



Seismic reconstruction using FWI with reciprocity misfit functional and dual-sensors data

Florian Faucher, Giovanni Alessandrini, Hélène Barucq, Maarten V. de Hoop, Romina Gaburro, Eva Sincich

► To cite this version:

Florian Faucher, Giovanni Alessandrini, Hélène Barucq, Maarten V. de Hoop, Romina Gaburro, et al.. Seismic reconstruction using FWI with reciprocity misfit functional and dual-sensors data. Applied Inverse Problems Conference, Jul 2019, Grenoble, France. hal-02184215

HAL Id: hal-02184215

<https://hal.science/hal-02184215>

Submitted on 16 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Seismic reconstruction using FWI with reciprocity misfit functional and dual-sensors data

Florian Faucher¹,

Giovanni Alessandrini², H      Barucq¹, Maarten V. de Hoop³,
Romina Gaburro⁴ and Eva Sincich².

AIP 2019, Grenoble, France

July 11th, 2019



¹Project-Team Magique-3D, Inria Bordeaux Sud-Ouest, France.

²Dipartimento di Matematica e Geoscienze, Universit   di Trieste, Italy.

³Department of Computational and Applied Mathematics and Earth Science, Rice University, Houston, USA

⁴Department of Mathematics and Statistics, Health Research Institute (HRI), University of Limerick, Ireland.

Overview

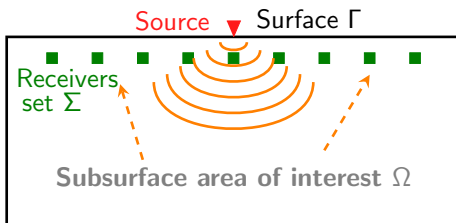
- 1 Introduction
- 2 Time-Harmonic Inverse Problem, FWI
 - Dual-sensors data
 - Iterative reconstruction algorithm
- 3 Reconstruction procedure using dual-sensors data
- 4 Numerical experiments
 - Experiments for acoustic media
 - Comparison of misfit functions
 - Changing the numerical acquisition with \mathcal{I}_g
 - Extension toward elasticity
- 5 Conclusion

Plan

1 Introduction

Seismic exploration inverse problem

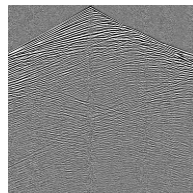
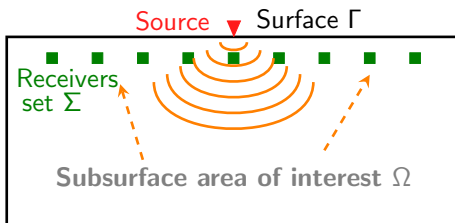
Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



- ▶ Reflection (back-scattered) partial data,
- ▶ **nonlinear, ill-posed inverse problem.**

Seismic exploration inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



- ▶ Reflection (back-scattered) partial data,
- ▶ **nonlinear, ill-posed inverse problem.**

Plan

2 Time-Harmonic Inverse Problem, FWI

- Dual-sensors data
- Iterative reconstruction algorithm

Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters κ and ρ :

$$\begin{cases} -i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = -\nabla p(\mathbf{x}), \\ -i\omega p(\mathbf{x}) = -\kappa(\mathbf{x})\nabla \cdot \mathbf{v}(\mathbf{x}) + f(\mathbf{x}). \end{cases}$$

p : scalar pressure field,

\mathbf{v} : vectorial velocity field,

f : source term,

κ : bulk modulus,

ρ : density,

ω : angular frequency.

Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters κ and ρ :

$$\begin{cases} -i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = -\nabla p(\mathbf{x}), \\ -i\omega p(\mathbf{x}) = -\kappa(\mathbf{x})\nabla \cdot \mathbf{v}(\mathbf{x}) + f(\mathbf{x}). \end{cases}$$

p : scalar pressure field,

κ : bulk modulus,

\mathbf{v} : vectorial velocity field,

ρ : density,

f : source term,

ω : angular frequency.

The system reduces to the Helmholtz equation when ρ is constant,

$$(-\omega^2 c(\mathbf{x})^{-2} - \Delta)p(\mathbf{x}) = 0,$$

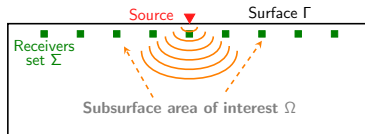
$$\text{with } c(\mathbf{x}) = \sqrt{\kappa(\mathbf{x})\rho(\mathbf{x})^{-1}}.$$

Measurements from dual-sensors

The quantitative inverse problem aims the recovery of the physical parameters from **surface field measurements**.

Dual-sensors record the pressure and vertical velocity:

$$\mathcal{F}(m = (\kappa, \rho)) = \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}})\};$$
$$\{v_n(\mathbf{x}_1), v_n(\mathbf{x}_2), \dots, v_n(\mathbf{x}_{n_{rcv}})\}.$$



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017 – 2018)

TopSeis acquisition (www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis)

Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

- ▶ d are the observed data,
- ▶ $\mathcal{F}(m)$ represents the simulation using an initial model m :



P. Lailly

The seismic inverse problem as a sequence of before stack migrations
Conference on Inverse Scattering: Theory and Application, SIAM, 1983



A. Tarantola

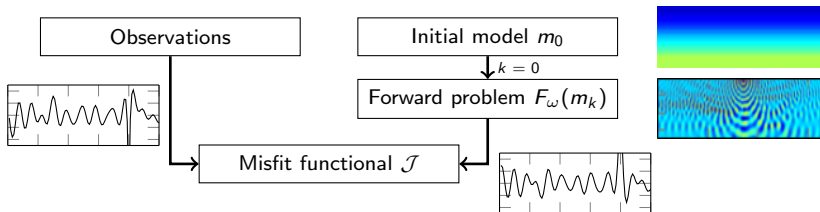
Inversion of seismic reflection data in the acoustic approximation
Geophysics, 1984



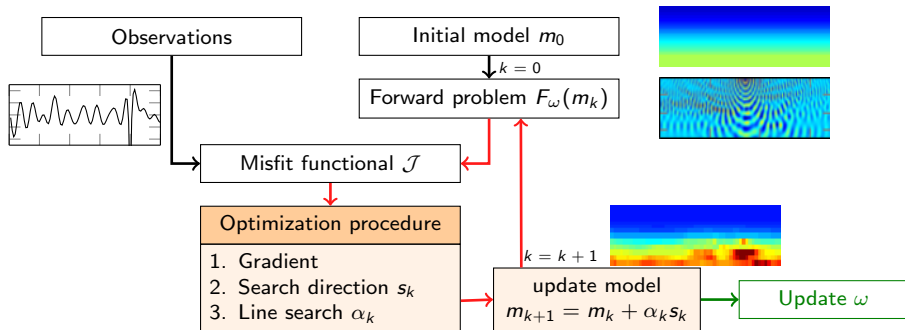
A. Tarantola

Inversion of travel times and seismic waveforms
Seismic tomography, 1987

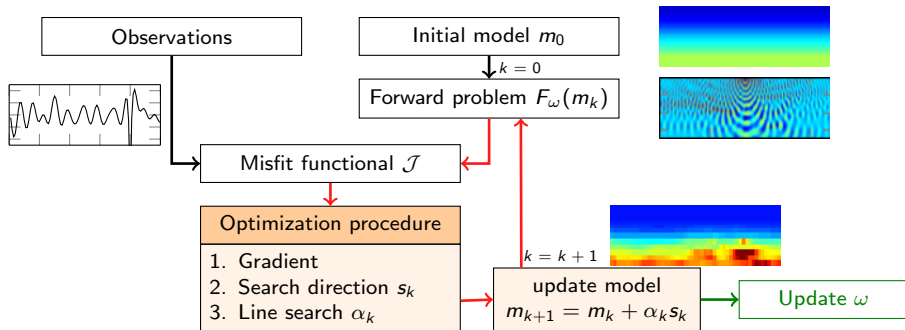
FWI, iterative minimization



FWI, iterative minimization



FWI, iterative minimization



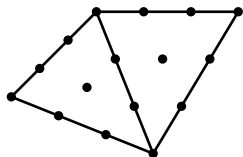
Numerical methods

- ▶ Adjoint-method for the gradient computation, L-BFGS method,
- ▶ **Hybridizable Discontinuous Galerkin** discretization method,
- ▶ elasticity, anisotropy, viscosity.

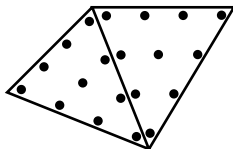
Hybridizable Discontinuous Galerkin discretization

Hybridizable Discontinuous Galerkin (HDG) discretization:

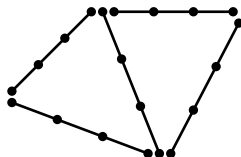
- ▶ **global matrix** with the **faces d.o.f.** only,
- ▶ **local problem** to have the volume solution on DG d.o.f.



Finite Element



Discontinuous Galerkin



HDG

- ▶ Global matrix needs **less memory** than FE and DG (order),
- ▶ the local problems are small and embarrassingly parallel,
- ▶ 1st order: same accuracy for p and \mathbf{v} ,
- ▶ topography, sub-surface shapes.

Plan

③ Reconstruction procedure using dual-sensors data

Iterative reconstruction with dual-sensors data

- ▶ Compare the pressure and velocity fields separately (L_2):

$$\mathcal{J}_{L_2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

Iterative reconstruction with dual-sensors data

- ▶ Compare the pressure and velocity fields separately (L_2):

$$\mathcal{J}_{L_2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

- ▶ Compare the **reciprocity-gap**:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

ESAIM: M2AN, 2019.

Reciprocity-gap waveform inversion

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

- Motivated by [Green's identity](#) (using variational formulation).

Reciprocity-gap waveform inversion

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

- ▶ Motivated by [Green's identity](#) (using variational formulation).
- ▶ [Reciprocity-gap functional](#) from inverse scattering with Cauchy data.



[R. Kohn and M. Vogelius](#)

Determining conductivity by boundary measurements II. Interior results
[Communications on Pure and Applied Mathematics, 1985.](#)



[D. Colton and H. Haddar](#)

An application of the reciprocity gap functional to inverse scattering theory
[Inverse Problems 21 \(1\), 2005, 383398.](#)



[G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich](#)

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
[ESAIM: M2AN, 2019.](#)



[T. van Leeuwen and W. A. Mulder](#)

A correlation-based misfit criterion for wave-equation traveltime tomography
[Geophysical Journal International, 2010](#)

Stability results

Lipschitz-type stability for the Helmholtz equation with partial data,

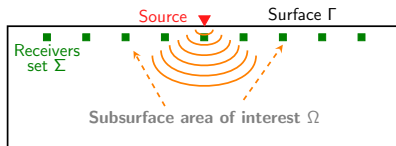
$$\|m_1 - m_2\| \leq \mathcal{C}(\mathcal{I}_{\mathcal{G}}(m_1, m_2))^{1/2},$$

Stability results

Lipschitz-type stability for the Helmholtz equation with partial data,

$$\|m_1 - m_2\| \leq \mathcal{C}(\mathcal{I}_{\mathcal{G}}(m_1, m_2))^{1/2},$$

- ▶ for piecewise linear parameters.
- ▶ Using back-scattered data from one side in a domain with free surface and absorbing conditions,



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data
[arXiv:1702.04222](https://arxiv.org/abs/1702.04222), 2017.



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

[ESAIM: M2AN](#), 2019.

Main feature

It allows the separation of numerical and observational sources:

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Main feature



It allows the separation of numerical and observational sources:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

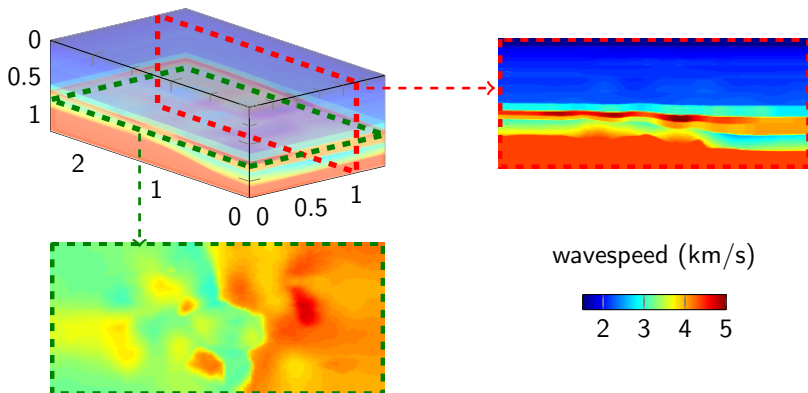
- ▶ s_1 is fixed by the observational setup,
- ▶ s_2 is **chosen** for the numerical comparisons,
- ▶ **arbitrary positions** of computational source,
- ▶ no need for a priori information on the observational source:
position and wavelet are not required,
- ▶ not possible with the traditional misfit.

4 Numerical experiments

- Experiments for acoustic media
- Comparison of misfit functions
- Changing the numerical acquisition with \mathcal{I}_G
- Extension toward elasticity

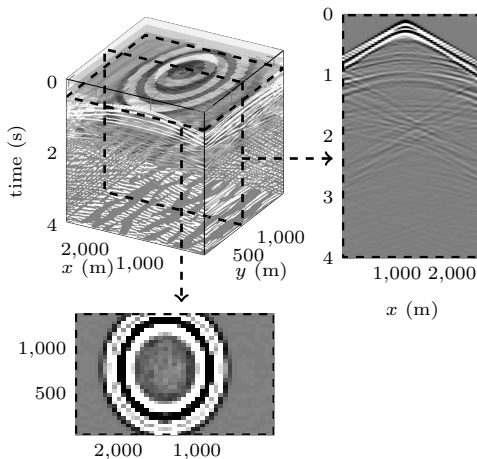
Experiment setup

3D velocity model $2.5 \times 1.5 \times 1.2\text{km}$ using dual-sensors data.



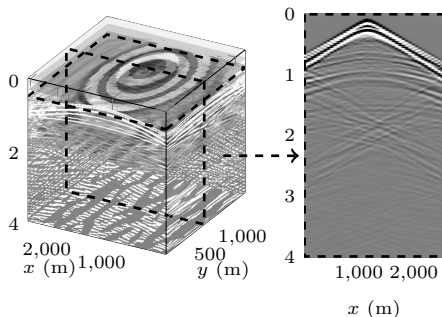
Experiment setup

We work with time-domain data acquisition.



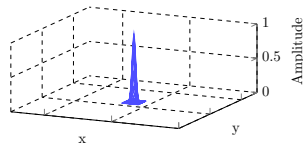
Experiment setup

We work with time-domain data (pressure and velocity).



Acquisition for the measures

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



For the reconstruction, we apply a Fourier transform of the time data.

Comparison of misfit functional

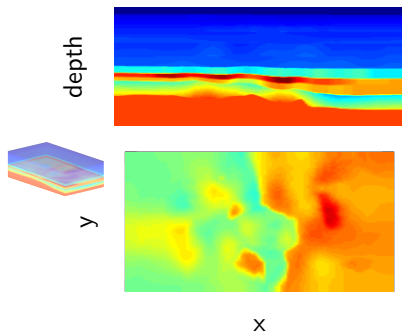
We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or $\mathcal{J}_{\mathcal{G}}$, frequency from 3 to 15Hz.

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

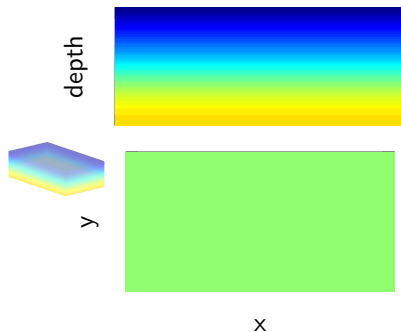
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{source} \sum_{source} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or \mathcal{J}_G , frequency from 3 to 15Hz.



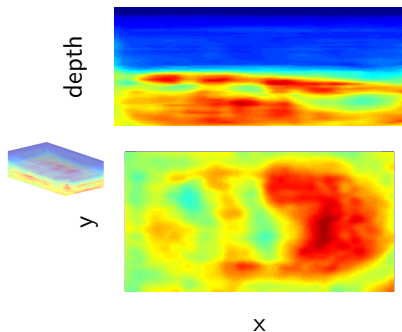
(a) True velocity



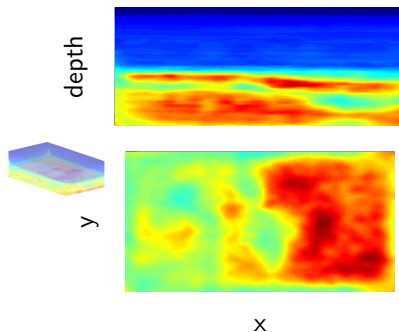
(b) Starting velocity

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or \mathcal{J}_G , frequency from 3 to 15Hz.



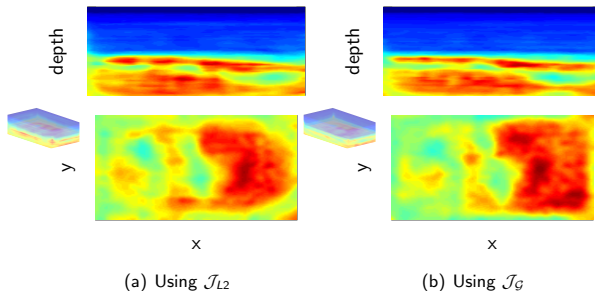
(a) Using \mathcal{J}_{L2}



(b) Using \mathcal{J}_G

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or \mathcal{J}_G , frequency from 3 to 15Hz.



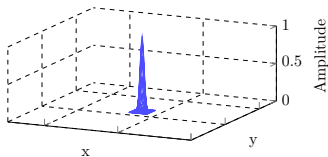
But the major advantage of \mathcal{J}_G is the possibility to consider alternative acquisition setup.

Experiment with different obs. and sim. acquisition

$$\min \mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Acquisition for the measures s_1

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,

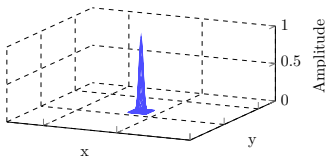


Experiment with different obs. and sim. acquisition

$$\min \mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

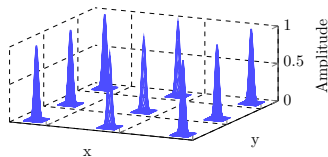
Acquisition for the measures s_1

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



Arbitrary numerical acquisition s_2

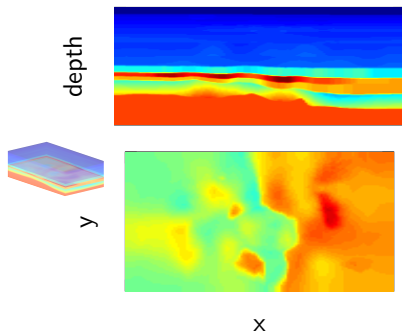
- ▶ **5 sources,**
- ▶ **80m depth,**
- ▶ **multi-point sources,**



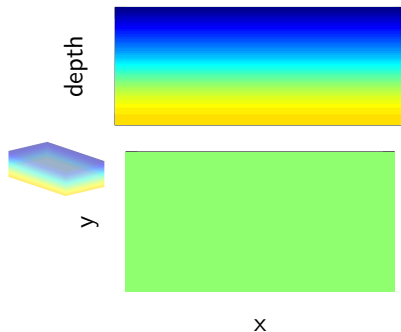
- ▶ No need to know observational source wavelet.
- ▶ Differentiation impossible with least squares types misfit.

Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size $2.5 \times 1.5 \times 1.2$ km,
Simulation using 5 sources only.



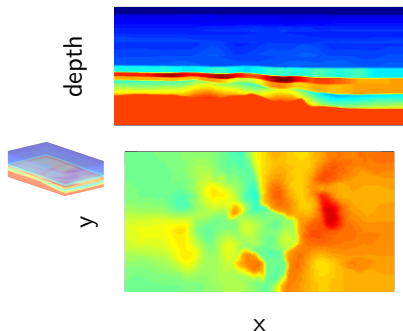
(a) True velocity



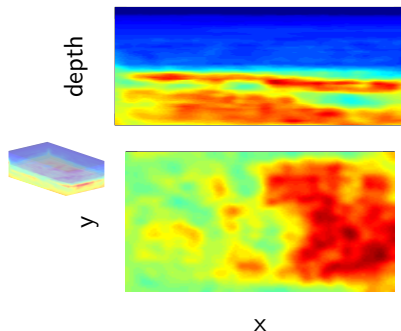
(b) Starting velocity

Experiment with different obs. and sim. acquisition

Frequency from 3 to 15 Hz, $2.5 \times 1.5 \times 1.2$ km,
Simulation using 5 sources only. -33% computational time.



(a) True velocity



(b) 15 Hz reconstruction

Reciprocity-gap for elasticity

Reciprocity-gap for elasticity

Wave propagation in elastic media

$$-\nabla \cdot \underline{\sigma}(\mathbf{x}) - \omega^2 \rho(\mathbf{x}) \mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}),$$

σ is the stress tensor; elastic isotropy, Lamé parameters λ and μ :

$$\underline{\sigma} = \lambda \text{Tr}(\underline{\epsilon}) I_d + 2\mu \underline{\epsilon}.$$

Reciprocity-gap for elasticity

Wave propagation in elastic media

$$-\nabla \cdot \underline{\sigma}(\mathbf{x}) - \omega^2 \rho(\mathbf{x}) \mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}),$$

σ is the stress tensor; elastic isotropy, Lamé parameters λ and μ :

$$\underline{\sigma} = \lambda \text{Tr}(\underline{\epsilon}) I_d + 2\mu \underline{\epsilon}.$$

Three (heterogeneous) parameters to characterize the medium:

- ▶ λ and μ , or $c_p = \sqrt{(\lambda + 2\mu)/\rho}$, $c_s = \sqrt{\mu/\rho}$
- ▶ Density ρ .

Increased computational requirement and the ill-posedness of the inverse problem.

Elastic reciprocity formula

In elasticity, reciprocity needs measurements of σ and \mathbf{u}

$$\mathcal{F}(m := (\lambda, \mu, \rho)) = \{ \mathbf{u}(\mathbf{x}) \mid_{\Sigma}, (\underline{\sigma}(\mathbf{x}) \cdot \mathbf{n}) \mid_{\Sigma} \}.$$

Elastic reciprocity formula

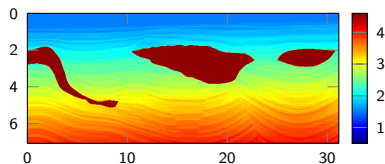
In elasticity, reciprocity needs measurements of σ and \mathbf{u}

$$\mathcal{F}(m := (\lambda, \mu, \rho)) = \{ \mathbf{u}(\mathbf{x}) \mid_{\Sigma}, (\underline{\sigma}(\mathbf{x}) \cdot \mathbf{n}) \mid_{\Sigma} \}.$$

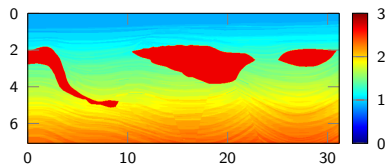
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_{\mathbf{u}}^{(s_1)T} \mathcal{F}_{\sigma \cdot \mathbf{n}}^{(s_2)} - d_{\sigma \cdot \mathbf{n}}^{(s_1)T} \mathcal{F}_{\mathbf{u}}^{(s_2)} \|^2.$$

Pluto model

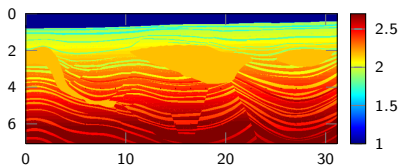
2D elastic models of size $31 \times 7\text{km}$.



(a) P-wave speed



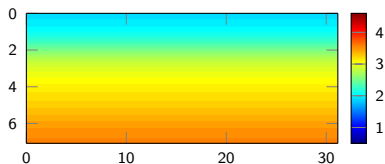
(b) S-wave speed



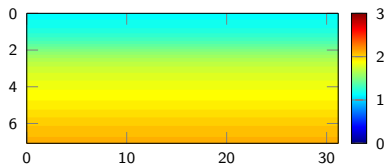
(c) Density

Pluto model

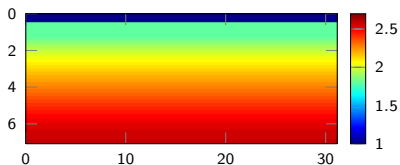
2D elastic models of size $31 \times 7\text{km}$.



(a) P-wave speed



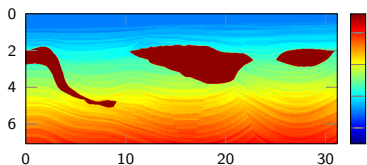
(b) S-wave speed



(c) Density

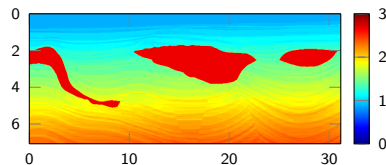
Reconstruction setup

- ▶ The density remains fixed; frequency from 0.50 to 7 Hz,
- ▶ Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.



Observational acquisition:

- ▶ 150 sources,
- ▶ 20 m depth.

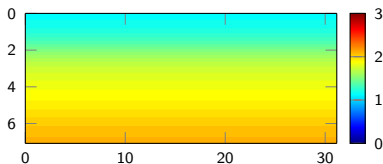
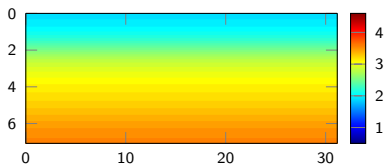
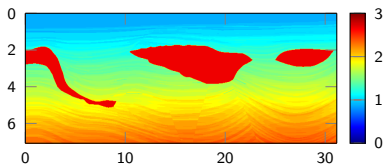
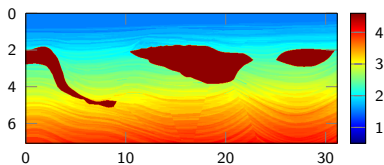


Computational acquisition:

- ▶ 30 sources,
- ▶ 10 m depth.

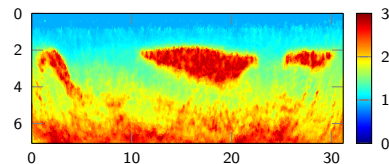
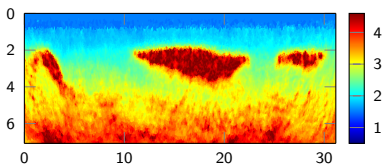
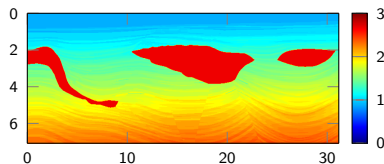
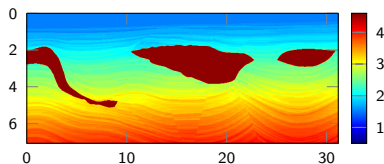
Reciprocity waveform inversion

- ▶ The density remains fixed; frequency from 0.50 to 7 Hz,
- ▶ Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.



Reciprocity waveform inversion

- ▶ The density remains fixed; frequency from 0.50 to 7 Hz,
- ▶ Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.



Plan

5 Conclusion

Conclusion



Quantitative time-harmonic inverse wave problem:

- ▶ Hybridizable Discontinuous Galerkin discretization, HPC,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ acoustic, elastic, anisotropy, 2D, 3D, attenuation,
- ▶ stability and convergence analysis.

Reciprocity-gap waveform inversion:

- ▶ minimal information on the acquisition setup,
- ▶ reduced computational cost,
- ▶ other applications (elasticity, seismology, helioseismology),
- ▶ perspective: design efficient setup; data (rotational seismology).

Conclusion



Quantitative time-harmonic inverse wave problem:

- ▶ Hybridizable Discontinuous Galerkin discretization, HPC,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ acoustic, elastic, anisotropy, 2D, 3D, attenuation,
- ▶ stability and convergence analysis.

Reciprocity-gap waveform inversion:

- ▶ minimal information on the acquisition setup,
- ▶ reduced computational cost,
- ▶ other applications (elasticity, seismology, helioseismology),
- ▶ perspective: design efficient setup; data (rotational seismology).

THANK YOU



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017 – 2018)

TopSeis acquisition (www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis)



P. Lailly

The seismic inverse problem as a sequence of before stack migrations

Conference on Inverse Scattering: Theory and Application, SIAM, 1983



A. Tarantola

Inversion of seismic reflection data in the acoustic approximation

Geophysics, 1984



A. Tarantola

Inversion of travel times and seismic waveforms

Seismic tomography, 1987



H. Barucq, H. Calandra, G. Chavent, F. Faucher

A priori estimates of attraction basins for velocity model reconstruction by time-harmonic Full Waveform Inversion and Data Space Reflectivity formulation

Research Report, 2019, <https://hal.archives-ouvertes.fr/hal-02016373/file/RR-9253.pdf> .



G. Alessandrini, S. Vessella

Lipschitz stability for the inverse conductivity problem

Adv. in Appl. Math., 2005



E. Beretta, M. V. de Hoop, F. Faucher, O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates

SIAM Journal on Mathematical Analysis, 2016



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

ESAIM: M2AN, 2019.



R. Kohn and M. Vogelius

Determining conductivity by boundary measurements II. Interior results

Communications on Pure and Applied Mathematics, 1985.



D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory

Inverse Problems 21 (1), 2005, 383398.



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data

arXiv:1702.04222, 2017.



M. Grote, M. Kray, U. Nahum

Adaptive eigenspace method for inverse scattering problems in the frequency domain

Inverse Problems, 2017.



M. Grote, U. Nahum

Adaptive eigenspace for multi-parameter inverse scattering problems

Computers & Mathematics with Applications, 2019.



H. Barucq, F. Faucher, O. Scherzer

Eigenvector Model Descriptors for Solving an Inverse Problem of Helmholtz Equation

arXiv preprint arXiv:1903.08991 (2019).