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# Mixed integer formulations for a routing problem with information collection in wireless networks

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## Abstract

We study a routing-collecting problem where a system of stations is considered. A vehicle is responsible for collecting information generated continuously in the stations and to deliver it to the base station. The objective is to determine the vehicle route and the collection operations, both physical and wireless, in order to maximize the amount of information collected during a time horizon. Three mixed integer programming models are introduced and a computational study is reported to compare the performance of a solver based on each one of the models.

*Keywords:* Routing, Wireless Networks, Mixed Integer Programming

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## 1. Introduction

Technological advances in network architectures add new features and applications to routing problems [6, 26, 17, 32, 27, 30, 23]. In this work, we are interested in the exact solution of a vehicle routing problem (VRP) for information collection in wireless networks. The new characteristic added to this well-know problem is the possibility of picking-up information via wireless transmissions.

A relevant application for this problem is to provide connection for difficult environments [26, 29, 23]. For example, Daknet [26] is a low-cost solution for providing web connection to small and remote villages. Daknet uses existing buses and local transport to carry messages and web connection to modest and isolated villages in which small kiosks are placed allowing people to send email and information in an off-line manner. A very similar case is to provide web connectivity for remote military stations which dispose of a set of vehicles to gather and deliver information [23]. Applications for this problem appears whenever one needs to provide vehicle routing strategies with wireless information transmission to the vehicles involved, as in the above examples, but also in underwater surveillance [8], environmental monitoring [33] or automated meter reading [31].

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In the context considered in this work [26, 33, 23], a unique base station is connected with the outside and a vehicle is responsible for collecting information from the other stations. The stations are equipped with technology capable of sending information via wireless connection to the vehicle when it is located in another sufficiently close station. Simultaneous transmissions are permitted. Time of transmission depends on the distance between stations, the amount of information transmitted, and other physical factors (e.g. obstacles along the way, installed equipment). Information to be sent outside of the network is continuously generated in each station at a constant rate. The VRP treated in this work looks for the vehicle route as well as for an efficient planning on how to collect information from stations, in order to minimize the total amount of remaining information at the nodes at the end of a finite time horizon  $T$ .

It is well-known [25] that a good mathematical model is crucial to solve a problem to optimality using exact methods such as branch-and-bound and branch-and-cut methods. The focus of this work is the investigation of Mixed Integer Linear Programming (MILP) formulations for this routing problem with wireless information transmission, aiming the efficient exact solution of the problem modeled.

### 1.1. Related works

First, we discuss related studies from the application perspective. Many works in the literature have address vehicle routing problems in the presence of wireless transmission with most of them focusing on the study of protocols/policies for routing and data collection. A smaller set of works addresses the development of vehicle routing strategies, most cases, in a two-phase manner (see [14]). When studying such routing and data collection protocols, normally, the authors consider that the vehicle route is set, i.e., the mobility of the vehicle is not controllable. That is frequently the case in applications taking advantage of an existing transportation infrastructure [32, 12, 9, 24]. In some situations, as in [11], the authors assume the vehicle route exist but they suppose vehicles can adjust their position (in order to receive information from some stations).

For applications where vehicle mobility is controlled, some authors address the design of the vehicle routing and the information collect planning in two-phases [16, 21, 15, 28]. First the route is designed and in the sequence, based on this finding, a collection planning/policy is established. The strategy adopted for the collection planning is used to conduct the vehicle route choice; for example, energy efficiency [21, 15] or single hop guarantee for all nodes [28]. The problem addressed in [5] looks for a vehicle routing that visits a set of nodes of the network needing repairing. However, the route designed is also used to collect information allowing the prediction of node failures. Thus, the vehicle routing changes the network architecture (once the nodes are repaired) as well as the belief of the network.

In [17], the authors were dedicated to the design of the vehicle routing together with the collection planning, but they assume an architecture is defined for the vehicle routing (cycle path or zig-zag path) and study the best placement for such architecture. To the best of our knowledge, the authors in [8] and [22] are the only ones to study the design of a vehicle route from scratch together with a wireless transmission planning.

In [8], they address an application of underwater wireless sensor networks for submarine monitoring. The authors considered a scenario with a set of surfacing nodes and underwater nodes and look for a routing to an autonomous underwater vehicle (AUV) during a given time period. The AUV must leave and return to a surface node while information generated by the underwater nodes is collected along the selected path. Opposite to our main assumption, the AUV must physically visit each station where information is collected. The value of information collected decreases with time and the strategy adopted by the authors is the maximization of the value of the information delivered to the surface nodes. The authors in [8] proposed an Integer Linear Programming (ILP) formulation able to solve the problem with up to 12 underwater nodes in a time that varies from a few hours to a few days.

The problem treated in [22] is a very close version of the VRP addressed in the present work: all but one assumption are the same. In [22] it is assumed that, once a station starts a transmission to the vehicle, all information accumulated in this station at that moment must be transmitted. This assumption is justified in situations where information is safer once it leaves the stations. However, the authors observed that this imposition contributes for the isolation of some stations. In our problem, transmission of partial information is allowed. Three different strategies adopted in the vehicle routing problem were investigated in [22]: maximizing the total amount of information extracted at the end of the time horizon  $T$ ; maximizing the average of the information in the vehicle at each time point; and maximizing the satisfaction of each station at the end of the time horizon  $T$ . A MILP formulation was introduced with three different objective functions being discussed. The authors presented computational experiments on randomly generated networks with up to 15 stations ( $|T| \in \{24, 28\}$ ), solved in one hour of computation time<sup>1</sup>. The results discussed in [22] showed the problem becomes more difficult as the total amount of information collected increases. The periodicity on remaining information at the stations (after a sequence of vehicle routings is solved) was studied for each strategy. The experiments were used to access how the maximization of one criteria affects the others and impacts the periodicity of the remaining information.

From the modeling perspective, our problem is related with the VRP problem but also with other classical network optimization problems: the classical traveling salesman problem (TSP); the prize collecting traveling salesman problem (PCTSP); and the inventory routing problem (IRP). Since only one uncapacitated vehicle is considered, the problem can be seen as a version of the Traveling Salesman Problem (TSP) [20]. However, opposite to most versions of the TSP, no constraint is imposed neither on the number nor on the frequency of visits to the stations. Notice that, depending on the position of the base station, the vehicle can be conducted to return several times to the base in order to have access to some stations. In the PCTSP (see [7]), a salesman gets a prize for every city visited. The goal is to find a minimum cost route including enough cities to collect a required amount of prize. As for the PCTSP, our problem assumes that not all the nodes need to be visited and the information collected can be regarded as a prize. Conversely, no routing costs are considered, the same

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<sup>1</sup>Using the IBM CPLEX Optimizer 12.6.1.0 solver on a server with 15 CPU's Intel ®Xeon (R) E5540@ 2.53Ghz X4, with 16 GB of RAM

station can be visited more than once, and the information collected (prize) depends on the time of the visits and on the information collected previously. Finally, in a IRP, the routing must be coordinated with the inventory management (see [13]). As for our problem, the same node can be visited multiple times and the amount of inventory that can be picked up or delivery depends on the time of the visit. However, in addition to the absence of routing costs, neither vehicle capacities nor inventory limits on the stations are considered.

## 1.2. Our contributions

Although the problem considered here has some similarities with classical problems (TSP, VRP, PCTSP and IRP), it also includes distinct characteristics making the study of the mathematical models a challenge. For classical related problems, several mathematical formulations have been proposed and compared (see, for instance, [18] for the TSP and [19] for the VRP). Adding new aspects to a problem can impact the mathematical modeling choices [10] and, as a consequence, the solution of the problem to optimality, or the design of matheuristics. Our work contributes to the development of different MILP formulations to the VRP with information collection in wireless networks.

We introduce three MILP formulations to the problem: one based on a time discretization, where each decision is measured in multiples of the time unit; and two formulations using continuous time and based on events. The two formulations based on continuous time differ in the type of events considered. One formulation assumes the events are visits to stations and transfer operations, and the other assumes that events are the vehicle stops. We will see that each such model proposed in this work has pros and cons making it more suitable for a particular instance. To the best of our knowledge these models have never been introduced for a VRP with information collection in wireless networks. Our contribution goes in the same direction as the one presented in [3] in which the authors compare discrete time models with continuous time models for a maritime IRP. However, as we discuss in the later sections, the conclusions are not coincident to the ones in [3] as the problems are different.

The paper is organized as follows. Section 2 formally describes the VRP being solved while notations and assumptions are presented. Three MILP formulations to the problem are presented in Section 3 with the discussion on how the models can be strengthened. Computational experiments are presented in Section 4. Finally, some conclusions and research directions are presented in Section 5.

## 2. Description of the problem

The wireless network is modeled by a directed graph  $D = (V, A)$ . The node set  $V$  represents the set of stations of the network and the arc set  $A$  represents the directed paths connecting pairs of stations in  $V$ . The base station is regarded as node 1. Weights  $t_{ij}$  and  $d_{ij}$  are associated with each arc (path)  $(i, j) \in A$  representing, respectively, the travel time from node (station)  $i$  to node (station)  $j$  and the distance among these nodes (stations). Let  $T = \{1, 2, \dots, m\}$  be the time horizon considered divided in  $m$  time periods. At the beginning of the time horizon, each node  $j \in V \setminus \{1\}$  contains an amount  $Q_j$  of information.

For each node  $j \in V \setminus \{1\}$  information is generated at a rate of  $r_j$  units per time period in  $T$ . Thus, the amount of information at node  $j$  at each time period  $k \in T$ , denoted by  $q_{jk}$ , depends on the elapsed time from the last extraction (either physically or by radio), i.e.,

$$q_{jk} = \begin{cases} Q_j + kr_j, & \text{if node } j \text{ has not been visited before time period } k, \\ (k - t_{last})r_j, & \text{otherwise, where } t_{last} \text{ is the time of the last extraction.} \end{cases}$$

Only the base node is properly equipped to send information outside the network; see Figure 1 for an illustration of the problem. A unique vehicle is in charge of collecting information from all the stations in  $V \setminus \{1\}$  and of transporting it to the base node. There is no capacity limit associated with the vehicle. At the beginning of the time horizon, the vehicle is located at the base node and at the end of the time horizon, it must return to the base node. Multiple visits are allowed to each node in  $V$ . Information can only be transferred to the vehicle once it is located in one of the stations in  $V$ , i.e., no information transfer is allowed while the vehicle is moving on an arc  $(i, j) \in A$ . Figure 1 shows the vehicle located at station and information transfer (dotted red lines) occurs.

Wireless transmission is used to transfer information from a node  $j \in V$  to the vehicle located in a node  $i \in V$ . Wireless transmission is only possible for close enough nodes. Let  $r_{cov}$  be the coverage radius, i.e., a maximum distance allowing wireless transmission between two points. A node  $j$  can wireless transfer its information to (the vehicle in) node  $i$  whenever  $d_{ij} \leq r_{cov}$ . We define the set of nodes that can send information to node  $i$  as  $Range(i) = \{j \in V : d_{ji} \leq r_{cov}\}$ . For the example depicted in Figure 1,  $Range(5) = \{4, 7\}$ . We make the same physical and technical assumptions as in [22]. Thus, we assume transmission speed inversely proportional to the square of the distance between nodes depending on two additional factors: the amount of information transmitted and physical factors (as equipments used or obstacles between nodes). Let  $\alpha_{ji}$  be a parameter representing the physical limitations of sending information among nodes  $j$  and  $i$ . The amount of information that can be sent per time unit from node  $j$  to node  $i$  is  $\frac{1}{\alpha_{ji}(1 + d_{ji}^2)}$ ; see Section 2 from [22]. As we have mentioned in the introduction, different from [22], we assume nodes are free to transfer only part of their information to the vehicle.

Simultaneous transmissions are possible but two limitations are imposed. Parameter  $M$  denotes the maximum number of nodes that can transfer information simultaneously to the vehicle at each time period. That means at most  $M$  nodes can send information to the vehicle simultaneously. For the example depicted in Figure 1,  $M \geq 3$ . Also, a parameter  $R$  denotes the maximum amount of information that can be transferred in each time period. During each time period, the total amount of information that the vehicle (located in a node  $i$ ) receives from all other nodes cannot exceed  $R$  units.

In this case, the simultaneous information transfer finishes only when each individual wireless transmission finishes. As a consequence, the time of a simultaneous transmission corresponds to the highest time among individual wireless transmissions.

The version of the VRP treated in this paper, called Wireless Transmission VRP (WT-VRP), consists of finding a feasible routing for the vehicle (i.e., a routing leaving at the

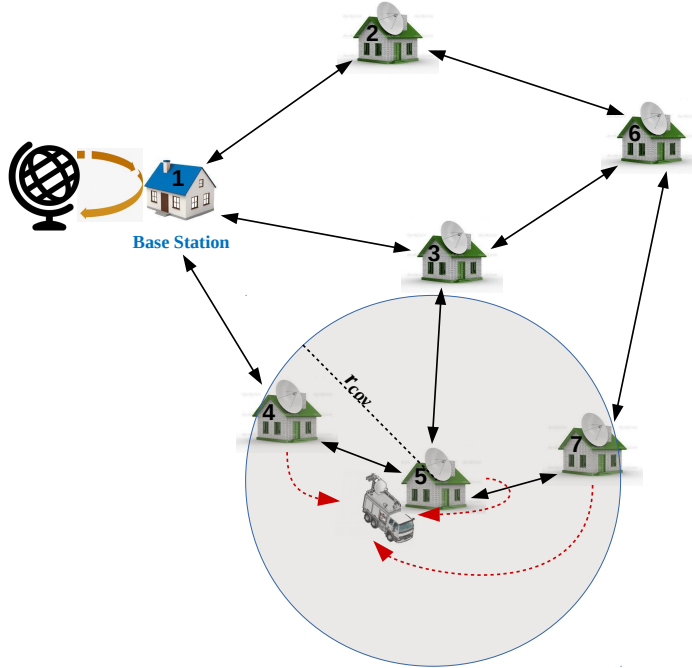


Figure 1: An illustration of the wireless transfer vehicle routing problem with 7 stations.

beginning and returning at the end to the base node) and an efficient planning for collecting information from nodes belonging to  $V \setminus \{1\}$ . The criterion for measuring the efficiency of a collect planning is the total amount collected.

### 3. Mathematical models

Next, we introduce three MILP formulations to the WT-VRP. First, a time discrete model (Section 3.1) is developed where each decision is a multiple of the time unit. Second, an event model is proposed (Section 3.2) where visits to stations and transfer operations are considered as events. Finally, another event model is presented (Section 3.3) in which the considered events are the vehicle stops.

#### 3.1. Discrete time model

Discrete time models have been used for related problems as maritime inventory routing problems (see [1, 2]). In this model the time horizon is discretized in a number of time periods  $T = \{1, 2, \dots, m\}$ . We assume that, at each time period in  $T$ , the vehicle is either traveling or waiting at a node and this behavior is modeled by the following two sets of binary variables.

For each  $(i, j) \in A$  and  $k \in T$ , let

$$x_{ijk} = \begin{cases} 1 & \text{if the vehicle crosses the arc } (i, j) \text{ (going directly from node } i \text{ to node } j) \text{ and arrives} \\ & \text{at the end of time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

For each  $j \in V$  and  $k \in T$ , let

$$z_{jk} = \begin{cases} 1 & \text{if the vehicle is waiting at node } j \text{ during time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

A third set of binary variables controls the wireless transmissions occurring at each time unit. For each pair of vertices  $j \in V$ ,  $i \in \text{Range}(j)$  and  $k \in K$ , let

$$\theta_{jik} = \begin{cases} 1 & \text{if node } j \text{ sends information to node } i \text{ during time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, continuous variables are used to describe the amounts of information at the nodes and the amounts of information transmitted. For each  $j \in V$  and  $k \in \{0\} \cup T$ , let  $q_{jk}$  be the amount of information in node  $j$  at the end of time period  $k$ . For each  $j \in V$ ,  $i \in \text{Range}(j)$  and  $k \in T$ , let  $f_{jik}$  be the amount of information transmitted from node  $j$  to node  $i$  during time period  $k$ .

The Discrete Time (DT) model follows.

$$\text{Minimize} \quad \sum_{j \in V} q_{jm} \quad (1)$$



$$\text{s.t. } \sum_{(1,j) \in A} x_{1jt_{1j}} = 1, \quad (2)$$

$$\sum_{(j,1) \in A} x_{j1m} = 1, \quad (3)$$

$$\sum_{j \in V} z_{jk} + \sum_{(i,j) \in A} x_{ijk} \leq 1, \quad \forall j \in V, \forall k \in T, \quad (4)$$

$$z_{jk} + \sum_{(i,j) \in A} x_{ijk} = \sum_{(j,p) \in A} x_{jp(k+t_{jp})} + z_{j(k+1)}, \quad \forall j \in V, \forall k \in T, \quad (5)$$

$$\sum_{j \in \text{Range}(i)} \theta_{jik} \leq M z_{ik}, \quad \forall i \in V, k \in T, \quad (6)$$

$$f_{jik} \leq \frac{\theta_{jik}}{\alpha_{ji}(1 + d_{ji}^2)}, \quad \forall j \in V, \forall i \in \text{Range}(j), \forall k \in T, \quad (7)$$

$$\sum_{j \in \text{Range}(i)} f_{jik} \leq R, \quad \forall i \in V, k \in T, \quad (8)$$

$$q_{jk} = q_{j,k-1} + r_j - \sum_{i \in \text{Range}(j)} f_{jik}, \quad \forall j \in V, k \in T | k > 1, \quad (9)$$

$$q_{j0} = Q_j, \quad \forall j \in V, \quad (10)$$

$$q_{jk} \geq 0, \quad \forall j \in V, k \in T, \quad (11)$$

$$f_{jik} \geq 0, \quad \forall j \in V, i \in \text{Range}(j), k \in T, \quad (12)$$

$$\theta_{jik} \in \{0, 1\} \quad \forall j \in V, i \in \text{Range}(j), k \in T, \quad (13)$$

$$z_{jk} \in \{0, 1\}, \quad \forall j \in V, k \in T, \quad (14)$$

$$x_{ijk} \in \{0, 1\}, \quad \forall (i, j) \in A, k \in T. \quad (15)$$

The objective function (1) minimizes the total amount of information remaining at the nodes at the end of the time horizon  $T$ , i.e., at time period  $m$ . Constrains (2)–(5) are the *Routing Constraints* assuring, together with binary conditions (15), each feasible solution of the model corresponds to a valid route of the vehicle. Equations (2) and (3) ensure that the vehicle starts and ends its route at the base node. Inequality (4) ensures that at most one of the following cases can occur at time period  $k$ : the vehicle arrives at a node or the vehicle is waiting at a node to receive information. Equations (5) ensure that if either the vehicle arrives at node  $j$  or it is waiting at this node at time period  $k$ , then, at the next time period  $k+1$ , either it travels to a neighbor node or it keeps waiting at node  $j$  (see Figure 2). As routing variables are time-indexed, equations (4) and (5) guarantee the continuity of the vehicle route over the time horizon  $T$ . Inequalities (6) and (8) are variable upper bound constraints imposing the *Transfer Constraints*. Constraints (6) guarantees that at most  $M$  nodes send information to the vehicle simultaneously. Also, this inequality ensures that, whenever variable  $\theta_{jik}$  is positive, for some  $j \in V$ ,  $i \in \text{Range}(j)$  and  $k \in K$  (i.e. at time period  $k$ , a node  $j$  sends information to the vehicle at node  $i$ ) then  $z_{ik}$  must be one (i.e. the vehicle must be located at  $i$  at time period  $k$ ). Similarly, inequalities (7) ensure that the

maximum amount of information sent from node  $j$  to node  $i$ , at a time period  $k$ , is obeyed. Additionally, this set of inequalities ensure that if  $f_{jik}$  is positive, then the binary variable  $\theta_{jik}$  must be one. Equations (8) ensures that during each time period the maximum amount of information that can be transferred to a node  $i$  cannot exceed  $R$ . Constraints (9) and (10) are the *Amount of information Constraints*. Equations (10) set the initial amount of information at each node. Equations (9) are the equilibrium constraints for the amount of information at each node. They impose that the amount of information at a node in time period  $k$  is equal to the amount information in the time  $k - 1$  plus the rate of that node minus the amount information extracted in the previous time period. Finally, inequalities (11)–(15) establish the domain of the variables.

**Example 3.1.** Consider an example with six nodes, where node 1 is the base station, and with  $m = 30$  time periods. The information generation rates for nodes 2 to 6 are given respectively by 3, 4, 2, 2 and 4. The parameters  $R$  and  $M$  are set to 20 and 3, respectively. We define  $\alpha_{ji} = 1/20$  for  $j = i$  and  $\alpha_{ji} = 1/6$ , otherwise. The initial amount of information at each node is zero. The following matrices are considered.

$$d = \begin{bmatrix} 0 & 4 & 5 & 4 & 7 & 7 \\ 4 & 0 & 2 & 4 & 3 & 6 \\ 5 & 2 & 0 & 2 & 2 & 1 \\ 4 & 4 & 2 & 0 & 5 & 1 \\ 7 & 3 & 2 & 5 & 0 & 2 \\ 7 & 6 & 1 & 1 & 2 & 0 \end{bmatrix} \quad t = \begin{bmatrix} 0 & 4 & \infty & 4 & \infty & \infty \\ 4 & 0 & 2 & \infty & 3 & \infty \\ \infty & 2 & 0 & 2 & \infty & 1 \\ 4 & \infty & 2 & 0 & \infty & 1 \\ \infty & 3 & \infty & \infty & 0 & 2 \\ \infty & \infty & 1 & 1 & 2 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

where  $d$  is the distance matrix,  $t$  is the travel times matrix, and  $W$  is the matrix indicating whether  $j \in \text{Range}(i)$  ( $W_{ji} = 1$ ) or  $j \notin \text{Range}(i)$  ( $W_{ji} = 0$ ). For example, node 2 can receive information from nodes 2, 3 and 5, while node 3 can receive information from nodes 2, 3, 4, 5, 6.

The optimal solution obtained with the DT model is depicted in Figure 2. The vehicle leaves the base station (node 1) at the beginning of the time horizon and arrives at node 2 at the end of period 4. It stays in node 2 during time periods 5 and 6, receiving information from nodes 2, 3 and 5. Next, the vehicle moves to node 3 where it spends one time period to receive information from nodes 2, 3 and 6. Then it moves to node 6 to receive information from nodes 3, 4 and 6. At the end of time period 12 the vehicle moves to node 5 where it stays for four periods. It receives information from nodes 3 and 5 during four time periods and from nodes 2 and 6 during two time periods. Then the vehicle moves again to node 6 where it stays for two periods, receiving information from nodes 3, 5 and 6. At the end of time period 22 it moves to node 4 to receive information from nodes 3, 4 and 6 during three time periods. Finally, the vehicle returns to the base station.

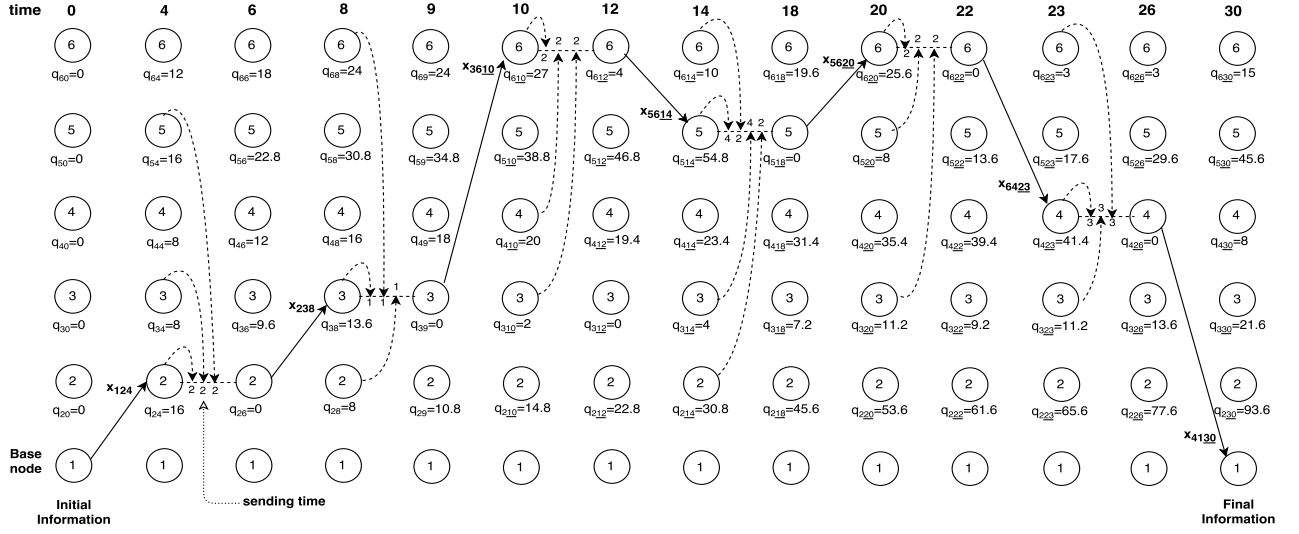


Figure 2: Optimal solution of the instance in Example 3.1 obtained with the Discrete Time model for  $m = 30$ . Objective function value = 183.8.

*Strengthening the model.* Here we discuss several enhancements that allow to tighten the model, that is, to derive a new model whose linear relaxation value is closer to the optimum value. Exact methods based on the linear relaxation such as branch-and-bound and branch-and-cut will have smaller enumeration trees when the model is tight [34].

The first enhancement is to disaggregate inequalities (6) as follows:

$$\theta_{jik} \leq z_{ik}, \quad \forall i \in V, j \in \text{Range}(i), k \in T. \quad (16)$$

Notice that inequalities (16) cannot replace inequalities (6) since they do not ensure that at most  $M$  nodes send information to the vehicle simultaneously. Hence, the two sets of inequalities are considered in the enhanced model.

Another improvement is to replace inequalities (8) by the following variable upper bound constraints.

$$\sum_{j \in \text{Range}(i)} f_{jik} \leq R z_{ik}, \quad \forall i \in V, k \in T. \quad (17)$$

Next we define a set of valid inequalities that impose a limit on the amount transferred, for a subset of time periods. Let

$$t^* = \min_{(i,j) \in A} \{t_{ij}\}$$

denote the minimum traveling time between nodes. The following proposition establishes an inequality based on the fact that, during the subset of time periods  $\ell \leq t^* + 1$ , only one node can be visited.

**Proposition 3.1.** *For  $l \leq t^* + 1$ , the following inequality is satisfied by each feasible solution*

of the DT model.

$$\sum_{s=1}^{l+1} \sum_{j \in V} \sum_{i \in \text{Range}(j)} f_{jis} \leq \sum_{j \in V} Q_j + \max_{\chi \in V} \{r_\chi\} l \quad (18)$$

**Proof** The proof is straightforward.

The DT model has  $\mathcal{O}(\kappa m)$  variables and constraints, where  $\kappa = \sum_{i \in V} |\text{Range}(i)|$ . When  $r_{cov}$  is larger than the greatest distance between two nodes, the size becomes  $\mathcal{O}(|V|^2 m)$ , while in the opposite case, when  $r_{cov}$  is smaller than the minimum distance, its size becomes  $\mathcal{O}(|V| m)$ . Hence, the main components with impact on the number of variables and constraints are the number of nodes (stations), the number of time periods considered, and the number of possible pairs of nodes for wireless transfer.

The DT model provides detailed information on the visits, normally leading to tight models [3]. However, as the model depends on the time discretization, and since a fine discretization may be required to model the traveling times and the transfer operations, it tends to increase with the increase of the time horizon.

### 3.2. Node event model

The DT model includes many variables that are null in each possible solution since some nodes are not visited, and those that are visited receive few visits during the time horizon  $[0, m]$ . In order to avoid the use of so many null variables, we introduce a flow model where only *events* are modeled, see [3]. Two types of events are considered. A first set of events is denoted by  $\Delta^r$  and includes all the possible physical node visits, which are defined by a pair  $(i, n)$  that represents the  $n^{\text{th}}$  visit of the vehicle to the node  $i$ . The second set of events, denoted by  $\Delta^w$ , include events  $(j, k)$  representing the  $k^{\text{th}}$  information transfer from node  $j$  to a neighbour node. For each visit to a given node  $i$ , at most one transfer operation can be made from each node in  $\text{Range}(i)$ . Let  $\tau_i, \omega_j$  represent the maximum number of allowed visits to node  $i$  and the maximum number of allowed transfers from station  $j$ , respectively. Thus,  $\Delta^r = \{(i, n) : i \in V, n \in \{1, \dots, \tau_i\}\}$  and  $\Delta^w = \{(j, k) : j \in V, k \in \{1, \dots, \omega_j\}\}$ .

A feasible vehicle route is defined by a combination of events from  $\Delta^r$  and  $\Delta^w$ .

Next, we define the set of binary variables to be used in this section. For each pair  $(i, n), (j, \eta) \in \Delta^r$ , with  $(i, j) \in A$ ,

$$x_{inj\eta} = \begin{cases} 1 & \text{if the vehicle goes directly from node visit } (i, n) \text{ to node visit } (j, \eta), \\ 0 & \text{otherwise.} \end{cases}$$

For each  $(j, \eta) \in \Delta^r$ ,

$$w_{j\eta} = \begin{cases} 1 & \text{if the node visit } (j, \eta) \text{ belongs to the vehicle route,} \\ 0 & \text{otherwise.} \end{cases}$$

For each  $(i, k) \in \Delta^w$ ,

$$z_{ik} = \begin{cases} 1 & \text{if at least } k \text{ transfers occur from node } i, \\ 0 & \text{otherwise.} \end{cases}$$

For each  $(j, k) \in \Delta^w$  and  $(i, n) \in \Delta^r$ ,

$$\theta_{jkin} = \begin{cases} 1 & \text{if the } k^{\text{th}} \text{ transfer from node } j \text{ to the vehicle occurs at its } n^{\text{th}} \text{ stop at node } i, \\ 0 & \text{otherwise.} \end{cases}$$

The following set of continuous and integer variables will be also necessary. For  $(j, k) \in \Delta^w$  and  $(i, n) \in \Delta^r$ ,

$f_{jkin}$  : amount of information sent during the  $k^{\text{th}}$  transfer from node  $j$  to the vehicle, occurred at its  $n^{\text{th}}$  stop at node  $i$ .

$\xi_{jkin}$  : duration, in number of time periods, of the  $k^{\text{th}}$  transfer from node  $j$  to the vehicle, occurred at its  $n^{\text{th}}$  stop at node  $i$ .

For  $(i, n) \in \Delta^r$ ,

$\gamma_{in}$  : duration, in number of time periods, of the  $n^{\text{th}}$  visit to node  $i$ .

$t_{in}^r$  : time period at which the node visit  $(i, n)$  starts.

For  $(j, k) \in \Delta^w$ ,

$q_{jk}$  : amount of information in node  $j$  at the beginning of the  $k^{\text{th}}$  information transfer.

$t_{ik}^w$  : time period at which the  $k^{\text{th}}$  transfer from node  $i$  starts.

The Node Event (NE) model is as follows.

$$\text{minimize } \left\{ \sum_{i \in V} Q_i + m \sum_{i \in V} r_i - \sum_{(j, \eta) \in \Delta^w} \sum_{(i, n) \in \Delta^r} f_{j\eta in} \right\} \quad (19)$$

$$\sum_{j \in V \setminus \{1\}} x_{11j1} = 1, \quad (20)$$

$$\sum_{(j,\eta) \in \Delta^r} x_{j\eta 12} = 1, \quad (21)$$

$$\sum_{(j,n) \in \Delta^r | (i,j) \in A} x_{injn} = w_{in}, \quad \forall (i,\eta) \in \Delta^r, \quad (22)$$

$$\sum_{(j,n) \in \Delta^r | (j,i) \in A} x_{jnin} = w_{in}, \quad \forall (i,\eta) \in \Delta^r, \quad (23)$$

$$w_{in} \leq w_{i,\eta-1}, \quad \forall (i,\eta) \in \Delta^r, \eta > 1, \quad (24)$$

$$\sum_{(j,k) \in V^w | j \in \text{Range}(i)} f_{jkin} \leq R\xi_{in}, \quad \forall (i,n) \in \Delta^r, \quad (25)$$

$$f_{jkin} \leq q_{jk} + r_j(\xi_{jkin} - 1), \quad \forall (j,k) \in \Delta^w, (i,n) \in \Delta^r, j \in \text{Range}(i), \quad (26)$$

$$f_{jkin} \leq \frac{\xi_{jkin}}{\alpha_{ji}(1 + d_{ji}^2)}, \quad \forall (i,n) \in \Delta^r, (j,k) \in \Delta^w, j \in \text{Range}(i), \quad (27)$$

$$\sum_{(j,k) \in \Delta^w | j \in \text{Range}(i)} \xi_{jkin} \leq M \gamma_{in}, \quad \forall (i,n) \in \Delta^r, \quad (28)$$

$$\xi_{jkin} \leq m\theta_{jkin}, \quad \forall (j,k) \in \Delta^w, (i,n) \in \Delta^r, j \in \text{Range}(i), \quad (29)$$

$$\sum_{k | (j,k) \in \Delta^w} \theta_{jkin} \leq w_{in}, \quad \forall (i,n) \in \Delta^r, j \in \text{Range}(i), \quad (30)$$

$$z_{jk} \leq \sum_{i \in V} z_{i,k-1}, \quad \forall (j,k) \in \Delta^w, k > 1, \quad (31)$$

$$z_{jk} = \sum_{(i,n) \in \Delta^r} \theta_{jkin}, \quad \forall (j,k) \in \Delta^w, \quad (32)$$

$$q_{jk} = Q_j + r_j t_{jk}^w - \sum_{(i,n) \in \Delta^r | i \in \text{Range}(j)} \sum_{\ell=1}^{k-1} f_{j\ell in}, \quad \forall (j,k) \in \Delta^w, \quad (33)$$

$$t_{j\eta}^r \geq t_{in}^r + \gamma_{in} + t_{ij} - (m + t_{ij})(1 - x_{inj\eta}), \quad \forall(i, n), (j, \eta) \in \Delta^r, (j, i) \in A \quad (34)$$

$$t_{i1}^r \geq \sum_{i|(i,1) \in \Delta^r} t_{i1} x_{11i1}, \quad \forall(i, 1) \in \Delta^r, \quad (35)$$

$$t_{j\eta}^r + \gamma_{j\eta} + t_{j1} x_{j\eta 12} \leq m, \quad \forall(j, \eta) \in \Delta^r, \quad (36)$$

$$t_{jk}^w \geq t_{in}^r - m(1 - \theta_{jkin}), \quad \forall(i, n) \in \Delta^r, (j, k) \in \Delta^w, \quad (37)$$

$$t_{jk}^w + \xi_{jkin} \leq t_{in}^r + \gamma_{in} + m(1 - \theta_{jkin}), \quad \forall(i, n) \in \Delta^r, (j, k) \in \Delta^w, \quad (38)$$

$$x_{j\eta in} \in \{0, 1\}, \quad \forall(j, \eta), (i, n) \in \Delta^r, (j, i) \in A, \quad (39)$$

$$\theta_{jkin} \in \{0, 1\}, \quad \forall(j, k), (i, n) \in \Delta^r, \quad (40)$$

$$q_{jk} \in \mathbb{R}^+, \quad \forall(j, k) \in \Delta^w, \quad (41)$$

$$f_{jkin} \in \mathbb{R}^+, \quad \forall(j, k), (i, n) \in \Delta^r, \quad (42)$$

$$\gamma_{jk} \in \mathbb{Z}^+, \quad \forall(j, k) \in \Delta^w, \quad (43)$$

$$\xi_{jkin} \in \mathbb{Z}^+, \quad \forall(j, k), (i, n) \in \Delta^r, \quad (44)$$

$$t_{jk}^r \in \mathbb{Z}^+, \quad \forall(i, n) \in \Delta^r, \quad (45)$$

$$t_{in}^w \in \mathbb{Z}^+, \quad \forall(j, k) \in \Delta^w. \quad (46)$$

The objective function (19) minimizes the amount of information kept in the nodes at the end of the time horizon. This amount is computed by removing the extracted information from the total information generated through the entire time horizon. Constraints (20)–(24) are the *Routing Constraints*. Equations (20) and (21) ensure, respectively, the vehicle leaves and ends its route in the node base 1. Equations (22) and (23) ensure the flow conservation, stating that if the  $\eta^{th}$  visit to node  $i$  occurs, then there must exist an arc entering and leaving that node. Constraints (24) state that if the  $k^{th}$  visit to node  $j$  occurs, so the previous  $k - 1^{th}$  visit must have occurred. The set of *Information Transfer Constraints* is defined by constraints (25)–(33). Constraints (25) limit the transfer amount considering the maximum transfer quantity per period. Constraints (26) ensure that the amount that can be transferred cannot exceed the information available at the corresponding node. Constraints (27) limit the transfer amount taking into account the transfer rate. Constraints (28) ensure that during the  $k^{th}$  visit of node  $i$ , the total duration of all transfers to node  $i$  cannot exceed the maximum allowed number of transfers per period,  $M$ , times the duration of the visit. Constraints (29) link the variables indicating the duration of the transfer operations (variables  $\xi_{jkin}$ ) to the binary variables  $\theta_{jkin}$  indicating whether a transfer occurs. Constraints (30) ensure that an information transfer occurs only if a visit occurs. Constraints (31) state that if the  $k^{th}$  transfer occurs so the previous  $k - 1^{th}$  must have occurred. Constraints (32) relate the binary transfer variables. Constraints (33) define the amount of information at each node at the beginning of each information transfer. Constraints (34)–(38) are the *Time Constraints*. The start time of each  $k^{th}$  visit is defined by constraints (34) and (35). Constraints (34) takes into account the start time of the previous visit plus the traveling time between the two locations and the time spent on the last visit (this inequality is redundant whenever  $x_{inj\eta} = 0$ ). Notice that constraints (34) ensure there are no subtours resulting from the solution satisfying the Routing Constraints (20)–(23),

which guarantees the continuity of the vehicle route until it returns to the base station. Constraints (35) restrict the start time of the first visit. Constraints (36) force all the visits to end early enough so the vehicle can return to the base station before the end of the time horizon. Constraints (37) and (38) relate the start and end times of an information transfer from node  $j$  to node  $i$ , with the start and end times of the visit to node  $i$ . Notice that these inequalities are redundant whenever  $\theta_{jkin} = 0$ . Finally, constraints (39) - (46) define the variables domain.

**Example 3.2.** Figure 3 depicts the solution of the instance used in Example 3.1, with the solution representation of the NE model. In this figure, two different set of nodes represent the two different types of events: circles represent events in  $\Delta^r$  while squares represent events in  $\Delta^w$ . For instance, we can see that station 6 is visited twice while the other stations are visited once; during the event  $(6, 2) \in \Delta^r$  (second visit to station 6), occur the events  $(3, 5), (5, 2), (6, 4) \in \Delta^w$  corresponding to transfer operations.

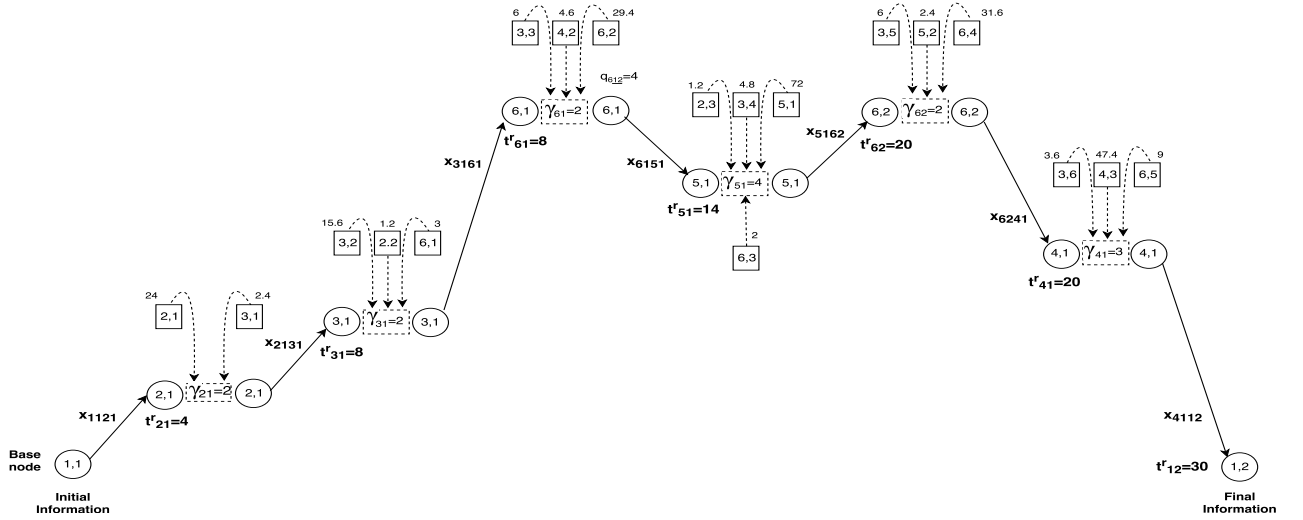


Figure 3: Optimal solution of the instance in Example 3.1 obtained with the Node Event model.

*Strengthening the model.* The NE model can be strengthened with a valid inequality that bounds the transfer amount  $\sum f_{jmin}$  with the maximum transfer rate  $R$  times the number of periods the vehicle can receive information, i.e., the number of periods the vehicle is not traveling.

**Proposition 3.2.** The following inequality is satisfied by each feasible solution of the NE model.

$$\sum_{(j,k) \in \Delta^w} \sum_{(i,n) \in \Delta^r: j \in \text{Range}(i)} f_{jmin} \leq R(m - \sum_{(i,n) \in \Delta^r} \sum_{(j,\eta) \in \Delta^r: (i,j) \in A} t_{ij} x_{injn}). \quad (47)$$

**Proof** The proof is straightforward.

Model NE has  $\mathcal{O}(\sum_{i \in V} \sum_{j \in \text{Range}(i)} \tau_i \omega_j + \sum_{(i,j) \in A} \tau_i T_j)$  variables and constraints.



### 3.3. Vehicle event model

Typically, a vehicle route includes only a small number of nodes visited. Since the events in the NE model were defined on the set of nodes, most of the variables in this model will have null value. In this section we define a set of events associated with the vehicle: each event is a vehicle stop. This formulation resembles the layered formulation for the vehicle routing problems, see [4] and the references therein. Let  $N = \{1, \dots, \hat{N}\}$  denote the set of possible events where  $\hat{N}$  is an upper bound on the number of events (for instance, this upper bound can be computed taking into account the distance between stations and considering one time period for information extraction at each visit). The new routing variables indicate the node visited at the  $k^{\text{th}}$  vehicle stop, indexed by the event  $k \in N$ .

Next we define the new set of binary variables. For each  $i \in V, k \in N$ ,

$$x_{ik} = \begin{cases} 1 & \text{if the } k^{\text{th}} \text{ vehicle event occurs at node } i, \\ 0 & \text{otherwise.} \end{cases}$$

The following continuous and integer variables are also defined. For each  $k \in N$ ,

$$\begin{aligned} t_k &: \text{time period at which the } k^{\text{th}} \text{ event begins,} \\ \gamma_k &: \text{time (in time periods) spent at the } k^{\text{th}} \text{ event.} \end{aligned}$$

For each  $j \in V, k \in N$ ,

$$q_{jk} : \text{amount of information in node } j \text{ at the beginning of event } k.$$

For each  $i, j \in V, k \in N$ ,

$$\begin{aligned} \xi_{jik} &: \text{duration (in time periods) of the information transfer from node } j \text{ to node } i \text{ at event } k, \\ f_{jik} &: \text{amount of information transmitted from node } j \text{ to node } i \text{ during event } k. \end{aligned}$$

The Vehicle Event (VE) model is as follows.

$$\text{Minimize } \left\{ \sum_{i \in V} (Q_i + mr_i) - \sum_{i \in V, j \in \text{Range}(i), k \in N} f_{jik} \right\} \quad (48)$$

$$\sum_{j:(1,j) \in A} x_{j1} = 1, \quad (49)$$

$$\sum_{k \in N} x_{1k} = 1, \quad (50)$$

$$\sum_{j \in V} x_{jk} \leq 1, \quad \forall k \in N, \quad (51)$$

$$x_{jk} \leq \sum_{i:(i,j) \in A} x_{i,k-1}, \quad \forall j \in V, k \in N, \quad (52)$$

$$x_{jk} \leq 1 - \sum_{\ell=1}^{k-1} x_{1\ell}, \quad \forall j \in V \setminus \{1\}, k \in N, \quad (53)$$

$$t_k \geq t_{k-1} + \gamma_{k-1} + t_{ij}(x_{i,k-1} + x_{jk} - 1), \quad \forall (i,j) \in A, k \in N, \quad (54)$$

$$t_1 \geq \sum_{j:(1,j) \in A} t_{1j} x_{1j}, \quad (55)$$

$$t_k \leq m, \quad \forall k \in N, \quad (56)$$

$$q_{jk} = Q_j + r_j t_k - \sum_{\ell=1}^{k-1} \sum_{i \in \text{Range}(j)} f_{jil}, \quad \forall j \in V, k \in N, \quad (57)$$

$$f_{jik} \leq q_{jk} + r_j (\xi_{jik} - 1), \quad \forall j \in V, i \in \text{range}(j), k \in N, \quad (58)$$

$$f_{jik} \leq \frac{\xi_{jik}}{\alpha_{ji}(1 + d_{ji}^2)}, \quad \forall j \in V, i \in \text{range}(j), k \in N, \quad (59)$$

$$\sum_{j \in \text{Range}(i)} f_{jik} \leq R\gamma_k, \quad \forall i \in V, k \in N, \quad (60)$$

$$\sum_{i \in \text{Range}(j)} \xi_{jik} \leq \gamma_k, \quad \forall j \in V, k \in N, \quad (61)$$

$$\sum_{j \in V, i \in \text{Range}(j)} \xi_{jik} \leq M\gamma_k, \quad \forall k \in N, \quad (62)$$

$$\xi_{jik} \leq m x_{ik}, \quad \forall i \in V, j \in \text{Range}(i), k \in N, \quad (63)$$

$$f_{jik} \geq 0, \quad \forall i \in V, j \in \text{Range}(i), k \in N, \quad (64)$$

$$q_{jk} \geq 0, \quad \forall j \in V, k \in N, \quad (65)$$

$$t_k \in \mathbb{Z}^+, \quad \forall k \in N, \quad (66)$$

$$\gamma_k \in \mathbb{Z}^+, \quad \forall k \in N, \quad (67)$$

$$\xi_{jik} \in \mathbb{Z}^+, \quad \forall i \in V, j \in \text{Range}(i), k \in N, \quad (68)$$

$$x_{ik} \in \{0, 1\}, \quad \forall i \in V, k \in N. \quad (69)$$

The objective function (48) minimizes the amount of information kept in the nodes at the end of the time horizon. Constraints (49)–(53) are the *Routing Constraints*. Equations (49)

and (50) ensure, respectively, the vehicle leaves and arrives at the base node. Inequalities (51) state that at most one visit labeled  $n$  is made. Inequalities (52) ensure that, if the  $k^{\text{th}}$  visit is made to node  $j$ , then the  $k - 1^{\text{th}}$  visit occurred in one of the predecessors of node  $j$ . Constraints (53) ensure that all the routing variables are null after the vehicle has returned to the base node. Constraints (54)–(56) are the *Time Constraints*. Inequalities (54) impose that the start time of the  $k^{\text{th}}$  visit takes into account the start time of the previous visit, the time spent on the last visit and the traveling time between the two locations visited. Constraints (55) restrict the start time of the first visit while constraints (56) force all the visits to start during the time horizon (this includes the last visit which is the return to the base station). [As in the previous model, time constraints \(54\) avoid subtours and, together with the set of Routing Constraints, guarantee the continuity of the vehicle route until it returns to the base station.](#) The *Information Transfer Constraints* are constraints (57)–(63). Constraints (57) define the amount of information at each node at the beginning of each visit. Constraints (58) ensure that the amount that can be transferred cannot exceed the information available at the corresponding node. Constraints (59) limit the transfer amount taking into account the transfer rate, while constraints (60) limit the transfer amount considering the maximum transfer quantity per period. Constraints (61) ensure that, during a visit to node  $i$ , the time used to transfer information from each node  $j$  to node  $i$ , cannot exceed the time the vehicle has spent at node  $i$ . Constraints (62) ensure that during each visit, the total transfer time to node  $i$  cannot exceed the maximum number of transfers per period,  $M$ , times the duration of the visit. Constraints (63) link the transfer variables to the routing variables, ensuring that a node  $j$  can transfer information to a node  $i$  during the  $k^{\text{th}}$  visit if the  $k^{\text{th}}$  visit occurred at node  $i$ . Finally, Constraints (64)–(69) define the variables domain.

**Example 3.3.** *Figure 4 depicts the solution of the instance used in Example 3.1, with the solution representation of the VE model. A node  $(i, k)$  in this network representation of a solution is associated with the event  $k \in N$  occurring at node  $i \in V$ . A dotted line from a node  $(j, k)$  to a node  $(i, k)$  represents an information transfer occurring from  $j$  to  $i$  during the  $k^{\text{th}}$  visit. For instance, we can see that the 5<sup>th</sup> visit of the vehicle occurs at node 6 to receive information from nodes 3, 5 and 6.*

Model VE has  $\mathcal{O}(\kappa\hat{N})$  variables and constraints, where  $\kappa = |\sum_{i \in V} \text{Range}(i)|$ . As for the NE model, the size of the VE model depends on the size of the event set, i.e., from  $\hat{N}$ . Let  $\varphi(\hat{N})$  denote the value of the objective function of the vehicle event model considering a maximum of  $\hat{N}$  events occur. The following statements hold true.

- (i)  $\varphi$  is non increasing in the maximum number of events, that is,  $N_1 > N_2$  then  $\varphi(N_1) \leq \varphi(N_2)$ .
- (ii)  $\exists N \in \mathbb{N}$  such that  $\varphi(N_1) = \varphi(N)$ ,  $\forall N_1 > N$ .
- (iii) Let  $N^*$  denote the lowest  $N$  satisfying statement (ii), that is, the lowest possible value for the number of visits that gives the optimal solution. If  $N' < N^*$ , then  $\varphi(N')$  gives an upper bound on the optimal value  $\varphi(N^*)$ .

Clearly,  $N^*$  is not known. By underestimating it the model becomes easier to solve but the solution cost will increase. Overestimating  $N^*$  may lead to longer running times.

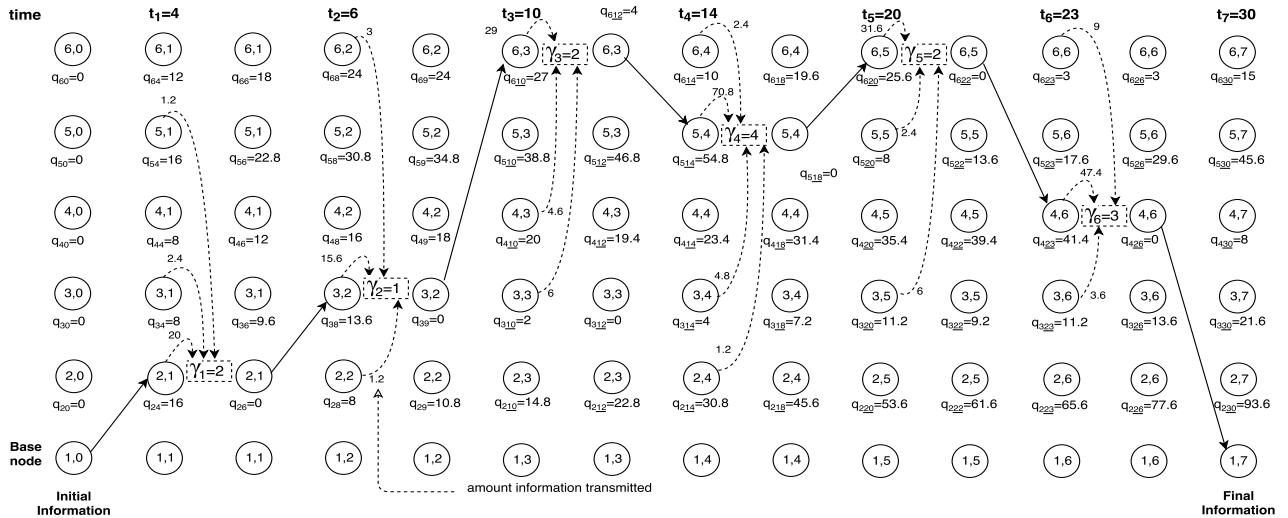


Figure 4: Optimal solution of the instance in Example 3.1 obtained with the Vehicle Event model.

#### 4. Computational tests

In this section, we report the computational tests conducted to evaluate the three models presented in the previous section. All the results were performed using a server with 15 CPU's Intel  $\text{\textcircled{R}}$ Xeon (R) E5540@ 2.53Ghz X4, with 16 GB of RAM. To solve the several MILP models, the IBM CPLEX Optimizer 12.6.1.0 solver was used with a time limit equal to 3600 seconds.

A set of instances was randomly generated as described in [22]. The vertices in  $V$  are located on a square grid of length  $\ell = 8$ . The base station is located on the bottom left vertex and the remaining stations are placed randomly on a square of length  $\ell' = 6$  in the upper right of the grid. The distance matrix is given by the euclidean distance between the stations. The graph edges are selected randomly. In order to obtain a certain graph density  $d$ , starting from a complete graph, edges are removed randomly, while ensuring connectivity, until the desired graph density is obtained. In this work, we generate instances varying  $|V|$  in  $\{8, 10, 12, 20\}$  and with  $d = 0.4$ . We considered the values of  $m \in \{72, 120, 240\}$  and the information generation rates  $r_j$  are randomly generated in the interval  $[1, 5]$ . The values of  $\alpha_{ij}$  were randomly generated in  $\{1/12, 1/13, 1/14\}$  if  $i = j$  and in  $\{1/5, 1/6, 1/7\}$  otherwise. The following values parameters were set:  $r_{cov} = 4$ ,  $R = 20$  and  $M = 3$ .

First, in Table 1, we compare the size of the three models for the combination of the parameters defining the instances and the models used in our experiments. For model NE, the values  $\tau, \omega$  in the top of the four columns represent, respectively, the maximum number of allowed visits to and the maximum number of allowed transfers from each station. We can observe that DT model is the largest model, while NE model is the smallest one. The VE model is an intermediate model in terms of size. Next, we will describe the results obtained with each one of the three models.

In Table 2, we report the computational results obtained with the DT model. The table is split into two parts accordingly to the time horizons. For each part, the first three

$ V $	$m$		DT Model	NE Model				VE Model		
				$\tau = 1, \omega = 2$	$\tau = 1, \omega = 3$	$\tau = 2, \omega = 2$	$\tau = 2, \omega = 3$	$\hat{N} = 7$	$\hat{N} = 8$	$\hat{N} = 9$
10	50	variables	7101	374	524	765	1038	743	849	955
		constraints	4142	597	838	1236	1682	1678	1927	2176
20	10	variables	14201	374	524	765	1038	743	849	955
		constraints	8292	597	838	1236	1682	1678	1927	2176
	50	variables	33701	1769	2543	3709	5200	3655	4177	4699
		constraints	16032	3002	4273	6230	8696	8168	9365	10562
	100	variables	67401	1769	2543	3709	5200	3655	4177	4699
		constraints	32082	3002	4273	6230	8696	8168	9365	10562

Table 1: Number of variables and constraints used in each model for different parameters. For the NE model, the values of  $\tau$  and  $\omega$  are, respectively, the maximum number of allowed visits and the maximum number of allowed transfers, assumed the same for each station.

columns give the number of stations ( $|V|$ ), the density of the graph,  $d$  (since the information generation process may generate graphs with density slightly different from 0.4), and the size of the time horizon  $m$ . The fourth column gives the total amount of information generated during the time horizon. The fifth column gives the value of the best feasible solution found, that is, the amount of information remaining in the nodes at the end of time horizon  $T$ . The following three columns give: (Cpu) is the running time (in seconds); (Gap) is the final integrality gap reported by the solver at the end of running time; and (Nodes) is the number of nodes of the branch-and-bound algorithm. An instance with \*\*\* in the (Cpu) and with (Gap) superior to zero is not solved to optimality in the time limit.

The results show that for  $m = 72$  all except one instance with  $|V| \in \{10, 15, 20\}$  are solved to optimality within the one hour time limit. When  $m$  is increased, the number of solved instances decreases. For  $m = 240$ , even when  $|V| = 8$ , no instance is solved to optimality with final gap arriving to 28%.

Next, in Table 3 we report the results obtained with the NE model. The four first columns are similar to the ones of Table 2. The following four sets of columns give the same information as the corresponding ones in Table 2; namely the best feasible solution value (best sol), the running time (Cpu), the final gap (Gap) and the number of nodes (Nodes). Again, the values  $\tau, \omega$  in the top of the four multicolumns represent, respectively, the maximum number of allowed visits to and the maximum number of allowed transfers from each station.

The results in Table 3 show that the model can only be solved to optimality for very small values of  $\tau$  and  $\omega$ . The average values of the best solutions obtained with the NE Model on the set of instances with  $|V| = 8$ ,  $m = 240$  are compared in Figure 5 with the average value of the best solutions found by the VE Model.

Finally, in Table 4, we report the results obtained using the VE Model. The meaning of the columns is the same as for the two previous tables. The last four sets of columns are grouped accordingly to the maximum number of vehicle events  $\hat{N}$ . The bold numbers mean that the corresponding value  $N$  is probably  $N^*$ : the objective value did not change for  $N > \hat{N}$ .

Table 2: Results obtained with the Discrete Time model.

$ V $	$d$	$m$	Total	best sol.	Cpu	Gap	Nodes	$ V $	$d$	$m$	Total	best sol.	Cpu	Gap	Nodes
10	0.42	72	2392	1676.64	63.69	0	6809	10	0.42	120	3832	2453.21	443.63	0	24423
			2220	1538.64	81.23	0	8589				3564	2216.67	515.01	0	28669
			1744	1128.11	57.55	0	8685				2752	1550.53	748.32	0	206814
			2006	1337.79	96.86	0	11171				3206	1876.88	***	3.64	261685
			1797	1127.11	146.49	0	18012				2853	1581.25	***	0.28	254528
			2073	1413.24	85.64	0	9085				3321	2063.8	515.77	0	23966
			2286	1620.57	106.2	0	11851				3678	2407.19	505.97	0	29222
			1952	1250.58	107.18	0	32751				3104	1726.78	636.26	0	24311
			2339	1671.6	102.15	0	9232				3731	2423.72	483.33	0	25211
			1734	1083.79	99.88	0	11483				2742	1422.04	752.43	0	42202
15	0.4	72	3808	3059.5	271.19	0	8477	15	0.4	120	5477	4086.59	***	2.47	21649
			3205	2468.17	487.51	0	27490				5638	4280.33	***	0.7	25301
			2753	2051.69	277.88	0	12014				4915	3602.7	***	0.7	31131
			3112	2390.6	203.24	0	10785				5140	3758.86	***	3.85	22132
			2738	2059.6	248.73	0	11993				4899	3637.02	***	1.84	22524
			2876	2173.72	***	2.18	360579				5068	3633.39	***	0.45	29757
			3213	2473.69	139.26	0	6694				5616	4138.39	***	0.23	24494
			3669	2912.56	212.09	0	9233				5626	4258.6	***	0.6	28796
			3112	2324.19	259.58	0	6046				6224	4791.89	***	1.76	29075
			2797	2123.41	192.35	0	11900				6234	4811.39	3382.54	0	45284
20	0.4	72	5149	4301.5	1001.32	0	151612	8	0.42	240	5216	2385.33	***	6.39	97091
			4574	3838.6	1033.39	0	16072				4976	2037.34	***	10.34	87142
			4239	3528.7	507.27	0	11944				5700	2895.91	***	4.7	100844
			4924	4167.57	483.79	0	9344				3764	1242.85	***	28.32	74207
			4657	3904.39	963.63	0	10171				4960	2297.1	***	7.09	71848
			5097	4315.39	1094.51	0	21540				4965	2190.77	***	8.42	55845
			4928	4180.5	905.1	0	26529				4977	2127.55	***	15.57	46945
			4175	3448.59	587.1	0	12830				5910	2896.44	***	3.71	110735
			5650	4844.29	410.95	0	8962				5223	2572.06	***	7.5	87638
			5007	4239.89	451.94	0	10770				5448	2700.38	***	5.65	114550

\*\*\* time limit: 3600 sec.

Table 3: Results obtained with the Node Event model. The values of  $\tau$  and  $\omega$  are, respectively, the maximum number of allowed visits and the maximum number of allowed transfers, assumed the same for each station.

V	d	m	Total	$\tau = 1 \ \omega = 2$				$\tau = 1 \ \omega = 3$				$\tau = 2 \ \omega = 2$				$\tau = 2 \ \omega = 3$			
				best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes
10	0.42	72	2392	1724.19	8.54	0	31450	1721.78	28.54	0	172659	1697.38	***	295.49	1149888	1758.86	***	290.6	929083
			2220	1966.41	0.85	0	646	1966.41	1.83	0	768	1601.85	***	272.52	1194195	1581.05	***	303.23	789375
			1744	1159.59	4.76	0	18794	1155.89	20.81	0	95069	1149.74	***	287.46	2025411	1166.71	***	322.3	1208221
			2006	1341.99	13.44	0	39759	1337.8	31.84	0	116451	1366.89	***	310.23	1298188	1357.49	***	332.79	717590
			1797	1168.92	10.01	0	41684	1160.55	39.14	0	283311	1168.92	***	310.43	2049193	1203.48	***	324.45	1101298
			2073	1424.42	10.49	0	30478	1413.7	32.45	0	153478	1426.85	***	277.62	1513668	1437.75	***	306.21	581914
			2286	1656.6	5.59	0	20531	1656.07	18.39	0	94219	1665.86	***	265.26	1800753	1661.51	***	293.46	794190
			1952	1262.28	10.41	0	30769	1250.58	22.4	0	87726	1284.89	***	271.44	1697986	1250.58	***	306.25	1035625
			2339	1680.4	11.02	0	34742	1671.6	34.81	0	139798	1680.4	***	274.55	1268322	1721.29	***	299.58	677665
			1734	1085.61	7.58	0	33221	1083.8	52.09	0	537229	1085.61	***	317.49	1350680	1097.39	***	331.25	1029658
15	0.4	72	3808	3116.19	**	156.49	1575724	3102	***	189.56	1229423	3129.90	***	266.80	722639	3083.19	***	297.82	316754
			3205	2471.59	**	126.6	1791434	2468.19	***	0.2	2588204	2530.29	***	282.39	771799	2523.38	***	299.28	362158
			2753	2051.69	**	179.04	1601614	2073.99	***	256.7	772934	2111.42	***	278.41	1381449	2200.1	***	296.77	364035
			3112	2394.47	**	142.19	1704099	2446.69	***	225.83	900202	2446.39	***	272.34	887673	2457.59	***	298.95	580941
			2738	2061.09	**	106.7	2133380	2071.79	***	228.2	986031	2142.02	***	268.10	746412	2089.71	***	308.15	615913
			2876	2190.5	1313.04	0	1254393	2211.21	***	166.47	1146607	2280.23	***	269.12	1081001	2282.35	***	298.27	520333
			3213	2473.69	***	91.44	1545789	2533.69	***	139.5	1114749	2509.69	***	288.28	1120249	2678.04	***	295.55	350887
			3669	3007.84	***	154.48	2402603	3006.35	1857.92	0	2977964	3057.86	***	271.66	1174708	3100.4	***	285.7	438971
			3112	2329.7	***	92.23	2044297	2324.4	***	131.22	1256214	2511.29	***	266.06	1138662	2468.39	***	294.15	399204
			2797	2123.41	1990.16	0	2049539	2127.94	**	108.63	1840108	2174.51	***	273.14	1456530	2239.27	***	297.04	509990
10	0.42	120	3832	2499.63	17.59	0	110862	2482.61	344.1	0	4142786	2534.36	***	284.97	1956508	2542.55	***	331.34	804639
			3564	3137.7	0.95	0	654	3137.7	1.47	0	480	2269.92	***	288.32	1328581	2286.49	***	343.7	812161
			2752	1564.61	31.7	0	498781	1555.61	57.25	0	593308	1612.34	***	338.78	1646827	1613.97	***	375.98	1267842
			3206	1884.73	19.03	0	96745	1876.88	95.86	0	694556	1939.38	***	346.34	1151266	1923.56	***	379.44	750328
			2853	1590.94	9.26	0	49145	1580.01	76.96	0	694533	1616.08	***	340.98	2382438	1617.72	***	375.25	1129317
			3321	2079.21	22.02	0	172610	2070.82	180.89	0	1977811	2079.21	***	310.36	1628539	2138.64	***	327.01	627780
			3678	2433.1	20.54	0	224393	2422.67	84.69	0	792202	2444.45	***	306.22	1148285	2479.52	***	316.8	853670
			3104	1763.53	17.09	0	89928	1738.93	78.33	0	589494	1763.53	***	310.9	2057964	1752.72	***	369.19	880102
			3731	2434.51	29.58	0	339080	2423.72	232.72	0	3082140	2434.51	***	289.3	1658307	2495.78	***	336.14	654701
			2742	1474	545.26	0	12937921	1448.57	***	0.8	22348921	1470.29	***	390.93	1095967	1480.79	***	392.28	842943
8	0.42	240	5216	2579.93	2.06	0	14880	2568.52	4.02	0	37643	2457.39	1677.24	0	10470801	2365.66	***	32.95	8250829
			4976	2298.28	2.6	0	25218	2231.32	27.81	0	420200	2171.69	1891.87	0	16504873	2107.25	***	46.09	6450978
			5700	3122.77	1.93	0	14116	3104.08	3.61	0	26239	2982.94	1070.25	0	3981748	2950.09	***	28.81	3987428
			3764	1444.09	2.35	0	15559	1426.74	13.28	0	150225	1355.23	1818.86	0	13223040	1273.45	***	71.07	3283058
			4960	2495.54	1.97	0	15098	2460.07	4.38	0	44873	2371.44	2113.4	0	10704389	2338.73	***	39.28	8278251
			4965	3784.35	0.27	0	0	3784.64	0.27	0	0	2285.32	***	270.53	2539042	2242.18	***	316.28	1944104
			4977	2398.77	2.05	0	12741	2357.5	7.46	0	77238	2198.97	557.91	0	3896604	2164.77	***	37.31	7336873
			5910	4704.82	0.26	0	0	4704.82	0.49	0	0	2956.79	2550.54	0	7059293	2907.28	***	27.42	6743235
			5223	2661.92	2.78	0	22127	2624.35	13.02	0	178347	2640.83	***	14.11	9554160	2619.68	***	229.23	2290740
			5448	2765.27	3	0	23710	2730.17	21.31	0	284750	2765.37	***	1.76	22104402	2711.08	***	36.26	6839767

\*\*\* time limit : 3600 sec.





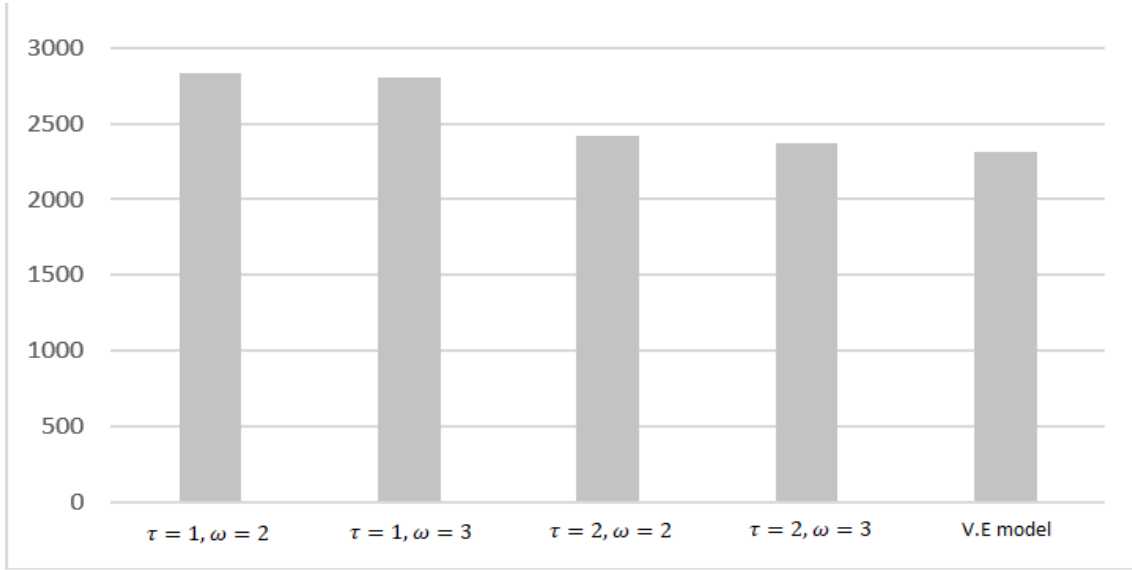


Figure 5: Average values of the best solutions obtained using NE model on the set of with  $|V| = 8$ ,  $m = 240$ .

As depicted in Figure 6, for the particular case with  $|V| = 15$  and  $m = 120$ , running times increase when  $\hat{N}$  increases. However, as it can be seen from Figure 7 that depicts the average amount of information at the end of time  $T$  at all nodes (for the same instances of Figure 6), the solution quality also increases when  $\hat{N}$  increases, until the value of  $N^*$  is obtained.

## 5. Conclusions

A routing-collecting problem where a vehicle collects information from a given set of stations is considered. Three mixed integer programming models are introduced. The first model DT is based on a time discretization, where each decision is a multiple of the time unit. The second model NE is an event model, where the visits to stations and transfer operations are modeled as events. Finally, the third model VE considers the vehicle stops as events. A computational study based on randomly generated instances was conducted to compare the three models. The DT model presents a high number of both variables and constraints while the NE and VE models need to be fed with parameters for the maximum number of events permitted. The results show that the NE model is always the worst and that the best model (DT or VE) depends on the instance. For shorter time horizons the DT model performs well, while for longer time horizons the VE is usually faster. The performance of the VE model is better when the optimal number of vehicle stops can be estimated. For future research, both DT and VE formulations can benefit from a study of the polytope defined by each formulation: the continuous relaxation of each model could be strengthened and the polyhedral study could be the base for the implementation of branch-and-cut methods to the WT-VRP. Moreover, the derivation of fast heuristics is important when the size of instances become large.

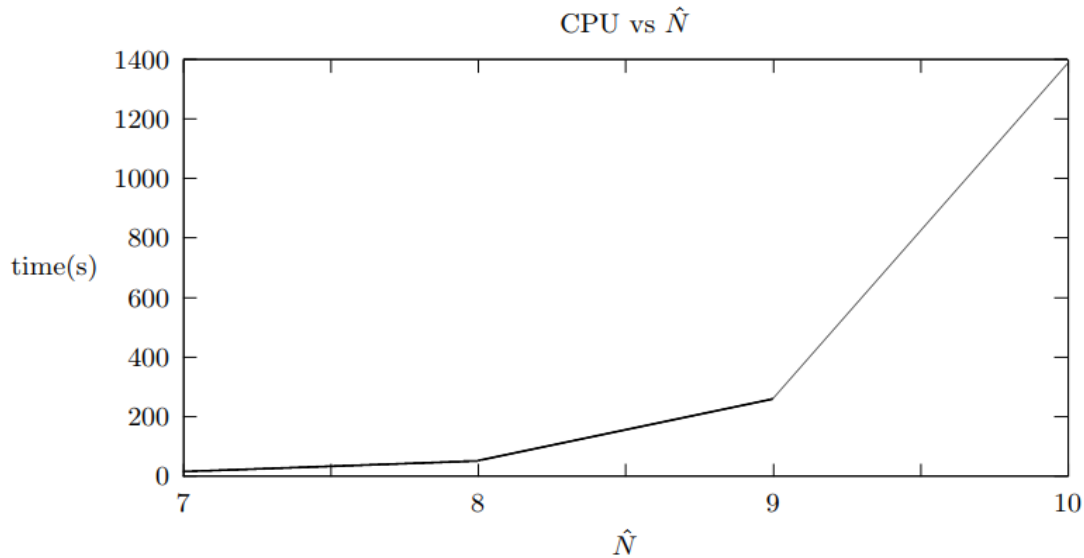


Figure 6: Average running times obtained by the Vehicle Event Model on the instances with  $|V| = 15$  and  $m = 120$ , for different values of  $\hat{N}$ .

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amount of information vs N

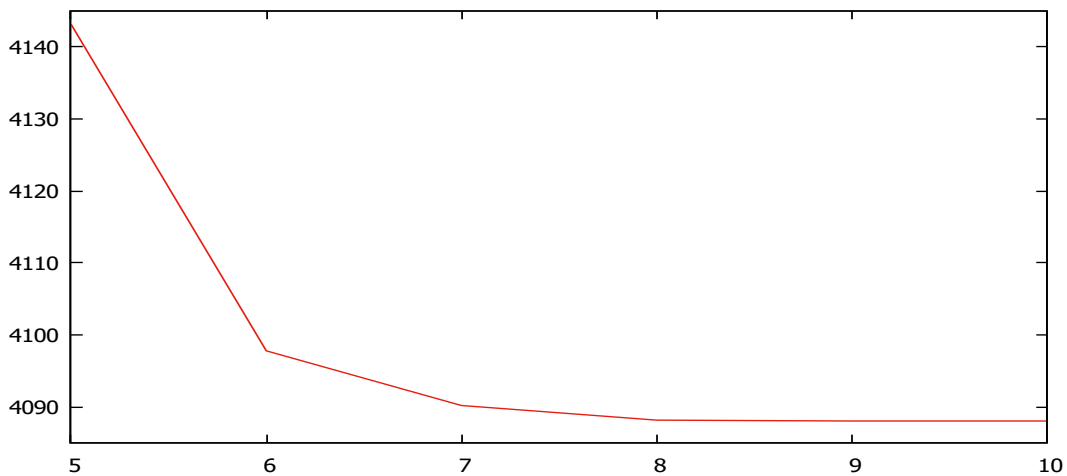


Figure 7: Average amount of information at time  $T$  at all nodes using the Event Vehicle Model with  $|V| = 15, T = 120$  versus different values of  $N$ .

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