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An Efficient DOA Estimation Method for Co-Prime Linear Arrays

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ABSTRACT Direction-of-arrival (DOA) estimation with a co-prime linear array, composed of two uniform linear arrays with inter-element spacing larger than half-wavelength of incoming signals, has been investigated a lot thanks to its high-resolution performance. For better computational efficiency, one class of methods treat the co-prime linear array as two sparse uniform linear subarrays. From each of them, high-precision but ambiguous DOA estimation is obtained, and the ambiguities are eliminated according to the co-prime property. However, the existing methods of this kind suffer from the insufficient reliability and high complexity. In this paper, the potential problems associated with the DOA estimation with two co-prime subarrays are discussed, and a reliable and efficient DOA estimation method is proposed. For each subarray, the true DOAs are treated as their equivalent angles and the pair matching of them is accomplished by exploring the cross-correlations between the equivalent signals associated with the equivalent angles. Compared with other existing methods, the proposed method is able to achieve a better estimation performance in all situations, in terms of accuracy and complexity.

INDEX TERMS Ambiguity, co-prime linear arrays, DOA estimation, grating angles, pair matching errors.

I. INTRODUCTION

Direction-of-arrival (DOA) estimation is one of the most crucial problems in radar, wireless communication and other applications [1]–[3]. Numerous DOA estimation algorithms, such as multiple signal Classification (MUSIC) [4] and Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT) [5], have been proposed for uniform linear arrays (ULAs). However this array geometry is not optimal due to the small array aperture and the possible mutual coupling effect between adjacent sensors. Recently, co-prime arrays have become a research focus and drawn lots of attention [6], [7]. A co-prime array can be regarded as a superposition of two ULAs with inter-element spacing larger than half-wavelength of incoming signals. Therefore, a larger array aperture can be achieved, and consequently a higher resolution and a better estimation performance can be obtained.

There are two main research orientations for the DOA estimation with co-prime linear arrays, which are

difference-coarray-based methods and subarray-based methods. The difference-coarray-based methods try to increase the number of consecutive covariance lags in a virtual half-wavelength spacing ULA coarray such that the degrees of freedom (DOFs) can be greatly increased [6]–[9]. In the subarray-based methods, the co-prime array is treated as two sparse uniform linear subarrays. From each of them, high-precision but ambiguous DOA estimation is obtained, and the ambiguities are eliminated according to the co-prime property [10]–[15]. The difference-coarray-based methods can increase the number of detectable incoming signals, but they require a great number of snapshots, which makes the algorithms computationally complex. In contrast, separately dealing with two uniform subarrays, the subarray-based methods sacrifice the DOFs, but can directly and efficiently exploit the uniform property. Consequently, the DOA estimation can be accomplished with low-complexity methods, which is more practical in some real applications. The subarray-based methods will be considered in this paper.

Many DOA estimation methods have been proposed in this research orientation. A MUSIC-based method is proposed

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in [11]. By dividing the co-prime array into two ULAs, and finding the common peaks of their MUSIC-spectrums, the DOAs can be uniquely obtained and the ambiguities caused by the large inter-element spacing can be eliminated based on the co-prime property. But the complexity caused by the step of peak-searching is high. Another method is proposed in [12], which can reduce the computational complexity by limiting the peak-searching region. However, since it also involves the step of peak-searching, the computational burden is still heavy. Besides, the methods in [11] and [12] suffer from the problem of pair matching errors when the number of incoming signals is greater than one. A low complexity method based on ESPRIT is proposed in [13]. Without spectral searching, the complexity is significantly reduced. The matching errors are eliminated by beamforming-based techniques, and the true DOAs are estimated uniquely. Similarly, another method for fixing the pair matching errors problem is proposed in [14]. Based on Root-MUSIC, it has low complexity. By exploiting the relationship between the directional matrices of the two subarrays, the pair matching of the estimated angles can be achieved automatically, and the ambiguities can be mitigated one by one. However, because of the large inter-element spacing, when two or more source signals come from a set of specific angles, for which they have exactly a same directional vector for one subarray, the directional matrix of this subarray will be rank deficient, it is then challenging to find the true DOAs for all the above mentioned methods. These specific angles are called grating angles, and this problem is called grating angles problem, which is discussed in [15], where a joint singular value decomposition (JSVD) based method is proposed. Thanks to the JSVD algorithm, the grating angles can be differentiated and the pair matching can be accomplished automatically. Nevertheless, since a “beamforming-like” method with spectral searching is involved, the performance of this method is limited by the searching step and high complexity.

In this paper, an efficient DOA estimation method is proposed. For each subarray, the true DOAs are mapped into the equivalent angles corresponding to a virtual traditional half-wavelength spacing ULA. From the perspective of accuracy and efficiency, after estimating the number of the equivalent signals, the ESPRIT method is performed and two sets of equivalent angles can be estimated from the two subarrays, respectively. Then the associated equivalent signals can be recovered. By exploring the cross-correlations between the equivalent signals recovered from the two subarrays, the pair matching of the equivalent angles is accomplished. Consequently, based on the relationship between a DOA and its equivalent angles, two sets of candidate DOAs are recovered for each pair of matched equivalent angles, and the corresponding true DOA is uniquely determined by finding the common element. Compared with other existing methods, the proposed method is able to achieve a better estimation performance in all situations, in terms of accuracy and complexity. Simulation results are provided to show the performance of the proposed method.

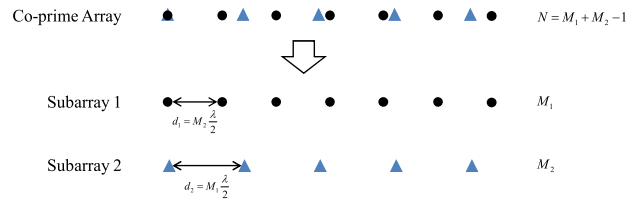


FIGURE 1. System model of a co-prime linear array.

Notations: in this paper, bold lowercase letters and bold capital letters symbolize vectors and matrices, respectively. Superscript $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^+$ denote the transpose, complex conjugate and pseudo-inverse operator, respectively. $|\cdot|$ denotes the modulus operator and \mathbf{I}_M stands for the identity matrix with dimension $M \times M$.

II. SYSTEM MODEL

Consider a co-prime linear array composed of two uniform linear subarrays, having M_1 and M_2 sensors, with inter-element spacing $d_1 = M_2 \frac{\lambda}{2}$ and $d_2 = M_1 \frac{\lambda}{2}$, respectively, where λ is the wavelength of incoming signals and M_1 and M_2 are two co-prime integers. The first sensor is shared by the two subarrays and set as the reference point. The total number of sensors is $N = M_1 + M_2 - 1$. FIGURE 1 shows the situation where $M_1 = 7$ and $M_2 = 5$.

Suppose that there are K (K is supposed to be known and $K < \min\{M_1, M_2\}$) uncorrelated, far-field and narrowband signals coming from directions $\{\theta_1, \theta_2, \dots, \theta_K\}$, respectively, with $-90^\circ < \theta_k < 90^\circ$ and $1 \leq k \leq K$. The signal received at the i^{th} subarray is

$$\begin{aligned} \mathbf{x}_i(t) &= \sum_{k=1}^K \mathbf{a}_i(\theta_k) s_k(t) + \mathbf{n}_i(t) \\ &= \mathbf{A}_i \mathbf{s}(t) + \mathbf{n}_i(t) \end{aligned} \quad (1)$$

where $\mathbf{A}_i = [\mathbf{a}_i(\theta_1) \mathbf{a}_i(\theta_2) \dots \mathbf{a}_i(\theta_K)]$ is the directional matrix of the i^{th} subarray, with directional vector $\mathbf{a}_i(\theta_k) = [1 \ e^{jM_i \pi \sin \theta_k} \dots e^{j(M_i-1)M_i \pi \sin \theta_k}]^T$, $i, \tilde{i} \in \{1, 2\}$ and $i \neq \tilde{i}$; $\mathbf{s}(t) = [s_1(t) \ s_2(t) \dots \ s_K(t)]^T$ denotes the source signals vector with $s_k(t)$ the signal transmitted by the k^{th} source and received at the reference sensor, and $\mathbf{n}_i(t)$, which is assumed to be independent from the source signals, is the white Gaussian noise vector with zero-mean and covariance matrix $\sigma^2 \mathbf{I}_{M_i}$, with σ^2 the noise power.

Due to the property of sinusoid function, for the signal coming from θ_k and impinging on the i^{th} subarray, there exists a unique angle denoted as $\theta_{i,k}^{\text{map}}$ ($-90^\circ < \theta_{i,k}^{\text{map}} < 90^\circ$), satisfying

$$\sin \theta_{i,k}^{\text{map}} = M_i \sin \theta_k + 2n_{i,k} \quad (2)$$

where $n_{i,k}$ is an integer with $-\frac{M_i+1}{2} < n_{i,k} < \frac{M_i+1}{2}$.

Because of the property of the complex exponential function, the directional vector associated to this signal can be re-written as $\mathbf{a}_i^{\text{map}}(\theta_{i,k}^{\text{map}}) = [1 \ e^{j\pi \sin \theta_{i,k}^{\text{map}}} \dots \ e^{j(M_i-1)\pi \sin \theta_{i,k}^{\text{map}}}]^T$. Therefore, $\theta_{i,k}^{\text{map}}$ can be considered as the mapped angle on a

virtual half-wavelength spacing ULA of the true DOA θ_k on the i^{th} subarray. Consequently, the received signal model of the i^{th} subarray can be considered as K source signals coming from K mapped angles $\theta_{i,k}^{\text{map}}$ which impinge on a M_i -element virtual half-wavelength spacing ULA. Equation (1) can then be re-written as

$$\begin{aligned} \mathbf{x}_i(t) &= \sum_{k=1}^K \mathbf{a}_i^{\text{map}}(\theta_{i,k}^{\text{map}}) s_k(t) + \mathbf{n}_i(t) \\ &= \mathbf{A}_i^{\text{map}} \mathbf{s}(t) + \mathbf{n}_i(t) \end{aligned} \quad (3)$$

where $\mathbf{A}_i^{\text{map}} = [\mathbf{a}_i^{\text{map}}(\theta_{i,1}^{\text{map}}) \mathbf{a}_i^{\text{map}}(\theta_{i,2}^{\text{map}}) \dots \mathbf{a}_i^{\text{map}}(\theta_{i,K}^{\text{map}})]$ is the mapped directional matrix of the i^{th} subarray, and the set of the K mapped angles associated to the K DOAs for the i^{th} subarray is defined as $\Theta_i^{\text{map}} = \{\theta_{i,1}^{\text{map}}, \theta_{i,2}^{\text{map}}, \dots, \theta_{i,K}^{\text{map}}\}$. Unlike the method in [11], which performs DOA estimation with the original signal model (1) and deals with the ambiguities directly, considering the potential problems discussed in Section III, the mapped angle $\theta_{i,k}^{\text{map}}$ in (2) and the virtual half-wavelength spacing ULA signal model (3) will serve as the basis for the proposed method.

III. PROBLEM FORMULATION

In this section, the three main problems associated to the DOA estimation with two co-prime subarrays, including ambiguity, pair matching errors and grating angles problem, will be described.

A. AMBIGUITY

Because of the large inter-element spacing, for the signal coming from θ_k and impinging on the i^{th} subarray, only the mapped angle $\theta_{i,k}^{\text{map}}$ can be obtained rather than the true DOA θ_k after DOA estimation. According to (2), each mapped angle $\theta_{i,k}^{\text{map}}$ corresponds to M_i candidate angles. The m^{th} candidate angle $\theta_{i,k}^{\text{cand},m}$ can be recovered by

$$\theta_{i,k}^{\text{cand},m} = \arcsin\left(\frac{1}{M_i}(\sin \theta_{i,k}^{\text{map}} - 2n_{i,k}^m)\right) \quad (4)$$

which is directly deduced from (2), with $n_{i,k}^m$ the value of $n_{i,k}$ associated with the m^{th} candidate angle $\theta_{i,k}^{\text{cand},m}$.

One of the candidate angles recovered by (4) is the true DOA θ_k , and the others are ambiguous angles. This problem is called ambiguity. According to the co-prime property of M_1 and M_2 , the true DOA θ_k can be uniquely determined by finding the common angle in the two sets of candidate angles recovered from the mapped angles $\theta_{1,k}^{\text{map}}$ and $\theta_{2,k}^{\text{map}}$, which are obtained from the two subarrays respectively [11], [12].

B. PAIR MATCHING ERROR

In the step of ambiguities elimination, the K common elements (the true DOAs) among the candidate angles recovered from all the mapped angles in Θ_1^{map} and Θ_2^{map} should be found. However, in the situation of multiple incoming signals, there may also exist common angles in the candidate angles recovered from the mapped angles of different sources in

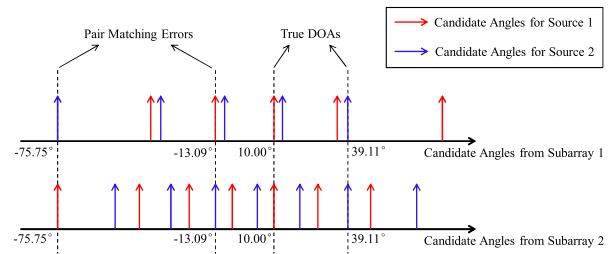


FIGURE 2. Pair matching errors.

different subarrays, resulting in more than K common angles, and consequently pair-matching errors occur. For example, consider the situation where $M_1 = 7$, $M_2 = 5$, and two signals come from $\theta_1 = 10.00^\circ$ and $\theta_2 = 39.11^\circ$. The candidate angles obtained from the two subarrays and associated to the two sources respectively are shown in FIGURE 2. It can be seen that besides the two true DOAs, there exist two other common candidate angles, -13.09° and -75.75° , recovered from the mapped angles associated to different sources in different subarrays, resulting in pair-matching errors.

Therefore, the mapped angles estimated from the two subarrays associated to a common source should be pair matched, such that for each of the K pairs of matched angles, two sets of candidate angles can be recovered, and the associated true DOA can be obtained by finding the common element among them without pair matching errors [13], [14].

C. GRATING ANGLES PROBLEM

When some signals come from a set of distinct angles, which belong to a common candidate angles set, or in other words, which are grating angles to each other, their mapped angles will be the same. Consequently, their associated directional vectors will be identical and the directional matrix of this subarray will be rank deficient. It will result in difficulties for the subsequent steps like DOA estimation and ambiguities elimination for the methods proposed in [13], [14], which are based on the full rank property of the two directional matrices.

To provide a better understanding, let's consider the situation where $M_1 = 7$ and $M_2 = 5$, and three signals come from $\theta_1 = 10.00^\circ$, $\theta_2 = 27.35^\circ$ and $\theta_3 = 35.01^\circ$, respectively. Their mapped angles for the 1st subarray and the 2nd subarray are $\Theta_1^{\text{map}} = \{\theta_{1,1}^{\text{map}}, \theta_{1,2}^{\text{map}}, \theta_{1,3}^{\text{map}}\} = \{60.25^\circ, 17.29^\circ, 60.25^\circ\}$ and $\Theta_2^{\text{map}} = \{\theta_{2,1}^{\text{map}}, \theta_{2,2}^{\text{map}}, \theta_{2,3}^{\text{map}}\} = \{-51.67^\circ, -51.67^\circ, 0.92^\circ\}$ respectively. It can be seen that θ_1 and θ_3 are grating angles to each other for the 1st subarray because they have the same mapped angle. For the same reason, θ_1 and θ_2 are grating angles to each other for the 2nd subarray. Then the directional matrices of the two subarrays \mathbf{A}_i ($i \in \{1, 2\}$) in (1) are rank deficient and the DOA estimation methods proposed in [13] and [14], which are based on the full rank assumption of \mathbf{A}_1 and \mathbf{A}_2 , cannot correctly work. For this antenna array configuration, the grating angles problem also occurs in many other situations. As an example with three incoming signals,

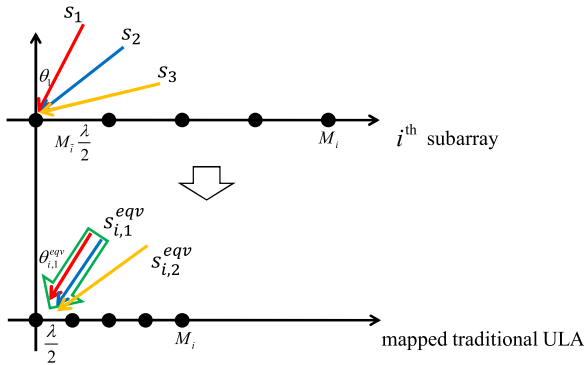


FIGURE 3. Equivalent system model of a subarray.

when $\{\theta_1, \theta_2, \theta_3\} = \{20.00^\circ, 38.88^\circ, 47.90^\circ\}$, $\{\theta_1, \theta_2, \theta_3\} = \{30.00^\circ, 51.79^\circ, 64.16^\circ\}$ and many other configurations, the phenomenon occurs. It is a real problem which cannot be ignored [15].

IV. PROPOSED DOA ESTIMATION METHOD

Considering the grating angles problem or the rank deficiency of the directional matrices, in this section, an equivalent system model is introduced. Then an efficient DOA estimation method is proposed. Compared with the existing methods [11], [13]–[15], the proposed method can deal with any situations with higher accuracy and lower complexity.

A. EQUIVALENT SYSTEM MODEL

When some signals come from a set of angles, which are grating angles to each other for one subarray, the mapped angles of them are the same, or in other words, these signals seem to come from a “same” direction to the virtual half-wavelength spacing ULA. In this situation, the received signal model of the i^{th} subarray can be regarded as K_i equivalent signals $s_{i,l}^{eqv}(t)$ coming from K_i different equivalent angles $\theta_{i,l}^{eqv}$, ($1 \leq K_i \leq K$ and $1 \leq l \leq K_i$), impinging on a virtual half-wavelength spacing ULA, as shown in FIGURE 3. Let’s define the set of the K_i equivalent angles of the i^{th} subarray as $\Theta_i^{eqv} = \{\theta_{i,1}^{eqv}, \theta_{i,2}^{eqv}, \dots, \theta_{i,K_i}^{eqv}\}$. Note that $\Theta_i^{eqv} \subseteq \Theta_i^{map}$, and without grating angles problem, we have $K_i = K$ and $\Theta_i^{eqv} = \Theta_i^{map}$, and the equivalent system model is identical to the system model introduced in Section II. In practice, the number of equivalent signals K_i can be estimated by Akaike Information Criterion (AIC) or Minimum Description Length (MDL) methods [16]. When the grating angles problem occurs, some of the equivalent signals $s_{i,l}^{eqv}(t)$ should be a combination of some original signals $s_k(t)$, as shown in FIGURE 3.

The signals observed at the i^{th} subarray can then be re-written as

$$\begin{aligned} \mathbf{x}_i(t) &= \sum_{l=1}^{K_i} \mathbf{a}_i^{map}(\theta_{i,l}^{eqv}) s_{i,l}^{eqv}(t) + \mathbf{n}_i(t) \\ &= \mathbf{A}_i^{eqv} \mathbf{s}_i^{eqv}(t) + \mathbf{n}_i(t) \end{aligned} \quad (5)$$

where $\mathbf{A}_i^{eqv} = [\mathbf{a}_i^{map}(\theta_{i,1}^{eqv}) \mathbf{a}_i^{map}(\theta_{i,2}^{eqv}) \dots \mathbf{a}_i^{map}(\theta_{i,K_i}^{eqv})]$ is the equivalent directional matrix, and $\mathbf{s}_i^{eqv}(t) = [s_{i,1}^{eqv}(t) s_{i,2}^{eqv}(t) \dots s_{i,K_i}^{eqv}(t)]^T$ is the equivalent source signal vector of the i^{th} subarray.

B. DOA ESTIMATION

Based on the equivalent system model, after estimating the number of equivalent source signals K_i , the ESPRIT method can be performed and two sets of equivalent angles can be obtained from the two subarrays, denoted as $\hat{\Theta}_1^{eqv} = \{\hat{\theta}_{1,1}^{eqv}, \hat{\theta}_{1,2}^{eqv}, \dots, \hat{\theta}_{1,K_1}^{eqv}\}$ and $\hat{\Theta}_2^{eqv} = \{\hat{\theta}_{2,1}^{eqv}, \hat{\theta}_{2,2}^{eqv}, \dots, \hat{\theta}_{2,K_2}^{eqv}\}$, respectively. To achieve the pair matching of the equivalent angles, the equivalent source signals vectors $\mathbf{s}_i^{eqv}(t)$ of the two subarrays should be recovered. The pair matching of the equivalent angles is achieved by exploring the cross-correlations between their associated equivalent signals.

Based on the equivalent angles estimated previously, an estimated equivalent directional matrix can be constructed for each subarray as follows

$$\hat{\mathbf{A}}_i^{eqv} = [\mathbf{a}_i^{map}(\hat{\theta}_{i,1}^{eqv}) \mathbf{a}_i^{map}(\hat{\theta}_{i,2}^{eqv}) \dots \mathbf{a}_i^{map}(\hat{\theta}_{i,K_i}^{eqv})] \quad (6)$$

with the estimated mapped directional vector

$$\mathbf{a}_i^{map}(\hat{\theta}_{i,l}^{eqv}) = [1 \quad e^{j\pi \sin \hat{\theta}_{i,l}^{eqv}} \quad \dots \quad e^{j(M_i-1)\pi \sin \hat{\theta}_{i,l}^{eqv}}]^T \quad (7)$$

Then the equivalent source signals of the i^{th} subarray can be recovered by

$$\hat{\mathbf{s}}_i^{eqv}(t) = (\hat{\mathbf{A}}_i^{eqv})^+ \mathbf{x}_i(t) \quad (8)$$

where $\hat{\mathbf{s}}_i^{eqv}(t) = [\hat{s}_{i,1}^{eqv}(t) \hat{s}_{i,2}^{eqv}(t) \dots \hat{s}_{i,K_i}^{eqv}(t)]^T$ denotes the obtained equivalent source signals vector.

In order to study the cross-correlations between the equivalent source signals of the two subarrays got by (8), $K_1 \times K_2$ cross-correlations can be estimated by

$$\hat{r}_{p,q} = \frac{1}{J} \sum_{t=1}^J \hat{s}_{1,p}^{eqv}(t) (\hat{s}_{2,q}^{eqv}(t))^* \quad (9)$$

where $1 \leq p \leq K_1$, $1 \leq q \leq K_2$ and J is the number of snapshots.

Since an equivalent source signal may be a combination of some original source signals, if a common original signal is contained in two equivalent source signals $\hat{s}_{1,p}^{eqv}(t)$ and $\hat{s}_{2,q}^{eqv}(t)$, the modulus of the cross-correlation between them $|\hat{r}_{p,q}|$ would be a large value. Otherwise, it turns out to be a small value. On the other hand, thanks to the co-prime property between M_1 and M_2 , two distinct DOAs with same mapped angle for one subarray have necessarily different mapped angles for the other subarray [15], [17]. Therefore, in the $K_1 \times K_2$ cross-correlations, there exist K cross-correlations with large modulus corresponding to the K original sources. By finding the K cross-correlations with largest modulus, the K pairs of matched angles can be found. Similarly to (4),

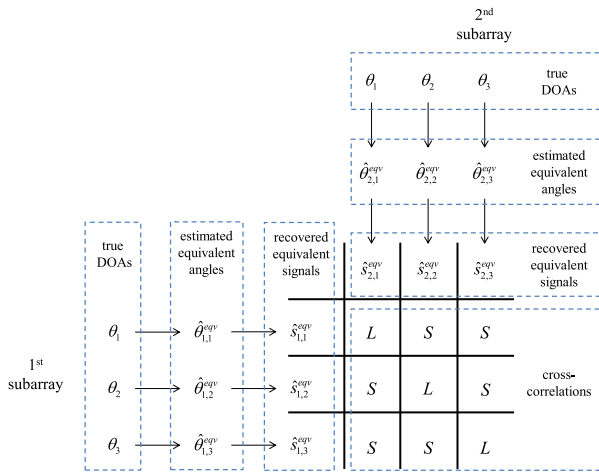


FIGURE 4. Processing flow chart for a normal situation.

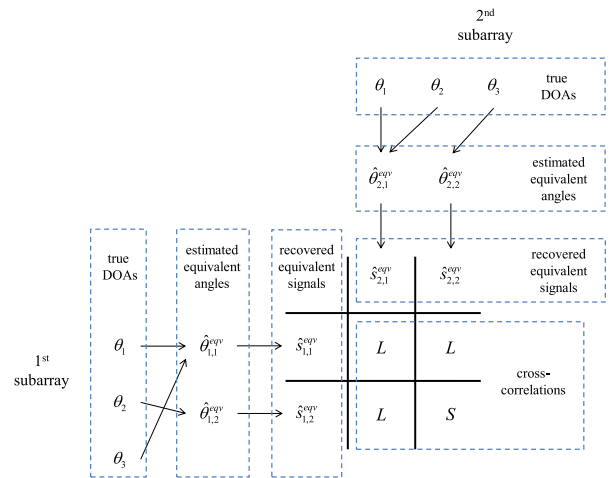


FIGURE 5. Processing flow chart for a grating angles problem situation.

for each pair of matched equivalent angles, two sets of candidate angles can be recovered by

$$\theta_{i,k}^{cand,m} = \arcsin\left(\frac{1}{M_i}(\sin \hat{\theta}_{i,k}^{eqv} - 2n_{i,k}^m)\right) \quad (10)$$

and the true DOA θ_k can be obtained by finding the common angle among them.

As a matter of illustration of this principle, the processing flow charts of the proposed method for a normal situation and a grating angles problem situation are depicted in FIGURE 4 and FIGURE 5 respectively, where “L” stands for a large value and “S” stands for a small value. It is assumed that three signals impinge on the co-prime array from $\{\theta_1, \theta_2, \theta_3\}$; in the grating angles problem situation (FIGURE 5), θ_1 and θ_3 are grating angles for the 1st subarray, and θ_1 and θ_2 are grating angles for the 2nd subarray. It can be seen that the proposed method can overcome the rank deficiency caused by grating angles problem, and the estimation results can be pair matched automatically. Finally, two sets of candidate angles can be recovered from each pair of matched equivalent angles, and the common element among them can be found to obtain the true DOAs. The main steps of the proposed method can be summarized in TABLE 1.

V. SIMULATION AND ANALYSIS

To assess the performance of the proposed method in every situation, firstly, the proposed method is compared with the method in [11] in a pair matching errors situation. Then, in a grating angles problem situation, it is compared with the methods in [13] and [14], which solve the pair matching errors. Finally, in order to assess the accuracy and complexity performance of the proposed method, it is compared with the method in [15], which also considers the pair matching errors and grating angles problem.

A. RELIABILITY COMPARISON

To show the superiority of the proposed method in pair-matching error situations, consider the situation

TABLE 1. Main steps of the proposed method.

Step 1	Estimate the numbers of equivalent source signals received at the two subarrays K_1 and K_2 .
Step 2	Perform the ESPRIT method to estimate the two sets of equivalent angles $\hat{\Theta}_1^{eqv}$ and $\hat{\Theta}_2^{eqv}$ from the two subarrays respectively.
Step 3	Construct the two equivalent directional matrices $\hat{\mathbf{A}}_1^{eqv}$ and $\hat{\mathbf{A}}_2^{eqv}$ of the two subarrays with $\hat{\Theta}_1^{eqv}$ and $\hat{\Theta}_2^{eqv}$, respectively (6).
Step 4	Recover the equivalent source signals $\hat{\mathbf{s}}_1^{eqv}(t)$ and $\hat{\mathbf{s}}_2^{eqv}(t)$ impinging on the two subarrays (8).
Step 5	Calculate the cross-correlations between $\hat{\mathbf{s}}_1^{eqv}(t)$ and $\hat{\mathbf{s}}_2^{eqv}(t)$ (9), and achieve the pair matching of the estimated equivalent angles by finding the K cross-correlations with the largest modulus.
Step 6	For each of the K pairs of matched angles, generate two sets of candidate angles (10). The estimation of the true DOA θ_k ($k = 1, 2, \dots, K$) is given by the unique common angle between them.

mentioned in Section III-B, where $M_1 = 7, M_2 = 5$, and two signals come from $\theta_1 = 10.00^\circ$ and $\theta_2 = 39.11^\circ$ respectively. The reliability comparison of the proposed method and the method in [11] is shown in FIGURE 6, with 10 independent runs, in which the signal-to-noise ratio (SNR) is 0dB and the number of snapshots is 200. It can be seen that because the method in [11] only finds out the common elements in the candidate angles estimated from the two subarrays without pair matching, the estimation results may be ambiguous. In contrast, the proposed method can achieve the pair matching of the equivalent angles of the same source in different subarrays by exploring the cross-correlations between the equivalent signals, and the performance remains remarkable and stable.

To emphasize the superiority of the proposed method in grating angles problem situations, consider again the situation

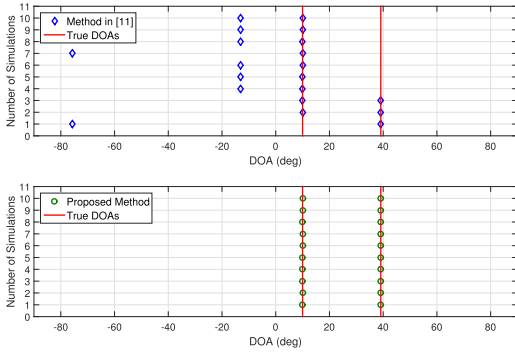


FIGURE 6. Reliability comparison in the pair matching errors situation.

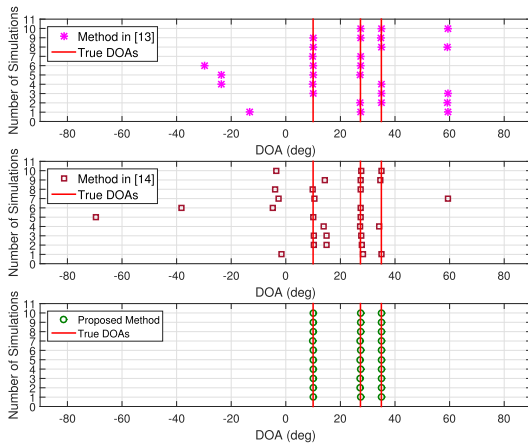


FIGURE 7. Reliability comparison in the grating angles problem situation.

mentioned in Section III-C, where $M_1 = 7$, $M_2 = 5$, and three signals come from $\theta_1 = 10.00^\circ$, $\theta_2 = 27.35^\circ$ and $\theta_3 = 35.01^\circ$ respectively. The reliability comparison of the proposed method with the methods in [13] and [14] is shown in FIGURE 7, with 10 independent runs, in which SNR is 0dB and the number of snapshots is 200. It is obvious that although the methods in [13] and [14] can overcome the pair matching errors with beamforming-based methods and the relationship between the directional matrices of the two sub-arrays, they ignore the fact that the directional matrices would be rank deficient due to the grating angles problem, and their performance cannot remain stable. In contrast, thanks to the equivalent system model, the equivalent directional matrices are full rank, and the correctly matched equivalent angle pairs can be found by exploring the cross-correlations between the equivalent signals. Thus it can work correctly in such situations.

B. ACCURACY COMPARISON

To assess the DOA estimation performance of the proposed method, the Root Mean Square Error (RMSE) is used as performance measurement, which is defined as

$$RMSE = \sqrt{\frac{1}{KQ} \sum_{k=1}^K \sum_{q=1}^Q (\hat{\theta}_{q,k} - \theta_k)^2} \quad (11)$$

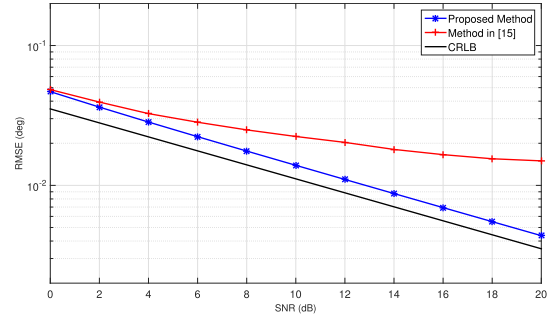


FIGURE 8. RMSE comparison versus SNR in the normal situation.

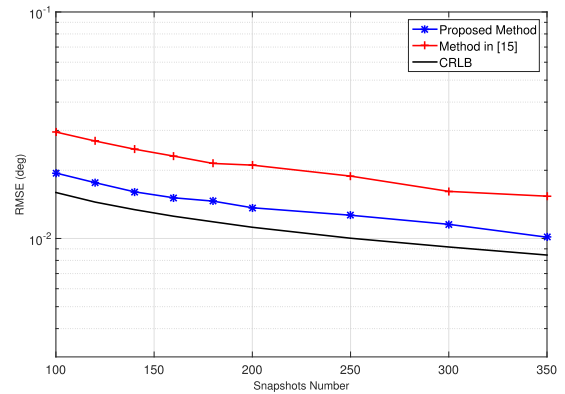


FIGURE 9. RMSE comparison versus snapshots number in the normal situation.

with K the number of incoming signals, Q the number of Monte Carlo trials, and $\hat{\theta}_{q,k}$ the estimate of the true DOA θ_k at the q^{th} Monte Carlo trial. $Q = 500$ is used, and a co-prime linear array with $M_1 = 7$ and $M_2 = 5$ is considered. The Cramér-Rao lower bound (CRLB) for this co-prime array geometry is also given as a benchmark [18].

The RMSE performance of the proposed method and the method in [15] is compared in a normal situation, where two signals are assumed to come from $\theta_1 = 10.00^\circ$ and $\theta_2 = 40.00^\circ$, and a grating angles problem situation, where three signals are assumed to impinge from $\theta_1 = 10.00^\circ$, $\theta_2 = 27.35^\circ$ and $\theta_3 = 35.01^\circ$, versus SNR (snapshots number is 200) and snapshots number (SNR is 10dB). FIGURE 8–FIGURE 11 illustrate the obtained results. Because the peak-searching in the method in [15] is performed in the sine domain, the searching step is chosen as 0.001 to obtain a precise estimation. It can be seen that both methods can achieve a remarkable performance in grating angles problem situations, but since a “beamforming-like” method is utilized, the accuracy of the method in [15] is limited. On the contrary, based on the ESPRIT method, the proposed method can acquire a better estimation result, and its RMSE curves are closer to the CRLB.

C. COMPLEXITY COMPARISON

According to Section IV-B, the proposed method requires the covariance matrices estimation, eigenvalue decomposition

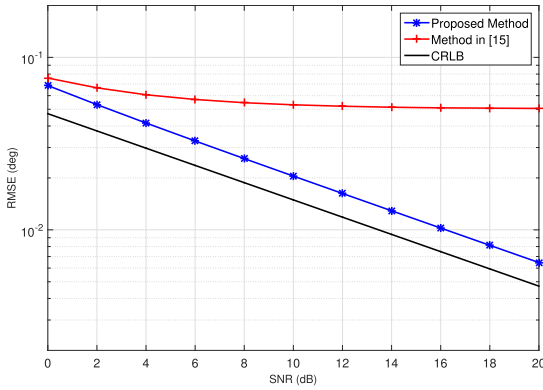


FIGURE 10. RMSE comparison versus SNR in the grating angles problem situation.

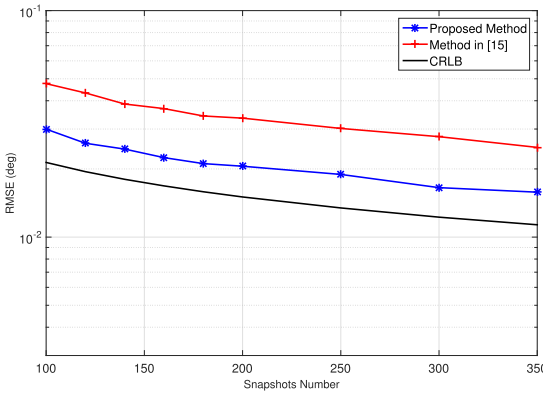


FIGURE 11. RMSE comparison versus snapshots number in the grating angles problem situation.

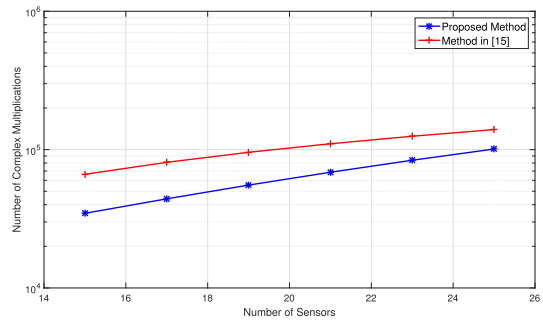


FIGURE 12. Complexity comparison versus sensors number.

of the covariance matrices, equivalent signals recovery and cross-correlation computation. The resulting complexity is given as $O((M_1^2 + M_2^2)J + (M_1^3 + M_2^3) + 5K^2(M_1 + M_2) + 6K^3 + (M_1 + M_2)KJ + K^2J)$. For the method in [15], it requires the cross-covariance matrix estimation, singular value decomposition of the cross-covariance matrix and peak-searching, with the order of complexity $O(M_1M_2J + 4M_1M_2^2 + M_2^3 + 3M_1K^2 + 2K^3 + \frac{2M_1K}{M_2 \times sch})$, where J is the number of snapshots and sch is the searching step length. The complexity comparison versus the total number of sensors $(M_1 + M_2 - 1)$ is given in FIGURE 12, with $K = 2, J = 200$. The searching step length is set as 0.0001 to achieve a similar RMSE performance between the two algorithms. It can be observed

that without peak-searching, the proposed method has lower computational complexity, thus the DOA estimation can be accomplished more efficiently.

For the other methods in [11], [13], [14], their practicability is limited by the pair matching errors or grating angles problem. Therefore, their performance in terms of accuracy and complexity is less significant in the case of real applications.

VI. CONCLUSION

In this paper, the existing problems associated to the DOA estimation with co-prime linear arrays, including ambiguity, pair matching errors and grating angles problem, are discussed. Based on the equivalent system, a reliable and efficient DOA estimation method is proposed. True DOAs are mapped into their corresponding equivalent angles, and after their estimation, the corresponding equivalent signals can be recovered from the received signals. Then the pair matching of the equivalent angles can be achieved by exploring the cross-correlations between the equivalent signals. Simulation results show that the proposed method is able to achieve a better estimation performance than other existing methods, in terms of accuracy and complexity.

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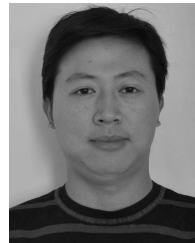


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