



**HAL**  
open science

# The effect of liquefaction-induced damage on an embankment: “virtual” laboratory tests

Christina Khalil, Fernando Lopez-caballero

## ► To cite this version:

Christina Khalil, Fernando Lopez-caballero. The effect of liquefaction-induced damage on an embankment: “virtual” laboratory tests. 3rd meeting of EWG Dams and Earthquakes. An international Symposium, May 2019, Lisboa, Portugal. hal-02177243

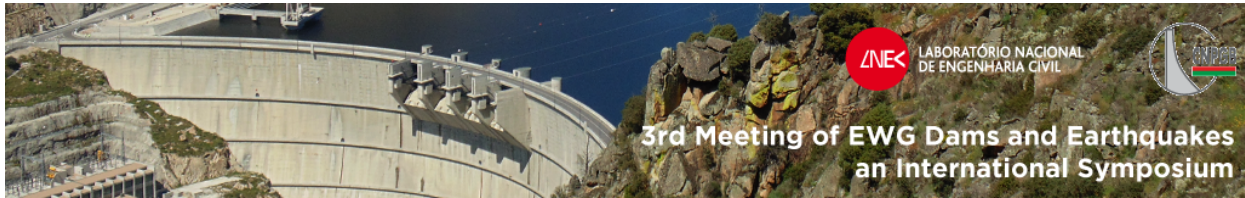
**HAL Id: hal-02177243**

**<https://hal.science/hal-02177243>**

Submitted on 22 Nov 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# THE EFFECT OF LIQUEFACTION-INDUCED DAMAGE ON AN EMBANKMENT: “VIRTUAL” LABORATORY TESTS

Christina Khalil<sup>1</sup> and Fernando Lopez-Caballero<sup>2</sup>

MSS-Mat CNRS UMR 8579 Laboratory, CentraleSupélec, Paris-Saclay University,  
8-10 rue Joliot Curie, Gif-sur-Yvette, France  
E-mail: [christina.khalil@centralesupelec.fr](mailto:christina.khalil@centralesupelec.fr)  
[fernando.lopez-caballero@centralesupelec.fr](mailto:fernando.lopez-caballero@centralesupelec.fr)

**Keywords:** liquefaction; virtual; laboratory tests; earthquake engineering practice

**Abstract.** *In earthquake engineering practice, the site effect simulation is evaluated based on the geometry and stratigraphy of the site, the characterization of the soil properties and the ground motions records. Usually, to identify the soil characterization, laboratory or in-situ tests take place (i.e. triaxial tests, direct shear tests, CPT or SPT ...). In order to simulate these tests and in case of absence of experimental tools, numerical models can represent the response of the soil. Liquefaction, till our days, is considered as one of the most complex behaviors that happen to the soil due to sudden and severe shakings. The aim of this paper is to conduct “virtual” laboratory tests on soil samples extracted from the foundation of an embankment. They serve to a better representation of the realistic case of the soil taking into account complex input data and to a comparison with the laboratory tests for the purpose of understanding the global soil response. The numerical model used is an elasto-plastic multi-mechanism model to represent the soil behavior.*

## 1 INTRODUCTION

The site effects are known as the local ground response, basin effects, and surface topographic effects. In practice, either empirical models or wave propagation analysis are used to quantify these effects. It is also necessary to know i) the geometry and stratigraphy of the site; ii) the characterization of the soil properties from geotechnical or geophysical tests; iii) ground motions recorded at the site of interest (i.e. seismic hazard) and iv) the choice of soil material model [1,2, among others]. Laboratory and in-situ tests serve to characterize the soil properties. They collect data based on definite experimental or field conditions that in some cases could not reflect the realistic case. Hence, they represent a large range of uncertainties. For example, the cyclic stress-based liquefaction resistance is influenced by factors such as the soil fabric, the age, the stress-strain history [3]. These factors can be destroyed by sampling and are difficult to replicate in the laboratory [3]. In addition, in-situ tests collect data from sites that are gently sloped or not geographically complicated, so the data will be affected by site conditions or restrictions [4].

---

<sup>1,2</sup> MSS-Mat CNRS UMR 8579 Laboratory, CentraleSupélec Paris-Saclay University,

Therefore, numerical models not only help to understand the global response of the soil even if there exist a big number of input variables, but also, they serve to validate the laboratory or in-situ tests.

Otherwise, liquefaction is one of the most devastating and complex behaviors that affect the soil. When induced by seismic shakings, liquefaction is related to the volume change of the soil and to the buildup of the pore water pressure [5]. Under both static and cyclic loadings, it is considered as one of the major causes of damage for earth structures and foundations [6]. It is well known that not all the soil types are susceptible to liquefaction [3]. The soil gradation, the particle shape, the density, all are factors that influence the soil to be susceptible to liquefaction. In addition to the evaluation of the disturbance severity that initiated or triggered the liquefaction of the soil.

This paper aims to simulate virtual tests in order to find the behavior of the soil and identify the liquefaction apparition. At the beginning, a soil sample is extracted from the foundation of an embankment, and typical triaxial tests were simulated. They serve as the laboratory data input of this study. Consistency with the laboratory experiments was taken into consideration by the type of loading and experiments conditions applied. Hence, regular and irregular loading were simulated on the sample and the liquefaction resistance of the sample was identified. These ideas are developed in Sections 3 and 4. In Section 5, a study on a 1D soil column taken at free field is conducted in order to compare the behavior of the soil after the propagation of the earthquake. For this purpose, 447 real input ground motions were injected on the column and the soil behavior was tested in addition to the liquefaction resistance. Finally, an accuracy between the laboratory and the finite element methods is conducted based on statistical approach. The finite element calculations of this study were performed using the GEFDyn code [7].

## 2 NUMERICAL MODEL AND GEOMETRY

### 2.1 Numerical model

A constitutive model for soils should be able to well represent the volume change in case of drained conditions or the distribution of the excess pore water pressure in case of undrained conditions [7]. The use of numerical modelling best describes the non-linear soil behavior under cyclic loading. A good modelling takes into consideration essential properties: necessary data, an appropriate constitutive model and adequate parameters, in addition to the method that solves boundary value problems [8]. The *Ecole Centrale Paris ECP* elastoplastic multi-mechanism model (also known as Hujeux model) is the one chosen for this study and it deals with the effective stresses. The non-linearity of this model is represented by four coupled elementary plastic mechanism: three plane-strain deviatoric plastic strain mechanism in three orthogonal planes ( $k$  - planes) and an isotropic plane to take into account normal forces. For the sake of brevity, the details of this numerical model will not be developed, hence, more information are provided in [7] and [8].

### 2.2 Geometry

The geometry of the virtual model, as shown in Figure 1, consists of a soil foundation of 10 m. The shallow layer is composed of a contractive loose to medium sand of 4 m followed by a 6 m layer of a dense sand. The bedrock at the bottom of the dense sand is 5 m and has the shear wave velocity  $V_s = 1000$  m/s. The water table is at 1 m below the ground surface. The “virtual” experimental laboratory tests were conducted first on a soil sample that has the same properties

as the shallow saturated layer. Second, a model of 1D column at free field, is taken into consideration from which a soil sample extracted from the middle and bottom of this layer (i.e. 2.5 m and 4 m below the ground surface). The effect of the embankment is not taken into consideration in this paper.

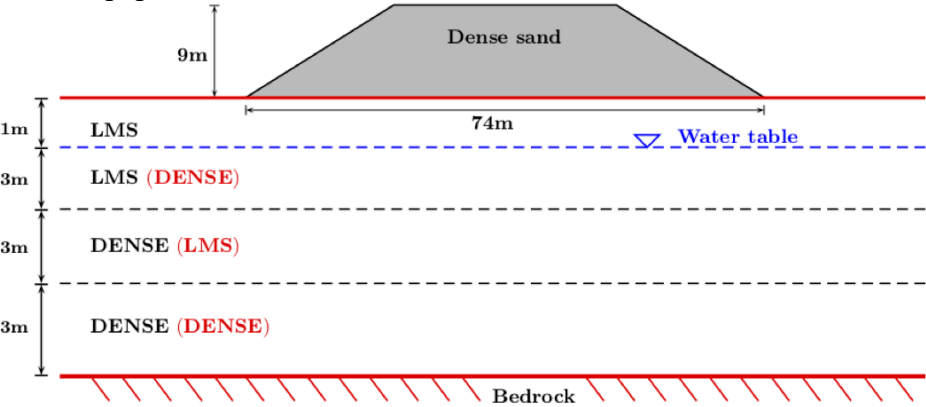


Figure 1: Model geometry

### 3 LABORATORY TESTS SIMULATIONS

In the development of the constitutive models, an identification of the effective stress path of the soil is important for analyzing its behavior. To do so, two approaches of liquefaction apparition are developed: the “Critical State” approach and the “Collapse Surface” approach; they are based on the effective stress path of the soil subjected to drained or undrained tests. In the drained tests, the separation between the compressive and dilative regions of the soil is called: “Characteristic Line” whereas in the undrained tests, it is called “Transformation Line” or “Critical State Line”. The transformation phase of the soil occurs at the point where the stress path turns its direction in stress-strain space [9]. This point is considered critical because the soil sample has to go through at least once in order for it to reach a liquefied state. Nevertheless, different approaches have been conducted to quantify the liquefaction susceptibility. The Critical State approach is the cyclic resistance of the soil defined in terms of the required cyclic stress under which a specified amount of axial strain is developed in a given number of cycles [10]. Based on the study of [11], this amount is determined by the occurrence of 5% double-amplitude axial strain in which the state of liquefaction would be adopted. It should be mentioned that the occurrence of 2% or 3% double-amplitude axial strain is also acceptable based on other studies like [12, 10, among others]. Whereas, the second approach for liquefaction prediction is the Collapse Surface approach which defines the peak points of undrained effective stress paths surface in terms of the major and minor stresses [13]. This surface represents the limit of stability of the soil above which liquefaction will occur under load-controlled conditions, in some studies it is nominated as “Instability Line” [14,15, among others].

Similar to any common laboratory test, first, monotonic triaxial tests are conducted in order to understand the type of the material and estimate its behavior once subjected to different types of loading. Hence, drained and undrained monotonic triaxial tests were conducted (better saying, simulated) on the tested material. Notice that the triaxial test has been a preferred method to determine the soil parameters because its apparatus is widely available, and the sample behavior will always be determined because of the minimization of the non-uniformities in case of a

contractive or dilative soil [13]. Different initial values of the mean effective stresses were chosen ( $p'_0 = 20, 50$  and  $70$  kPa).

From the conducted test, the stress-strain path represented by the major and minor stresses (i.e.  $s' = (\sigma'_1 + \sigma'_3)/2$  and  $t = (\sigma'_1 - \sigma'_3)/2$ ) from the undrained monotonic triaxial tests is shown in Figure 2(a), and the variation of the volumetric strain from drained monotonic triaxial tests is shown in Figure 2(b).

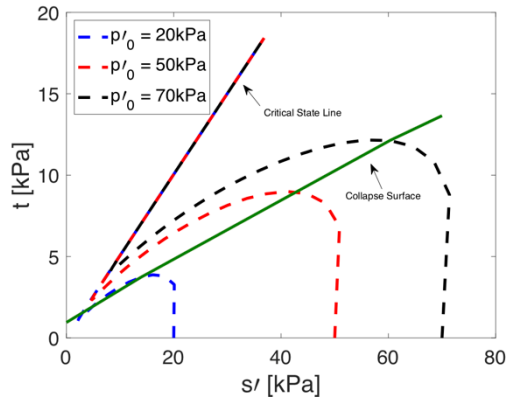


Figure 2(a): The stress path of the tested material

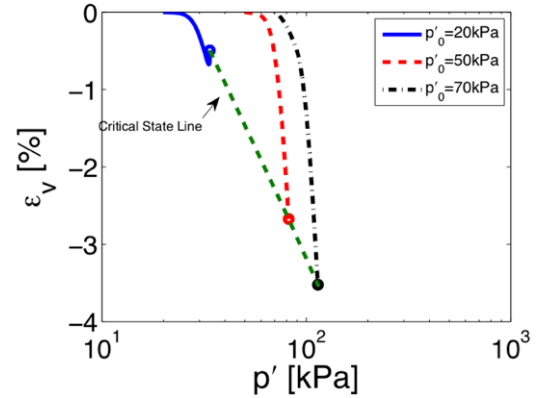


Figure 2(b): The variation of the volumetric strain with respect to the mean effective stress

It can be seen from Figure 2(a) that the stress path shows a peak value until it reached the Critical State Line (CSL). From Figure 2(b), the volumetric strain decreases to reach the CSL. From these "experimental" results, it can be viewed that the tested material is a "normally consolidated" sand. It is true that the consolidation nomination is only applied for clay soils and is difficult to adopt for sands except for some cases [16] but in this study, the nomination of *loose* and *dense* sand refers to high or low relative density  $D_r$  as will be shown later in this section. In Figure 2(a), the collapse surface passes by the maximum stresses for each initial confining pressure. It designates the failure initiation state of the material; above this line, the material is not stable and may be subjected to liquefaction.

Moreover, in practice, the liquefaction charts are used to quantify the liquefaction apparition. These charts are characterized by the severity of the earthquake loading and the soil resistance for liquefaction [17]. The level of the loading is characterized by the cyclic stress ratio which is taken to be the ratio of the maximum cyclic shear stress to the initial effective confining pressure (i.e.  $CSR = \frac{q_{cyc}}{2p'_0}$ ) [18,3, among others]. Whereas the soil resistance can be identified by field measurements (i.e. N-values of the SPT test, q-values of the CPT test and shear wave velocity) or laboratory tests (triaxial tests; monotonic or cyclic, drained or undrained). In the case of regular loading, which is best compatible with the laboratory experiments, different values of the cyclic shear stress were considered in order to find the soil resistance (i.e. 8, 9, 10, 11 and 12 kPa). For the sake of brevity only, one initial confining pressure is considered ( $p'_0 = 50$  kPa). The cyclic resistance is represented in the graphs as the cyclic stress ratio versus the number of cycles  $N_f$  that generates an axial strain of 2% double-amplitude [11]. The results are shown in red curve in Figure 3.

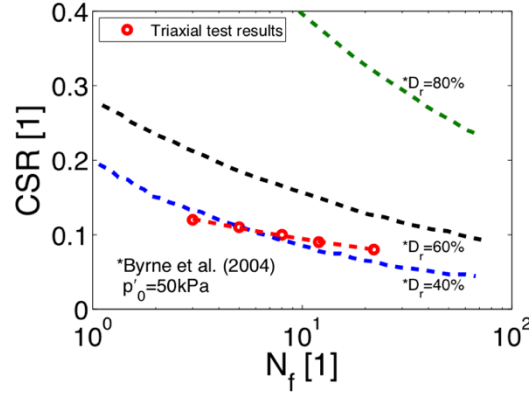


Figure 3: The Variation of the Cyclic Stress Ratio CSR as function of the number of cycles  $N_f$

In Figure 3, a reference to an experimental study [19] conducted on Nevada sand for different values of relative densities is represented. Comparing their results to the ones simulated in this study, the cyclic resistance of the tested material lies approximately on the curve of  $D_r = 40\%$ . Hence, the tested material is shown to behave as a loose sand, so it has a high tendency to generate liquefaction. The results of the cyclic resistance for regular loading will be used in the proceeding of this paper as a boundary between the combination of the loading parameter and the liquefaction resistance parameter in order to characterize the liquefaction apparition [3].

#### 4 EFFECT OF IRREGULAR CYCLIC LOADING

Liquefaction happens due to strong shakings; therefore, the effect of the earthquakes is important to consider. Against the usual laboratory tests that only deal with sinusoidal loading, application of irregular loading represents more realistically the effect of earthquake motions by the use of "virtual" laboratory tests [21]. For this purpose, this section will develop the effect of irregular cyclic loading on the tested sand by conducting triaxial undrained tests and by applying different loading with earthquake shapes. An example of the used type of load is represented in Figure 4. In order to compare an irregular earthquake-induced loading with laboratory loading conditions, a conversion factor from a series of irregular cycles to uniform cycles is required. Hence, 60% of the maximum shear stress value are used in this case [13, 3, 4 among others]. The values of the shear stresses were chosen accordingly (i.e. 16, 17, 18, 19 and 20 kPa). Based on the theory, liquefaction happens due to the rapid increase of the pore water pressure [13, 3, 5, among others] or when the axial strain reaches 2% or 3% double amplitude (DA) [12, 10 among others]. Accordingly, a representation of the generation of the excess pore water pressure  $\Delta u$  and the variation of the axial strain  $\varepsilon_1$  was found as function of the number of cycles  $N_{cyc}$  of the regular loading applied (Figure 5). The initial confining pressure used for this case is also  $p'_0 = 50$  kPa. For the sake of brevity, one tested motion will serve to analyze the soil behavior of this section and one value of the cyclic shear stress will be shown ( $q_{cyc} = 20$  kPa).

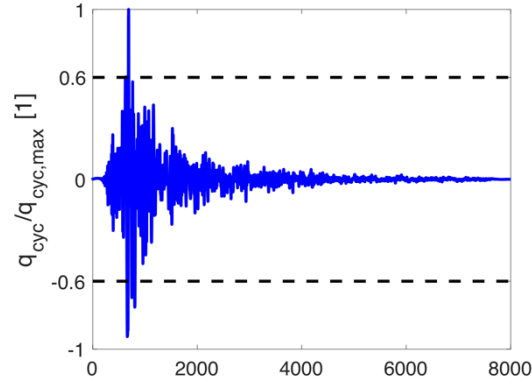


Figure 4: An example of an applied loading with earthquake shape

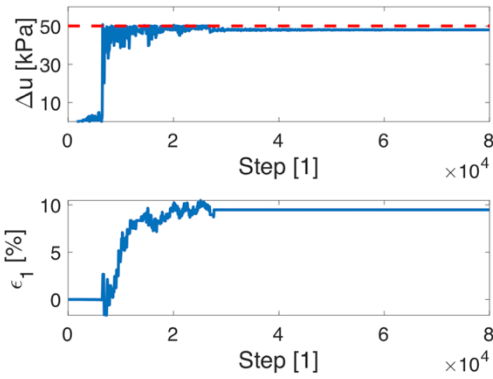


Figure 5: The variation of  $\Delta u$  (top) and  $\epsilon_1$  (bottom) with respect to  $N_{cyc}$  of the irregular loading

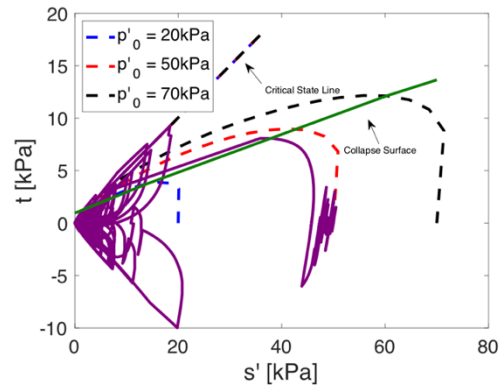


Figure 6: The stress path of the irregular loading

It can be seen from Figure 5 that the excess pore water pressure  $\Delta u$  increases at the beginning of the loading until it reaches the same value of the initial confining pressure of 50 kPa (i.e. the excess pore water pressure ratio  $r_u$  is equal to 1). As for the evolution of the axial strain, at the beginning of the loading, it was zero until it reaches a certain number of cycles where it shows values of 2% DA which corresponds also to the generation of the excess pore water pressure. Hence, the tested sand shows liquefaction for the presented value of cyclic shear stress and this is confirmed by the stress paths shown in Figure 6. This path shows clearly that the specimen enters an instability phase once it reaches the collapse line after which hysteretic loops starts to occur until failure is reached.

In order to identify the level of loading for all the tested motions, the cyclic stress ratio is represented in terms of the number of cycles. In the case of irregular loading, the number of cycles that generates liquefaction is different from that of regular loading (i.e. the one designated as  $N_f$  in the previous section). From the viewpoint of counting cycles in fatigue [21], the number of cycles of the irregular loading  $N_{equiv}$  is considered. Also in this case, the cyclic stress ratio becomes  $CSR^* = \frac{q_{cyc}}{2p'_0}$  with  $q_{cyc} = 0.6 q_{cyc,max}$ . The results are shown in Figure 7. The black dots designate the non-liquefied cases on the opposite of the blue dots that designate the liquefied cases. The red curve on Figure 7 is the boundary generated by the experimental data results developed in Section 3.



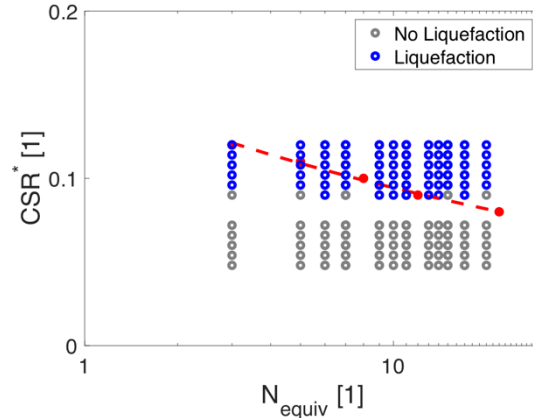


Figure 7: Cyclic stress ratio CSR\* for the case of irregular loading

From Figure 7, it can be seen that the increase in the cyclic stress affects in the generation of the liquefaction in a way that the specimen tends to liquefy. The cases that are below the boundary did not liquefy, whereas for the ones above it: there are cases that liquefied and other did not. It can be partially concluded that the laboratory tests under-estimate the soil response because the cases that were predicted to liquefy, did not do as such in the actual scenario. It should be noted that the experimental tests can also be complicated by the specimen non uniformity which causes large range of uncertainty [3]. To be more consistent with the realistic case, the behavior of the soil will be studied based simulated *in-situ* soil response. A study on a soil column will take place in the next section. Moreover, the soil response will be compared to the experimental and field data.

## 5 *IN-SITU* RESPONSE - STUDY ON A COLUMN

In the previous sections, the behavior of a soil sample was studied based on simulated laboratory experiments. The effect of irregular loading was taken into consideration in order to better approach the realistic case of a soil subjected to earthquake, and hence to characterize the liquefaction apparition. In this section, the behavior of the soil sample is simulated from a 1D wave propagation in a column. The finite element code used for the simulation is a 2D coupled modelling with GEFDyn Code [7] using a dynamic approach derived from the  $u - p_w$  version of the Biot's generalized consolidation theory [22]. The FE element model is composed of quadrilateral isoparametric elements (0.5 m x 0.5 m). For the boundary condition of the dynamic phase, only vertically incident shear waves are introduced in the column. For the bedrock's boundary condition, paraxial elements simulating "deformable unbounded elastic bedrock" have been used [23]. To take into account the dynamic effect, 447 different ground motions were injected at the bottom of the column so that the waves will propagate all along the soil layers. For the sake of brevity only, the behavior of the soil will be studied for one tested ground motion as an example. The response of all the tested motions will be represented in the proceedings of this section. The tested soil sample was extracted from the bottom of the liquefied layer, so at 4m below the ground surface.



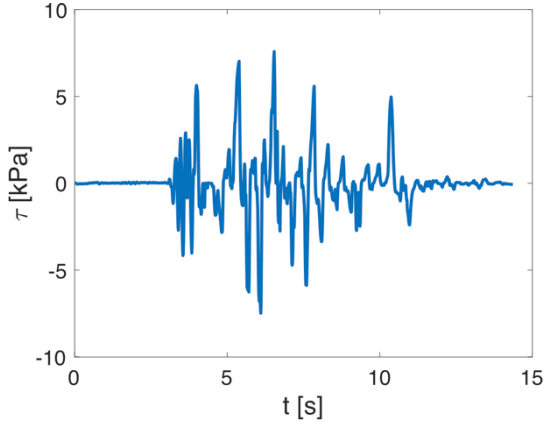


Figure 8: Obtained irregular time history of shear stress

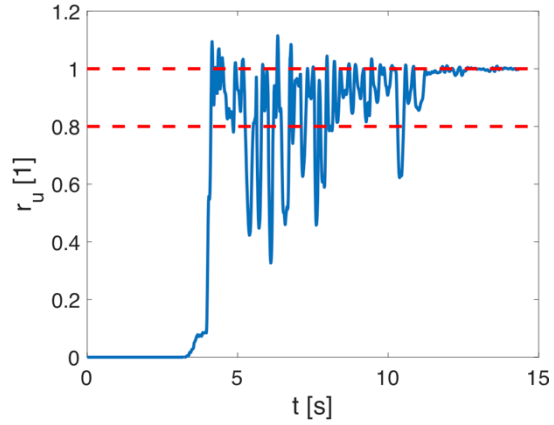


Figure 9: Excess pore water pressure time history

Figure 8 shows the variation of the obtained shear stress  $\tau$  during the earthquake occurrence. It shows irregular behavior. Figure 9 shows the variation of the excess pore water pressure ratio  $r_u$ . To take into consideration the cyclic mobility and the true liquefaction, it is assumed that liquefaction appears when  $r_u > 0.8$  [24]. For the ground motion taken as an example, it can be seen that liquefaction occurs.

Characterizing the liquefaction resistance of the soil and taking into account all the tested ground motions, the cyclic stress ratio approach is developed for this case as well. Laboratory tests show that the cyclic shear stress required to trigger liquefaction increases at high effective confining pressures [3]. [25] and [4] suggested correction factors that take into account the effect of the initial shear stress (i.e.  $K_\alpha$ ) and the effective overburden pressure (i.e.  $K_\sigma$ ). The field corrected cyclic stress ratio for the case of this study is shown as a green curve in Figure 10. This curve will be considered as the new boundary for the coming tested cases. Studying the soil response at the bottom of the liquefied layer, the cyclic stress ratio was found for the 447 ground motions and is shown also in Figure 10. It was calculated similar to the case of irregular loading described in Section 4, in addition to the same definition of the number of cycles (i.e.  $N_{equiv}$ ). Liquefaction apparition was identified based on the excess pore water pressure ratio. The motions that did not liquefy are represented in black and the ones that did, are shown in blue.

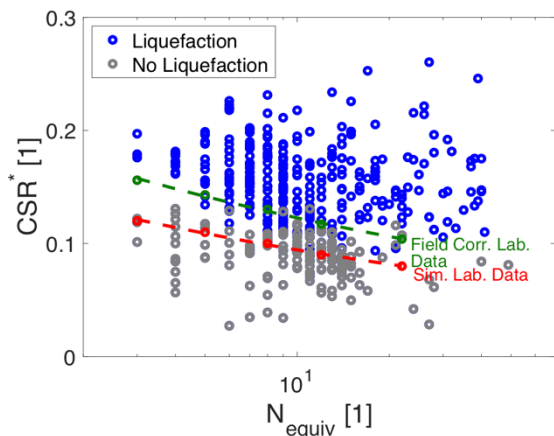


Figure 10: The cyclic stress ratio of the tested motions

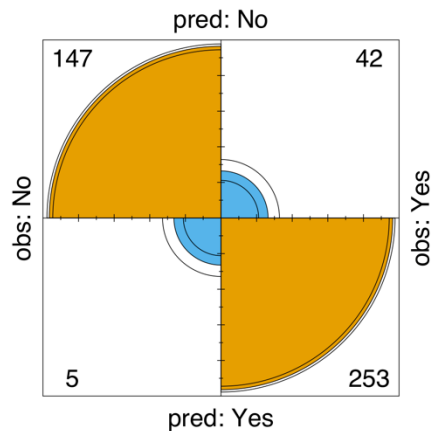


Figure 11: The confusion matrix of the tested motions

It can be seen from Figure 10 that there exist some cases where liquefaction did not occur above the field corrected boundary. In addition to some cases where liquefaction occurs below the boundary. Hence, accuracy of the laboratory tests cannot be confirmed. In order to better

understand it, in addition to an understanding of the global response of the soil regarding the tested ground motions, confusion matrices were drawn and shown in Figure 11. The purpose of these matrices in the presented case of this paper, is to summarize the performance of the soil based on both the actual observations (i.e. finite element methods) and the predicted observations (i.e. laboratory tests). From Figure 11, 147 cases are not supposed to liquefy based on the predicted laboratory test and actually, they did not liquefy. These cases are called the True Negative (TN) results. Also, 253 cases are supposed to liquefy, and they liquefied based on the actual observations. Hence, they are the True Positive (TP). On the contrary, 5 cases located above the field corrected boundary did not liquefy based on the actual observations and they are called the False Positive (FP) data. And finally, the False Negative (FN) data (i.e. 42 cases) are the ones that were not supposed to liquefy since their cyclic stress ratio is below the field corrected boundary, but based on the actual observations, they liquefied. The FN data are considered as a dangerous case for the decision making. They under-estimated the response of the soil because the soil that did not liquefy in the laboratory, had a different behavior when being in-situ. The FP data are also not beneficial for the decision maker since in the laboratory the soil sample liquefied whereas when the wave propagated in the realistic case, the soil did not liquefy. This means that unnecessary precautions could be set in field if there was a reference to laboratory tests only.

Accuracy between the laboratory and the finite element method in this case, is 89.49% (i.e.  $(TN+TP)/(TN+TP+FN+FP)$ ). This will lead to a partial conclusion that the laboratory tests, in some cases, under-estimate the response of the soil. This last one will be accurate when attached to numerical models for a better understanding of the soil global response.

## 6 CONCLUSIONS

Virtual laboratory tests were simulated in this paper via an elastoplastic multi-mechanism soil behavior model with the help of a 2D finite element code (GEFDyn). These tests are helpful to validate the laboratory tests in order to better understand the global response of the soil.

At the beginning of this paper, the behavior of a soil sample extracted from the foundation of an embankment was found based on simulated triaxial tests. To be consistent with the realistic cases that the loading applied on the soil are not regular, loadings with earthquake shaped were simulated on the soil sample. Based on this case, the change in the behavior of the soil was analyzed. In the final section, in-situ tests were simulated on a 1D column after the propagation of 447 real input motions. The behavior of the soil at the bottom of the liquified layer was analyzed accordingly. At the end of this section an accuracy study between the laboratory and the finite element methods.

The results have shown that the studied soil sample is very loose and hence, it has an ability to liquefy rapidly. Subjected to regular loading, the cyclic resistance of the sample created a boundary that was used as a reference to characterize the liquefaction apparition. It was shown that based on irregular loading, liquefaction did not appear below the boundary whereas above it, the response depends on the severity of the irregular load.

Concerning the in-situ simulations, the results of the soil resistance show that below a field corrected boundary, the soil may liquefy whereas above it, the response will also depend on the severity of the earthquake.

As a conclusion, the laboratory tests in some cases, under-estimate the soil response but they are somehow accurate with the finite element models. Hence, for a decision making and to a good estimation of the soil response, experimental and numerical tests will be perfect.

## 7 ACKNOWLEDGMENT

This work, within the ISOLATE project, benefited from French state funding managed by the National Research Agency reference under program Mobility and Sustainable Urban Systems (DS06) 2017 reference No. ANR-17-CE22-0009. The research reported in this paper has been supported in part by the SEISM Paris Saclay Research Institute.

## REFERENCES

- [1] Lopez-Caballero, F., Razavi, A. M. F., & Modaressi, H. (2007). Nonlinear numerical method for earthquake site response analysis I—elastoplastic cyclic model and parameter identification strategy. *Bulletin of Earthquake Engineering*, 5(3), 303.
- [2] Foerster, E., & Modaressi, H. (2007). Nonlinear numerical method for earthquake site response analysis II—case studies. *Bulletin of Earthquake Engineering*, 5(3), 325-345.
- [3] Kramer, S. (2005). L.(1996). Geotechnical Earthquake Engineering. *Pren-tice Hall, New Jersey*.
- [4] Youd, T. L., & Idriss, I. M. (2001). Liquefaction resistance of soils: summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils. *Journal of geotechnical and geoenvironmental engineering*, 127(4), 297-313.
- [5] Ueng, T. S., Wu, M. C., Lin, C. Y., & Yu, R. Y. (2000, January). Pore water pressure changes in sands under earthquake loading. In *Proceedings of the 12th World Conference on Earthquake Engineering, Auckland, New Zealand* (Vol. 30).
- [6] Papadopoulou, A. I., & Tika, T. M. (2016). The effect of fines plasticity on monotonic undrained shear strength and liquefaction resistance of sands. *Soil Dynamics and Earthquake Engineering*, 88, 191-206.
- [7] Aubry, D., Hujeux, J. C., Lassoudiere, F., & Meimon, Y. (1982, September). A double memory model with multiple mechanisms for cyclic soil behavior. In *Proceedings of the Int. Symp. Num. Mod. Geomech* (pp. 3-13).
- [8] Lopez-Caballero, F., Modaressi, A., & Elmi, F. (2003). Identification of an elastoplastic model parameters using laboratory and in-situ tests. In *Deformation Characteristics of Geomaterials/Comportement Des Sols Et Des Roches Tendres* (pp. 1202-1209). CRC Press.
- [9] Ishihara, K., Tatsuoka, F., & Yasuda, S. (1975). Undrained deformation and liquefaction of sand under cyclic stresses. *Soils and foundations*, 15(1), 29-44.
- [10] Pan, K., & Yang, Z. X. (2018). Effects of initial static shear on cyclic resistance and pore pressure generation of saturated sand. *Acta Geotechnica*, 13(2), 473-487.
- [11] Ishihara, K. (1993). Liquefaction and flow failure during earthquakes. *Geotechnique*, 43(3), 351-451.
- [12] Cubrinovski, M. (2011). Seismic effective stress analysis: Modelling and application.
- [13] Sladen, J. A., D'hollander, R. D., & Krahn, J. (1985). The liquefaction of sands, a collapse surface approach. *Canadian Geotechnical Journal*, 22(4), 564-578.
- [14] Lade, P. V. (1994). Instability and liquefaction of granular materials. *Computers and Geotechnics*, 16(2), 123-151.

- [15] Lade, P. V., & Ibsen, L. B. (1997, October). A study of the phase transformation and the characteristic lines of sand behavior. In *Proc. Int. Symp. on Deformation and Progressive Failure in Geomechanics, Nagoya* (pp. 353-359).
- [16] Biarez, J., & Hicher, P. Y. (1994). *Elementary mechanics of soil behavior: saturated remoulded soils*. AA Balkema.
- [17] Sassa, S., & Yamazaki, H. (2016). Simplified liquefaction prediction and assessment method considering waveforms and durations of earthquakes. *Journal of Geotechnical and Geoenvironmental Engineering*, 143(2), 04016091.
- [18] Seed, H. B., & Idriss, I. M. (1971). Simplified procedure for evaluating soil liquefaction potential. *Journal of Soil Mechanics & Foundations Div.*
- [19] Byrne, P. M., Park, S. S., Beaty, M., Sharp, M., Gonzalez, L., & Abdoun, T. (2004). Numerical modeling of liquefaction and comparison with centrifuge tests. *Canadian Geotechnical Journal*, 41(2), 193-211.
- [20] Kim, S. I., Park, K. B., Park, S. Y., Hwang, S. J., Lee, J. H., & Choi, J. S. (2005, December). Effects of irregular dynamic loads on soil liquefaction. In *16th International Conference on Soil Mechanics and Geotechnical Engineering: Geotechnology in Harmony with the Global Environment, ICSMGE 2005*.
- [21] Niesłony, A. (2009). Determination of fragments of multiaxial service loading strongly influencing the fatigue of machine components. *Mechanical Systems and Signal Processing*, 23(8), 2712-2721.
- [22] ZIENKIEWICZ, C. (1991). The finite element method; solid and fluid mechanics. *Dynamics and non-linearity*, 2, 219.
- [23] Modaressi, H., & Benzenati, I. (1994). Paraxial approximation for poroelastic media. *Soil Dynamics and Earthquake Engineering*, 13(2), 117-129.
- [24] Rapti, I., Lopez-Caballero, F., Modaressi-Farahmand-Razavi, A., Foucault, A., & Voldoire, F. (2018). Liquefaction analysis and damage evaluation of embankment-type structures. *Acta Geotechnica*, 13(5), 1041-1059.
- [25] Seed, H. B. (1981). Earthquake-resistant design of earth dams.