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A cost model for predictive maintenance based on risk-assessment

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Résumé – La complexité croissante des systèmes industriels amène les décideurs publics ou privés à optimiser le cycle de vie d’un système, notamment en ce qui concerne ses opérations de maintenance. Dans cet article, nous proposons une approche d’optimisation et de planification de la stratégie de maintenance tenant compte à la fois des coûts des opérations et des risques liés à la défaillance du système. La nouveauté de l’approche proposée réside dans une intégration, dans la fonction objectif que nous minimisons, de l’ensemble de coûts de maintenance ainsi que des risques financiers, environnementaux et humains pouvant être causés par une éventuelle défaillance du système. Pour cela, nous nous fondons sur la durée de vie utile restante (RUL) du système en tant qu’indicateur de l’état de santé du système. Les variables de décision sont alors : un seuil critique de RUL en-deça duquel le composant est remplacé et un pas d’inspection donnant la régularité avec laquelle le système est inspecté.

Abstract - The growing complexity of industrial systems is driving public and private decision-makers to optimize the life cycle of a system, particularly with regard to maintenance operations. In this article, we propose an approach to optimize and plan the maintenance strategy taking into account both the costs of operations and the risks associated with system failure. The novelty of the proposed approach lies in an integration, in the objective function that we minimize, of the set of maintenance costs as well as the financial, environmental and human risks that could be caused by a possible system failure. This is based on the system’s remaining useful life (RUL) as an indicator of the health status of the system. The decision variables are then: a critical threshold of RUL below which the component is replaced and an inspection step giving the regularity with which the system is inspected.

Mots clés – maintenance basée sur les risques, maintenance prévisionnelle, durée de vie résiduelle, modèle coût, optimisation.

Keywords – risk-based maintenance, predictive maintenance, remaining useful life, cost model, optimization.

1 INTRODUCTION

Maintenance is the combination of all technical, administrative, and managerial actions during the life cycle of a system aiming to retain it in, or restore it to a state in which it can perform its required function [Afnor, 2018]. We distinguish different types of maintenance such as corrective maintenance which is a maintenance performed after detection of a failure [Afnor, 2018] and preventive maintenance which is performed at predetermined intervals intended to reduce the probability of failure or degradation of the system [Afnor, 2018]. These two types of maintenance have major drawbacks: corrective maintenance generates additional costs and significant system downtime [Lee et al., 2006], [Lesobre et al., 2014], [Palem, 2013], while preventive maintenance does not allow an optimal system operation [Le, 2016]. In order to overcome these drawbacks, predictive maintenance has emerged as a solution to reduce the system downtime and the cost of maintaining the system. Predictive maintenance is based on a regular monitoring of the system in order to evaluate the health state of its components. Predictive maintenance is carried out following a forecast derived from repeated analysis or known characteristics and evaluation of the significant parameters of the degradation of the system [Afnor, 2018]. Usually, we use the Remaining Useful Life (« RUL »), defined as the expected length of time left for the system before it falls down, as a measure of the the system’s health state.

Through predictive maintenance, it is now possible for industrials to estimate the RUL of the system as one among other measures used to predict the failure time of the system, so that industrials are able to maintain the system before it falls down. However, in practice, once the RUL or other indicator of the health state of the system is determined, industrials face several issues related to the decision making process: what is the best time to perform predictive maintenance ? and how can we optimize the total cost of maintenance ?

Besides, a sudden failure of the system may result in major accidents, causing damages to human and to the environment. The
main challenge for industrials is to quantify the impacts of these probable accidents in order to implement a cost-efficient maintenance strategy [Khan et Haddara, 2003]. In this paper, we try to answer the previous issues by proposing an approach allowing to identify the cost-optimal strategy for predictive maintenance, taking into considerations the possible impacts that a failure accident may cause on the system and on its surroundings.

2 State of the art

The introduction of new maintenance strategies such as predictive maintenance has been motivated mainly by the increase in the complexity of industrial systems and by the importance of the impact that maintenance may have on environment, equipment security, human safety and economic profitability. For instance, more recently, we have witnessed the use of risk measures in maintenance. In this paper, and in first approximation, risk is considered as the product of the probability of failure occurring to the system and the consequence of failure to the system and its surrounding, is then adopted as an index measure to clarify priority in risk maintenance technologies. Alternative criteria for taking into account decision maker’s attitude to risk is an avenue for future research.

The Risk-Based Maintenance (RBM) is a technique for identifying, characterizing, quantifying and evaluating the loss from a failure event in order to plan a maintenance action [Khan et Haddara, 2003]. RBM was first deployed in the chemical engineering and petroleum refining fields. For instance, the work of [Aller et al., 1995] and [Reynolds, 1995] was used to develop a risk-based inspection policy for equipment owned by Brunei Shell Petroleum [Hagemeijer et Kerkveld, 1998]. A simple RBM was used by [Dey et al., 1998] to maintain a cross-country pipelines. [Nessim et Stephen, 1998] developed a quantitative risk-analysis model for maintenance budgeting, while [Dey, 2001] developed a more general framework for risk-based inspection and maintenance for cross-country pipelines.

According to [Khan et Haddara, 2003], the risk-based methodology can be broken down in three main modules: risk determination which consists of risk identification and risk estimation, risk evaluation which consists of risk aversion and risk acceptance analysis and maintenance planning considering risk factors. This approach was successfully applied to a Heating Ventilation and Air-conditioning (HVAC) system [Khan et Haddara, 2003].

[Sakai, 2010] has defined a general procedure for RBM. In this general procedure, data are collected and used to evaluate the risk of each part of the system under study. The risk evaluation will be the basis to rank priority for part inspection. As a consequence, mitigation measures are proposed and the operation is iterated from the beginning if problems are detected [Sakai, 2010].

A more recent work on RBM combined with Bayesian network to model the risk and its associated uncertainty is developed by [Leoni et al., 2018] and successfully applied to a case study of Natural Gas Reduction and Measuring Station in Italy. Finally, the work of [Jaderi et al., 2019] considers both traditional RBM and fuzzy RBM for the risk analysis of petrochemical assets failure.

The review of literature on RBM shows that the risk measure can be used as a criterion to plan maintenance. However, in practice, it is not evident to define the risk acceptance level which is the basis of the RBM methodology. Besides, the notion of risk cannot be excluded from the notion of monetary loss as the ultimate goal of industrials is to reduce the overall cost of maintenance.

Another approach widely used in literature to plan maintenance is the approach of cost optimization. This approach aims at identifying the maintenance strategy minimizing the total cost of maintenance. For example, [Vaurio, 1999] developed a cost model taking into account finite repair, maintenance durations and costs due to testing, repair, maintenance and lost production or accidents. The objective of the maintenance optimization is to minimize the total cost rate by proper selection of two intervals: one for inspections and one for replacements [Vaurio, 1999]. [Maillard et Pollock, 2002] analyzed predictive maintenance policies for systems exhibiting 2-phase behavior: the phase of new condition and the phase of worn condition, and presented cost-minimizing policies, to determine when monitoring should take place. [Zou et al., 2007] and [You et al., 2010] developed a sequential imperfect preventive maintenance policy and determined the optimal preventive maintenance schedule that minimizes the cost rate in the life cycle of the system or in the long run, while [Van Horenbeek et Pintelon, 2013] developed a dynamic predictive maintenance policy for complex multi-component systems aiming at minimizing the long-term mean maintenance cost per unit time. Finally, a recent work on predictive maintenance decision-making method based on cyber manufacturing and mission reliability state was developed by [He et al., 2018].

The review of literature on maintenance cost optimization methods shows that the concept of risk in cost optimization has not been tackled yet, although the occurrence of a failure on a system may have onerous consequences for industrials [Khan et Haddara, 2003].

To deal with this gap, we propose in this paper an original approach allowing to combine the notion of risk with the cost of maintenance for optimal maintenance strategy identification in terms of economic profit.

3 Cost model

3.1 Assumptions

In developing the predictive maintenance strategy, some assumptions are addressed as below:

1. The system under study is a single component.
2. The system under study is part of a whole complex system, which has a duration of exploitation known beforehand, noted \( D \).
3. A perfectly reliable inspection is applied regularly on the system (figure 1). The inspection gives an information on the state of health of the system. For instance, the inspection gives a real estimation of the RUL of the system. After simulations, the RUL is the expected interval of time the system is likely to operate before it requires replacement. The RUL of the system can be expressed by the following equation:
\[ RUL(t) = E[T - t | T > t] = \int_{t}^{\infty} (u - t) f(u) \, du \frac{1}{S(t)} \]

with \( T \) the time of failure of the system, \( f \) the failure density function of the system and \( S \) the survival function of the system.

4- The inspection does not affect the system’s performance.
5- A first inspection is required in the early life of the system, but the health of the system is supposed not to require replacement because it is a new one (figure 1). Once the system attains \( D \), there is no use to perform inspection and the system can be replaced by a new one.

\[ \sum f + n \] of inspection and economic loss

In our methodology, costs can be divided in two types

3.2 Typology of costs

In our methodology, costs can be divided in two types as shown in figure 2: costs related to maintenance composed of: cost of predictive maintenance, cost of corrective maintenance and cost of inspection and economic loss due to maintaining the system.

The economic loss includes cost of operating loss and cost of indirect loss (figure 2).

3.3 Cost of predictive maintenance

In our methodology, the decision to preventively maintain the system is not systematic. In fact, in some cases, corrective maintenance may be preferable to predictive maintenance.

The cost of predictive maintenance during the time cycle \( D \), denoted by \( C_{pm} \), can be described by the following equation [He et al, 2018]:

\[ C_{pm} = (C_p + C_i) \sum_{i=1}^{N_{in} - 1} N_i \quad (Eq. 1) \]

where \( C_p \) is the cost of a predictive replacement of the system, \( C_i \) is the cost of installation for maintenance (fixed cost), \( N_{in} \) is the total number of inspections (the first one does not cause any replacement) and \( N_i \) is a binary decision variable in the inspection interval \([i, i+1]\) which takes the value of 1 in case of preventive maintenance (the \( RUL \) is under the threshold value \( RUL_{lim} \)) and 0 elsewhere.

3.4 Cost of corrective maintenance

The cost of corrective maintenance for the \( i^{th} \) inspection is paid only if there is no predictive replacement and if there is a failure before the next inspection (inspection \( i+1 \)). Thus, the cost of corrective maintenance during the time cycle \( D \), denoted by \( C_{cm} \), can be described by the following equation [He et al, 2018]:

\[ C_{cm} = \sum_{i=1}^{N_{in} - 1} (C_c + C_i) \cdot (1 - N_i) \int_{T > t_i}^{i+1} f(t) \, dt \quad (Eq. 2) \]

where \( C_c \) is the cost of a corrective replacement of the system.

3.5 Inspection cost

The inspection process is done regularly on the system to evaluate the \( RUL \) of the system under study (figure 1). We stipulate that inspection is required in the early life of the system. According to figure 1, the step of inspection \( \theta \) is linked to the number of inspections \( N_{in} \) per cycle \( D \) according to the following equation:

\[ N_{in} \cdot \theta = D \]

which means:

Figure 1. Inspection procedure

Figure 2. Typology of costs for maintenance strategy optimization

Key words:
1: inspection \( n^i (i = 1 \ldots N_{in}) \)
2: total number of inspections
3: duration of exploitation of the global system

Costs
- Cost of predictive maintenance
- Cost of corrective maintenance
- Inspection cost
- Operating loss
- Indirect loss
- Environmental loss
- Human loss
- Financial loss

Cost of predictive maintenance scenario
- Predictive maintenance scenario: the \( RUL \) of the system attains some threshold value called \( RUL_{lim} \) under which the system is considered as deteriorated: the system is replaced by a new one before the inspection \( i+1 \).
- Non-predictive maintenance scenario: in this scenario, the system is not replaced by predictive maintenance. In such case, the system can fail or not:
  - The system fails before the next inspection \( i+1 \) knowing that he was operating at inspection \( i \), the system is then predictively replaced by a new one. The probability of occurrence of this scenario is given by
    \[ \int_{l,T > t_i}^{i+1} f(t) \cdot dt = \frac{1}{S(t_i)} \int_{T > t_i}^{i+1} f(t) \cdot dt \]
  - The system operates normally until the next inspection \( i+1 \) with the complementary probability of occurrence \( 1 - \int_{T > t_i}^{i+1} f(t) \cdot dt \).

Cost of corrective maintenance scenario
- The system will be replaced by a new one.
- Predictive maintenance scenario: the system attains some threshold value called \( RUL_{lim} \) under which the system is considered as deteriorated: the system is replaced by a new one before the next inspection \( i+1 \).
- Non-predictive maintenance scenario: in this scenario, the system is not replaced by predictive maintenance. In such case, the system can fail or not:
  - The system fails before the next inspection \( i+1 \) knowing that he was operating at inspection \( i \), the system is then predictively replaced by a new one. The probability of occurrence of this scenario is given by
    \[ \int_{l,T > t_i}^{i+1} f(t) \cdot dt = \frac{1}{S(t_i)} \int_{T > t_i}^{i+1} f(t) \cdot dt \]
  - The system operates normally until the next inspection \( i+1 \) with the complementary probability of occurrence \( 1 - \int_{T > t_i}^{i+1} f(t) \cdot dt \).

Cost of inspection

Cost of installation for maintenance
\[ \frac{N_{in}}{\theta} = D \quad (Eq. 3) \]

Thus, the total cost of inspection \( C_{in} \) per time cycle \( D \) can be expressed by the following equation:

\[ C_{in} = \frac{D}{\theta} \cdot c_{in} \quad (Eq. 4) \]

where \( c_{in} \) is the cost of one inspection.

### 3.6 Cost of operating loss

Generally the failure of the system causes in loss of operation capacity [He et al., 2018]. Besides, the system may require to shut down the system for security measures which leads to loss of the system’s operation capacity. The cost of operating loss \( C_{o} \) contains the cost of operating loss from predictive maintenance and the cost of operating loss from corrective maintenance. This can be modeled by the following equation:

\[ C_{o} = D_p \cdot c_{p} + \sum_{i=1}^{N_{pc}} c_{pc}^{(i)} + \sum_{i=1}^{N_{cr}} c_{cr}^{(i)} \int_{T > t_i} f(t) \cdot dt \cdot c_{o} \cdot D_e \quad (Eq. 5) \]

where \( D_p \) is the duration of a predictive replacement, \( D_e \) is the duration of a corrective replacement and \( c_{ot} \) is the system downtime cost per unit of time.

As expected before, the first term of the equation corresponds to the cost of operating loss due to corrective maintenance and the second term of the equation corresponds to the cost of operating loss due to predictive maintenance.

### 3.7 Indirect loss cost

In reality, the loss of operation capacity may affect negatively the customer satisfaction, which indirectly brings economic loss to the company such as reduced orders caused by diminished company standing and other factors [He et al., 2018].

Besides, the occurrence of a failure event can have negative effects on human health and environment. As for example, a failure of the system may cause the emission of toxic chemicals harmful for human health as well as for the environment [Haddara, 2003].

Thus, the indirect loss cost includes the following terms:
- the financial risk \( R_f \) caused by reduction of customer orders.
- the human risk \( R_h \) caused by human loss (injury, disease or death) from a failure event.
- the environment or ecological risk \( R_e \) caused by environmental degradation due to emission of pollutants.

The indirect loss cost, denoted by \( C_{il} \), can then be represented by the equation below:

\[ C_{il} = R_f + R_h + R_e \quad (Eq. 6) \]

In the following section (section 4), we describe how we measure the financial risks in section 4.1, the environmental risks in section 4.2 and the human risks in section 4.3.

### 4 Risk Assessment

The underpinnings of the risk-based methodology come from the identification of failure scenarios, their consequences and the probabilities of their occurrence. A failure scenario is a sequence of events which may lead to a system’s failure. It may be a single event or a combination of sequential events [Khan et Haddara, 2003]. In the context of predictive maintenance, estimation of the likelihood of system’s failure is based on inspections and simulations. To evaluate the risk of a failure scenario, we use the classical definition of the risk: a risk can be defined, in first approximation - which does not address risk related behaviour of the decision maker - by the following set of duplets for a predefined failure scenario [Khan et Haddara, 2003]:

\[ Risk = \text{probability of failure} \times \text{consequence of failure} \]

#### 4.1 Financial risks

The literature review presents a wide variety of financial measures allowing to evaluate the financial performance of an industry: for example, growth rates used at their most basic level to express the annual change in a variable as a percentage, profit margins used to evaluate the monetary gain left over after accounting for the cost of goods sold or the average revenue per user [Ngobo et Ramaroson, 2005]. In our study, we recognize the use of the churn rate as a financial measure as it allows to model a financial loss [Crie, 1996]. The basic definition of churn rate is the proportion of customers that a business loses during a given period of time [Crie, 1996].

In our study, we assume that the business loses \( x\% \) of customers in case of predictive maintenance and \( y\% \) of customers in case of corrective maintenance. As one may notice, \( y\% \) is superior to \( x\% \) because the failure of the system is negative for consumers.

We know that during the period of time \( D \) in \( \left( \sum_{i=1}^{N_{pc}} N_i \right) \) of cases, the system is predictively maintained and in \( \left( \sum_{i=1}^{N_{cr}} (1 - N_i) \right) \) of cases, the system is correctly maintained.

Thus, the expected financial risks \( R_f \) can be evaluated by the following equation:

\[ R_f = M \cdot C \left( x \times \left( \sum_{i=1}^{N_{pc}} N_i \right) + y \times \left( \sum_{i=1}^{N_{cr}} (1 - N_i) \right) \right) \quad (Eq. 7) \]

where \( M \) is the number of potential customers at the beginning of the period \( D \), and \( C \) is the cost of loss of one customer for the business.

#### 4.2 Environmental risks

A failure scenario may cause damages to environment by emission of harmful pollutants. For a failure scenario \( i \), we consider:
- \( n \): the total number of chemicals emitted during failure scenario \( i \).
- \( P = (P_1, P_2, ..., P_n) \): the probability emission of pollutants, so that \( P_j \) is the probability of emission of chemical \( j \) during the failure scenario \( i \).
- \( V = (V_1, V_2, ..., V_n) \): the volume of emission of pollutants, so that \( V_j \) is the volume of emission of chemical \( j \) during the failure scenario \( i \).
- \( q = (\rho_1, \rho_2, \ldots, \rho_n) \): the density vector of chemicals, so that \( \rho_j \) is the density value of chemical \( j \) emitted during the failure scenario \( i \).
- \( D_a = (D_{a1}, D_{a2}, \ldots, D_{an}) \): the cost of damage per tonne emission of pollutants, so that \( D_{aj} \) is the cost of damage per tonne emission of chemical \( j \) during the failure scenario \( i \).

There are several methods to evaluate the cost of damage of pollutants. For example, disability-adjusted life years (DALYs) is used to evaluate the environmental impact on human health by measuring the reduced quality of life due to illness in years [Gao et al., 2015]. Environmental burden of disease (EBD) assess the disease burden attributable to environmental risk factors [Prüss-Ustün et al., 2003], while the CAFE-CBA methodology is an approach for cost-benefit analysis used by the clean air for Europe (CAFE) program in order to quantify the damage of some chemicals to crops and to human health [Holland et Pye, 2005].

The environmental risks \( R_e \) of a failure scenario \( i \) by taking into account the expected number of failures can then be evaluated by the following equation:

\[
R_e = \left( \sum_{j=1}^{n} p_j \times v_j \times \rho_j \times D_{aj} \right) \sum_{i=1}^{N_{e-1}} (1 - N_i) \int_{T_e}^{T_{e+1}} f(t) \, dt \quad \text{(Eq. 8)}
\]

where \( D_{aj} \) is the cost of damage per tonne emission of pollutant \( j \) by considering one the previously described methods for damage cost evaluation.

4.3 Human risks

It is difficult to evaluate risks outside the financial domain. Risks to life and health are usually different from financial risks and thus, cannot be evaluated in terms of money. To deal with this issue, economists have introduced the concept of Value of Statistical Life (VSL). The VSL is the most frequent terminology to refer to the trade-off rate between fatality risks and money [Kip Viscusi, 1993], [Kip Viscusi et Aldy, 2003], [Kip Viscusi, 2004], [Machina et Kip Viscusi, 2013]; it reflects the worker’s willingness to pay to accept risk and to pay for more safety. The terminology of VSL emphasizes the probabilistic aspect of the valuation because at the time of a decision, the lives that will be saved are only probabilistically known [Shelling, 1968], [Machina and Kip Viscusi, 2013]. The VSL has attractive properties: according to [Machina et Kip Viscusi, 2013], it provides a cardinal measure of the value of life rather than an ordinal measure and it is applied to estimate the willingness-to-pay value as well-as the willingness-to-accept value to risk changes. Let’s note \( p_i^d \) the death probability of the person \( j \) in case of occurrence of failure scenario \( i \). The human risks \( R_h \) of a failure scenario \( i \) by taking into account the expected number of failures can be evaluated by the following equation:

\[
R_h = \left( \text{VSL} \cdot \sum_{j=1}^{n} p_j^d \right) \sum_{i=1}^{N_{e-1}} (1 - N_i) \int_{T_e}^{T_{e+1}} f(t) \, dt \quad \text{(Eq. 9)}
\]

where \( n \) is the total number of persons that are possibly being impacted by the failure scenario \( i \).

By way of similarities, this method can be applied to evaluate the risk of human injuries: we may consider different levels of injuries with their corresponding compensation costs.

In practice, experts are not able to evaluate with certainty the injury/death probability from a failure scenario. They are not able neither to evaluate the probability of emission of chemicals. Thus, methods for uncertainty reduction are used to estimate approximately these probabilities (appendix 1).

5 COST MODEL BASED ON RISK ASSESSMENT

In this section, we describe the optimization process to follow in order to identify the optimal strategy for predictive maintenance. The objective function to optimize is defined in section 5.1, the decision variables are described in section 5.2 and the constraints are described in section 5.3. Finally, a synthesis of the steps of the optimization process is described in section 5.4.

5.1 Objective function

The objective function that we want to minimize is the total cost of maintenance during the period of time \( D \). This objective function is given by the following equation:

\[
\text{Objective function} = C_{pm} + C_{cm} + C_{in} + C_{ol} + C_{il} \quad \text{(Eq. 10)}
\]

5.2 Decision variables

The decision variables that we need to determine by the process of optimization are the following:

- \( RUL_{lim} \): the limit of the RUL that indicates that a predictive maintenance should be performed on the system. It means:
  - If \( RUL_{system} \leq RUL_{lim} \), then the system should be maintained.
  - If \( RUL_{system} > RUL_{lim} \), then the system operates normally and does not need to be maintained.
- \( \theta \): the inspection step, i.e. the interval between two consecutive inspections.

5.3 Constraints

- **Positivity constraints**

The different costs should be positive:

\[
\begin{align*}
C_{pm} & \geq 0 \\
C_{cm} & \geq 0 \\
C_{in} & \geq 0 \quad \text{(Eq. 11)} \\
C_{ol} & \geq 0 \\
C_{il} & \geq 0
\end{align*}
\]

and the decision variables need to verify the following constraints:

\[
\begin{align*}
N_i & \text{ binary, } i = 1..N_{in} - 1 \\
\theta & \geq 0
\end{align*}
\]

- **Constraints on the inspection process**

The system requires at least one inspection: one inspection at the early life of the system. This can be translated by the following inequality:

\[
N_{in} \geq 1 \quad \text{(Eq. 13)}
\]

- **Constraints on the system’s availability**

We must ensure the availability of the system which means that the maintenance action should be negligible comparing
to the operating time of the system. In other words, the duration of both predictive and corrective replacement should be too small in comparison with the inspection step $\theta$. 

$$
\begin{align*}
D_p & \leq \epsilon \theta \\
D_c & \leq \epsilon \theta 
\end{align*} 
\quad (Eq. 14)
$$

with $\epsilon$ a sufficiently small number.

5.4 Synthesis

The inspection $i$ of the system gives data on the health state of the system, in particular a measurement of the RUL at time $t_i$ (time of inspection $i$). At time $t_i$, we need to decide whether predictive maintenance should be performed on the system or not. Therefore, we proceed by:

- assessing the different risks caused by a possible failure of the system
- evaluating the different costs related to maintenance
- identifying the decision variables that minimize the global cost of maintenance

Once the decision variables are evaluated, we are confronted with several possibilities:

- **Case 1**: the decision variable $N$ is equal to $I$, thus a predictive replacement of the system should be performed and the $RUL_{lim}$ is superior to the RUL of the system. In this case, the process must be reset once the system is replaced by a new one.
- **Case 2**: the decision variable $N$ is equal to $0$, thus there is no predictive replacement of the system. However, the system may fail or not before the next inspection (inspection $i+1$):
  - The system does not fail before inspection $i+1$: the system continues to operate normally and inspection $i+1$ is performed at time $t_i + \theta$.
  - The system fails before inspection $i+1$: a corrective replacement of the system should be performed and the process is reset once the system is replaced by a new one.

Figure 3 represents the flowchart of the global approach developed in this paper.

![Figure 3. Flowchart of the optimization process for predictive maintenance planning](image)

6 CONCLUSION AND FUTURE PERSPECTIVES

Maintenance aims at increasing the availability of a system, taking into account issues related to safety and to environment, and considering as well the problem of cost optimization. Risk assessment tries to answer the following questions:

- what can cause the system to fail?
- what are the possible impacts of system failure?
- how probable does it occur?

while cost optimization tries to answer the following questions:

- in regards with risk assessment, what are the different costs of maintenance?
- how to minimize the total cost of maintenance, in other words: what is the best time to perform inspection/maintenance in order to minimize the total cost of maintenance?

We tried in this paper to answer the previous questions by proposing an original approach for maintenance strategy optimization considering risk assessment. However, this approach is subject to several possible improvements:

- **uncertainty in the probability estimation**: in fact, it is not evident for the decision maker to evaluate with certainty the probability of a possible damage caused by a failure scenario. Thus, a method for uncertainty reduction may be integrated to our approach (appendix 1).
- **decision maker’s attitude to risk and uncertainty**: in reality, whether the decision maker is averse or not to risk or uncertainty has a tremendous impact on the optimization process. It may be interesting to integrate to our approach the attitude to risk and uncertainty of the decision maker (see example in appendix 2).

Our future work will try to develop these two points as a possible way to improve the approach described in this paper.

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9 APPENDIX 1 : UNCERTAINTY REDUCTION IN PROBABILITY ESTIMATION

Suppose that for a failure scenario \( i \), the total number of persons that are possibly being physically impacted by the failure of the system is known. We note \( n \) the total number of physically impacted people. All that experts know is that the probability of death of the \( j^{th} \) person lies in the interval \( P_j^d = [p_{j}^{d}, u_{j}^{d}] \subset [0,1] \).
where \( p_j^d \) and \( \overline{p_j^d} \) are respectively the minimum and maximum possible value of death probability of the \( j^{th} \) person [Aspinall et Cooke, 2013], [Ben Abdallah et Destercke, 2015]. The uncertainty \( U_j^d \) on the value of \( p_j^d \) can then be expressed as follows:

\[
U_j^d = \overline{p_j^d} - p_j^d
\]

We assume that the uncertainty \( U_j^d \) is reducible via expert elicitation and that experts are able to answer correctly all the questions about the input [Ben Abdallah et Destercke, 2015]. Thus, the expert knowledge are exploited to reduce the uncertainty \( U_j^d \) on the value of death probability \( p_j^d \) to a some desired level \( s_j^d \) [Ben Abdallah et Destercke, 2015].

- **Expert elicitation method:**
  We use simple questions that do not require high cognitive effort such as: « is \( p_j^d \leq \alpha_j \) ? » with \( \alpha_j \) in \( p_j^d \). Such a query is denoted by \( Q_j^{a_j} \) and the set of possible queries is denoted by \( Q = \{ Q_j^{a_j} , je\{1,\ldots,n\}, \alpha_j \in p_j^d \} \). The set of possible answers \( A \) is binary \{yes, no\}. When a question \( Q_j^{a_j} \) is asked and an answer \( a \in A \) is collected, \( p_j^d \) remains the same if \( k \neq j \), while if \( k = j \), \( p_j^d \) is updated to \( p_j(Q_j^{a_j}, A) \) as follows [Ben Abdallah et Destercke, 2015]:

\[
p_j(Q_j^{a_j}, A) = \begin{cases} p_j^d \cap [\alpha_j, \infty) & \text{if } k = j, A = Yes \\ p_j^d \cap (-\infty, \alpha_j] & \text{if } k = j, A = No \end{cases}
\]

which satisfies \( p_j(Q_j^{a_j}, A) \in p_j^d \) for whatever \( Q_j^{a_j} \in Q \) and \( a \in A \).

\( U_j^d(\overline{p_j^d}) \) is then the uncertainty reduction induced by query \( Q_j \): this is a typical problem of decision making under uncertainty where the decision is the value of \( \alpha_j \), the uncertain event is the answer to the question and the outcome that we want to maximize is the uncertainty reduction on the output \( p_j^d \) [Ben Abdallah et Destercke, 2015].

The method of expert elicitation is used to reduce uncertainty. As one may expect, this method can also be applied to reduce uncertainty on the estimation of probability of chemical emission or on the estimation of probability of human injury, because experts are not able to measure these probabilities with certainty.

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### Appendix 2: Decision Making Criteria Under Uncertainty

Decision under uncertainty refers to the problem of choosing the best decision \( d \) among a set of possible alternatives \( D \) which desired outcomes (or utilities), \( U(d,e) \), depend on an uncertain event \( e \) in a set of events \( E \). The objective of the decision maker is to maximize the outcome under incomplete knowledge [Aspinall et Cooke, 2013], [Ben Abdallah et Destercke, 2015]. The decision theory proposes several rules that model rational behaviour the decision maker may use to evaluate the alternatives and make choices. In what follows, uncertainty is not totally reduced and probabilities of each event are not well known:

- **The rule of maximax:** it describes extreme optimistic attitude and selects the alternatives with the best-case outcome [Hwang et Yoon, 1981]. This rule can be represented by the following equation:

\[
a_j^* = \arg \min_{a_j \in p_j^d} \min_{\alpha_j \in p_j^d} \left( U(p_j(Q_j^{a_j}, No), U(p_j(Q_j^{a_j}, Yes)) \right)
\]

- **The rule of maximin (Wald’s criterion):** it suggests to select the alternatives with the worst case outcome. It reflects a cautious attitude [Wang et Boutilier, C., 2003], [Viappiani et Kroer, 2013]. This rule can be expressed by the following equation:

\[
a_j^* = \arg \min_{a_j \in p_j^d} \max_{\alpha_j \in p_j^d} \left( U(p_j(Q_j^{a_j}, No), U(p_j(Q_j^{a_j}, Yes)) \right)
\]

- **Hurwicz’s criterion:** it is a trade-off between the strategies above. It uses a set of balancing coefficients \( p \) and \( q \) that satisfies \( p + q = 1 \). They reflect respectively the decision maker’s degree of optimism and pessimism [Hwang et Yoon, 1981].

\[
a_j^* = \arg \min_{a_j \in p_j^d} \left( p \cdot \min_{\alpha_j \in p_j^d} \left( U(p_j(Q_j^{a_j}, No), U(p_j(Q_j^{a_j}, Yes)) \right) + q \cdot \max_{\alpha_j \in p_j^d} \left( U(p_j(Q_j^{a_j}, No), U(p_j(Q_j^{a_j}, Yes)) \right) \right)
\]