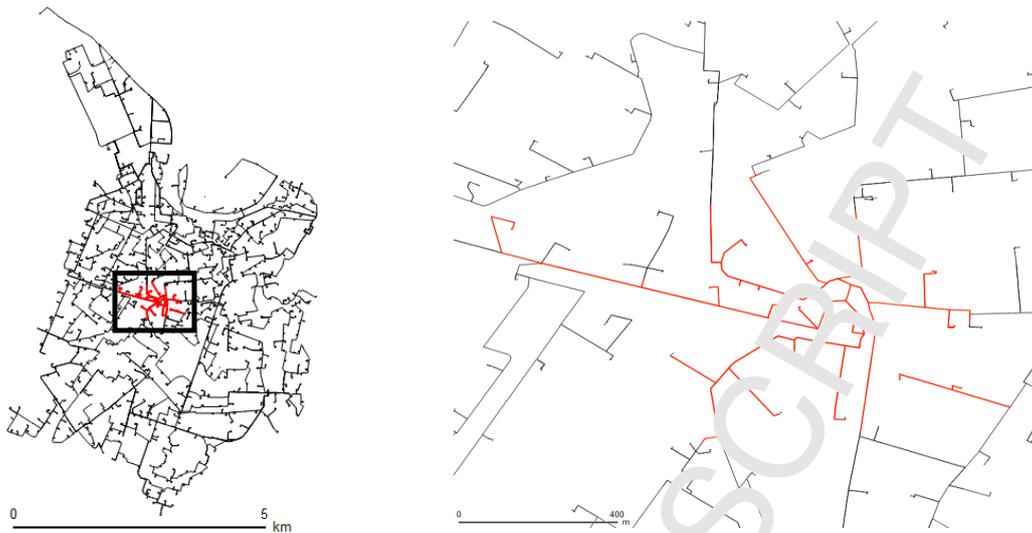


Fig 5: Grenoble's medium voltage power grid after processing: (left) Grenoble city (right) Grands boulevards district

To assess the effects of data correction, we analyze the fractal behavior of the power network of Grenoble under its original form and cleaned one.

In Fig 6, we observe the scaling behavior over a wide range of scales, from 0 to 900m, the radius of Grenoble being about 5 km. Over 100m, the correction has a small impact on the results. But we notice in the zoom shown in Fig 7 that the cleaning process changes dramatically the results at small scales for the power grid. For box sizes between 5 and 40m, the behavior is different as the data processing deals mostly with parallel cables whose distance are situated at this scale. The corrected data show a smoother transition at small scales meaning that the shape of the network is more homogeneous across scales. The merger of parallel cables decreases the scaling behavior between non-processed and processed data at very small scales as there is no more cables with sizes smaller than 20 m in between.

For the processed data, at very small scales, there is no bifurcation and the power network can be seen as nearly linear (scaling value is close to one). For higher values of  $\epsilon$ , the curve increases in a linear fashion until reaching a stable scaling behavior around 1.7. This is due to the appearance of bifurcations at this scale ensuring the necessary coverage of the distributed territory.

For these sizes, the box counting method is still relevant, the network being more uniform than the built-up areas. Fig 8 shows one example of network coverage using the box counting method.

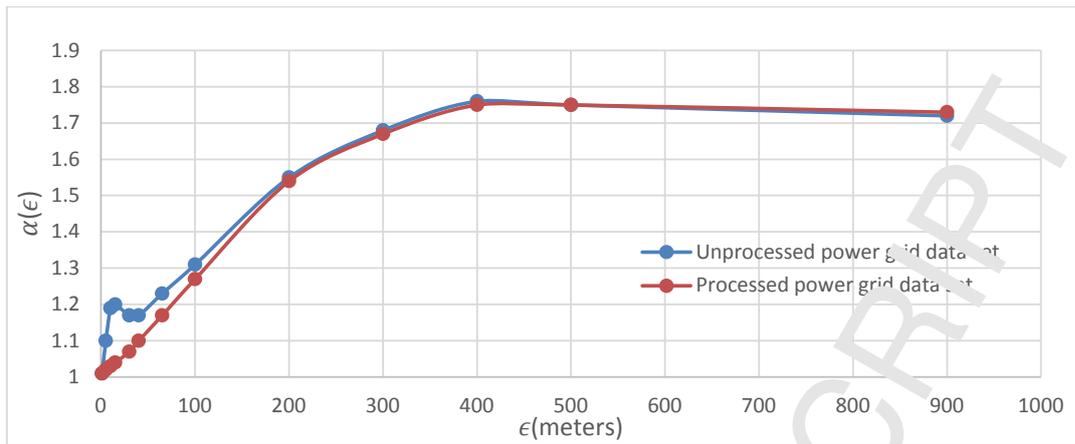


Fig 6: Overview of the fractal behavior of the power grids using unprocessed data versus processed data

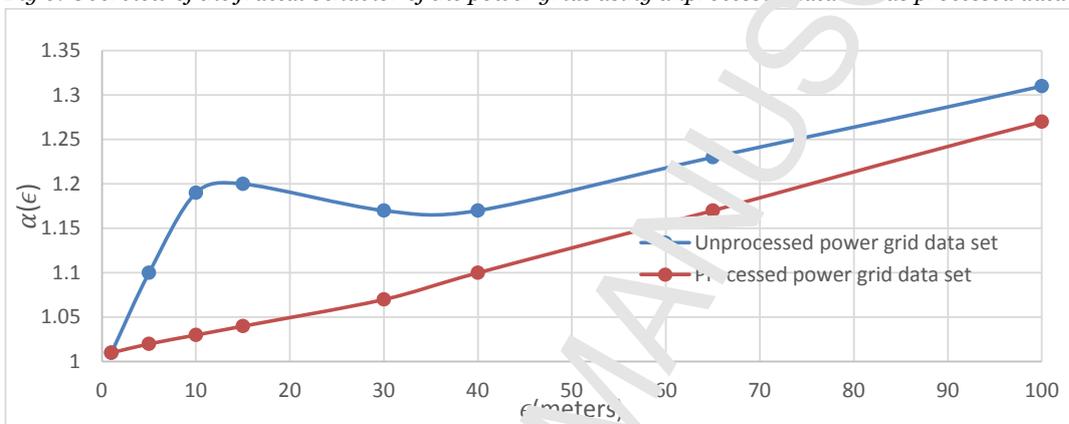


Fig 7: Small scales view of the Fractal behavior of the power grids using unprocessed data versus processed data

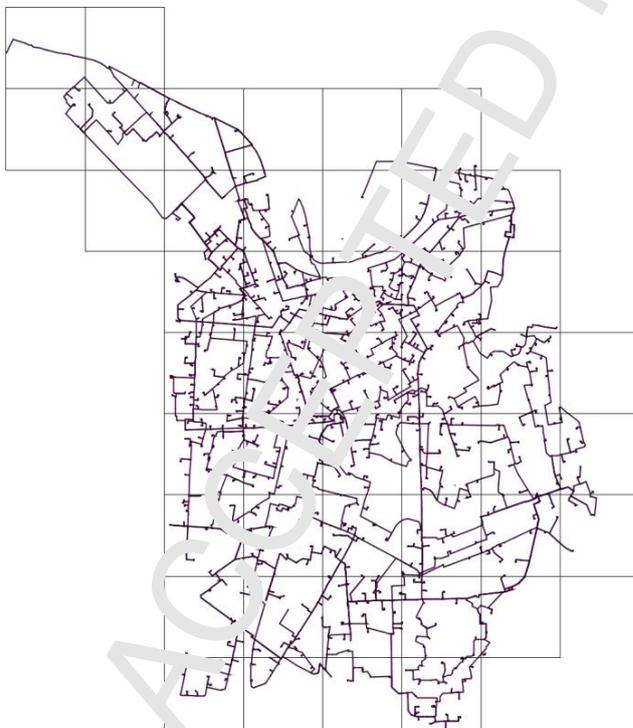


Fig 8: Example of network coverage with boxes of size 1024 m

### 3.3. Road network

The road network of Grenoble is also available in the IGN open data. The data consists of all the paths within the city including parking lots, railways and pedestrian paths in parcs ...

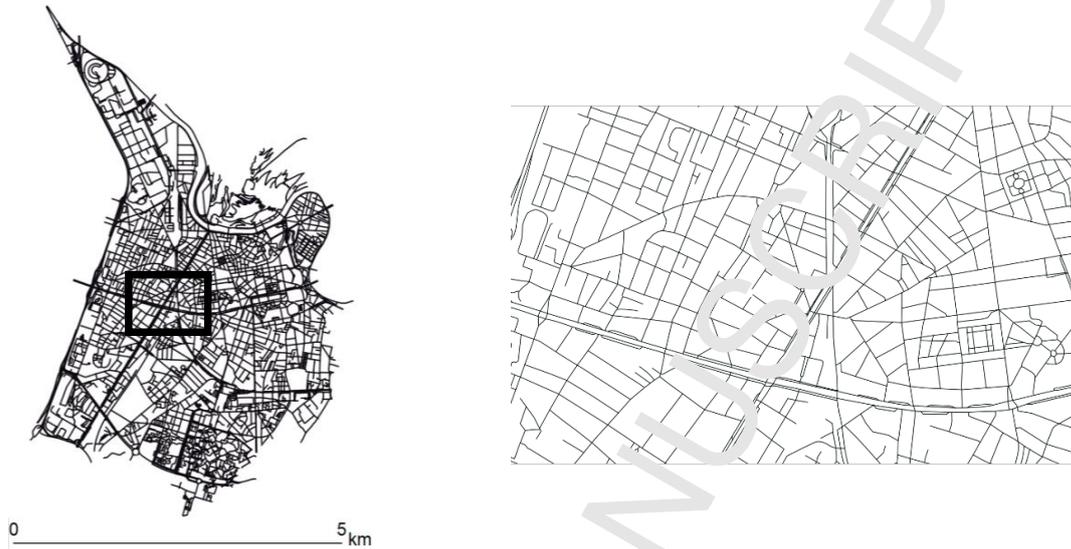


Fig 9: Grenoble's road network: (left) Grenoble city (right) Grands boulevards district

The road data set was processed manually as well. As we are studying the medium voltage network, part of this data, e.g. pedestrian paths and parking lots unlikely to be used for power lines, was unnecessary and was removed.

The processed data set describes both single or multiple driving directions roads as a single line as shown in Fig 10.



Fig 10: Grenoble's road network after processing: (left) Grenoble city (right) Grands boulevards district

In this case, the cleaning process did not lead to any noticeable changes in the fractal behavior (Fig 11). Here, in the case of the city of Grenoble, deleting pedestrian paths and parking lots from the data set does not seem to change the scaling behavior, as

the city is rather medium size (18.44 km<sup>2</sup>). It means that even at small scales, the road network is dominated by the main routes and not by paths or parking lots. The results would be different in the case of rural areas where rural tracks and paths are more prominent. Moreover, we notice that the scaling behavior becomes stable between 1.6 and 1.7 (Fig 11), which is, for high  $\epsilon$ , very similar to the fractal behavior of the power grid (Fig 6).

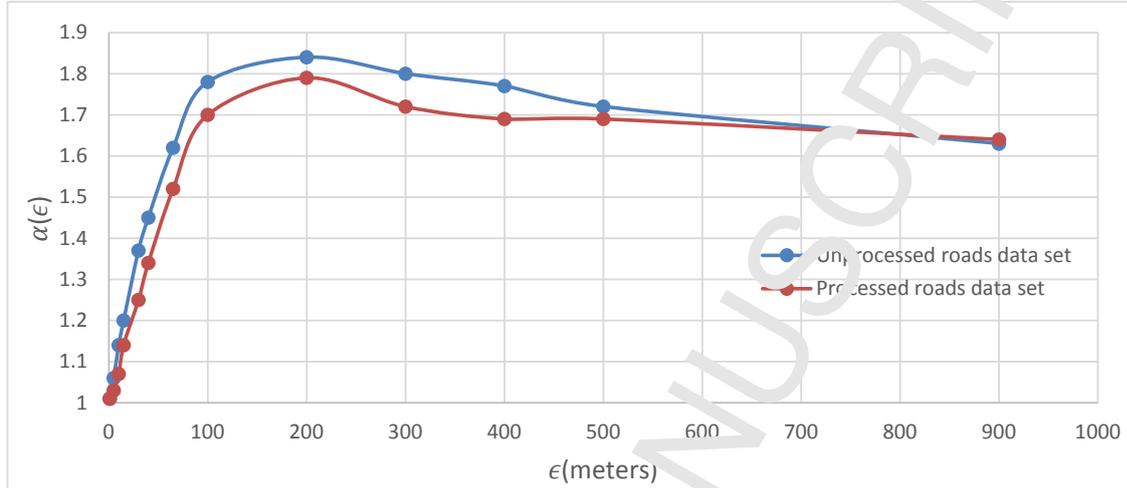


Fig 11: Overview of the fractal behavior of the road network using unprocessed data versus processed roads data set

#### 4. Global concordance analysis

In this section, we will compare the fractal behavior of both urban transportation networks at hands to the built-up areas within the whole city. Finally, the human flow via the road network is confronted with the power flow via the power system.

##### 4.1. Power grid versus buildings

The concordance indicator between buildings and the power grid is defined by:

$$P_{build/powergrid}(\alpha_i) = \frac{\alpha_{build}(\epsilon_i)}{\alpha_{powergrid}(\epsilon_i)} \quad (4)$$

In Fig 12, at small scale (lower than 20m), the ratio between the dimension of the buildings and the power grid decreases quickly from 1.9 to 1.5. At these scales, the built-up area dimension is close to 2 ( $\epsilon$  much lower than building size), whereas the power grid is almost linear, being smaller than ramification distances. At larger values of  $\epsilon$ , the concordance indicator decreases a lot until reaching a value close to 1. It is due to the increase of the grid's dimension resulting from its ramification to deliver electricity all over the covered area while minimizing its length.

It is worth noting that low values of the concordance parameter mean that the network is overdeveloped compared to the built-up distribution. Historically, the power grid developed in Grenoble was mainly a 5.5 kV network but has expanded very quickly with the construction of new buildings in the 1960s to host the winter Olympics. This expansion added a 20 kV layer, making the network hypertrophied. The local DSO is in

the process of switching from a mixed 20 kV and 5.5 kV network to an only 20 kV network by 2020, which is expected to fit more the expectations of planners from a concordance point of view. One of the planner's objective is indeed to ensure a coverage of the urban territory at the minimal cost, meaning the maximization of the concordance indicator.

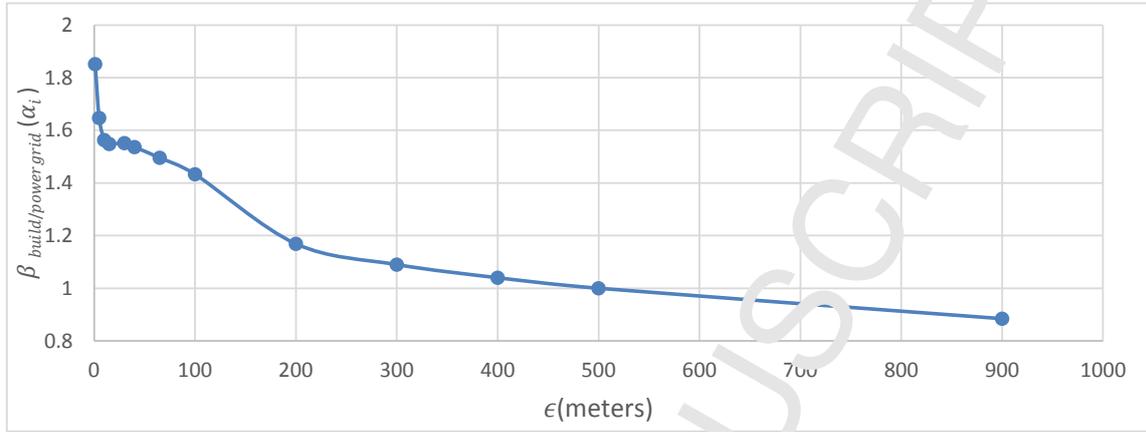


Fig 12: Concordance analysis between the power grid and building.

#### 4.2. Roads versus buildings

The concordance indicator is defined as follows:

$$\beta_{build/road}(\alpha_i) = \frac{\alpha_{build}(\epsilon_i)}{\alpha_{road}(\epsilon_i)} \quad (5)$$

We notice that the concordance between roads and buildings (Fig 13) has a similar variation to the concordance between the power grid and buildings (Fig 12). However, the indicator decreases until reaching 1 faster than the power grid. From an urbanistic point of view, it means that for distances higher than 100 m, the road network and built-up spaces follow the same type of spatial distribution while the same logic becomes only true starting from 300 m for the power grid.

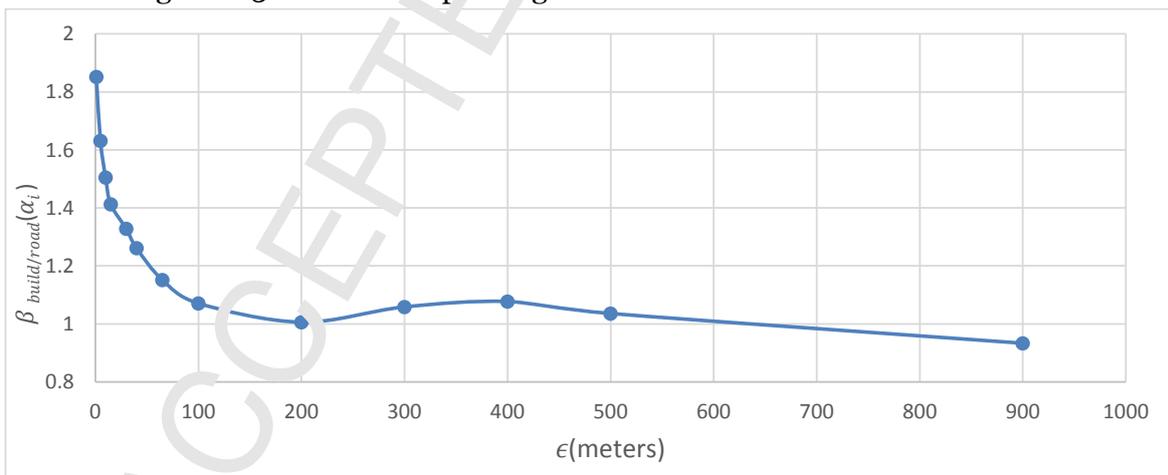


Fig 13: Concordance analysis between the road network and buildings

#### 4.3. Power grid versus road network

According to (Eq 3) we use the ratio defined by:

$$\beta_{road/power\ grid}(\alpha_i) = \frac{\alpha_{road}(\varepsilon_i)}{\alpha_{power\ grid}(\varepsilon_i)} \quad (6)$$

At very local scales, this ratio is close to 1 (Fig 14) because both behaviors are linear. For  $\varepsilon$  between 0 and 100m, we notice a high rate of increase of  $\beta$  because the scaling behavior of the power grid remains almost linear ( $\varepsilon$  is smaller than its ramification distance) whereas the road network is already developed for enabling house accesses (Fig 9). For higher values of  $\varepsilon$ , both networks become more and more concordant. This is a consequence of the hypertrophy of the power grid as mentioned in the previous section.

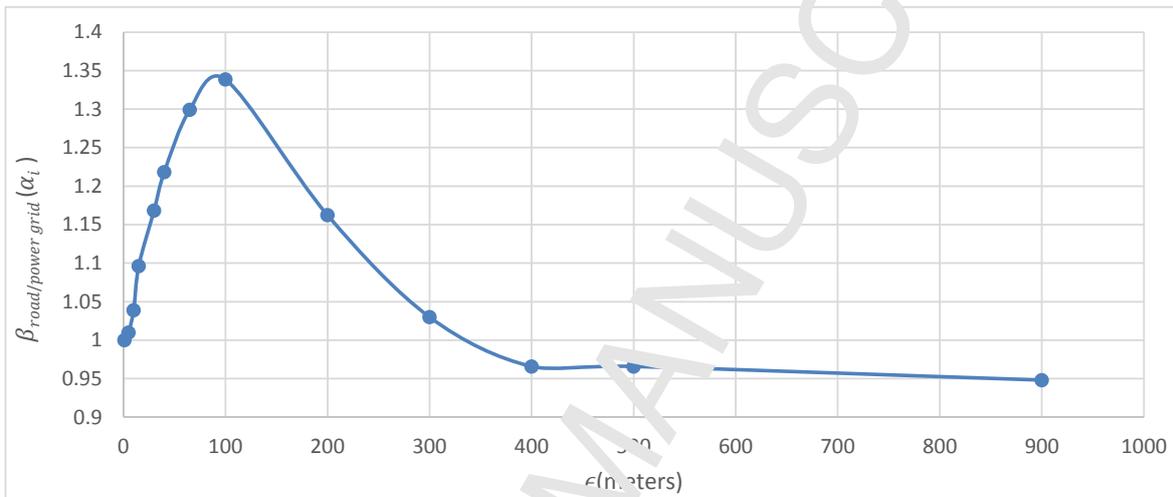


Fig 14: Concordance analysis between the power grid and road network

## 5. Local concordance analysis

Instead of analyzing the whole city, an assessment of local concordances is performed to determine whether the spatial configuration of some city areas impact a lot the results.

The “presqu’île” district, also known as the scientific polygon includes a significant number of research centers between the rivers Isère and Drac. It hosts the European Synchrotron Radiation Facility and the French Atomic Energy Commission. Downtown Grenoble represents the city’s old town, which is a pedestrian zone, associated with the squares Victor Hugo and Grenette. It hosts the main shopping area of the city, as well fast-food chains, bars, restaurants. It is also the biggest hub for public transportation in the city. The “Villeneuve” district, also known as the Olympic district was constructed in the early 1960s in order to host the athletes competing in the 1968 Grenoble’s winter Olympics and the journalists covering the event. It is characterized by its park covering 14 hectares (10% of the district’s area).

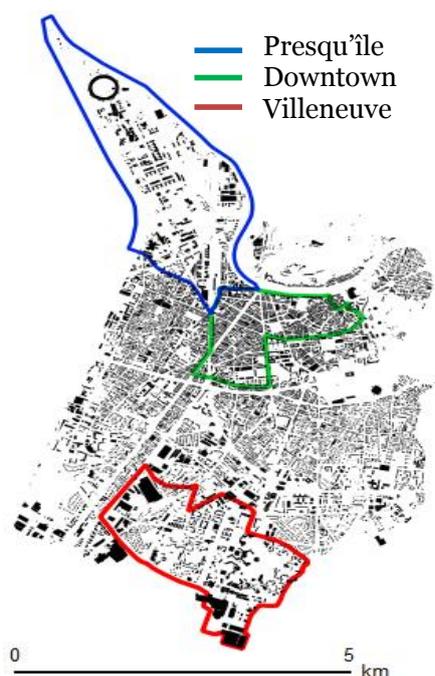


Fig 15: Definition of districts for local concordance analysis

In Fig 16, for small sizes, the buildings distribution has in all cases the same behavior as overall and takes over the power grid. However, we notice that the value of the concordance indicator is higher for downtown Grenoble, which could be explained by the higher density of the buildings in this area, followed by Villeneuve and then Presqu'île which is characterized by its strong industrial/research activity hence bigger and more dispersed buildings. For distances over 100 m, the ratio is closer to 1, meaning that the both fractal behavior are similar power grid and built-up space are more in concordance.

In Fig 17, we notice that the road network's concordance to the built-up areas exhibits a similar behavior in both Presqu'île and Downtown. It is interesting to notice, that the district's main activity is not correlated with the function of the road network, which gives access to the built-up areas. For Villeneuve, we notice a plateau at 1.2 between 50 and 100 m, which could be explained by the presence of a huge park in this area.

In Fig 18, we notice that for very local scales, both networks are concordant. However, their scaling behavior starts to move away from one another up 80 m. Afterwards, the concordance indicator starts decreasing again, meaning that the scaling behaviors become similar again. We notice that, beyond 100m, the power grid is less developed than the road network in downtown Grenoble, compared to both other districts. This could be explained by the history of Villeneuve and Presqu'île districts. They were constructed between the 1950s and 1960s and are characterized by longer/larger boulevards and fewer small streets than downtown Grenoble.

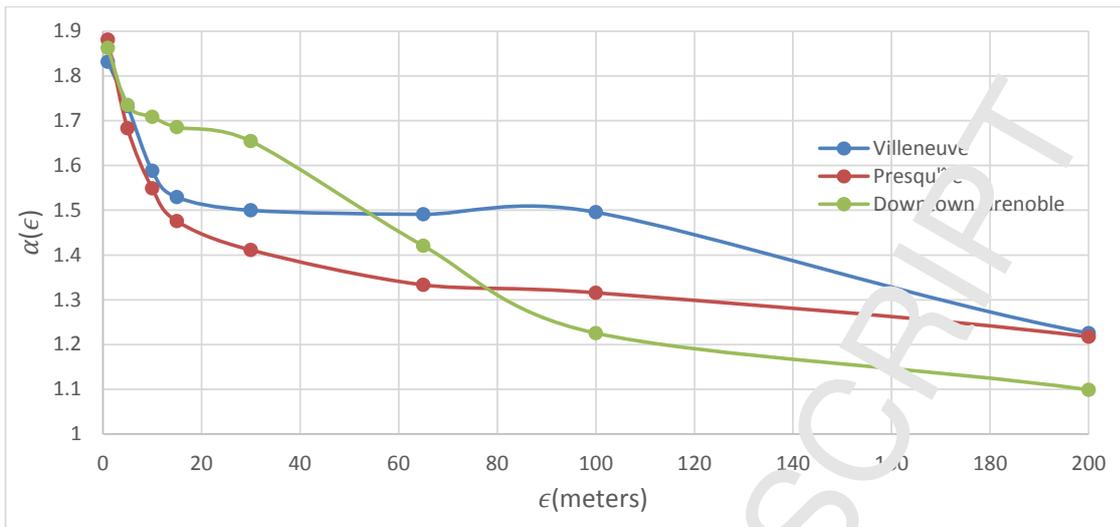


Fig 16: Concordance analysis between the power grid and buildings

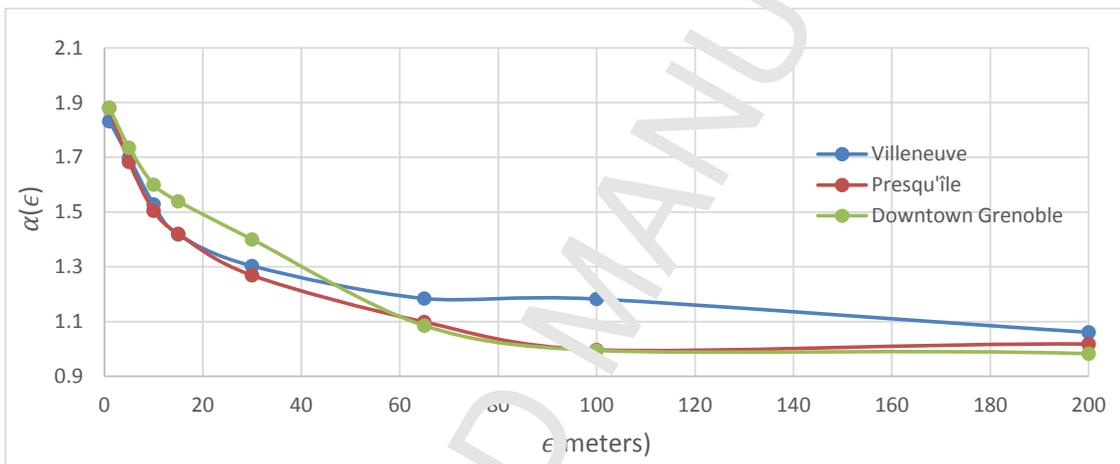


Fig 17: Concordance analysis between the road network and buildings

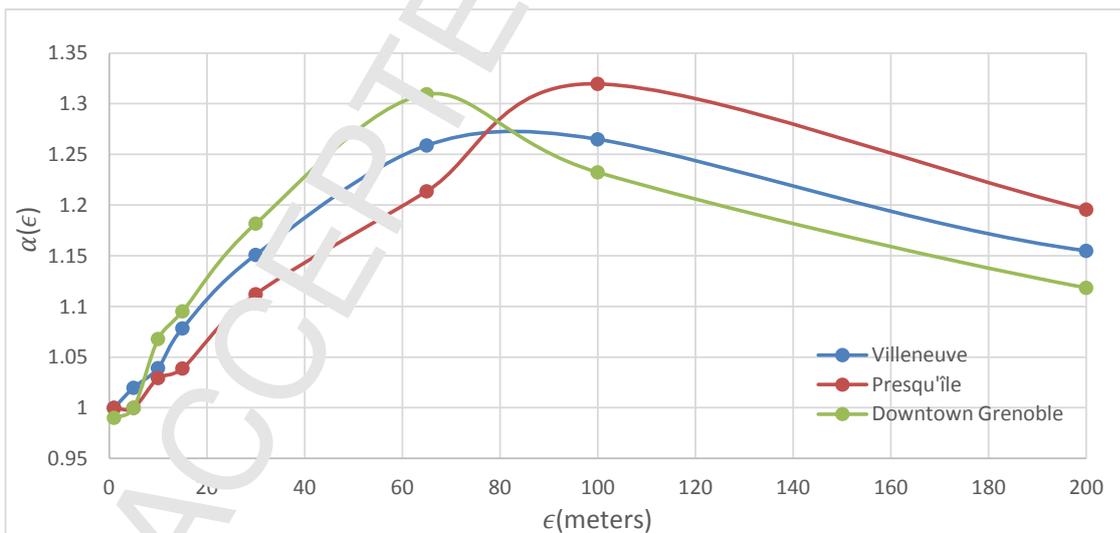


Fig 18: Concordance analysis between the power grid and road network

## 6. Conclusion

In this paper, we analyzed and compared the fractal dimension of the power grid, the road network and the buildings of the city of Grenoble, France. We showed the importance of using carefully data sets from different sources and how it could sometimes make major differences ahead of further use. The box counting method was chosen to identify the fractal behavior but should be used carefully as one can identify a maximum value of box size that should not be exceeded to have a suitable coverage depending on the type of system at hands. It is undeniable that an important limitation of fractal analysis is that arriving at an empirically determined fractal dimension does not necessarily prove that a pattern is fractal. However, carrying out a fractal analysis is valuable in understanding the complexity of the structure of the power grid and its relations with urban fabrics.

We first noted that the power grid shows a scale invariant behavior for a box coverage above 400 m. The power network is a subset of the road network and their coverage of the built-up areas is identical at large scales. . We also identified the fractal behavior of different districts within the city. Grenoble being dense and having a territory which is constrained by mountains, the local and global conformance behaviors are similar. These behaviors are mainly determined by the gradual evolution of Grenoble's urban fabric and give an indicator of how optimal this evolution is.

## 7. Acknowledgments

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