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# Representation of Multiple Agent Preferences

## A Short Survey

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**Abstract.** Different types of graphical representation for local preferences have been proposed in the literature. Graphs may be directed or not. Modeling may be quantitative or qualitative. Principles for extending local preferences to complete configurations may be based on different independence assumptions. Some extensions of such graphical representation settings to multiple agent preferences have been proposed, with different ways of handling agents: they may be viewed just as a set of individual agents, or described in terms of attribute values inducing a partition of the set of agents in terms of subcategories, or they may be reduced to some anonymous statistical counting. The fact that preferences pertain to multiple agents raises the question of either working with a collective graphical representation or aggregating individual preferences, the preferences of each agent being then represented as a graph. Moreover the multiple agent nature of the representation enriches the types of preference queries that can be addressed. The purpose of this short note is to start with a brief survey of the main graphical preference models found in the literature, such as CP-nets,  $\pi$ -pref nets, GAI networks, and to discuss their multiple agent extensions in an organized way, with a view to understand how the different representation options could be combined when possible.

**Keywords:** Multiple agent model · Preference model  
Graphical representation · Possibilistic network

## 1 Introduction

The idea of representing individual local preferences in a graphical manner, together with an independence assumption for being able to compare complete solutions of decision problems, is attractive since it combines the benefits of a compact representation with the easiness of elicitation. Several models, which significantly differ, have been proposed and developed along this line: in the following we shall consider the main ones, CP-nets [11,12], GAI networks [15] and  $\pi$ -pref nets [2,4], and some of their variants. General surveys can be found

in [3, 18]. The graphical representation of the collective preferences of a group of agents is clearly of interest, and several multiple agent extensions of the previous settings have been proposed: mCP-nets [19], PCP-nets [9], mGAI networks [14] and ma  $\pi$ -pref nets [5]. This short paper intends to provide a survey and a discussion of these models going from qualitative to quantitative ones.

The paper is organized as follows. In the next section, we first review the different options underlying the graphical representation of individual preferences before providing a synthetic survey of the existing proposals. Section 3 first discusses how sets of agents can be handled, either on an individual basis, or in a collective anonymous manner, or in terms of subcategories described in terms of attribute values. Existing multiple agent representation proposals are then surveyed, and new options are also proposed.

## 2 Single Agent Preference Representation

Consider an agent expressing his/her preferences on the solutions of a decision problem, each solution being described by a set of features. A solution is thus represented by an instantiation of all the features, what is called a *configuration* in the following. Due to their combinatorial nature, complete configurations (also called outcomes) are difficult to compare. Then, models for preference representation rank configurations on the basis of (i) (conditional) local preferences pertaining to each feature domain and (ii) independence assumptions. This section provides a short description of the main graphical models for representing single agent preferences, comparing them on the basis of their underlying principles.

### 2.1 Building Principles of a Graphical Representation of Preferences

The basic idea is to represent local preferences of an agent by means of an acyclic graph. Some models represent preferences with *directed* structures, other ones with *undirected* ones. Within these structures, we can distinguish two main categories: quantitative models, where preferences are associated with numerical values, and purely ordinal models, where conditional preference statements are expressed by comparisons between feature instantiations and only the preference order between values matters. As a result, the preference relation between outcomes can be either complete, when all configurations can be compared, or partial, where some comparisons cannot be made. Each setting relies on a specific independence property between variables that allows us to construct the preference relation between configurations from the preference graph.

### 2.2 A Brief Review of the Main Graphical Preference Models for a Single Agent

**CP-Nets.** A Conditional Preference network (CP-net for short) [11, 12] represents strict partial preference orderings of an agent under the form of local

comparisons between values of each variable conditioned by other (parent) variables. It uses a directed acyclic graph  $G = (V, E)$ , where nodes in the set  $V = \{X_1, \dots, X_n\}$  represent variables (features) and  $E$  is a set of arcs. An arc from  $X_j$  to  $X_i$  expresses that the preference between values of variable  $X_i$  depends on the values of parent variable  $X_j$ . Each variable  $X_i \in V$  is then associated to a conditional preference table  $CPT(X_i)$  in the context of  $X_i$ 's parents. CP-nets are based on the *ceteris paribus* principle, which enables the preference between values  $x_i$  and  $x'_i$  of variable  $X_i$  in the context of an instantiation of its parents to be extended to complete configurations assuming the other variables take the same values for those configurations. Let  $par(X_i) = u$  be an instantiation of the set of  $X_i$ 's parents,  $d(X_i) = d$  an instantiation of the set of  $X_i$ 's children, and  $n(X_i) = n$  an instantiation of other  $X_j$ 's. Actually, a direct preference comparison (denoted by  $\succ$ ) between two complete configurations can only be done for those that differ by a single flip of one variable, in the context of an instantiation of its parents. In other words, due to *ceteris paribus* independence, we can conclude that  $\forall d, \forall n, ux'dn \succ uxdn$  if and only if  $x'$  is preferred to  $x$  in the context  $u$  (denoted  $u : x' \succ x$ ).

**$\pi$ -pref Nets.** A possibilistic preference network [2,4,6] is a directed graph sharing the same structure as a CP-net. However, preferences are no longer represented by an ordering (of the form  $u : x' \succ x$ ), but with a *symbolic* conditional possibility distribution over the domain of each variable in  $V$ . It means a possibility distribution where possibility degrees, taking values between  $]0, 1]$ , are not instantiated. Let us consider a variable  $X_i$  taking  $x$  and  $x'$  as possible values and having  $U_i$  as parents. If we are in a case of a strict preference  $x \prec x'$  for an instantiation  $u_i$  of variables  $U_i$ , then  $\pi(x|u_i) = \alpha < \pi(x'|u_i) = \beta$ . In contrast, if  $x \sim x'$  then  $\pi(x|u_i) = \pi(x'|u_i) = \alpha \leq 1$  (in case of a binary-valued variable,  $\pi$  being normalized,  $x \sim x'$  entails that the two configurations have a possibility 1, and  $x \prec x'$  entails  $\beta = 1$ ).

In the spirit of possibilistic belief networks [7,8], the degree of satisfaction of each configuration is computed as the product of symbolic weights using the chain rule associated to the product-based conditioning in possibility theory [2], namely,  $\pi(X_1, \dots, X_n) = \prod_{i=1, \dots, n} \pi(X_i|u(X_i))$ . As an illustration, consider preferences described in terms of three binary variables  $X_1, X_2, X_3$ , based on a  $\pi$ -pref net of the form  $X_1 \rightarrow X_2 \rightarrow X_3$ . Let  $\pi(X_1 X_2 X_3) = \pi(X_3|X_2)\pi(X_2|X_1)\pi(X_1)$ . Suppose  $\pi(x_1) = \pi(x'_1) = 1$ ,  $\pi(x_2|x_1) > \pi(x'_2|x_1) = \gamma$ ,  $\pi(x'_2|x'_1) > \pi(x_2|x'_1) = \alpha$ ,  $\pi(x_3|x_2) > \pi(x'_3|x_2) = \beta$ . Then  $\pi(x_2|x_1) = 1$  and when comparing configurations  $x_1 x_2 x'_3$  and  $x'_1 x_2 x'_3$ , we note that using the chain rule,  $\pi(x_1 x_2 x'_3) = \beta$  and  $\pi(x'_1 x_2 x'_3) = \alpha\beta$ , and we can see that  $\beta > \alpha\beta$  whatever the values of  $\alpha$  and  $\beta$ , which means that  $x_1 x_2 x'_3 \succ x'_1 x_2 x'_3$ . It would be equivalent to compare the vectors of the form  $(\pi(X_3|X_2), \pi(X_2|X_1), \pi(X_1))$ , here  $(1, 1, \beta)$  with  $(1, \alpha, \beta)$  by means of the Pareto ordering (in fact its symmetric version [2]).

The two partial orderings of configurations respectively obtained from a  $\pi$ -pref net and from a CP-net built on the basis of the same preference statements of an agent, do not contradict each other [4]. Constraints between symbolic weights of the  $\pi$ -pref net can be added so as to recover comparisons produced

by the *ceteris paribus* property. As a consequence,  $\pi$ -pref nets look more flexible than CP-nets since they leave complete freedom for adding relative priorities between possibilistic weights. However, if no constraints are added, configurations may remain incomparable while they can be compared in the sense of the CP-net. Moreover, since  $\pi$ -pref nets offer the opportunity to switch from symbolic weights to instantiated numerical ones, this representation can be viewed as being halfway between qualitative models such as CP-nets and quantitative ones. Although CP-nets share the same graphical structure and level of simplicity as  $\pi$ -pref nets, they do not have the same expressive power. In fact, CP-nets are based on *ceteris paribus* independence, while  $\pi$ -pref nets rely on a Markov independence property namely, each variable  $X_i$  is independent from the remaining non-children nodes ( $N$ ) in the context of its parents ( $U$ ). With *ceteris paribus* independence, we can say that  $uxdn \succ ux'dn$ , i.e., the parent-dependent preference between local configurations of one variable are completed with the same instantiation of the other variables, while with Markov-based nets, when flipping a variable  $X$  from  $x$  to  $x'$  in the context  $u$ , first we choose the best instantiations for all variables that depend on the value of  $X$ , and next, we instantiate the other variables in the same manner in all possible ways; see [2] for an example.

**GAI-Nets.** Generalized Additive Independence (GAI) networks [15] are graphical quantitative models for representing preferences expressed by means of utilities. A GAI-net is composed of two components. The first is a graphical structure defined by an *undirected* graph  $G = (C, E)$  where  $C$  denotes a set of cliques and  $E$  denotes the corresponding set of edges. Each clique  $C_j \in C$ , is a set of variables s.t.  $C_j \subseteq V$  and  $\cup_{i=1}^k C_i = V$ . Each edge refers to overlapping cliques, and is labeled by a separator  $S_{ij} = C_i \cap C_j \neq \emptyset$ . The second component is a set of local numerical utility functions associated to cliques.

GAI-nets are based on a generalized additive independence decomposition [15]. This property allows us to associate to each clique of attributes a utility function and then to sum them in order to compare the different configurations. Then a *total* ordering between possible instantiations can be obtained. Thus the utility of a configuration  $\sigma$  can be expressed as the sum of partial utilities associated with clique configurations, namely  $u(\sigma) = \sum_{j=1}^k u_j(\omega_{C_j})$ . This is a form of decomposition that allows interaction between some variables, while preserving independence between other ones. In fact, the variables in  $C_i \setminus S_{ij}$  are considered independent from the variables in  $C_j \setminus S_{ij}$  when variables in  $S_{ij}$  are instantiated. A total preorder between outcomes is obtained.

**UCP-Nets.** Utility functions can be added to CP-nets in order to represent preferences in quantitative terms. In this context, a Utility CP-net (UCP-net for short) [10] is a DAG with quantified utility preferences associated to nodes. This representation combines some aspects of both CP-nets and GAI-nets. UCP-nets share the same graphical structure as CP-nets, and constraints between utility values are added so as to obey the *ceteris paribus* principle. However, unlike CP-nets, this model obtains a total preorder between

configurations. Moreover, by adding utilities to CP-nets, the expressiveness and computational power are enhanced. Comparing different outcomes comes down to comparing their respective utility functions, additively computed as  $u(x_1, \dots, x_n) = \sum_{i=1}^n u_i(x_i | \text{par}(X_i))$ . This function is very similar to the chain rule in Bayesian networks [17], namely  $p(x_1, \dots, x_n) = \prod_{i=1}^n p(x_i | \text{par}(X_i))$ , up to a logarithmic transformation.

### 3 Multiple Agent Preference Networks

In the existing studies on multiple agent representations, groups of agents are represented differently. In the simplest approaches, agents are represented as a set of individuals regardless of their characteristics; this is the case of several multiple agent models. Other representations view groups of agents as a whole and their preferences are summarized in statistical terms. By contrast, one may also describe subgroups of agents in terms of attribute values, e.g., gender, age, etc. Yet, only a single model has been proposed along this line, which are the  $\pi$ -pref nets, presented at the end of this section.

#### 3.1 A Survey on Multiple Agent Preference Models

**MCP-Nets.** A multiple agent CP-net [19] is a collection of  $m$  distinct CP-nets with graphs  $(V_i, E_i), i = 1, \dots, m$ , reflecting the preferences of  $m$  agents: each CP-net thus represents individual preferences. Graphically, an mCP-net is just the juxtaposition of  $m$  single agent CP-nets. Different ways of deriving collective preferences between configurations have been studied, using a collection of dominance (voting) semantics (Pareto, majority, Condorcet winner, etc.) [16].

Individual CP-nets in an mCP-net share a common set of nodes  $V_C \subseteq V$ , such that every variable in  $V$  is informed by at least one agent. A particular agent is allowed not to use all variables in  $V$  when stating his/her preferences. When an agent does not define his preferences over the domain of a variable in  $V_i$ , this is interpreted as an incomparability situation. It could be also interpreted as indifference between values of this variable. Interestingly, both cases are treated alike in  $m$ -CP nets. Moreover, even if preferences are explicitly provided about  $X$ , an agent might neglect dependencies between variables. This case can be represented by a CP-net associated to some empty conditional preference tables. However, such cases have not been fully explored yet.

**Probabilistic CP-Nets.** PCP-nets [9] represent collective preferences of a set of agents. Formally, a PCP-net is as usual a DAG  $G = (V, E)$ , where conditional preferences over variable domains are replaced by conditional probabilities assigned to each local preference. These probabilities reflect the proportions of agents that share these local preferences in their personal CP-nets. The preference relation over all possible configurations is replaced by a probability distribution on them.

For reasoning with this model, voting semantics can be used in order to determine the most preferred configuration. However, these methods are sequential and proceed locally. PCP-nets offer the alternative to aggregate the set of CP-nets specific to each agent into a single structure in order to perform globally. To proceed, another structure is used, called *induced* CP-net. A CP-net induced from a PCP-net  $P$  is a network that has the same variables, with the same corresponding domains, but with a subset of  $P$ 's edges. Each induced CP-net has its associated probability, computed by taking the product of the orderings probabilities chosen in  $P$ . For example, let us consider a PCP-net over 2 variables, where  $p(x_1 \succ x'_1) = 0.6$  means that 60% of agents prefer  $x_1$  to  $x'_1$ , hence  $p(x'_1 \succ x_1) = 0.4$ ,  $p(x'_2 \succ x_2 | x_1 \succ x'_1) = 0.7$ ,  $p(x_2 \succ x'_2 | x_1 \succ x'_1) = 0.3$ ,  $p(x'_2 \succ x_2 | x'_1 \succ x_1) = 0.2$ ,  $p(x_2 \succ x'_2 | x'_1 \succ x_1) = 0.8$ . An induced CP-net can be derived by assuming  $x'_1 \succ x_1$ ,  $x'_2 \succ x_2$  in context  $x_1$ ,  $x_2 \succ x'_2$  in context  $x'_1$  with the following probability:  $0.4 \times (0.7 \times 0.8) = 0.224$ . To determine the most probable optimal configuration, we have to compute the sum of probabilities of induced CP-nets that have each configuration  $\sigma$  optimal. On top of comparing two or different configurations, this model allows to deal with a new query: finding the optimal configuration of the most probable induced CP-net [13].

**Multiple Agent GAI Nets.** Multiple agent GAI nets are an extension of GAI-nets for representing collective preferences and reasoning with them. In this case, preferences are expressed by means of utility vectors, one component per agent. In order to compare different utility vectors, voting semantics can be used. The most widely used is Pareto optimality. However, this semantics reflects a partial weak order that leaves many configurations incomparable. This is easily overcome by means of aggregation operations. In this context, one such aggregation function is the Choquet integral. It is an aggregation function that associates weights to subset of agents (modelling their possible interactions) and then proceeds to a piecewise linear aggregation [14].

**Multiple Agent  $\pi$ -pref Nets.** The extension of  $\pi$ -pref nets to multiple agent preferences has been proposed recently [5], by combining  $\pi$ -pref nets with the multi-agent counterpart of possibilistic logic [1]. An *ma- $\pi$ -pref net* shares the same graphical structure as  $\pi$ -pref nets, consequently, as CP-nets. Each node is associated to a multiple agent possibility distribution, where  $(x_i \succ x'_i, \alpha | A)$  ( $A$  is a subset of agents) is interpreted as: at least all agents in  $A$  prefer  $x_i$  to  $x'_i$  with minimal priority degree  $\alpha$ . However, this model may display some contradictions inside subsets of agents, which leads to a non-normalized possibility distribution. Moreover, the independence property in this representation is yet to be studied. This model is of interest, since it allows for expressing new types of query, such as finding the configuration that best satisfies a given category of agents. This model uses logical descriptions of classes of agents, that can be simplified via a propositional logic machinery.



## 4 Perspectives

The above considerations suggest a number of research lines:

- Regarding single agent preference network representations, there is a need for a precise comparison of independence notions at work in CP-nets, GAI networks, UCP-nets and  $\pi$ -pref nets as well as the types of preference graphs between solutions obtained from these independence assumptions. This is related to the study of translations of directed structures into non-directed ones, well-known with quantitative representations, especially Bayesian networks.
- On the issue of multiple agent representations, we have seen that a set of agents can be represented in extension (as a list of agents) or in intension (describing subclasses of agents by means of suitable attributes). In the first case, some approaches compute statistics, evaluating the proportion of agents preferring one option to another. In other approaches, voting methods are used to compare options at the collective level. Finally, one may be interested in computing preferences of subgroups of agents from the known preferences of other subgroups described by attribute values. The three approaches are not mutually exclusive, since one may wish to go from a statistical description of the preferences of subsets of agents to a description of agents having those preferences, and conversely.
- Finally, it is interesting to consider open issues in the state of the art of multiple agent graphical preference representations. Is it possible to envisage applying the various representations of groups of agents to all graphical structures. For instance, we may consider a probabilistic GAI-net, where one would count the proportion of agents that prefer one option to another based on their utility values for each agent.

**Table 1.** Conditional tables of a Probabilistic ma  $\pi$ -pref net

$N(b_1 \succ b_2)$	$N(b_2 \succ b_1)$	$N(. \cdot)$	$b_1$	$b_2$
$0.8 \setminus F \cap Y, 0.7$	$0.6 \setminus M, 0.9$	$c_1 \succ c_2$	$0.3 \setminus F \cap O, 0.7$	$0.1 \setminus O, 0.8$
		$c_2 \succ c_1$	$(0.5 \setminus F, 1) \wedge (0.85 \setminus M \cap O, 0.9)$	$0.8 \setminus M \cap Y, 0.7$

Especially, one could hybridize PCP-nets and ma- $\pi$ -pref nets, the former summarizing the latter if we have knowledge about the number of agents having properties used in the ma- $\pi$ -pref net. One could also make inferences involving both frequentist and logical computations, using probabilistic inference patterns, for instance, knowing the proportion of agents in group  $A$  that prefer  $p$  to its negation and of group  $B$  that prefer  $\neg p \vee q$  (to its negation), describe the set of agents that prefer  $q$  to its negation and the proportion thereof.

**Example of a Probabilistic ma  $\pi$ -pref Net.** A group described by their gender: male (M) or female (F) and their age: old (O) or young (Y) along with

proportions of agents for each category. Agents express their preferences over car colors ( $c_1$  or  $c_2$ ) in the context of their make ( $b_1$  or  $b_2$ ). Consider the multiple agent conditional preference tables in Table 1. A statement of the form  $(\alpha \setminus C, \gamma)$  associated to a preference  $x_i \succ x'_i$ , means that at least  $\alpha\%$  of agents in category  $C$  are satisfied with  $x_i \succ x'_i$  with priority (necessity) at least  $\gamma$ . For instance,  $(0.8 \setminus M \cap Y, 0.7)$  reads at least 80% of young men prefer  $x_i$  to  $x'_i$  with priority 0.7.

## 5 Conclusion

First, this paper has summarized the most important properties of different preference representation models, with some comparison between them. Several lines of research are proposed for further extensions. A new model for representing multiple agent possibilistic preferences under uncertainty has been suggested. Several queries for this model are yet to be analyzed and processed. We might also think of bridging the gap between this new model with Probabilistic CP-nets.

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