HAL
open science

Passive control of an harmonically excited vibrating beam using a vibro-impact nonlinear energy sink

Etienne Gourc, Sébastien Seguy, Guilhem Michon, Alain Berlioz

To cite this version:

Etienne Gourc, Sébastien Seguy, Guilhem Michon, Alain Berlioz. Passive control of an harmonically excited vibrating beam using a vibro-impact nonlinear energy sink. 21ème Congrès Français de Mécanique, 26-30 août 2013, Bordeaux, France, Aug 2013, Bordeaux, France. hal-02183201

HAL Id: hal-02183201

https://hal.science/hal-02183201v1

Submitted on 1 Jul 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Passive control of an harmonically excited vibrating beam using a vibro-impact nonlinear energy sink

E. Gourc[a], S. Seguy[a], G. Michon[b], A. Berlioz[c]

a. *Université de Toulouse, ICA, INSA*
b. *Université de Toulouse, ICA, ISAE*
c. *Université de Toulouse, ICA, UPS*

Abstract :

Recently, it has been demonstrated that Vibro-Impact Nonlinear Energy Sink (VI-NES) can be used efficiently to mitigate vibration under transient loading. In this paper, the behavior of an harmonically excited Clamped-Free (C-F) beam with an embedded VI-NES is investigated theoretically. Due to the small mass ratio between the beam and the flying mass of the NES, the equation of motion are analyzed using the method of multiple scales. Theoretical prediction are validate by numerical simulation.

Mots clefs : Energy pumping ; Vibro-impact Nonlinear Energy Sink

1 Introduction

Targeted Energy Tranfer (TET) – aslo called energy pumping – has been widely studied during the last decade. It consists in the addition of a small mass essential nonlinear oscillator – called Nonlinear Energy Sink (NES) – to a primary structure to protect. Properly designed, the NES may lead to efficient vibration absoption. Transient TET has been extensively studied. In [1, 2] it has been demonstrated that the main phenomena allowing TET is based on the 1 : 1 resonance capture. Under periodic forcing, the introduction of an asymptotic procedure [3] partitioning slow-fast dynamics has shown that in addition to periodic regimes, system with NES can exhibit beating response, refered as Strongly Modulated Response (SMR) [4]. Such a response has been experimentally observed in [5]. All the afore mentionned studies deal with discrete primary system and NES with cubic stiffness nonlinearity. Recent studies have shown that piecewise linear system can be used as NES [6]. The main advantage of this kind of system over cubic type NES is that they are often easier to built and also easier to integrate on a real structure. Vibro-impact type NES have been studied in [7, 8], however, these study concentrated around numerical simulations. In a recent study, the invariant manifold approach has been extended to VI-NES under transient loading [9]. The use of classic NES with cubic nonlinearity to mitigate vibration on an impulsively loaded hinged-hinged beam has been sutied numerically in [10] and theoretically in [11]. In this paper, an harmonically excited Clamped-Free (C-F) beam with an embedded VI-NES is investigated. The first section is devoted to the presentation of the mechanical model. In the second section, the equations are analysed using the method of multiple scales in the regime of 1 : 1 resonance with the first bending mode of the beam. Numerical simulations are presented in the next section. Finaly, concluding remarks are addressed.

2 Description of the model

The system considered herein consist of an harmonically excited C-F beam with an embedded VI-NES. The dynamic of the beam is governed by linear Euler-Bernoulli theory. The system is depicted in Fig. 1 and the equations of motion in non-dimensional form are given by :

$$\begin{aligned} &\ddot{u} + u'''' = \epsilon G(t)\delta(x-d) + U\delta(x-b)\textstyle\sum_n \delta(t-t_n) \\ &\epsilon\ddot{y} = -U\textstyle\sum_n \delta(t-t_n) \end{aligned} \tag{1}$$

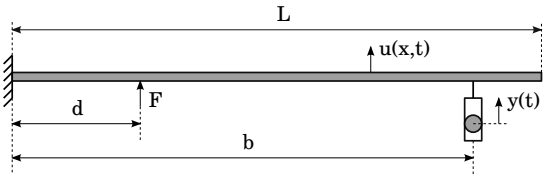

FIGURE 1 – Schema of the system

with

$$|u(b,t_n) - y(t_n)| = 1 \tag{2}$$

where $\epsilon \ll 1$ represents the mass ratio between the beam and the free mass of the VI-NES. The partial differential equations 1 are now discretized. The modal functions satisfies the orthogonality condition $\int_0^1 \phi_i(x)\phi_j(x)dx = \delta_{i,j}$. The discretized equations of motion are written as :

$$\begin{array}{l} \ddot{q}_j + \omega_j^2 q_j + \epsilon\lambda_j \dot{q}_j = \epsilon G(t)\phi_j(d) + U\phi_j(b)\sum_n \delta(t-t_n) \\ \epsilon\ddot{y} = -U\sum_n \delta(t-t_n) \end{array} \tag{3}$$

And the impact condition Eq. (2) is rewritten as :

$$|\sum_j \phi_j(b)q_j(t_n) - y(t_n)| = 1 \tag{4}$$

Non-smooth modeling technique are used to describe the impact between the NES and the beam. The impact is considered as an infinite force applied to the system, with an opposite sign due to the action reaction principle. U is the ammount of momentum transfered during impact and satisfies the following impact constitutive relation :

$$\dot{u}(b,t_n^+) - \dot{y}(t_n^+) = -r\left[\dot{u}(b,t_n^-) - \dot{y}(t_n^-)\right], \quad u(x,t_n^+) = u(x,t_n^-), \quad y(t_n^+) = y(t_n^-) \tag{5}$$

Where the superscripts $-$ and $+$ denotes time immediately before and after impact. Equation (3) can be integrated numerically until an impact occurs. The pre-impact velocities are known, but the post-impact velocities needs to be determined. To this end, Eq. (3) is integrated around a small time instant t_n yielding :

$$\begin{array}{l} \dot{q}_j(t_n^+) - \dot{q}_j(t_n^-) = \phi_j(b)U \\ \dot{y}(t_n^+) - \dot{y}(t_n^-) = -\dfrac{U}{\epsilon} \end{array} \tag{6}$$

The system has $N+1$ equations and $N+2$ unknowns. The missing equation is obtained using Eq. (5) :

$$\sum_j \phi_j(b)\dot{q}_j(t_n^+) - \dot{y}(t_n^+) = -r[\sum_j \phi_j(b)\dot{q}_j(t_n^+) - \dot{y}(t_n^+)] \tag{7}$$

System Eq. (6, 7) is solved for U giving :

$$U = -\epsilon\frac{(1+r)\left(\sum_j \phi_j(b)\dot{q}_j(t_n^-) - \dot{y}(t_n^-)\right)}{1+\epsilon\sum_j \phi_j(b)^2} \tag{8}$$

The post-impact velocities are then computed using Eq. (6).

3 Theoretical analysis

The response regimes of the system are investigated in the vicinity of the resonance of the first bending mode. Then $G(t)$ in Eq. (3) is given by :

$$G(t) = G\sin\left((1+\epsilon\sigma)t\right) \tag{9}$$

New variables are introduced as follow :

$$v_j(t) = q_j(t) + \epsilon\phi_j(b)y(t) \tag{10}$$

Substituting Eq. (9, 10) into Eq. (3) gives :

$$\begin{array}{l} \ddot{v}_j + \omega_j^2\left(v_j - \epsilon\phi_j(b)y\right) + \epsilon\lambda_j\left(\dot{v}_j - \epsilon\phi_j(b)\dot{y}\right) = \epsilon\phi_j(d)G\sin\left((1+\epsilon\sigma)t\right) \\ \ddot{y} = (1+r)\left(\frac{\sum_j \phi_j(b)\dot{v}_j}{1+\epsilon\sum_j \phi_j(b)^2} - \dot{y}\right)\sum_n \delta(t-t_n) \end{array} \tag{11}$$

And the impact condition Eq. (4) is rewritten as :

$$|\sum_j \phi_j(b)v_j - (1+\epsilon\sum_j \phi_j(b)^2)y| = 1 \tag{12}$$

Due to the smallness of the mass ratio ϵ, it can be used as a small parameter in multiple scales expansion in the form :

$$\begin{array}{l} v_j(t;\epsilon) = v_{j0}(T_0,T_1) + \epsilon v_{j1}(T_0,T_1) + \dots \\ y(t;\epsilon) = y_0(T_0,T_1) + \epsilon y_1(T_0,T_1) + \dots \end{array} \tag{13}$$

where $T_i = \epsilon^i t$. Substituting Eq. (13) into Eq. (11) gives at order ϵ^0 :

$$\begin{array}{l} D_0^2 v_{j0} + \omega_j^2 v_{j0} = 0 \\ D_0^2 y_0 - (1+r)(\sum_j \phi_j(b)D_0 v_{j0}^- - D_0 y_0^-)\sum_n \delta(T_0 - T_{0_n}) = 0 \end{array} \tag{14}$$

The solution of equation of Eq. (14) can be expressed in the following form [9] :

$$v_{j0} = A_j(T_1)\sin(\omega_j T_0 + \theta_j(T_1)), \quad y_0 = \frac{2}{\pi}B(T_1)\arcsin(\cos(\omega_1 T_0 - \eta(T_1))) \tag{15}$$

According to Eq. (15), impact occurs at $T_0 = (k\pi - \eta)/\omega_j$ $(k = 0, 1, \dots)$. Substituting Eq. (15) into the second equation of Eq. (14) and integrating around a small time instant during impact gives :

$$-\frac{4B}{\pi} - (1+r)(\sum_j \phi_j(b)A_j\omega_j\cos(\frac{\omega_j}{\omega_1}\eta + \theta_j) - \frac{2B}{\pi}) = 0 \tag{16}$$

Multiple scales expansion of the first equation of Eq. (11) at order ϵ^1 is given by :

$$D_0^2 v_{j1} + \omega_j^2 v_{j0} = \phi_j(d)G\sin(\omega_1 T_0 + \sigma T_1) + \omega_j^2\phi_j(b)y_0 - 2D_0D_1v_{j0} - \lambda_j D_0 v_{j0} \tag{17}$$

In order to identify terms that produce secular terms in Eq. (17), the solution for y_0 in Eq. (15) is expanded in Fourier series giving :

$$y_0 = \frac{8}{\pi^2}B\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^2}\cos\left((2k-1)(\omega_1 T_0 - \eta)\right) \tag{18}$$

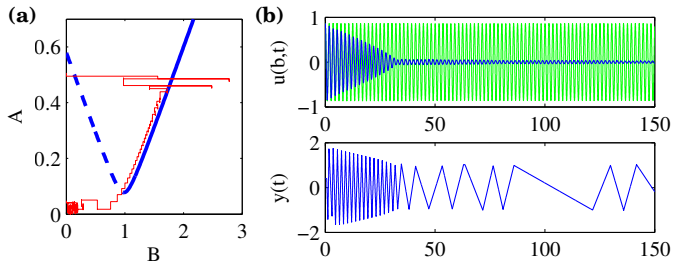

FIGURE 2 – Illustration of transient resonance capture. (a) : SIM, Red line : projection of numerical integration on the SIM. (b) : Numerical simulation, Green line : uncoupled system, Blue line : coupled system.

Substituting Eq. (15, 18) into Eq. (17), assuming that no internal resonance occurs, the condition of elimination of secular terms gives :

$$\begin{aligned}&\text{for } j=1\\&2D_1A\omega_1\sin(\theta_1)+2A_1\omega_1\cos(\theta_1)D_1\theta_1+\phi_1(d)G\cos(T_1\sigma)+\lambda_1A_1\omega_1\sin(\theta_1)+\tfrac{8}{\pi^2}\omega_1^2\phi_1(b)B\sin(\eta)=0\\&-2D_1A_1\omega_1\cos(\theta_1)+2A_1\omega_1\sin(\theta_1)D_1\theta_1+\phi_1(d)G\sin(T_1\sigma)-\lambda_1A_1\omega_1\cos(\theta_1)+\tfrac{8}{\pi^2}\omega_1^2\phi_1(b)B\cos(\eta)=0\end{aligned} \tag{19}$$

$$\begin{aligned}&\text{for } j\geq 2\\&2D_1A_j\omega_j\sin(\theta_j)+2A_j\omega_j\cos(\theta_j)D_1\theta_j+\lambda_jA_j\omega_j\sin(\theta_j)\\&-2D_1A_j\omega_j\cos(\theta_j)+2A_j\omega_j\sin(\theta_j)D_1\theta_j-\lambda_jA_j\omega_j\cos(\theta_j)\end{aligned} \tag{20}$$

Equation (19) gives $A_j(T_1)=K\mathrm{e}^{-\frac{1}{2}\lambda_jT_1}$ for $j\geq 2$. Therefore as $T_1\to\infty$ Eq. (16, 12) are rewritten giving the following relations :

$$\begin{aligned}&\cos(\eta+\theta_1)=\frac{2B\Gamma}{\phi_1(b)A_1\pi},\quad \sin(\eta+\theta_1)=\frac{B-1}{\phi_1(b)A_1},\quad \text{with } \Gamma=\frac{r-1}{r+1}\\&\phi_1(b)^2A_1^2=\frac{4B^2\Gamma^2}{\pi^2}+(B-1)^2\end{aligned} \tag{21}$$

In fact, the last equation of Eq. (21) represents the Slow Invarian Manifold (SIM) of the problem. The SIM is divided into two branches. The left one is unstable while the right one is stable. The detailled study of the stability of the SIM is beyond the scope of this paper, however it can be shown using numerical integration, that periodic motion with two symetric impact per cycle takes place only on the lower part of the right branch. For higher amplitudes, the motion becomes non symetric and degenerate into motion with more than two impact per cycle.

From the equation of the SIM, we are also able to determine the minimum value of the amplitude of the first bending mode that allow TET. To this end, the derivative of the right hand side of the SIM equation is equated to zero :

$$B_{min}=\frac{\pi^2}{4\Gamma^2+\pi^2},\quad \phi_1(b)^2A_{1min}^2=\frac{4\Gamma^2}{4\Gamma^2+\pi^2} \tag{22}$$

An exemple of the SIM and transient resonance capture is presented in Fig. 2(a) for single mode discretization. Numerically simulation is presented in Fig. 2(b). Resonance capture between the beam and the VI-NES is clearly observed, and the energy is quickly dissipated by successive impact ($r<1$).

Equation (19) is now investigated. Substituting Eq. (21) into Eq. (19) and introducing $\gamma_1=T_1\sigma-\theta_1$ gives :

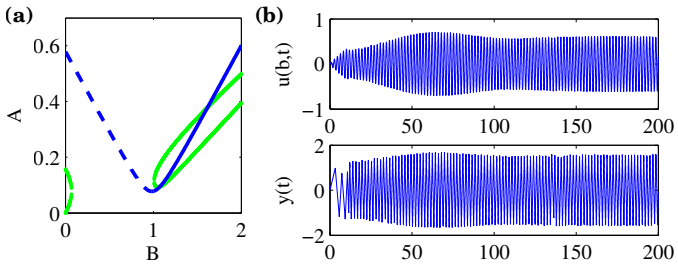

FIGURE 3 – Case of periodic solution for $G = 50$, $\sigma = -15$. (a) : SIM curves, (b) : Numerical integration.

$$\begin{aligned} D_1 A_1 &= \frac{8\omega_1 B^2 \Gamma}{\pi^3 A_1} - \frac{1}{2}\lambda_1 A_1 + \frac{\phi_1(d) G \sin(\gamma_1)}{2\omega_1} \\ D_1 \gamma_1 &= \frac{4\omega_1 B(B-1)}{A_1^2 \pi^2} + \frac{\phi_1(d) G \cos(\gamma_1)}{2 A_1 \omega_1} + \sigma \end{aligned} \tag{23}$$

In order to understand qualitatively the behavior of the system, we need to know the location of the fixed points on the SIM. Fixed points are computed by equating the derivative in Eq. (23) to zero. After rearanging gives :

$$\begin{aligned} \omega_1^2 \pi^6 (\lambda_1^2 + 4\sigma^2) A_1^4 - \pi^3 \left[32\omega_1^3 B(\sigma\pi(1-B) + B\Gamma\lambda_1) + \pi^3 \phi_1(d)^2 G^2 \right] A_1^2 \\ + 64\omega_1^4 B^2 \left[4B^2\Gamma^2 + \pi^2 (B-1)^2 \right] = 0 \end{aligned} \tag{24}$$

Solving Eq. (24) for A_1^2, two curve equations are obtained. These expressions is not displayed here due to their length. The fixed points are then obtained graphically by looking at the intersection of the SIM and the curves obtained via Eq. (24).

4 Description of some response regimes

In this section, some response regimes are presented for different values of system parameters. Equations of motion (3) are integrated using classical matlab ode45 alorithm and the impacts are located using the event location property. The modal basis is truncated to the third bending mode. For each numerical simulation, the following set of parameters and initial conditions is used :

$$\epsilon = 1\%, \quad d = \frac{1}{3}, \quad b = 0.9, \quad r = 0.65, \quad q_j(0) = \dot{q}_j(0) = y(0) = 0, \quad \dot{y}(0) = 0.4 \tag{25}$$

In Fig. 3(a), the SIM at the slow and superslow time scale is presented for $G = 50$ and $\sigma = -15$. In this case, the green curve intersect the stable branch of the SIM two times. The lower fixed point is unstable and the higher one is stable. If the initial conditions are in the domain of attraction of the SIM, $1:1$ resonance between the beam and the NES takes place, and the slow flow will be attracted by the stable fixed points. Thus motion will be periodic with two symetric impact per cycle. This theoretical prediction is confirmed by numerical integration in Fig. 3(b).

Another case is presented in Fig. 4(a) for the set of parameters in Eq. (25) and $G = 30$, $\sigma = 0$. Now the green curve cross the SIM one times on its unstable branch. The resulting fixed point is unstable. The only possible response for the system is Strongly Modulated Response (SMR). Numerically, it is observed in Fig. 4(b) that the free-mass engage successive resonance capture between the beam and the VI-NES, thus limitting the energy in the system.

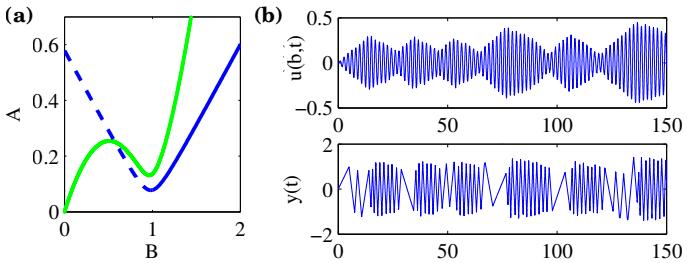

FIGURE 4 – Expemple of strongly modulated response for $G = 30$, $\sigma = 0$. (a) : SIM, (b) : Numerical integration.

5 Concluding remarks

The passive control of an harmonically excited beam using a Vibro-Impact Nonlinear Energy Sink (VI-NES) was investigated. The behavior of the system is studied in the vicinity of 1 : 1 resonance capture with the first bending mode of the beam. Equations of motion are analysed using the method of multiple scales. It is shown that in addition to periodic regimes with two symetric impact per cycle, the system can exhibit Strongly Modulate Response, where the VI-NES engage successive resonance capture with the beam. Numerical simulation are presented to confirm the existence of these types of response. This type of NES is easy to build and may be integrated in a real structure.

Références

[1] O. Gendelman, LI Manevitch, A.F. Vakakis, and R. M'CLOSKEY. Energy pumping in nonlinear mechanical oscillators : Part i : Dynamics of the underlying hamiltonian systems. *J. Appl Mech-T ASME*, 68(1) :34–41, 2001.

[2] A.F. Vakakis and O. Gendelman. Energy pumping in nonlinear mechanical oscillators : Part ii : Resonance capture. *J. Appl Mech-T ASME*, 68(1) :42–48, 2001.

[3] O.V. Gendelman. Bifurcations of nonlinear normal modes of linear oscillator with strongly nonlinear damped attachment. *Nonlinear Dyn.*, 37(2) :115–128, 2004.

[4] Y. Starosvetsky and OV Gendelman. Strongly modulated response in forced 2dof oscillatory system with essential mass and potential asymmetry. *Physica D*, 237(13) :1719–1733, 2008.

[5] E. Gourc, G. Michon, S. Seguy, and A. Berlioz. Design optimisation of a nonlinear energy sink embedded on a harmonically forced linear oscillator : theoretical and experimental developments. In *Proc. of ASME, IDETC 2012, VIB*. ASME, 2012.

[6] C.H. Lamarque, O.V. Gendelman, A. Ture Savadkoohi, and E. Etcheverria. Targeted energy transfer in mechanical systems by means of non-smooth nonlinear energy sink. *Acta Mech.*, 221(1) :175–200, 2011.

[7] F. Nucera, F. Lo Iacono, DM McFarland, LA Bergman, and AF Vakakis. Application of broadband nonlinear targeted energy transfers for seismic mitigation of a shear frame : Experimental results. *J. Sound Vib.*, 313(1) :57–76, 2008.

[8] A. Mohammad, N. Wierschem, D.D. Quinn, A.D. Vakakis, L.A. Bergman, and Spencer B.F. Numerical and experimental investigation of a highly effective single-sidedvibro-impact nonlinear energy sink for shock mitigation. *Int. J. Nonlinea Mech.*, 2013.

[9] OV Gendelman. Analytic treatment of a system with a vibro-impact nonlinear energy sink. *J. Sound Vib.*, 331 :4599â4608, 2012.

[10] F Georgiades and AF Vakakis. Dynamics of a linear beam with an attached local nonlinear energy sink. *Commun. Nonlinear Sci. Num. Sim.*, 12(5) :643–651, 2007.

[11] KV Avramov and OV Gendelman. On interaction of vibrating beam with essentially nonlinear absorber. *Meccanica*, 45(3) :355–365, 2010.