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# Seismic reconstruction using FWI with reciprocity misfit functional and dual-sensors data

Florian Faucher<sup>1</sup>,

Giovanni Alessandrini<sup>2</sup>, H el ene Barucq<sup>1</sup>, Maarten V. de Hoop<sup>3</sup>,  
Romina Gaburro<sup>4</sup> and Eva Sincich<sup>2</sup>.

**AIP 2019, Grenoble, France**

July 11<sup>th</sup>, 2019



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<sup>4</sup>Department of Mathematics and Statistics, Health Research Institute (HRI), University of Limerick, Ireland.

# Overview

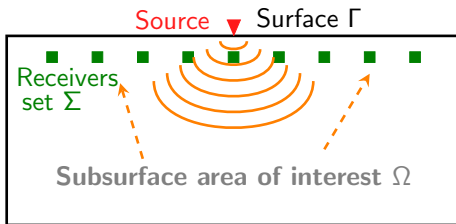
- 1 Introduction
- 2 Time-Harmonic Inverse Problem, FWI
  - Dual-sensors data
  - Iterative reconstruction algorithm
- 3 Reconstruction procedure using dual-sensors data
- 4 Numerical experiments
  - Experiments for acoustic media
  - Comparison of misfit functions
  - Changing the numerical acquisition with  $\mathcal{J}_G$
  - Extension toward elasticity
- 5 Conclusion

# Plan

## 1 Introduction

# Seismic exploration inverse problem

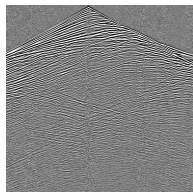
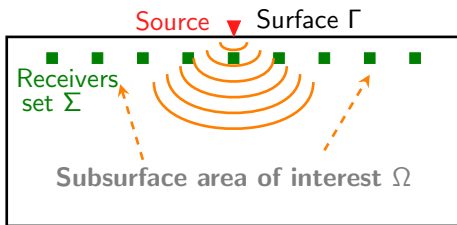
Reconstruction of subsurface Earth properties from seismic campaign:  
collection of **wave** propagation data at the surface.



- ▶ Reflection (back-scattered) partial data,
- ▶ **nonlinear, ill-posed inverse problem.**

# Seismic exploration inverse problem

Reconstruction of subsurface Earth properties from seismic campaign:  
collection of **wave** propagation data at the surface.



- ▶ Reflection (back-scattered) partial data,
- ▶ **nonlinear, ill-posed inverse problem.**

# Plan

- 2 Time-Harmonic Inverse Problem, FWI
  - Dual-sensors data
  - Iterative reconstruction algorithm

# Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters  $\kappa$  and  $\rho$ :

$$\begin{cases} -i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = -\nabla p(\mathbf{x}), \\ -i\omega p(\mathbf{x}) = -\kappa(\mathbf{x})\nabla \cdot \mathbf{v}(\mathbf{x}) + f(\mathbf{x}). \end{cases}$$

$p$ : scalar pressure field,

$\kappa$ : bulk modulus,

$\mathbf{v}$ : vectorial velocity field,

$\rho$ : density,

$f$ : source term,

$\omega$ : angular frequency.



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The system reduces to the Helmholtz equation when  $\rho$  is constant,

$$(-\omega^2 c(\mathbf{x})^{-2} - \Delta)p(\mathbf{x}) = 0,$$

$$\text{with } c(\mathbf{x}) = \sqrt{\kappa(\mathbf{x})\rho(\mathbf{x})^{-1}}.$$

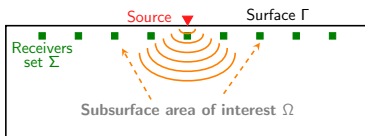
# Measurements from dual-sensors

The quantitative inverse problem aims the recovery of the physical parameters from **surface field measurements**.

**Dual-sensors** record the pressure and vertical velocity:

$$\mathcal{F}(m = (\kappa, \rho)) = \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}})\};$$

$$\{v_n(\mathbf{x}_1), v_n(\mathbf{x}_2), \dots, v_n(\mathbf{x}_{n_{rcv}})\}.$$



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017 – 2018)

TopSeis acquisition ([www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis](http://www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis))

# Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

- ▶  $d$  are the observed data,
- ▶  $\mathcal{F}(m)$  represents the simulation using an initial model  $m$ :



P. Lailly

The seismic inverse problem as a sequence of before stack migrations  
 Conference on Inverse Scattering: Theory and Application, SIAM, 1983



A. Tarantola

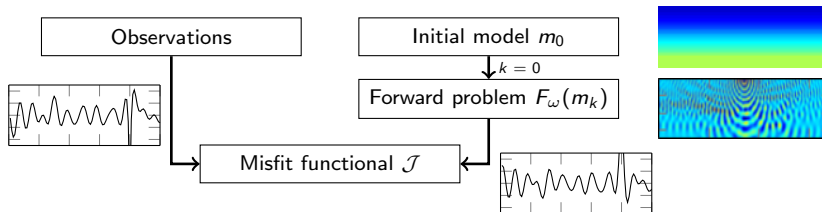
Inversion of seismic reflection data in the acoustic approximation  
 Geophysics, 1984



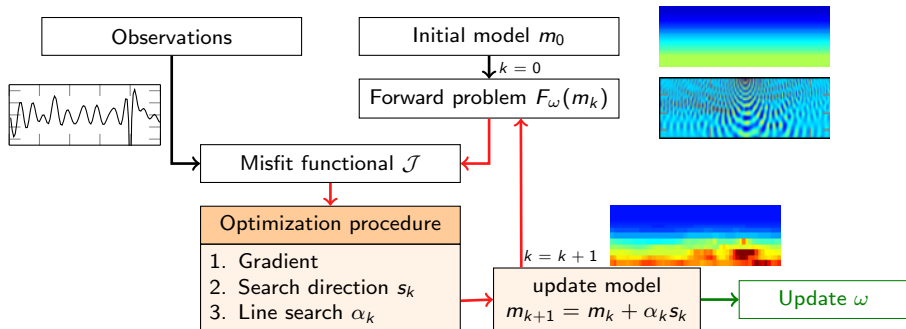
A. Tarantola

Inversion of travel times and seismic waveforms  
 Seismic tomography, 1987

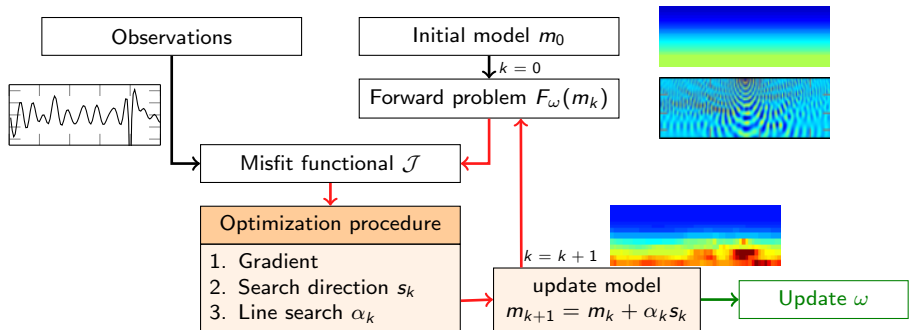
# FWI, iterative minimization



# FWI, iterative minimization



# FWI, iterative minimization



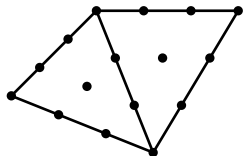
## Numerical methods

- ▶ Adjoint-method for the gradient computation, L-BFGS method,
- ▶ **Hybridizable Discontinuous Galerkin** discretization method,
- ▶ elasticity, anisotropy, viscosity.

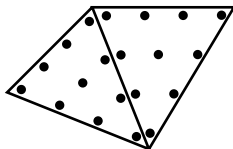
# Hybridizable Discontinuous Galerkin discretization

Hybridizable Discontinuous Galerkin (HDG) discretization:

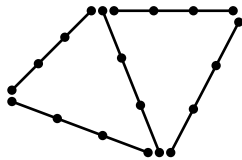
- ▶ **global matrix** with the **faces d.o.f.** only,
- ▶ **local problem** to have the volume solution on DG d.o.f.



Finite Element



Discontinuous Galerkin



HDG

- ▶ Global matrix needs **less memory** than FE and DG (order),
- ▶ the local problems are small and embarrassingly parallel,
- ▶ 1<sup>st</sup> order: same accuracy for  $p$  and  $\mathbf{v}$ ,
- ▶ topography, sub-surface shapes.

# Plan

## 3 Reconstruction procedure using dual-sensors data



# Iterative reconstruction with dual-sensors data



- ▶ Compare the pressure and velocity fields separately ( $L_2$ ):

$$\mathcal{J}_{L_2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

# Iterative reconstruction with dual-sensors data

- ▶ Compare the pressure and velocity fields separately ( $L_2$ ):

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- ▶ Compare the **reciprocity-gap**:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

ESAIM: M2AN, 2019.

# Reciprocity-gap waveform inversion

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

- ▶ Motivated by [Green's identity](#) (using variational formulation).

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- ▶ Motivated by [Green's identity](#) (using variational formulation).
- ▶ [Reciprocity-gap functional](#) from inverse scattering with Cauchy data.



R. Kohn and M. Vogelius

Determining conductivity by boundary measurements II. Interior results  
 Communications on Pure and Applied Mathematics, 1985.



D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory  
 Inverse Problems 21 (1), 2005, 383398.



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization  
 ESAIM: M2AN, 2019.



T. van Leeuwen and W. A. Mulder

A correlation-based misfit criterion for wave-equation traveltime tomography  
 Geophysical Journal International, 2010

# Stability results

Lipschitz-type stability for the Helmholtz equation with partial data,

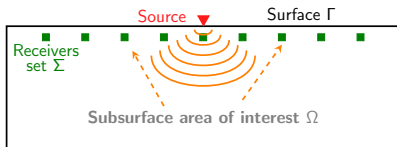
$$\|m_1 - m_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(m_1, m_2))^{1/2},$$

# Stability results

Lipschitz-type stability for the Helmholtz equation with partial data,

$$\|m_1 - m_2\| \leq \mathcal{C}(\mathcal{J}_G(m_1, m_2))^{1/2},$$

- ▶ for piecewise linear parameters.
- ▶ Using back-scattered data from one side in a domain with free surface and absorbing conditions,



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
arXiv:1702.04222, 2017.



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

ESAIM: M2AN, 2019.

# Main feature

It allows the separation of numerical and observational sources:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

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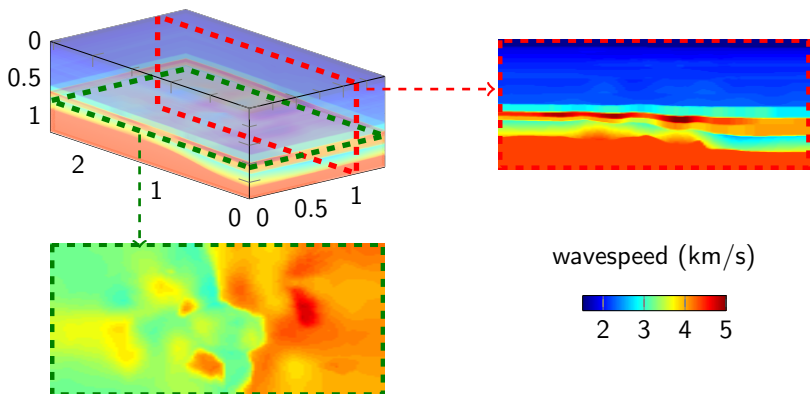
- ▶  $s_1$  is fixed by the observational setup,
- ▶  $s_2$  is **chosen** for the numerical comparisons,
- ▶ **arbitrary positions** of computational source,
- ▶ no need for a priori information on the observational source:  
**position and wavelet are not required**,
- ▶ not possible with the traditional misfit.



- 4 Numerical experiments
  - Experiments for acoustic media
  - Comparison of misfit functions
  - Changing the numerical acquisition with  $\mathcal{I}_G$
  - Extension toward elasticity

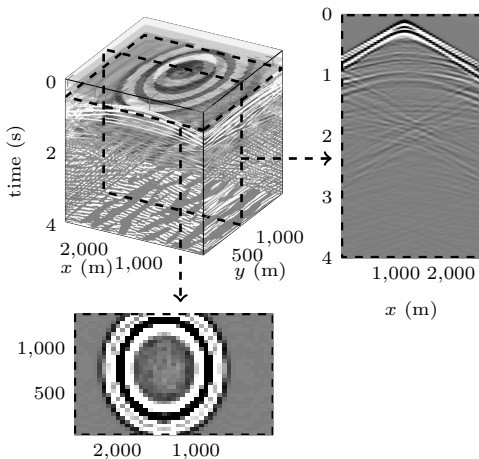
# Experiment setup

3D velocity model  $2.5 \times 1.5 \times 1.2\text{km}$  using dual-sensors data.



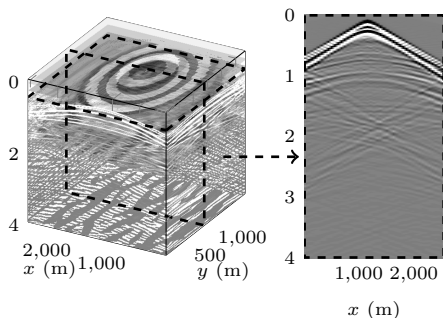
# Experiment setup

We work with time-domain data acquisition.



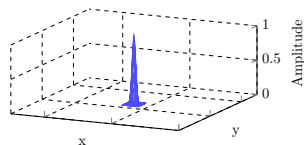
# Experiment setup

We work with time-domain data (pressure and velocity).



Acquisition for the measures

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



For the reconstruction, we apply a Fourier transform of the time data.

# Comparison of misfit functional

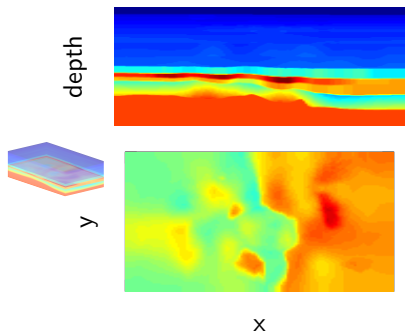
We respect the observational acquisition setup and perform the minimization of  $\mathcal{J}_{L2}$  or  $\mathcal{J}_G$ , frequency from 3 to 15Hz.

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

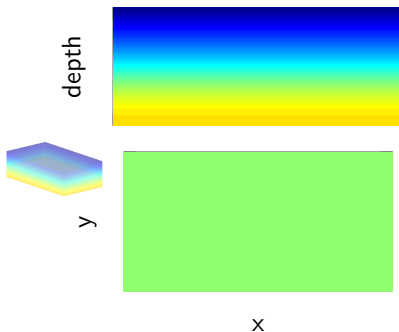
$$\mathcal{J}_G = \frac{1}{2} \sum_{source} \sum_{source} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

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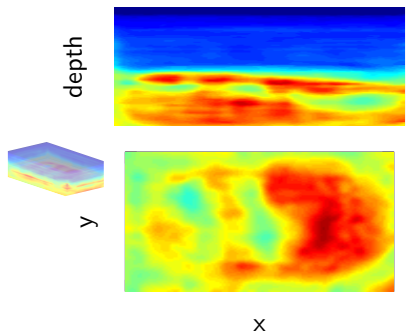
(a) True velocity



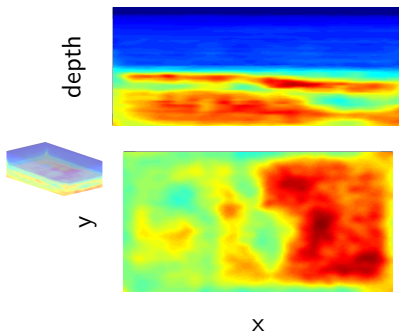
(b) Starting velocity

# Comparison of misfit functional

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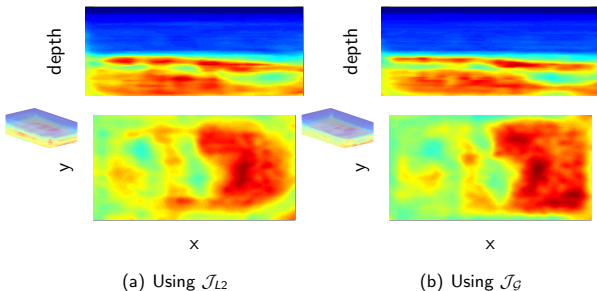
(a) Using  $\mathcal{J}_{L2}$



(b) Using  $\mathcal{J}_G$

# Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of  $\mathcal{J}_{L2}$  or  $\mathcal{J}_G$ , frequency from 3 to 15Hz.



**But the major advantage of  $\mathcal{J}_G$  is the possibility to consider alternative acquisition setup.**

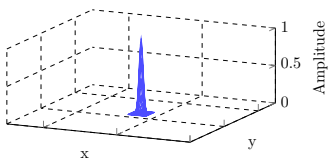


## Experiment with different obs. and sim. acquisition

$$\min \mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Acquisition for the measures  $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,

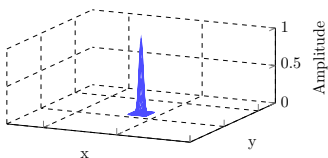


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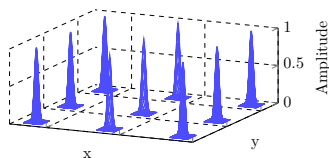
## Acquisition for the measures $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



## Arbitrary numerical acquisition $s_2$

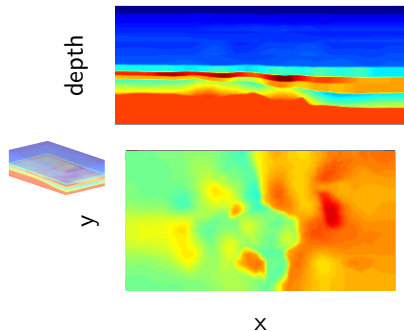
- ▶ **5 sources,**
- ▶ **80m depth,**
- ▶ **multi-point sources,**



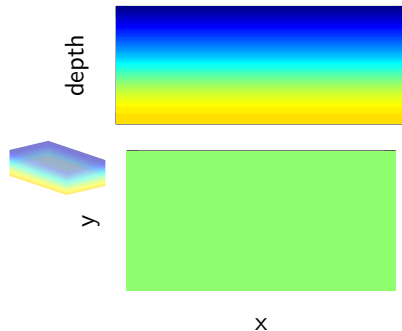
- ▶ No need to know observational source wavelet.
- ▶ Differentiation impossible with least squares types misfit.

# Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size  $2.5 \times 1.5 \times 1.2$  km,  
Simulation using 5 sources only.



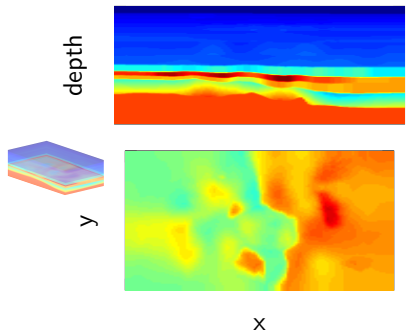
(a) True velocity



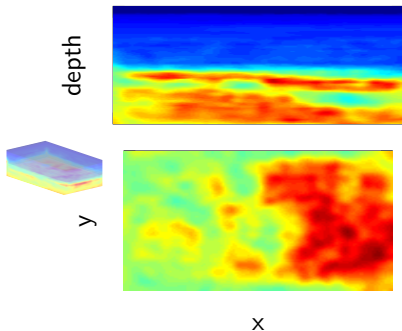
(b) Starting velocity

# Experiment with different obs. and sim. acquisition

Frequency from 3 to 15 Hz,  $2.5 \times 1.5 \times 1.2$  km,  
Simulation using 5 sources only. -33% computational time.



(a) True velocity



(b) 15 Hz reconstruction

# Reciprocity-gap for elasticity

# Reciprocity-gap for elasticity

Wave propagation in elastic media

$$-\nabla \cdot \underline{\sigma}(\mathbf{x}) - \omega^2 \rho(\mathbf{x}) \mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}),$$

$\sigma$  is the stress tensor; elastic isotropy, Lamé parameters  $\lambda$  and  $\mu$ :

$$\underline{\sigma} = \lambda \text{Tr}(\underline{\epsilon}) I_d + 2\mu \underline{\epsilon}.$$

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$$\underline{\sigma} = \lambda \text{Tr}(\underline{\epsilon}) I_d + 2\mu \underline{\epsilon}.$$

Three (heterogeneous) parameters to characterize the medium:

- ▶  $\lambda$  and  $\mu$ , or  $c_p = \sqrt{(\lambda + 2\mu)/\rho}$ ,  $c_s = \sqrt{\mu/\rho}$
- ▶ Density  $\rho$ .

Increased computational requirement and the ill-posedness of the inverse problem.

# Elastic reciprocity formula

In elasticity, reciprocity needs measurements of  $\sigma$  and  $\mathbf{u}$

$$\mathcal{F}(m := (\lambda, \mu, \rho)) = \{ \mathbf{u}(\mathbf{x}) |_{\Sigma}, (\underline{\sigma}(\mathbf{x}) \cdot \mathbf{n}) |_{\Sigma} \}.$$



# Elastic reciprocity formula

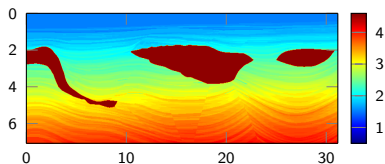
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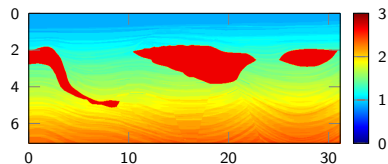
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_{\mathbf{u}}^{(s_1)T} \mathcal{F}_{\underline{\sigma} \cdot \mathbf{n}}^{(s_2)} - d_{\underline{\sigma} \cdot \mathbf{n}}^{(s_1)T} \mathcal{F}_{\mathbf{u}}^{(s_2)} \|^2.$$

# Pluto model

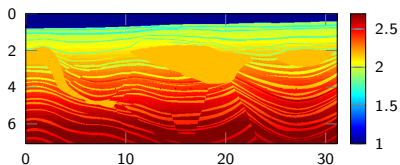
2D elastic models of size  $31 \times 7\text{km}$ .



(a) P-wave speed



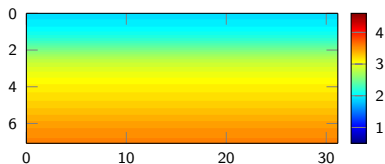
(b) S-wave speed



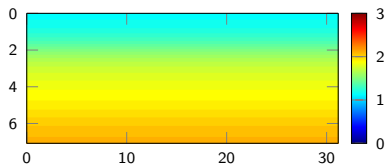
(c) Density

# Pluto model

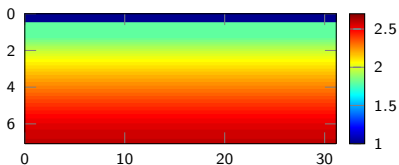
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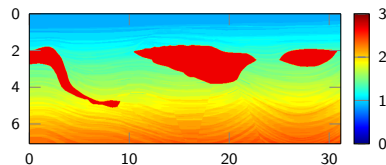
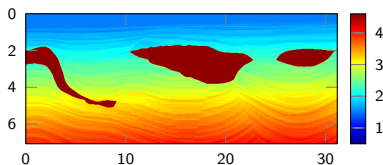
(b) S-wave speed



(c) Density

# Reconstruction setup

- ▶ The density remains fixed; frequency from 0.50 to 7 Hz,
- ▶ Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.



Observational acquisition:

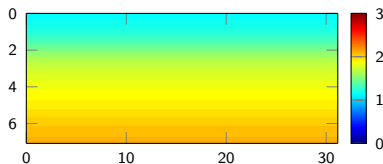
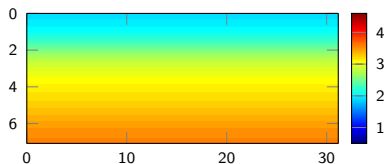
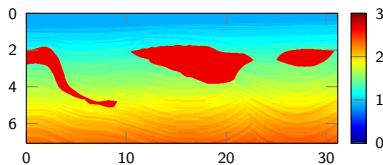
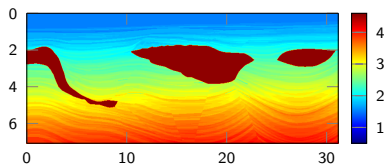
- ▶ 150 sources,
- ▶ 20 m depth.

Computational acquisition:

- ▶ 30 sources,
- ▶ 10 m depth.

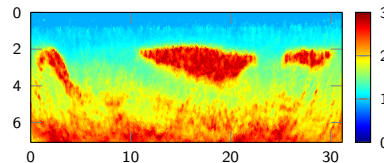
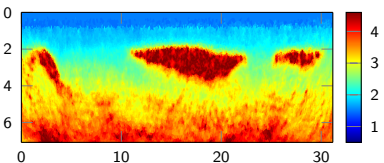
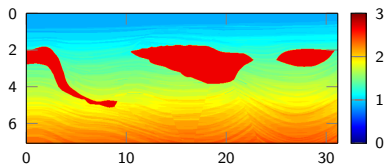
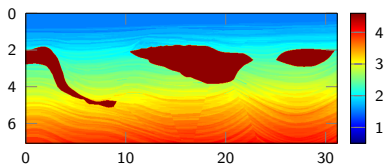
# Reciprocity waveform inversion

- ▶ The density remains fixed; frequency from 0.50 to 7 Hz,
- ▶ Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.



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# Plan

## 5 Conclusion

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## Quantitative time-harmonic inverse wave problem:

- ▶ Hybridizable Discontinuous Galerkin discretization, HPC,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ acoustic, elastic, anisotropy, 2D, 3D, attenuation,
- ▶ stability and convergence analysis.

## Reciprocity-gap waveform inversion:

- ▶ minimal information on the acquisition setup,
- ▶ reduced computational cost,
- ▶ other applications (elasticity, seismology, helioseismology),
- ▶ **perspective:** design efficient setup; data (rotational seismology).



# Conclusion

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THANK YOU



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