BASICS IN SOLAR RADIATION AT EARTH SURFACE -REVISED VERSION #2

Lucien Wald

BASICS IN SOLAR RADIATION AT EARTH SURFACE – REVISED VERSION #2

Lucien Wald

Lecture Notes

Edition 1 from 2018-01-03 – Revised 2019-07-06

MINES ParisTech, PSL Research University

O.I.E. – Observation, Impacts, Energy Center

Sophia Antipolis, France

TABLE OF CONTENTS

1 FOREWORD

The sun produces a vast amount of energy. The energy emitted by the sun is called solar energy or solar radiation. Despite the considerable distance between the sun and the earth, the amount of solar energy reaching the earth is substantial. At any one time, the earth intercepts approximately 180 10^6 GW. Solar radiation is the earth primary natural source of energy and by a long way. Other sources are: the geothermal heat flux generated by the earth interior, natural terrestrial radioactivity, and cosmic radiation, which are all negligible relative to solar radiation.

As a consequence, the solar radiation influences many aspects of the earth, including weather and climate, ocean, life on earth, agronomy and horticulture, forestry, ecology, oenology, energy, architecture and building engineering, or materials weathering.

These lecture notes intend to present the fundamentals in solar radiation at earth surface to a wide community. Its content originates from lectures given to students of master degree level or higher, engineers and researchers in climate, geophysics and environment sciences, life sciences, or energy. This document should be valuable to any engineer, scientist and practitioner.

The first edition was published on 2018-01-03. This edition is a revision achieved on 2019-07-06. There is nothing new; a few small mistakes have been corrected.

The solar radiation received at a given geographical site varies in time: between day-night due to the earth rotation and between seasons because of the earth orbit. At a given time it also varies in space, because of the changes in the obliquity of the solar rays with longitude and latitude. Notwithstanding the effects of the clouds and other atmospheric constituents, the solar radiation received at a given location and time depends upon the relative position of the sun and the earth. This is why both sun-earth geometry and time play an important role in the amount of solar radiation received at earth surface. A major part of this textbook is devoted to this matter. The geometry of the earth relative to the sun is described as well as its variation throughout the year. The concept of time is very important in solar radiation. It is detailed here and the notions of mean solar time and true solar time are dealt with. The apparent course of the sun in the sky is described; the solar zenithal, elevation and azimuthal angles are defined. These angles are identical at top of the atmosphere and earth surface; no change is introduced by the atmosphere. Equations are given in this part that can be easily introduced in e.g., a spreadsheet or a computer routine, to compute all quantities and reproduce the figures. Both horizontal and inclined surfaces are dealt with.

The amount of solar radiation that is intercepted by the earth varies because of variations in sun-earth distance and as far as the spectral distribution is concerned by day-to-day variations due to solar activity. The closer to the sun the earth, the greater the solar irradiance impinging on a plane normal to the sun rays and located at the top of the atmosphere. The total solar irradiance, often abbreviated in TSI, is the yearly average of this irradiance during a year integrated over the whole spectrum. The variations within a year amount to ± 3 % of the TSI. The spectral distribution of the extra-terrestrial radiation is such that about half of it lies in the visible part of the electromagnetic spectrum. It produces daylight and is well perceived by the human vision system. Other parts of it are in the near-infrared and ultraviolet ranges. A series of equations is offered to compute the extra-terrestrial total radiation for any instant and for any inclined surface.

During its path downwards to the earth surface, the constituents of the atmosphere deplete the incident solar radiation. On average, less than half of extra-terrestrial radiation reaches ground level. A good knowledge of the optical properties of the atmosphere is necessary to understand and model the depletion of the radiation. The description and modelling of the optical processes affecting the solar radiation within the atmosphere is called radiative transfer. The phenomena of scattering and absorption are presented and the effects of molecules, aerosols, gases and clouds on radiation are discussed. Several examples are given that illustrate atmospheric effects as a function of the solar zenithal angle and atmospheric optical properties.

Even when the sky is very clear with no clouds, approximately 20 % to 30 % of extra-terrestrial radiation is lost during the downwelling path by scattering and absorption phenomena by aerosols and molecules. The role of the clouds is of paramount importance: optically thin clouds allow a small proportion of radiation to reach the ground while optically thick clouds create obscurity by stopping the radiation downwards. The magnitude of the depletion of the radiation varies with wavelength and the spectral distribution of the solar radiation is modified as the radiation makes its path downwards. The spectral distribution is discussed for several different conditions.

The direct, diffuse and reflected components of the solar radiation at earth surface are defined. How to compute them on an inclined surface is briefly discussed and equations are provided.

The direct radiation is the radiation coming from the direction of the sun. Only direct radiation is present at the top of the atmosphere. On the contrary, the radiation at surface comprises a direct and a diffuse components, the sum being called the global radiation. If a tilted surface is under concern, then it may also receive a reflected component that is a part of the radiation reflected by the surrounding landscape. How to compute each component on an inclined surface is briefly discussed.

A last word. Readers may regret the lack of colours in graphs. It is true that colours greatly improve the legibility of graphs. The author knows from his experience that this document will be printed, and in part in places where colour printers are a luxury that not everyone may afford. Hence, it is his choice to avoid colours in this text.

2 GLOSSARY

2.1 ANGLES

Solar declination: The angle formed by the direction to the centre of the sun and the terrestrial equatorial plane.

Solar zenithal angle: The angle formed by the direction of the sun and the local vertical.

Solar elevation angle: The angle formed by the direction of the sun and the horizon.

Solar azimuthal angle, solar azimuth: The angle formed by the projection of the direction of the sun on the horizontal plane and the north.

2.2 RADIATION – GENERAL DEFINITIONS

Broadband irradiance: The irradiance integrated over a large spectrum. It is not well defined and is used as a current term to denote that the radiation is integrated over a certain range of wavelength. It may be used to denote the solar radiation measured by pyranometers, e.g. from 300 nm to 2200 nm. One may find *broadband UV* that opposes to *spectral UV*.

Irradiance: The power received per area; unit is W m^{-2}.

Irradiation: The energy received per area; unit is J m^{-2}. The unit Wh m^{-2} is commonly used in commercial metering of electrical energy but should be avoided.

Radiant energy: The amount of energy that is transferred by radiation. It is expressed in J (Joule).

Radiant flux: The time rate of flow of the radiant energy. It is expressed in W (Watt).

Radiance: The radiant flux per unit solid angle per unit area. Unit is W m^{-2} sr^{-1}.

Spectral distribution of the irradiance: The distribution of the irradiance as a function of the wavelength.

Total irradiance, irradiation: The irradiance, irradiation, integrated over the whole spectrum.

2.3 RADIATION AT THE TOP OF THE ATMOSPHERE

Extra-terrestrial radiation, irradiance or irradiation: the total radiation, irradiance and irradiation originating from the sun impinging on a horizontal surface located at the top of the atmosphere.

Total solar irradiance: The yearly average of the solar irradiance impinging on a plane normal to the sun rays and located at the top of the atmosphere. It is often abbreviated as TSI.

2.4 RADIATION AT GROUND LEVEL

Diffuse irradiation, irradiance: The downward scattered shortwave irradiation, irradiance, coming from the whole hemisphere, with the exception of the solid angle of the sun disc. This does not usually include the part of the irradiation reflected by the ground in the case of an inclined receiving plane.

Direct irradiation, irradiance: The shortwave irradiation, irradiance, coming from the solid angle of the sun disc.

Global irradiation, irradiance: The shortwave irradiation, irradiance, received at ground level; it is the sum of the direct, diffuse and reflected irradiations.

Reflected irradiation, irradiance: The irradiation, irradiance, reflected by the ground and impinging on an inclined receiving plane.

2.5 TIME AND TIME SYSTEMS

Summarization: The time interval during which a measurement is performed.

Time – legal time, local time, standard time, civil time, local clock time: The time used legally in a given country.

Time – mean solar time: The time determined locally by dividing the average duration of a rotation by 24 h. The mean solar time is equal to 12 h when the sun is at its highest, i.e. at zenith, as an annual average.

Time – true solar time: The time for which the sun is actually at its highest when it is 12 h. It depends on the day of the year and longitude of the site.

Time – Universal Time (UT): The mean solar time for the longitude 0°.

2.6 MISCELLANEOUS

Clearness index: Ratio of irradiation, irradiance, at ground level to extra-terrestrial irradiation, irradiance.

Linke turbidity factor: A quantity that conveniently describes atmospheric absorption and scattering of the incident light by the clear sky.

Optical depth: A measure of the extinction of the radiation during its vertical travel through a layer of molecules or particles or clouds, with a solar zenithal angle equal to 0.

Turbidity: The optical effects of the aerosols that attenuate the incident light.

3 THE SUN AS SEEN BY AN OBSERVER AT THE SURFACE OF THE EARTH

In summary

The earth describes an elliptical orbit, quasi-circular, counterclockwise, around the sun. The duration of an orbit is approximately 365.25 days, one day being 24 h, i.e. 86 400 s.

The distance between the earth and the sun varies around the year by ±1.7 % around a mean value equal to 1 astronomical unit (1 au = 1.496 10^8 km). The distance is the greatest between the 2 and 5 January (=1.017 au), and reaches its minimum between the 3 and 5 July (=1.017 au). The exact dates vary slightly depending on the year.

Because of the large distance between the sun and the earth, the sun appears as a small spot to an observer on the earth. Its apparent diameter is approximately 32 ±0.5' of arc angle, i.e. about half a degree. As a consequence, the sunrays can be considered as parallel as they hit the top of the earth atmosphere.

The equatorial plane of the earth inclines on the plane containing the orbit of the earth. The solar declination is the angle made by the line joining the sun to the earth and the equatorial plane. It varies with time from -23.45° to +23.45°. The solstices of December and June respectively are the two points in the earth orbit where these extreme values are attained. The equinoxes of March and September are the two points where the solar declination is null.

The solstices define the beginning of two seasons, respectively the astronomical winter which begins around 20-22 December depending on the year, and the astronomical summer which begins around 20-22 June. The astronomical spring begins around 19-21 March and the astronomical autumn around 21-24 September. Each astronomical season lasts three months, with a variable number of days.

The solar zenithal angle is the angle formed by the direction of the sun and the local vertical (zenith). The azimuth of the sun, or solar azimuthal angle, is defined as the angle between the projection of the direction of the sun on the horizontal plane and the north, and increasing clockwise.

The solar radiation received at a given geographical site varies in time: between day-night due to the earth rotation and between seasons because of the earth orbit. At a given time it also varies in space, because of the changes in the obliquity of the solar rays with longitude and latitude. It is governed by the specific astronomical situation of the earth on its orbit around the sun, its rotation around its polar axis and the location of this point on the earth. This is why both sun-earth geometry and time play an important role in the amount of solar radiation received at earth surface.

3.1 THE SUN-EARTH ASTRONOMY IN BRIEF

The earth describes an elliptical orbit with the sun at one of the foci (Figure 3.1). In this figure, the ellipse is exaggerated because in fact, the eccentricity of the orbit is small (0.01675), which means that the orbit is almost circular. The orbit is flat, and this plane is called the plane of the ecliptic. The sun is located at one of the centers of the ellipse, not at its center, which causes variations in the distance between the sun and the earth during a revolution. For the reader with knowledge in astronomy, I specify that for reasons of simplicity in the presentation, I confused in this figure, the solstices and the moments when the distance sun-earth reaches its extrema.

The direction of rotation is counterclockwise, or trigonometric, or direct. The duration of an orbit varies over time over long periods of time, and this variation can be neglected on the scale of a few centuries. The orbit is traveled in one year. The definition of the year is not unique: sidereal year, Julian, tropical ... but overall, it is about 365.25 days, each day having 86 400 s, i.e. 24 h of 3600 s each. There is therefore a gap between this duration and a regular year of 365 days, used in the Gregorian calendar, the one of every day in most countries in the world. The gap is overtaken every 4 years by adding one day (February 29th). This is called a leap year, which includes 366 days. Of course, when it comes to calendar and time, things are not so simple, and years are leap years only if the year is divisible by 4 and not divisible by 100, or if it is divisible by 400. Note that in climatology, we do not keep February 29 in the time series to have the same number of days for each year.

The earth rotates on itself. This rotation induces the notion of a mean solar day divided into 24 h of 60 min each, i.e. 86 400 s. More exactly, the earth turns on itself in 23 h 56 min and 4 s. During this time, the earth has also progressed in its revolution around the sun, and it takes an average of 3 min and 56 s to find the sun at the same point on the sky, a total of 24 hours. The time variables: year, day, and hour, are essential in order to compute the apparent position of the sun in the sky and, accordingly, the radiation at ground level that may be exploited.

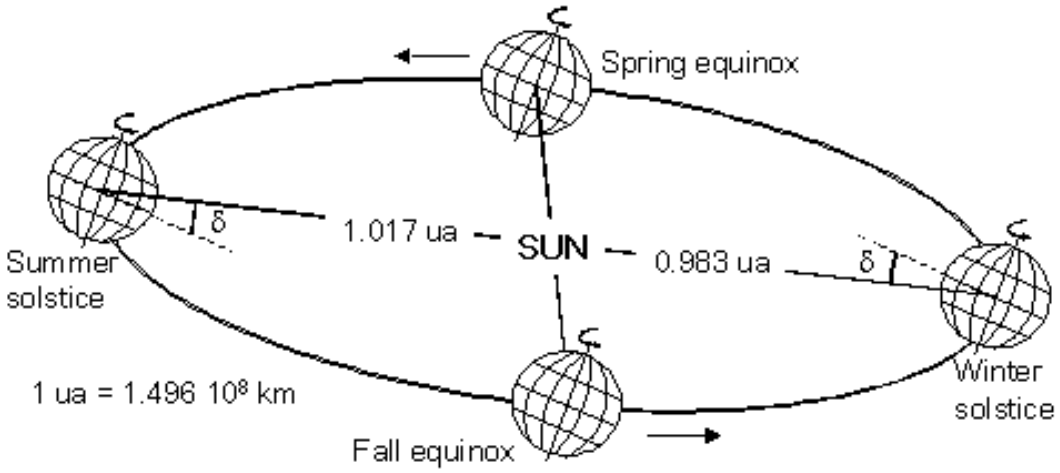

Figure 3.1. Schematic view of the earth orbit around the sun. The angle δ is the solar declination. Adapted from Perrin de Brichambaut and Vauge (1982)[1].

The speed of rotation of the earth on itself tends to slow down because of the movements of the tides in particular. This variation affects the duration of the solar day, which currently exceeds 86 400 s by fractions of a second. In the course of a year, one adds regularly, one second to the universal time, and therefore to the civil time used every day, in order to keep the duration the day of 24 hours in general.

The mean distance between the sun and the earth is approximately equal to $1.496\ 10^8$ km, exactly 149 597 870 700 m according to the decision of the International Astronomical Union in 2012. This distance is called 1 astronomical unit. Its unit is *ua* or *au*, the latter being the recommendation made in 2012 by the International Astronomical Union. The distance between the sun and the earth varies within a year (Figure 3.1), from a minimum of 0.983 au reached approximately on 2-5 January to a maximum of 1.017 au reached approximately on 2-5 July. The exact dates depend on the year.

A consequence of the large distance between the sun and the earth is that despite its formidable size, the sun appears as a small spot to an observer on the earth. The solid angle under which the sun appears is equal to $0.68\ 10^{-4}$ sr. Differently written, the apparent diameter of the sun is approximately 32 ± 0.5' of arc angle, i.e. half a degree.The sunrays can be considered as parallel as they hit the top of the earth atmosphere.

Figure 3.1 shows that the equatorial plane of the earth inclines by 0.4093 rad (23.45°) on the plane containing the earth orbit. In other words, the axis of the earth daily rotation passing by the two poles is inclined by this angle with respect to the orbit plane. This angle is also called the obliquity of the ecliptic.

Because of this inclination, the northern hemisphere is farther from the sun than is the southern hemisphere in December. Conversely, the northern hemisphere is closer to the sun than is the southern hemisphere in July.

As it can be seen in Figure 3.1, the direction of the solar rays is not always parallel to the equatorial plane. The angle composed by the direction to the sun and the equatorial plane is called the solar declination, represented by δ in Figure 3.1. The winter and summer solstices are defined as the two points in the earth orbit of the maximum solar declination in absolute value. The solstices define the beginning of two seasons, respectively the

[1] Perrin de Brichambaut Ch., Vauge Ch., 1982. Le Gisement Solaire. Published by Technique et Documentation (Lavoisier), Paris, France, 222 pp. [This book in French offers a practical description of solar radiation for solar engineering purposes].

astronomical winter which begins on 20-22 December and the astronomical summer which begins on 20-22 June.

The beginnings of the two other astronomical seasons are determined by the equinoxes that are the two points in earth orbit where δ is equal to 0. The astronomical spring begins about 19-21 March and the astronomical autumn about 21-24 September. Each astronomical season lasts three months, with a variable number of days.

On earth, these astronomical seasons have different names for each hemisphere: the boreal (north hemisphere) and austral (south hemisphere) seasons. For example, the astronomical summer corresponds to the boreal summer in the northern hemisphere, and to the austral winter in the southern hemisphere. See Table 3.1 for correspondence. These seasons differ from the meteorological seasons.

Astronomical season	Period (approximate)	Season in the northern hemisphere	Season in the southern hemisphere
Spring	20 Mar – 21 Jun	Boreal spring	Austral autumn
Summer	21 Jun – 23 Sep	Boreal summer	Austral winter
Autumn	23 Sep – 21 Dec	Boreal autumn	Austral spring
Winter	21 Dec – 20 Mar	Boreal winter	Austral summer

Table 3.1. Correspondence between astronomical, boreal and austral seasons

3.1.1 THE SUN-EARTH DISTANCE

The mean distance between the earth and the sun, r_0, is 1 au (Figure 3.1). This value is reached for spring and autumn equinoxes. The actual distance varies during the year. A number of mathematical expressions of this distance are available; they are usually expressed in terms of Fourier series type of expansion. In the following expression adopted from the European Solar Radiation Atlas[2], the actual distance r is a function of the number of the day in the year, d. d ranges from 1 (1st January) to 365, or 366 in case of a leap year. It is convenient to define the day angle j:

$$j = d\ 2\ \pi / 365.2422 \qquad (3\text{-}1)$$

j is expressed in rad. It is almost null on 1st January (0.0172), and is equal to π (180°) on 1st July and 2π (360°) on 31st December.

The ratio (r_0/r) is equal to:

$$r_0/r = \sqrt{(1+\varepsilon)} \qquad (3\text{-}2)$$

where ε is given by:

$$\varepsilon \approx 0.03344 \cos(j - 0.049) \qquad (3\text{-}3)$$

with an accuracy sufficient for many applications. ε varies between 0.97 et 1.03. Figure 3.2 exhibits the sun-earth distance as a function of the day in the year. The maximum is 1.017 au and is reached between 3-5 July. This moment is called aphelion. The smallest distance is 0.983 au and is reached between 2-5 January (perihelion). The exact dates vary slightly depending on the year. The variation of distance exhibits relative amplitude of 3.4 %. This seems weak, but it should not be neglected because the radiation received by the earth varies with the square of the sun-earth distance.

[2] Greif, J. J., Scharmer, K., Aguiar, R., Albuisson, M., Beyer, H.-G., Borisenkov, E. P., Bourges, B., Czeplak, G., Lund, H., Joukoff, A., Page, J. K., Terzenbach, U. and Wald, L., 2000. European Solar Radiation Atlas. Vol. 2: Database and Exploitation Software. Published for the Commission of the European Communities by Les Presses de l'Ecole, Ecole des Mines de Paris, France, 290 pp. [This book contains equations describing the sun position with respect to an observer can be found. Several models for converting global horizontal irradiance into diffuse and direct irradiances on an inclined plane surface are described. Equations in C are available at www.oie.mines-paristech.fr/Valorisation/Outils/Solar-Geometry/]

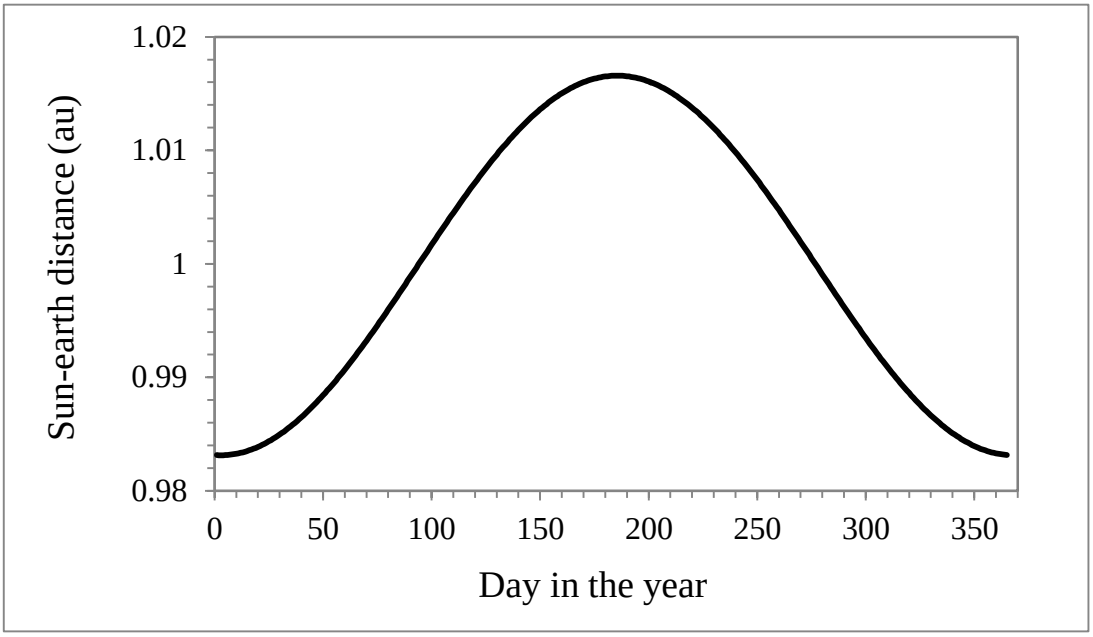

Figure 3.2. Sun-earth distance in au as a function of the day in the year

3.1.2 SOLAR DECLINATION

As already written, the solar declination δ is the angle made by the line joining the sun to the earth and the equatorial plane (Figure 3.1). The convention for counting the solar declination δ is the same as that for latitudes. Here the ISO 19115 convention is adopted[3]. Latitudes Φ are counted positive in the northern hemisphere and negative in the southern hemisphere. Longitudes λ are counted positive east of the meridian 0° and negative west of this meridian. Latitudes and longitudes are angles; they are usually expressed in degrees.

δ is the greatest for the summer solstice and is equal to +0.4093 rad (23.45°). It reaches its minimum for the winter solstice and is equal to −0.4093 rad (-23.45°). By definition, δ is equal to 0 for the equinoxes. δ is a function of the longitude λ. A mean daily value is accurate enough for many applications. The European Solar Radiation Atlas proposes a series of equations for computing the sun-earth geometry and the angles depicting the course of the sun in the sky as seen by an observer at earth surface. Accuracy in time is of order of a few minutes and these equations are suitable if one is focusing on hourly or daily sums of solar radiation. For a greater accuracy, other works should be preferred[4] [5].

Using the European Solar Radiation Atlas equations, δ can be computed from d, λ and the year y. Let n_0 represent the spring-equinox time expressed in days from the beginning of the year, i.e. the time in decimal day that elapses from 0.0 h on 1st January to the spring equinox at longitude 0° in the year y. Let t_{day} represent the

3 ISO 19115-1:2014. Geographic information -- Metadata -- Part 1: Fundamentals, 167 pp, International Standards Organization, Geneva, Switzerland, 2014.

4 Reda, I., Andreas, A., 2004. Solar position algorithm for solar radiation applications: Solar Energy, 76(5), 577–589. Corrigendum, 81, 838-838, 2007. [It describes solar position algorithm, called SPA. The uncertainty, in terms of standard deviation of solar azimuth and zenithal angles is stated to be within 5 µrad (0.0003 °, or 1") for a very large period from -2000 to 6000. Libraries in C, Matlab and Python that implement the SPA algorithm are available (respectively at: rredc.nrel.gov/solar/codesandalgorithms/spa, www.mathworks.com/matlabcentral/fileexchange/4605 and www.pysolar.org).]

5 Blanc P., Wald L., 2012. The SG2 algorithm for a fast and accurate computation of the position of the Sun. Solar Energy, 86, 3072-3083. [This algorithm SG2 is very fast. Maximum error is of order of 10", for a multi-decadal time period. Equations in C and Matlab are available at http://www.oie.mines-paristech.fr/Valorisation/Outils/Solar-Geometry/]

time, in days, from the spring equinox and ω_{day} the day angle counted from the spring equinox. If INT denotes the integer part of an expression, i.e., the left part before the decimal, these quantities are given by:

$$n_0 = 78.8946 + 0.2422\,(y - 1957) - INT[(y - 1957)/4] \quad (3\text{-}4)$$

$$t_{day} = -0.5 - \lambda / (2\,\pi) - n0 \quad (3\text{-}5)$$

$$\omega_{day} = (2\,\pi / 365.2422)\,(d + t_{day}) \quad (3\text{-}6)$$

The declination is then given by:

$$\delta = b_1 + b_2\,sin(\omega_{day}) + b_3\,sin(2\,\omega_{day}) + b_4\,sin(3\,\omega_{day}) + b_5\,cos(\omega_{day}) + b_6\,cos(2\,\omega_{day}) + b_7\,cos(3\,\omega_{day}) \quad (3\text{-}7)$$

where:

$b_1 = 0.0064979,\ b_2 = 0.4059059,\ b_3 = 0.0020054,\ b_4 = -0.0029880$

$b_5 = -0.0132296,\ b_6 = 0.0063809,\ b_7 = 0.0003508$

Figure 3.3 displays the variation of the solar declination as a function of the number of the day in the year for longitude 0° and for the year 2006. The declination is 0 rad for days 80 (21 March) and 266 (22 September), minimum (-0.4093 rad, -23.45°) for day 356 (22 December) and maximum (+0.4093 rad, +23.45°) for day 172 (21 June).

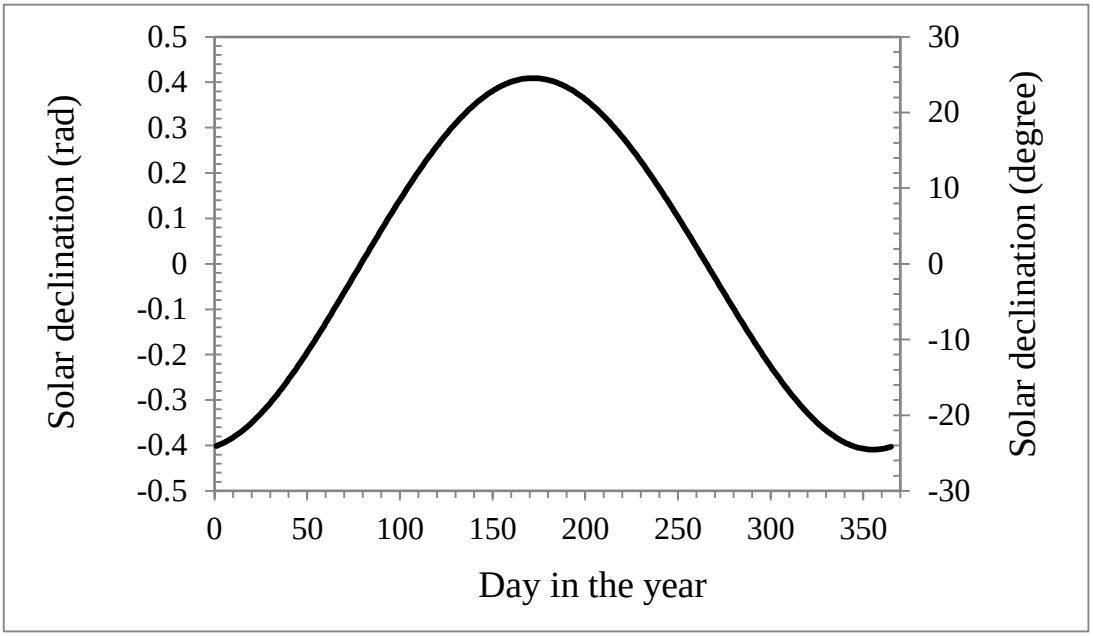

Figure 3.3. Solar declination (in rad) and its variation throughout the year. The year is 2006. Longitude is 0°.

Neglecting the longitude, i.e., making the calculations by setting λ to 0° in Eq. 3-5, induces an error in the solar declination that is less than 0.001 rad in absolute value. Such an error is acceptable in solar engineering calculations; the influence of the longitude can be neglected.

3.1.3 GEOCENTRIC AND GEOGRAPHIC COORDINATES

The equations are given here with the geocentric coordinates, i.e., by considering the earth as a perfect sphere. These geocentric coordinates are the latitude Φ_c and the longitude λ. The earth is not a perfect sphere and is slightly elongated in the equatorial plane. The radius to the poles R_{pole} is equal to 6 356.752 km, and is slightly less than the radius at Equator $R_{equator}$ which is 6 378.137 km.

Accordingly, there is a difference between the geocentric latitude Φ_c and the actual geographic latitude Φ. The relationship between both is:

$$\tan\Phi_c = (R_{pole} / R_{equator})^2 \tan\Phi \qquad (3\text{-}8)$$

Because (R_{pole} / $R_{equator}$) is less than 1, $\tan\Phi_c$ is less than $\tan\Phi$; Φ_c is closer to the equator than is the actual latitude Φ. The difference between these angles is null at the poles and the equator and maximum at geographic mid-latitudes $\pi/4$ (45°) and $-\pi/4$ (-45°), where it reaches 0.0033 rad (0.19°).

As a first approximation, the perimeter of the earth P along a longitude is

$$P = 2\,\pi\,[(R_{pole} + R_{equator})/2] \qquad (3\text{-}9)$$

The mean distance on the earth surface corresponding to 1° in latitude is equal to $P/360°$, i.e. 111 km. Hence, the maximum error of 0.19° corresponds to an error in the north-south distance of approximately 21 km (≈ 0.19*111 km). If this effect is to be taken into account, the geographic latitude Φ should be replaced in all equations by the geocentric latitude Φ_c.

3.2 THE SUN GEOMETRY PERCEIVED BY AN OBSERVER

From the point of view of an observer at ground level, the relative movements of the sun and earth are not perceived as described above. For this observer, the sun describes a course in the sky from east to west. The exact course depends upon the season and may be described by two angles: the solar zenithal angle and the solar azimuthal angle, or azimuth (Figure 3.4).

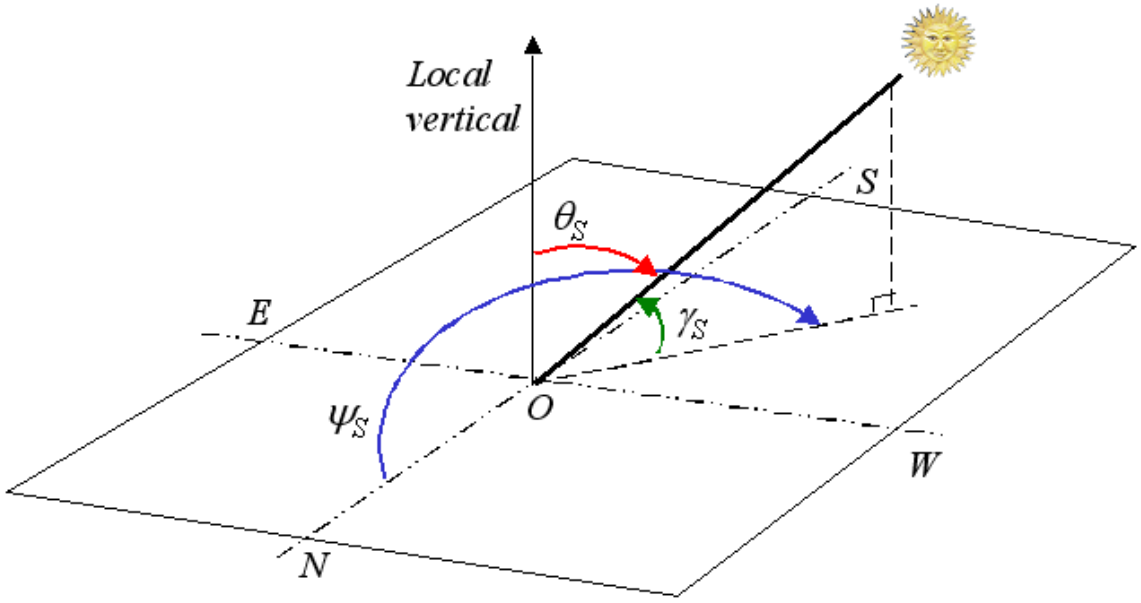

Figure 3.4. The various angles describing the position of the sun in the sky viewed from an observer O. θ_S is the solar zenithal angle, γ_S is the solar elevation angle and Ψ_S is the azimuth. N, E, S, and W denote respectively the north, east, south and west.

It should be noted that the atmosphere does not affect the perceived direction of the sun, except when the sun is very low on or below the horizon. Hence, the solar zenithal angle and the azimuth are the same at earth surface or at the top of the atmosphere and more generally for any altitude.

The solar zenithal angle θ_S is the angle formed by the direction of the sun and the local vertical (zenith). The solar elevation angle γ_S is the angle formed by the direction of the sun and the horizon. One is the complementary of the other, that is:

$$\pi/2 = \theta_S + \gamma_S \qquad (3\text{-}10)$$

In the following, the angle θ_S is mostly used in equations. To replace θ_S by γ_S, the following relationships may be used:

$$\cos\theta_S = \sin\gamma_S \quad (3\text{-}11)$$

$$\sin\theta_S = \cos\gamma_S \quad (3\text{-}12)$$

During daytime, the solar elevation angle γ_S varies from 0 when the sun is at the horizon to $\pi/2$ when the sun is at its highest, i.e. at zenith. For more accuracy when the sun is low, i.e., when γ_S is less than 10°, one should take into account the atmospheric refraction of the solar rays. The angular difference between the geometric elevation and the apparent elevation is 0.01454 rad (0.8333°, i.e. 50'); the sun appears lower than its actual position. This effect may be neglected for most cases, except for high latitudes.

It is very convenient to define the hour angle ω of the sun to describe its apparent movement. This angle is the arc angle comprised between the longitudinal plane of the point of interest *O* and the instantaneous position of the sun (Figure 3.5). In other words, it is the arc of trajectory of the sun counted from noon. ω is 0 when the sun is at its highest. ω is counted negative in the morning and positive in the afternoon.

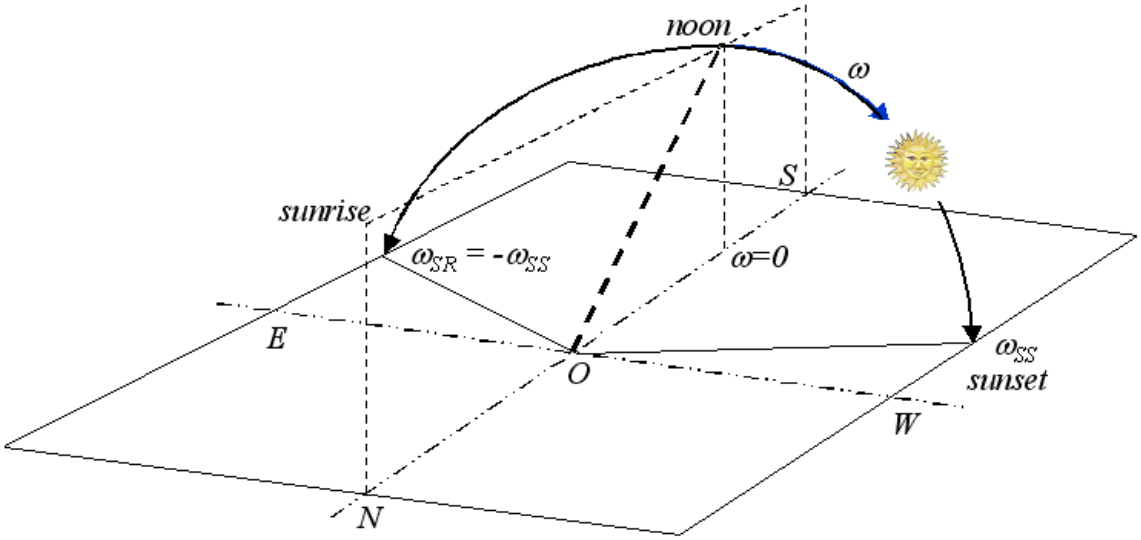

Figure 3.5. Scheme showing the hour angle ω of the sun for the observer O. ω_{ss} is the hour angle for sunset, ω_{sr} is that of sunrise ($\omega_{ss} = -\omega_{sr}$). Adapted from Perrin de Brichambaut and Vauge (1982, op. cited).

The *true solar time* (TST) is defined by the fact that it is 12:00 when the sun is at its highest for the day under concern, i.e. for ω=0. This highest position is called solar noon. Since the apparent trajectory of the sun during a day, from -π to +π, i.e. 2π, is uniformly described by ω in 24 h, ω is related to the true solar time t_{TST} (expressed in h) by:

$$\omega = (2\,\pi/24)\,(t_{TST} - 12) = (\pi/12)\,(t_{TST} - 12) \quad (3\text{-}13)$$

since the sun path of 2π is uniformly described in 24 h. Reciprocally, given ω, t_{TST} is given (in h) by:

$$t_{TST} = 12\,(1 + \omega/\pi) \quad (3\text{-}14)$$

The *solar zenithal angle* θ_S for a given t_{TST} is obtained by:

$$\cos\theta_S = \sin\Phi\sin\delta + \cos\Phi\cos\delta\cos\omega \quad (3\text{-}15)$$

where ω is given by Eq. 3-13, Φ is the latitude and δ the solar declination.

The azimuth of the sun Ψ_S, or *solar azimuthal angle*, is defined as the angle between the projection of the direction of the sun on the horizontal plane and a reference direction (Figure 3.4). There are several conventions for measuring Ψ_S. The simplest is to count Ψ_S clockwise from north where its value is *0*. Thus, it is π/2 (90°) for east, π (180°) for south and 3π/2 (270°) for west. This convention is that recommended by the international organization for standardization ISO in geographic information (ISO 19115) and is used in this text. The following equations link θ_S and Ψ_S:

$$\sin\Psi_S \sin\theta_S = -\cos\delta \sin\omega \qquad (3\text{-}16)$$

$$\cos\Psi_S \sin\theta_S = \sin\delta\cos\Phi - \cos\delta\sin\Phi\cos\omega$$

If θ_S is different from 0 and if the latitude Φ is different from $\pm\pi/2$, the solar azimuth Ψ_S is given by the following equations:

$$\cos\Psi' = [\sin\delta\cos\Phi - \cos\delta\sin\Phi\cos\omega] / \sin\theta_S \qquad (3\text{-}17)$$

$$\text{if } \sin\omega \leq 0, \text{ then } \Psi_S = \cos^{-1}(\cos\Psi')$$

$$\text{if } \sin\omega > 0, \text{ then } \Psi_S = 2\pi - \cos^{-1}(\cos\Psi')$$

The following equations may have an interest in further calculations implying the solar azimuthal angle:

$$\sin\Psi_S = -\cos\delta\sin\omega / \sin\theta_S \qquad (3\text{-}18)$$

$$\tan\Psi_S = -\sin\omega / [\sin\Phi\cos\omega - \cos\Phi\tan\delta] \text{ in northern hemisphere}$$

$$\tan\Psi_S = \sin\omega / [\sin\Phi\cos\omega - \cos\Phi\tan\delta] \text{ in southern hemisphere}$$

These equations stand provided that θ_S is not equal to *0* and not at the pole ($\Phi = \pm\pi/2$). When θ_S is null, the azimuth is undetermined; it may be set to π, i.e. to the south. At the poles, Ψ_S is undetermined for any position of the sun; it may be set to any value.

Several solar engineering publications have adopted another convention: the solar azimuth is measured from due south in the northern hemisphere and is positive towards the west. In this way, it behaves similarly to the solar hour angle ω: it is positive in the afternoon (west) and negative in the morning (east). Thus, the solar azimuth for solar engineers is equal to 0 to the south, $\pi/2$ to the west, π to the north and $-\pi/2$ to the east. In the southern hemisphere, the solar azimuth for solar engineers is counted from due north and still positive to the west. Thus, it is equal to 0 in the north, $\pi/2$ to the west, π to the south and $-\pi/2$ to the east. With this convention, the solar azimuth for solar engineers is equal to ($\Psi_S - \pi$) in the northern hemisphere and ($-\Psi_S$) in the southern hemisphere where Ψ_S is defined by Eq. 3-17 with the ISO convention.

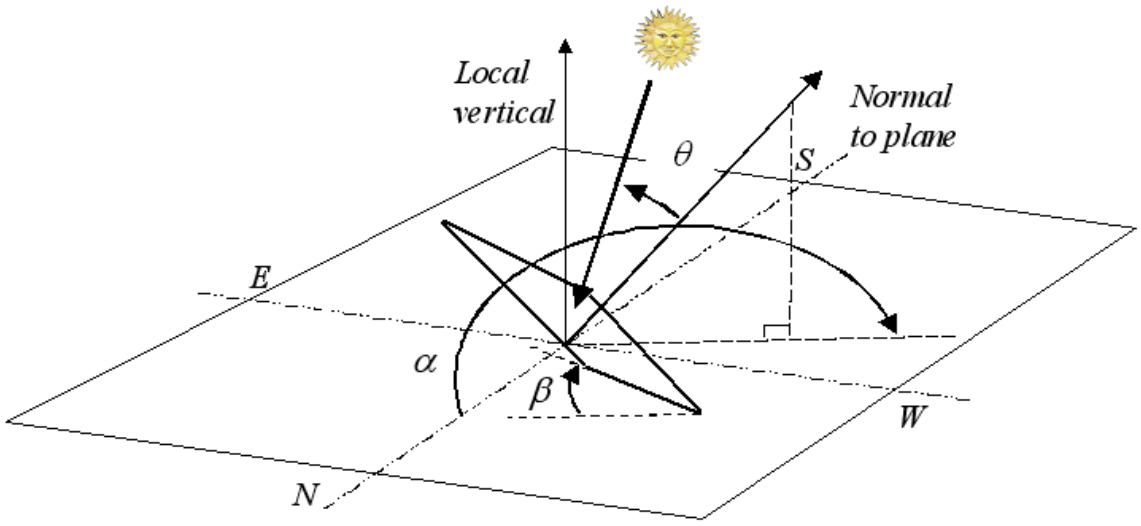

Figure 3.6. The inclination (or tilt) angle β and the azimuth α describe the inclined plane. θ is the incidence angle and is comprised between the normal to plane and the solar rays.

An inclined plane can be described with two angles: the inclination angle β, also called the tilt angle, and the azimuth α (Figure 3.6). The inclination angle β varies from 0 (horizontal plane) to $\pi/2$ (vertical plane). The azimuth of a plane is defined as the angle between the projection of the normal to this plane on the horizontal plane and a reference direction. It is also called orientation or aspect in the case of natural slopes. The convention for the azimuth α should be the same as for the solar azimuth Ψ_S.

Assume a solar engineer who has adopted the convention described above: the solar azimuth is measured from due south in the northern hemisphere and is positive towards the west. Then if the solar engineer describes the orientation of a photovoltaic panel (PV) as being 62°, i.e. the panel is orientated towards south-west, the azimuth is 242° (=180°+62°) in the ISO convention. If the orientation of the PV panel is -118°, i.e. the panel is orientated towards north-east, then it is 62° (=180°-118°) in the ISO convention. If the PV panel is located in the southern hemisphere with an orientation of 62°, i.e. the panel is orientated towards north-west, the orientation is 298° (=360°-62°) in the ISO convention. If the orientation of the PV panel is -118°, i.e. the panel is orientated towards south-east, the orientation is 118° in the ISO convention.

The angle of incidence θ of the solar rays is the angle formed by the normal to the plane and the rays (Figure 3.6). It is given by:

$$\cos\theta = \cos\beta\cos\theta_S + \sin\beta\sin\theta_S\cos(\Psi_S - \alpha) \qquad (3\text{-}19)$$

This equation and the following ones are valid whatever the convention for counting the azimuth.

For further calculations, it is convenient to express θ as an explicit function of ω. It will ease the computation of the radiation received by an inclined surface. By exploiting Eqs 3-15 to 3-19 and given that

$$\cos(\Psi_S - \alpha) = \cos\Psi_S\cos\alpha + \sin\Psi_S\sin\alpha \qquad (3\text{-}20)$$

we obtain

$$\cos\theta = A\cos\omega + B\sin\omega + C \qquad (3\text{-}21)$$

where

$$\begin{aligned} A &= \cos\delta(\cos\Phi\cos\beta - \sin\Phi\sin\beta\cos\alpha) \\ B &= -\cos\delta\sin\beta\sin\alpha \\ C &= \sin\delta(\sin\Phi\cos\beta + \cos\Phi\sin\beta\cos\alpha) \end{aligned} \qquad (3\text{-}22)$$

The integral of $\cos\theta$ during any period of time $[t_1, t_2]$ expressed by the hour angles ω_1 and ω_2, may be computed using Eqs 3-21 and 3-22:

$$\begin{aligned} \int_{\omega_1}^{\omega_2}\cos\theta\, d\omega &= \left[A\sin\omega - B\cos\omega + C\omega\right]_{\omega_1}^{\omega_2} \\ &= A(\sin\omega_2 - \sin\omega_1) - B(\cos\omega_2 - \cos\omega_1) + C(\omega_2 - \omega_1) \end{aligned} \qquad (3\text{-}23)$$

If the plane is horizontal ($\beta = 0$), then

$$\begin{aligned} A &= \cos\delta\cos\Phi \\ B &= 0 \\ C &= \sin\delta\sin\Phi \end{aligned} \qquad (3\text{-}24)$$

and the integral of $\cos\theta_S$ is:

$$\int_{\omega_1}^{\omega_2}\cos\theta_S\, d\omega = [A\sin\omega + C\omega]_{\omega_1}^{\omega_2} = A(\sin\omega_2 - \sin\omega_1) + C(\omega_2 - \omega_1) \qquad (3\text{-}25)$$

4 THE DIFFERENT TIME SYSTEMS AND DAYTIME

In summary

Time is an essential element in solar radiation. It should be known with care. Its encoding in data exchange should follow ISO standards.

The daily rotation of the earth on itself induces the notion of a mean solar day divided into 24 h of 60 min each. The time defined in this way is called the mean solar time (MST). The sun is approximately at its zenith when the mean solar time is equal to 12:00. Consequently, the mean solar time is not the same everywhere on the earth. It depends upon the longitude. A difference of 1 h corresponds to a difference of 15° in longitude.

The difference between the true solar time and the mean solar time is as a function of the day in the year. The maximum is reached on 31st October: the difference is equal to 0.276 h (≈17 min). The minimum is −0.242 h (≈15 min) on 13th February.

The reference for time is the mean solar time for the longitude 0°; it is called the Universal Time, abbreviated in UT. The legal time is the time used legally in a given country. It is also called local time, standard time, civil time or local clock time.

The daily rotation of the earth on itself determines daytime and night-time. Daytime is the duration of the day during which the sun is above the horizon, i.e. the period of time between the sunrise and the sunset. The night-time is the period of time comprised during the sunset and the sunrise when the sun is below the horizon. The duration of a day is 24 h; the sum of the durations of daytime and night-time is equal to 24 h.

The astronomical daytime is defined as the period of time between the sunrise and the sunset in the case there is no obstruction in the line of sight. If obstructions are present such as in cities or in case of marked relief, the actual daytime may differ from the astronomical daytime.

The sun-earth geometry has for long served as standards for defining time and its units. For the time being, the second is defined by electronic transitions in certain atoms in atomic clocks. The second is the international unit of time. Time is primordial for computing the sun-earth geometry and subsequently the solar radiation at the top of the atmosphere and at earth surface.

4.1 THE TRUE SOLAR TIME, MEAN SOLAR TIME, UNIVERSAL TIME AND LEGAL TIME

4.1.1 THE MEAN SOLAR TIME AND TRUE SOLAR TIME

A day is the time duration for one rotation; a day is divided into 24 h, i.e. 24 times 3600 s, as an average. The time defined in this way is called the *mean solar time* (MST), noted t_{MST}. The sun is approximately at its zenith when the mean solar time is equal to 12:00. Consequently, the mean solar time is not the same everywhere on the earth. It depends upon the longitude.

One hour corresponds to an angle of $(2\pi/24)$ (=15°), because the earth rotates 2π in 24 h. This is equivalent to saying that 1° in longitude corresponds to 4 min. Consequently, the difference in time between longitude 0 and longitude λ is given by $(24\lambda / 2\pi)$, expressed in h:

$$t_{MST}(\lambda) = t_{MST}(\lambda=0) + (24\lambda / 2\pi) \qquad (4\text{-}1)$$

Because the orbit of the earth is an ellipse, the earth angular speed varies slightly throughout the year. Combined with the rotation of the earth on itself, which is very regular, it results that the sun does not reach its highest position in the sky at 12:00 MST every day. The time of solar noon may differ from 12:00 MST by up to 17 min. The time determined every day by the actual position of the sun in the sky is called true solar time (TST), or local apparent time; it is represented by t_{TST}. It is important not to confuse this local apparent time with the legal local time. Solar noon is reached at 12:00 TST. The difference between t_{TST} and t_{MST} is sometimes called *time equation*; it is expressed in h and can be approximated by:

$$t_{TST} - t_{MST} = -0.128 \sin(j - 0.04887) - 0.165 \sin(2j + 0.34383) \qquad (4\text{-}2)$$

Figure 4.1 displays the variation of the difference between t_{TST} and t_{MST} as a function of the day in the year. The maximum is reached on 31st October: the difference is equal to 0.276 h (≈17 min). The minimum is equal to −0.242 h on 13th February. The difference is equal to 0 for days 16th April, 14th June, 31st August and 25th December. One notes a secondary maximum of 0.061 h (≈4 min) on 14th May and a secondary minimum of −0.106 h (≈6 min) on 27th July.

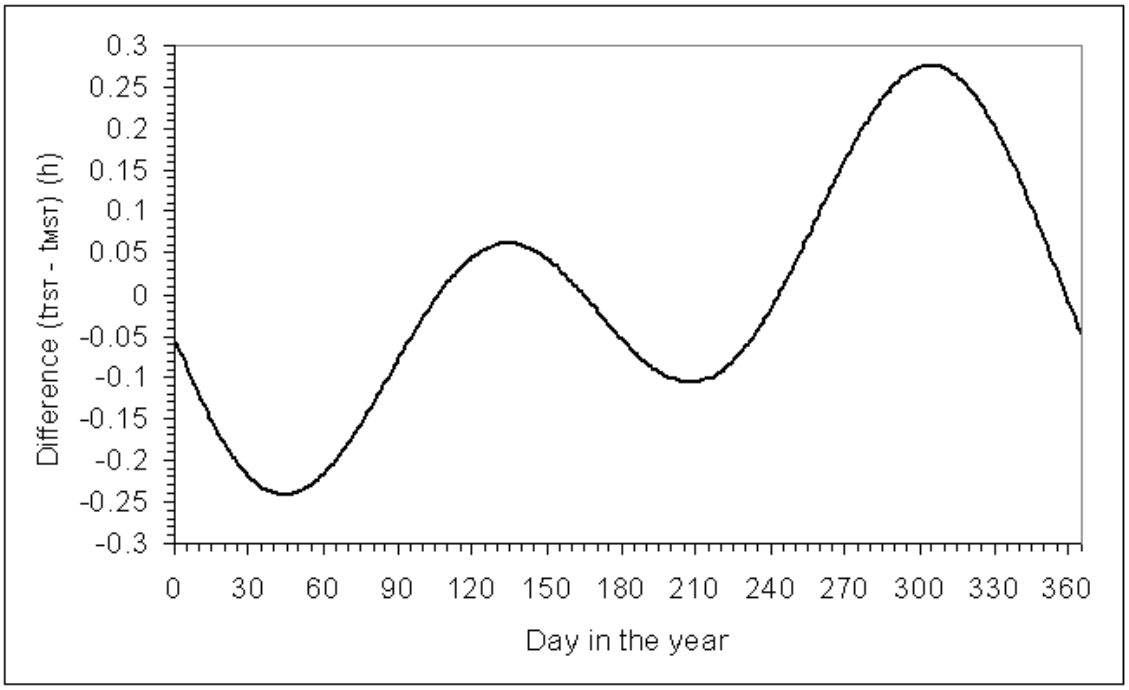

Figure 4.1. Difference between times in TST and MST as a function of the day in the year ($t_{TST} - t_{MST}$)

4.1.2 THE UNIVERSAL TIME AND LEGAL TIME

One hour corresponds to an angle of (2π/24) in radian (=15°), because the earth rotates 2π in 24 h. This is equivalent to saying that 1° in longitude corresponds to 4 min. The earth is divided into 24 time zones; each differs from the other by adding 1 h eastwards. The width of each time zone is 15° in longitude. Consequently, the difference in time between longitude 0° and longitude λ, expressed in rad, is given by $(24\lambda / 2\pi)$ h. If λ is expressed in degrees, then the difference in time is $(24\lambda / 360)$ h.

The reference for time is the mean solar time for the longitude 0°; it is called the Universal Time, abbreviated UT and represented by t_{UT}. More exactly, the time standard is the Coordinated Universal Time, abbreviated UTC. It is the basis of legal time and is derived from International Atomic Time. The difference between UT and UTC is slight, approximately 1 s per day. Hence, the UT time is adopted here.

Given a time t_{UT} and a location of longitude λ, the mean solar time t_{MST} is given in h by:

$$t_{MST} = t_{UT} + (24\lambda / 2\pi) \text{ if } \lambda \text{ is in rad or } = t_{UT} + (24\lambda / 360) \text{ if } \lambda \text{ is in degree} \qquad (4\text{-}3)$$

At longitude 0°, the mean solar time is equal to the Universal Time. At longitude 180°, it is equal to the Universal Time plus 12 h, and at longitude 120°, it is equal to the Universal Time plus 8 h.

The legal time, t_{legal}, is the time used legally in a given country. It is often called local time by the general public. Other terms are standard time, civil time and local clock time. The legal time at a given location is quite often the time of the time zone containing this location. However, the reality is more complex and is illustrated by a few examples below.

Brazil is comprised approximately between the longitudes -35° and -73°. The mean solar times at each extreme longitude are respectively t_{UT}-2.3 h and t_{UT}-4.9 h. The legal time has been set to t_{UT}-3 h for the whole country,

which corresponds to the longitude -45°. It could have been separated in two or three time zones. Russia spans approximately from 30° to 190° in longitude and is separated in eight time zones.

China spans approximately from 75° to 135° in longitude. The mean solar times at each extreme longitude are respectively t_{UT}+5 h and t_{UT}+9 h. It could have been separated in several time zones. The legal time has been set the same over the whole China and is t_{UT}+8 h, i.e. approximately the mean solar time at Beijing whose longitude is 116.36°. In this case, there may be considerable differences between the legal time and the mean solar time. Assume a legal time of 12.5 h. The corresponding UT time is t_{UT} = (12.5 – 8) h = 4.5 h. According to Eq. 4-3, the mean solar time is 12.26 h. The city of Chengdu, China is located more to the West, with a longitude of 104.06°. Though the legal time is the same as in Beijing, i.e. 12.5 h, the mean solar time at Chengdu is 11.44 h; it is equal to 10.34 h for the city of Urumqi whose longitude is 87.62°.

The Westernmost Africa countries such as Mauritania or Senegal could have adopted t_{UT}+1 h as the legal time given their longitudes. The legal time was set to UT instead. On the opposite, the Iberian Peninsula (Portugal and Spain) exhibits the same longitudes than Ireland and the United Kingdom. While Portugal, Ireland and the United Kingdom share the same time zone, Spain is 1 h ahead.

Several countries from mid- to high latitudes have adopted daylight saving time, which is usually one hour ahead with respect to the time zone during the local summer time. For example, legal time in Western Europe is equal to UTC+1 h, except in summer, when 1 h is added. To add to the complexity of handling legal time with respect to the UTC time, the dates of change for daylight saving time are not the same throughout the world.

Detailed information on time zone is easily found on the Web and is of great help in understanding and handling the time that is associated to a measure for example.

In summary, if one needs to compute the position of the sun for a given location at a given time, one must know accurately the true solar time. If the time is given in legal time, the following operations must be performed:

- convert the legal time for the location under concern into UT time t_{UT}, by taking into account the time zone and possibly the daylight saving time,
- then, apply Eq. 4-3 to obtain t_{MST},
- then, apply Eq. 4-2 to obtain t_{TST},
- then, use equations of the previous Chapter to compute the solar angles.

4.2 SUNRISE, SUNSET AND DAYTIME

The daily rotation of the earth on itself determines daytime and night-time. Daytime is the duration of the day during which the sun is above the horizon, i.e. the period of time between the sunrise and the sunset. The night-time is the period of time comprised during the sunset and the sunrise when the sun is below the horizon. The duration of a day is 24 h; the sum of the durations of daytime and night-time is equal to 24 h.

The astronomical daytime is defined as the period of time between the sunrise and the sunset in the case there is no obstruction in the line of sight. If obstructions are present such as in cities or in case of marked relief, the actual daytime may differ from the astronomical daytime.

4.2.1 THE SUNRISE AND SUNSET

Remember that the hour angle ω is 0 rad when the sun is at its highest, i.e. when t_{TST} =12 h. ω is counted negative in the morning and positive in the afternoon. The hour angles for sunrise (ω_{SR}, negative) and sunset (ω_{SS}, positive with $\omega_{SS} = -\omega_{SR}$) define the daytime. The following relationship holds (Eq. 3-14):

$$t_{TST} = 12\,(1 + \omega / \pi) \qquad (3\text{-}14)$$

where ω is in rad and t_{TST} in h. The sunset hour angle, ω_{SS}, is calculated by setting the solar zenithal angle θ_S to $\pi/2$ in Eq. 3-15 provided there is no obstruction:

$$\cos\theta_S = \sin\Phi \sin\delta + \cos\Phi \cos\delta \cos\omega \qquad (3\text{-}15)$$

ω_{SS} is the solution of this equation. It results

if $\Phi = \pi/2$, *if* $\delta > 0$ $\omega_{SS} = \pi$, *otherwise* $\omega_{SS} = 0$

if $\Phi = -\pi/2$, *if* $\delta > 0$ $\omega_{SS} = 0$, *otherwise* $\omega_{SS} = \pi$

if $(-\tan\Phi \tan\delta) \geq 1$ $\omega_{SS} = 0$ *(4-4)*

if $(-\tan\Phi \tan\delta) \leq -1$ $\omega_{SS} = \pi$

otherwise $\omega_{SS} = \arccos(-\tan\Phi \tan\delta)$

$\omega_{SS} = 0$ means that the sun is always below the horizon; $\omega_{SS} = \pi$ means that the sun is always above the horizon. The sunrise hour angle, ω_{SR}, is calculated from ω_{SS} by:

$$\omega_{SR} = -\omega_{SS} \quad (4\text{-}5)$$

The azimuth for sunrise Ψ_{SR}, respectively sunset Ψ_{SS}, is computed by means of Eq. 3-17 where θ_S is set to $\pi/2$. Ψ_{SR} is less than π, while Ψ_{SS} is greater than π.

The true solar times for sunrise t_{SR} and sunset t_{SS} are given in h by:

$$t_{SR} = 12\,(1 + \omega_{SR} / \pi) \quad (4\text{-}6)$$

$$t_{SS} = 12\,(1 + \omega_{SS} / \pi)$$

ω_{SS} is always equal to $\pi/2$ at the equator: the sunrise and sunset occur at the same time every day: t_{SR} is equal to 6 h and t_{SS} to 18 h. ω_{SS} is equal to $\pi/2$ on 21 March and 23 September for all latitudes, except the poles. On these days, t_{SR} is equal to 6 h and t_{SS} to 18 h for any latitude.

At the North Pole ω_{SS} is equal to 0 during a period of six months centred on the winter solstice, i.e. around 22 December: the sun is always below the horizon. ω_{SS} is equal to π during the other six months centred around 21 June: the sun is always above the horizon. It is the opposite at the South Pole.

The polar circles are particular latitudes with respect to the sunrise and sunset and hence daytime. North of the northern polar circle, there is at least one day per year for which ω_{SS} is π, i.e. the sun is always above the horizon, and reciprocally at least one day per year for which ω_{SS} is 0, i.e., the sun is always below the horizon. The period during which $\omega_{SS} = \pi$ is centred around 21 June and that during which $\omega_{SS} = 0$ is centred around 22 December. It is the opposite for the southern hemisphere. The latitudes of these polar circles are the complement to $\pi/2$ of the inclination angle (0.4093 rad, i.e. 23.45°) of the axis of the daily rotation of the earth on the orbit plane (Figure 3.1) and are equal to 1.1615 rad (+66.55°) for the northern polar circle and -1.1615 rad (-66.55°) for the southern polar circle.

There is no allowance for atmospheric refraction or angular size of the solar disc in the above equations. If these effects are to be taken into account, then the sunset hour angle ω_{SS} is now the solution of the equation below:

$$\sin(-0.014544) = \sin\Phi \sin\delta + \cos\Phi \cos\delta \cos\omega_{SS} \quad (4\text{-}7)$$

as the edge of the solar disk just appears, or disappears, on the horizon at sunrise, or sunset, when the solar elevation angle γ_S is 0.0145 rad (0.8333°). The sunrise hour angle is computed by Eq. 4-5. These effects may be neglected as a first approximation.

If one considers obstructions of the horizon, such as mountains, buildings or vegetation, the sunset and sunrise hour angles will correspond to a solar elevation angle greater than 0, or greater than −0.01454 rad if one considers the atmospheric refraction and the size of the solar disk. Assuming that the solar elevation angles for such an obstructed site are known and are noted γ_{SR} at sunrise and γ_{SS} at sunset, then the hour angles for sunrise ω_{SR} and sunset ω_{SS} are the solutions of the following equations:

$$\cos\Phi \cos\delta \cos(-\omega_{SR}) = \sin\gamma_{SR} - \sin\Phi \sin\delta \quad (4\text{-}8)$$

$$\cos\Phi \cos\delta \cos\omega_{SS} = \sin\gamma_{SS} - \sin\Phi \sin\delta$$

4.2.2 THE DAYTIME

The daytime S_{day} is given in h by:

$$S_{day} = (t_{SS} - t_{SR}) = (12 / \pi) (\omega_{SS} - \omega_{SR}) \quad (4\text{-}9)$$

If obstructions are not taken into account, then $(\omega_{SR} = -\omega_{SS})$. In this case, the daytime is also called the astronomical daytime and is noted S_0. It is given in h by:

$$S_0 = (24\ \omega_{SS} / \pi) \quad (4\text{-}10)$$

Figure 4.2 reports the astronomical daytime S_0 as a function of the day of the year for various latitudes. Note that the shapes of variations in S_0 are the same than those in ω_{SS} since S_0 is equal to ω_{SS} multiplied by a constant (Eq. 4-10). S_0 is always 12 h at the equator. For the days 21 March and 23 September, S_0 is equal to 12 h for all latitudes, except the poles. Outside the equatorial belt, S_0 changes considerably according to the time of year and latitude. For the northern hemisphere, the daytime is greater from April to September than from October to March. On the contrary, in the southern hemisphere, S_0 is greater from October to March than from April to September. At any latitude in northern hemisphere, except equator, the daytime is the greatest at the summer solstice and the smallest at the winter solstice. This is opposite in the southern hemisphere, the daytime is the smallest at the summer solstice and the greatest at the winter solstice.

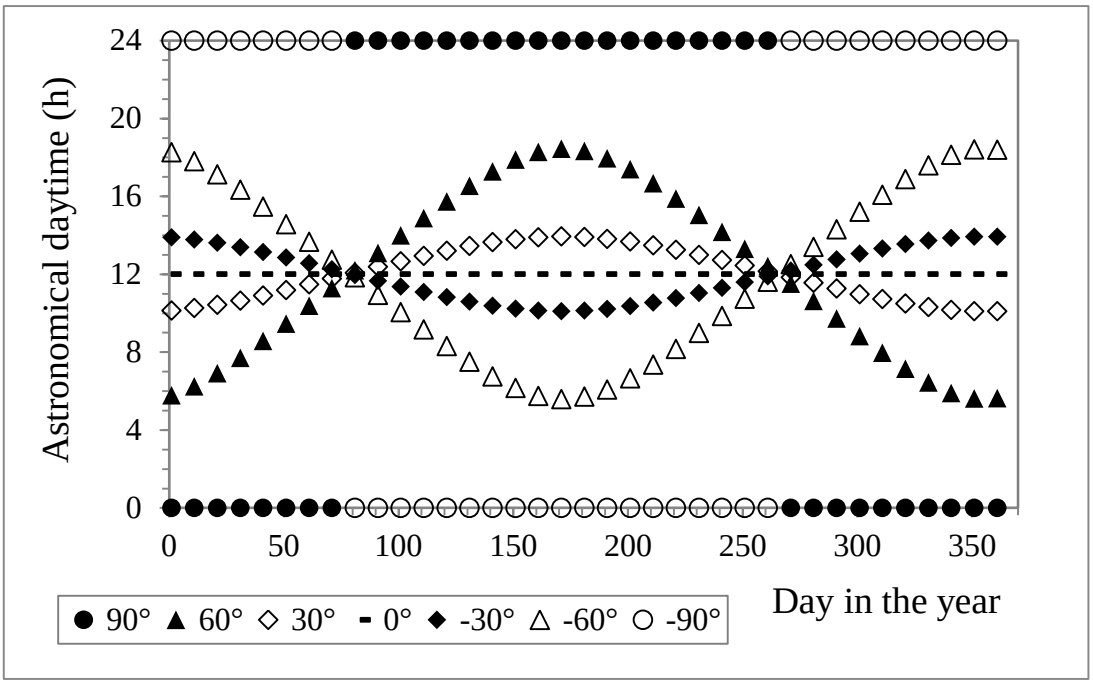

Figure 4.2. The astronomical daytime S_0 as a function of the day in the year and latitude.

At the poles, daytime is either 24 h or 0 h. The sun is always above the horizon during the six months centred around the summer solstice –the famous midnight sun- and always below the horizon during the six months centred around the winter solstice –the polar night.- The midnight sun –and reciprocally the polar night– can be experienced for latitudes north of the northern polar circle or south of the southern polar circle.

At the North Pole ω_{SS} is equal to 0 during a period of six months centred on the winter solstice, i.e. around 22 December: the sun is always below the horizon. ω_{SS} is equal to π during the other six months centred around 21 June: the sun is always above the horizon. It is the opposite at the South Pole.

Table 4.1 reports on the yearly mean of the astronomical daytime at various latitudes. At the equator, the mean is 12 h. The dissymmetry between the northern and southern hemispheres can be noted: the mean duration of the day is greater in the north than in the south. The mean reaches its maximum (12.30 h) at the North Pole; it

decreases towards the South Pole where it reaches a minimum of 11.70 h. The difference between these minimum and maximum is 0.6 h.

Latitude	Astronomical daytime (h)
90°	12.30
70°	12.20
60°	12.11
45°	12.06
30°	12.03
0°	12.00

Latitude	Astronomical daytime (h)
−90°	11.70
−70°	11.80
−60°	11.89
−45°	11.94
−30°	11.97

Table 4.1. The yearly mean of the astronomical daytime S_0 for various latitudes

4.3 ENCODING TIME IN ISO FORMAT

As already stated, time is an essential element in solar radiation. Its encoding for exchange of data is crucial. There are several numeric representations of dates and time in different countries or different domains of activities. The purpose of an international standard is to avoid confusion and the misinterpretation. The authors have adopted the ISO format on time[6] and strongly recommend it.

The ISO format is applicable when dates are given in the Gregorian calendar and times in the 24-h system. It includes

- calendar dates expressed in terms of calendar year, calendar month and calendar day of the month;
- ordinal dates expressed in terms of calendar year and calendar day of the year;
- week dates expressed in terms of calendar year, calendar week number and calendar day of the week;
- local time based upon the 24-h timekeeping system;
- Coordinated Universal Time (UTC) of day;
- local time and the difference from UTC;
- combination of date and time of day;
- time intervals;
- recurring time intervals.

There are two types of format: one with separators and one without separators. Only the latter is dealt with as it allows handling the many cases that are encountered when exchanging information on solar radiation. The standard may appear complicated as it has been designed for interchange between machines. Several popular computer languages offer libraries that cope with this ISO standard, such as *datetime* for Python.

4.3.1 ENCODING DATES AND WEEKS

The encoding of a date is *YYYY-MM-DD*, where the year *YYYY* is given with four characters, the month *MM* is encoded from 01 (January) to 12 (December) with two characters, and the day *DD* is the day in the month, from 00 up to 31 with two characters. For example,

- 31 October 2017 is encoded as 2017-10-31;
- 2 May 2017 is encoded 2017-05-02.

Reduced representations are allowed. For example,

- 2017 alone means the year 2017 without specifying a specific day in this year;
- 2017-10 means October 2017 without specifying a specific day in this month.

[6] ISO 8601:2004(E) Data elements and interchange formats – Information interchange – Representation of dates and times, 2014, Third edition, 40 p., International Standard Organization, Geneva, Switzerland. [ISO 8601 specifies numeric representations of date and time.]

One may need to express the date with the year *YYYY* and the number of the day in the year *DDD*. In this case, *DDD* is encoded with three characters and ranges from 001 to 365 or 366 in case of a leap year. For example,

- 2017-122 corresponds to 2 May 2017.

A week, or calendar week, is defined as the time interval of seven days starting with a Monday. The week number is the ordinal number which identifies a week within the year according to the rule that the first calendar week of a year is that one which includes the first Thursday of that year and that the last calendar week of a year is the week immediately preceding the first calendar week of the next year. In the encoding of a week, the week number *ww* is encoded with two characters starting from 01: *YYYY-Www*. For example,

- 2017-W03 means the third week of 2017 without specifying a specific day in this week;
- 2017-W01 denotes the first week of 2017 and starts on Monday 2 January. The 1st January 2017 was a Sunday and belongs to the week #52 of 2016.

If one wants to specify a day in the week, one may use the following encoding *YYYY-Www-d* where *d* is the number of the day in the week, starting from 1 (Monday) up to 7 (Sunday). For example,

- 2017-W03-3 means Wednesday of the third week of 2017, i.e. Wednesday 18 January 2017.
- 2017-W18-2 means Tuesday of the eighteenth week of 2017, i.e. Tuesday 2 May 2017.

4.3.2 ENCODING TIME

The encoding of a time is *hh:mm:ss*, where the hour *hh* ranges from 00 to 24, encoded with two characters, the minutes *mm* are encoded from 00 to 60 with two characters, and the seconds are encoded from 00 to 60 with two characters. For example,

- 13:34:21 means 13 h 34 min and 21 s;
- 13:34:00 means 13 h 34 min and 00 s;
- 13:34 means 13 h and 34 min irrespective of the number of seconds in this minute.

The standard states that 00:00:00 is the beginning of the day and 24:00:00 is the end of the day. The authors draw attention to the reader that this may not be the case in all implementations of the standard. In some cases, 24:00:00 may mean the beginning of the following day.

Representation of time with decimal fraction is possible. For example;

- 13:34:21,023 means 13 h 34 min, 21 s and 23 thousandths, if the decimal sign is the comma (preferred sign);
- 13:34:21.023 means 13 h 34 min, 21 s and 23 thousandths, if the decimal sign is the full stop;
- 13:34,023 means 13 h 34 min, and 23 thousandths of 1 min;
- fractional hour in decimal format is not allowed in this format, i.e. 13,023 (or 13.023) is not permitted.

It has been seen that several time systems may be found. The ISO standard accommodates for that. The *Z* letter indicates the UTC system. For example,

- 13:34:21 means 13 h 34 min and 21 s in an unspecified time system;
- 13:34:21Z means 13 h 34 min and 21 s in the UTC system;
- 13:34Z means 13 h and 34 min in the UTC system irrespective of the number of seconds in this minute.

If local time is used, it is possible to indicate the difference between the local time and UTC. The representation of the difference is appended to the representation of the local time. The difference between the time scale of local time and UTC shall be expressed in hours and minutes, or hours only independent of the accuracy of the local time expression. For example,

- 13:34:21+01:00 means 13 h 34 min and 21 s in the local time system, which is 1 h ahead of UTC;
- 13:34:21.023+01:00 means 13 h 34 min, 21 s and 23 thousandths in the local time system, which is 1 h ahead of UTC;
- 13:34:21-01:00 means 13 h 34 min and 21 s in the local time system, which is 1 h behind of UTC.

4.3.3 ENCODING DATE AND TIME OF DAY

A time point can be identified through a date and time of day expression. The elements are written in the following sequence:

- for calendar dates: year – month – day in the month – time designator – hour – minute – second – zone designator;
- for ordinal dates: year – day in the year – time designator – hour – minute – second – zone designator;
- for week dates: year – week designator – week – day in the week – time designator – hour – minute – second – zone designator.

T is the time designator. For example,

- 2017-05-02T13:34 is the encoding for 2 May 2017 at 13 h and 34 min (no time system specified);
- 2017-05-02T13:34:21 is the encoding for 2 May 2017 at 13 h 34 min and 21 s (no time system specified);
- 2017-05-02T13:34:21Z is the encoding for 2 May 2017 at 13 h 34 min and 21 s UTC;
- 2017-05-02T13:34:21-01:00 is the encoding for 2 May 2017 at 13 h 34 min and 21 s in the local time system, which is 1 h behind of UTC;
- 2017-122T13:34:21 is the encoding for the day 122 in the year (2 May 2017) at 13 h 34 min and 21 s (no time system specified);
- 2017-W18-2T13:34:21 is the encoding for the day 122 in the year (2 May 2017) at 13 h 34 min and 21 s (no time system specified);

Reduced representations of dates are not permitted. For example, it is not permitted to write 2017-05T13:34 nor 2017-W18T13:34 because the day is undefined.

4.3.4 ENCODING TIME INTERVALS AND DURATION

A time interval may be expressed by a start and an end. In this case, the starting time point will be separated from the ending time point by a solidus [/]. A typical representation is *YYYY-MM-DDThh:mm:ss/YYYY-MM-DDThh:mm:ss*. For example,

- 2017-05-02T13:34/2017-05-02T13:35 is the encoding for the time interval starting at 13 h 34 min and ending at 13 2 May 2017 with no specified time system;
- 2017-05-02/2017-05-03 is the encoding for the time interval starting 2 May 2017 and finishing on 3 May 2017 with no indication of time at the start and end;
- 2017-05/2017-07 is the encoding for the time interval starting May 2017 and finishing on July 2017 with no indication of day and time at the start and end.

A time interval may also be expressed by a start and a duration, or by a duration and an end.

The duration is encoded by a string of characters beginning by *P*. A typical format is *PYYYY-MM-DDThh:mm:ss*. For example,

- P0001-02-15T12:25:21 is the duration of 1 year, 2 months, 15 days, 12 h, 25 min, and 21 s;
- P0000-00-01 is the duration of 1 day.

An alternative representation for encoding duration is *PnnYnnMnnDTnnHnnMnnS*. For example,

- P1Y2M15DT12H25M21S is the duration of 1 year, 2 months, 15 days, 12 h, 25 min, and 21 s and is equivalent to P0001-02-15T12:25:21;
- P1D is the duration of 1 day and is equivalent to P0000-00-01;
- P10M is the duration of 10 months;
- PT10M is the duration of 10 min;
- PT1H30M is the duration of 1 h and 30 min;
- P2W is a duration of 2 weeks.

A time interval expressed by a start and a duration has a typical representation *YYYY-MM-DDThh:mm:ss/PYYYY-MM-DDThh:mm:ss* or *YYYY-MM-DDThh:mm:ss/PnnYnnMnnDTnnHnnMnnS*. For example,

- 2017-05-02T13:34/P0000-00-00T00:01 is the encoding for the time interval starting at 13 h 34 min and ending 1 min later 2 May 2017 with no specified time system;
- 2017-05-02T13:34/PT1M is an alternative encoding for the same time interval.

A time interval expressed by a duration and an end has a typical representation *PYYYY-MM-DDThh:mm:ss/YYYY-MM-DDThh:mm:ss* or *PnnYnnMnnDTnnHnnMnnS/YYYY-MM-DDThh:mm:ss*.

- P0000-00-00T00:01/2017-05-02T13:35 is the encoding for the time interval starting at 13 h 34 min and ending 1 min later at 13 h 35 min 2 May 2017 with no specified time system;
- PT1M/2017-05-02T13:35 is an alternative encoding for the same time interval.

5 THE SOLAR RADIATION AT THE TOP OF THE ATMOSPHERE

In summary

The amount of solar radiation that is intercepted by the earth varies because of variations in sun-earth distance and, to a much lesser extent, day-to-day variations in the spectrum due to solar activity. The closer to the sun the earth, the greater the solar irradiance impinging on a plane normal to the sun rays and located at the top of the atmosphere. The total solar irradiance is the yearly average of this irradiance during a year and integrated over the whole spectrum. The variations within a year amount to ± 3 % of the total solar irradiance.

The International Astronomical Union has recommended a value of 1361 W m^{-2} ± 1 W m^{-2} as the mean value for the solar cycle #23, i.e. for the period May 1996 to January 2008. Recent measurements in 2010 yield a total solar irradiance of 1362 W m^{-2}. Day-to-day changes in the solar irradiance impinging on a plane normal to the sun rays may reach 5 W m^{-2}, i.e. approximately 0.4 % of the total solar irradiance.

The spectral distribution of the solar irradiance is such that it is peaked around 550 nm and that about half of it lies in the visible and near-infrared parts of the electromagnetic spectrum. The interval covered by the spectral distribution of the solar irradiance is not very wide. Approximately 98 % of the power is found between 0.3 μm and 4 μm, and 99.9 % is located between 0.2 μm and 8 μm. The amount of radiation integrated over the whole spectrum is called total radiation or broadband radiation.

The spectral distribution of the total solar irradiance is actually not constant and varies in time. The visible and infrared parts of the spectrum are the least variable parts, while the UV (ultra-violet) part is highly variable.

A series of equations is offered to compute the solar irradiance impinging on any inclined surface, including horizontal, located at the top of the atmosphere for any instant and any location.

5.1 RADIANCE, IRRADIANCE AND IRRADIATION: DEFINITIONS AND UNITS

Several quantities are necessary to describe radiation. The radiant energy Q is the amount of energy that is transferred by radiation. It is expressed in J (Joule).

Power is an energy divided by a time interval Δt. The time interval may be called *summarization* in meteorology. The radiant flux F is the time rate of flow of the radiant energy:

$$F = Q / \Delta t \qquad (5\text{-}1)$$

The radiant flux is expressed in W (Watt). The downward flux is the flux received on the upper face of a surface; the upward flux is the flux received on the lower face of this surface.

The *radiance* L is the radiant flux per unit solid angle $d\omega$ per unit projected area of a surface $dA \cos\theta$:

$$L = d^2F / (dA \cos\theta \, d\omega) \qquad (5\text{-}2)$$

The radiance is expressed in W m^{-2} sr^{-1}. If the surface is horizontal, $L = d^2F / (dA \, d\omega)$.

The *irradiance* is defined as a power received per area. It is expressed in watt per square meter (W m^{-2}). It is represented by E in this text, as recommended by the S.I. (*système international d'unités*). It is equal to the integral of a radiance distribution on the hemisphere above the point of interest (downward irradiance) or below this point (upward irradiance). Solar irradiance means the irradiance originating from the sun. It is also called shortwave irradiance because its most energetic wavelengths are less than 2 μm.

The *irradiation* is the energy received per area; unit is Joule per square meter (J m^{-2}). The International Solar Energy Society recommends the symbol H. In commercial metering of electrical energy, a frequently used unit for irradiation is Watt-hour per square meter (Wh m^{-2}) though this should not be used in scientific and technical work since it is not part of the S.I. The conversion is defined by:

$$H = 1 \text{ Wh } m^{-2} = 3\,600 \text{ J } m^{-2} \qquad (5\text{-}3)$$

The conversion from irradiation into irradiance is performed by dividing irradiation by the duration of the measurement Δt, or summarization. Reciprocally, irradiance is converted into irradiation by multiplying by Δt:

$$E = H / \Delta t \quad (5\text{-}4)$$

It is important to always mention the summarization. For example, one should write "hourly solar irradiation" to denote the solar irradiation collected during 1 h. The daily solar irradiation is the amount of energy collected by a surface during the daytime, i.e. the duration between sunrise and sunset. The yearly solar irradiation is the solar irradiation collected during one full year.

If irradiance is used, one should write "hourly mean of irradiance" if the time scale of interest is 1 h. The case of day, and greater periods is different by convention in the climate domain. By convention, the daily mean of irradiance is the daily irradiation divided by the number of seconds in 24 h, i.e. 86400 s, irrespective of the actual daytime. The monthly mean of irradiance is the monthly irradiation divided by the number of days in this month and by the number of seconds in 24 h. The yearly mean of irradiance is the yearly irradiation divided by the number of days in this year and by the number of seconds in 24 h.

The radiative energy is in the form of radiative waves of different wavelengths. The distribution of the irradiance with the wavelength is called the *spectral distribution of the irradiance*. The *total irradiance* is the irradiance integrated over the whole spectrum.

The broadband solar irradiance is the irradiance integrated over a large spectrum. It is not well defined and is used as a current term to denote that the radiation is integrated over a certain range of wavelength. For example, it is used to denote the solar radiation measured by pyranometers, e.g. from 300 nm to 2200 nm. It may be used differently. For example, one may read in literature *broadband UV* that opposes to *spectral UV*.

5.2 THE ENERGY EMITTED BY THE SUN AND THE TOTAL SOLAR IRRADIANCE

The sun is the seat of thermonuclear processes and produces a vast amount of energy. The energy emitted by the sun is called solar energy or solar radiation. Despite the considerable distance between the sun and the earth, the amount of solar energy reaching the earth is substantial. It is the earth primary natural source of energy and by a long way. Other sources are: the geothermal heat flux generated by the earth interior, natural terrestrial radioactivity, and cosmic radiation, which are all negligible relative to solar radiation.

Solar radiation is a key factor controlling the climate of the earth. There is a global radiative equilibrium between the earth and extra-terrestrial space. It means that the part of the incoming solar radiation that is absorbed by the earth and its atmosphere is equal to the outgoing longwave radiation from the earth and its atmosphere. The solar radiation has a specific spectral distribution which is dealt with in the second part of this chapter.

5.2.1 THE TOTAL SOLAR IRRADIANCE

The amount of solar radiation that is intercepted by the earth varies because of variations in sun-earth distance and to a much lesser extent, day-to-day variations in the spectrum due to solar activity. Let E_{0N} denote the solar irradiance impinging on a plane normal to the sun rays located at the top of the atmosphere and integrated over the whole spectrum, i.e. the total irradiance. E_{0N} depends on the distance from the earth to the sun.

The total solar irradiance, often abbreviated in TSI, and noted E_{TSI}, is the yearly average of E_{0N} during a year. This term "total solar irradiance" may be confusing, especially because it does not bear any reference to the top of the atmosphere. However, it has been adopted here as it is widely used in scientific literature.

E_{0N} is a quadratic function of the distance between the sun and the earth:

$$E_{0N} = E_{TSI} (r_0/r)^2 \quad (5\text{-}5)$$

The closer the earth to the sun, the greater E_{0N}. It has been seen that the sun-earth distance r is given by:

$$r_0/r = \sqrt{(1+\varepsilon)} \quad (3\text{-}2)$$

where ε is given by Eq. 3-3 as a function of the day in the year. Hence:

$$E_{0N} = E_{TSI} (1 + \varepsilon) \quad (5\text{-}6)$$

ε is comprised between −0.03 and +0.03. In other words, E_{0N} is comprised between 0.97 E_{TSI} and 1.03 E_{0TSI}. Neglecting the change in the sun-earth distance, i.e. writing $E_{0N} \approx E_{TSI}$, yields a relative uncertainty on E_{0N} that is a function of the day in year and is comprised between -3 and +3 %.

The total solar irradiance has been called *solar constant* for many years as it was thought to be fairly invariant. The value of E_{TSI} has varied over the recent decades as the instrumentation was more and more accurate. In 1981, the World Radiometric Reference for E_{TSI} was 1370 W m^{-2} ± 6 W m^{-2}. Accurately measuring E_{TSI} is a difficult task. It is measured by space-borne sensors since 1979. Several values are found in literature; they are close to each other within a range of 5 W m^{-2}. A value of 1367 W m^{-2} was proposed in the years 2000. Recent measurements of E_{TSI} in 2010 yields 1362 W m^{-2} with an uncertainty of order of 2 W m^{-2} [7]. The International Astronomical Union has recommended a value of 1361 W m^{-2} ± 1 W m^{-2} as the mean value for the solar cycle #23, i.e. for the period May 1996 to January 2008[8].

The sun is an active star and its activity includes changes in the intensity of solar radiation and ejection of solar material and by its appearance. The solar activity exhibits a nearly periodic 11-year cycle, each cycle being characterized by the number and size of sunspots, flares, and other manifestations. The solar cycle has a limited influence on the total solar irradiance, of order of 0.1 %. In other words, average changes during a cycle are small and of order of 1 W m^{-2}. Day-to-day changes in E_{0N} are greater and may reach 5 W m^{-2}, i.e. approximately 0.4 % of the total solar irradiance[9].

5.2.2 THE SPECTRAL DISTRIBUTION OF THE SOLAR IRRADIANCE AT THE TOP OF THE ATMOSPHERE

Any object emits electromagnetic radiation, provided its temperature is above 0 K. The emitted spectral radiance is entirely determined by temperature and the emitting properties of the surface of the object. The laws of Kirchhoff and Planck describe this process. The radiance emitted by the sun is approximately that of a blackbody (i.e., a perfect radiative body) at a temperature of 5780 K. The emitted radiation spans over a very large spectrum, from X-rays to far infrared. Nevertheless, most of the emitted radiation exhibits wavelengths in the visible and near-infrared domain.

Let $E_{0N}(\lambda)$ and $E_{TSI}(\lambda)$ denote the value of E_{0N} and E_{TSI} at wavelength λ:

$$E_{0N} = \int_0^\infty E_{0N}(\lambda)\ d\lambda \qquad (5\text{-}7)$$

$$E_{TSI} = \int_0^\infty E_{TSI}(\lambda)\ d\lambda \qquad (5\text{-}8)$$

Figure 5.1 displays a typical spectral distribution $E_{TSI}(\lambda)$ of the total solar irradiance E_{TSI} for the wavelength intervals [200, 2000] nm (central graph) and [0, 4000] nm. The spectral distribution shows how much power is available for each wavelength. The surface under the curve is the irradiance integrated over the spectral range under concern.

It can be seen that the interval covered by the spectral distribution of the solar irradiance is not very wide. The irradiance is not negligible from 200 nm (0.2 μm) up to 4000 nm (4 μm). Table 5.1 reports the irradiance $E_{TSI}(\lambda)$ from Figure 5.1 integrated over various spectral ranges. Approximately 99 % of the total solar irradiance E_{TSI} is found between 200 nm and 4000 nm. Pyranometers usually measure between 280 nm and 2800 nm; this range represents approximately 97 % of E_{TSI}.

---

[7] Meftah M., Dewitte S., Irbah A., Chevalier A., Conscience C., Crommelinck D., Janssen E., Mekaoui S., 2014. SOVAP/Picard, a spaceborne radiometer to measure the total solar irradiance. Sol. Phys., 289, 1885-1899. doi:10.1007/s11207-013-0443-0.

[8] The XXIXth International Astronomical Union General Assembly 2015. Resolution B3 on recommended nominal conversion constants for selected solar and planetary properties. Available at https://www.iau.org/static/resolutions/IAU2015_English.pdf, last accessed on 2017-05-07.

[9] Kopp G., Lean J.L., 2011. A new, lower value of total solar irradiance: evidence and climate significance. Geophys. Res. Lett., 38, L01706. doi:10.1029/2010GL045777.

The spectral distribution is well-peaked around 550 nm. About half of the radiation is in the visible part of the spectrum [380, 780] nm. The range of the photosynthetically available radiation (PAR), i.e. the light that is available to plants for the photosynthesis, is 400 nm to 700 nm. This range accounts for 39 % of E_{TSI}.

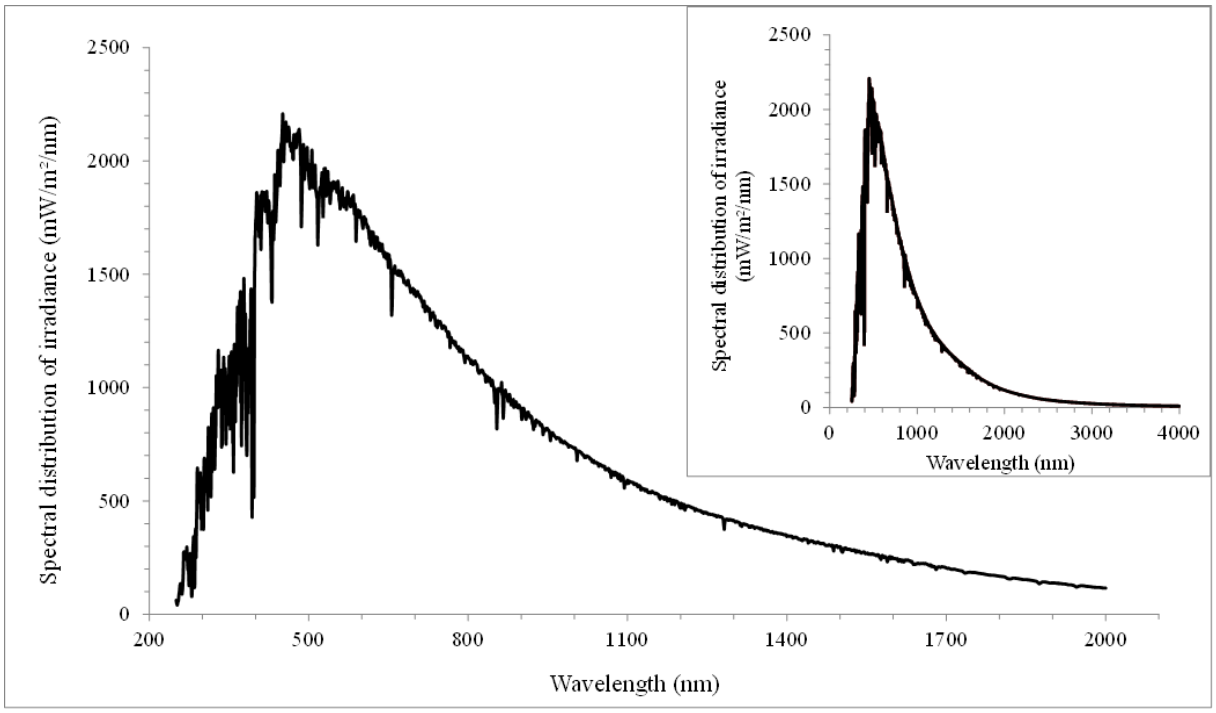

Figure 5.1. Typical spectral distribution of the solar irradiance at the top of the atmosphere at normal incidence for wavelengths from 200 nm to 2000 nm (central graph) and 0 nm to 4000 nm (upper right-hand corner).

Note that the definition of specific spectral intervals may vary according to the domain of application, especially in the visible. For example, in scientific literature visible may refer to the interval [380, 780] nm, or [400, 700] nm, or [400, 780] nm, or [400, 800] nm.

The spectrum of the solar irradiance is not as smooth as a Planck spectral curve. It exhibits several departs from a smooth curve particularly at wavelengths below 800 nm. These numerous "lines" are due to wavelength-dependent processes of absorption and emission in the sun. In these absorption lines, the sun does not emit as much as in the neighbouring wavelengths.

Spectral range, in nm	Irradiance (W m-2)	Fraction of the total irradiance (%)	Spectral range, in nm	Irradiance (W m-2)	Fraction of the total irradiance (%)
250 to 20 000	1361	100	280 to 315 (UV-B)	18	1
250 to 4 000	1353	99	315 to 400 (UV-A)	86	6
380 to 2 100	1206	89	250 to 400	108	8
400 to 1 100	909	67	380 to 780 (visible CIE)	663	49
1 100 to 4 000	336	25	400 to 700 (PAR)	536	39
1 100 to 20 000	344	25	400 to 800	663	49
4 000 to 20 000	8	<1	330 to 2 200 (typical spectral band of pyranometers)	1266	93

Table 5.1. Typical values of irradiance at the top of atmosphere at normal incidence for various spectral ranges, from Figure 5.1. PAR stands for photosynthetically available radiation. CIE stands for Commission internationale de l'éclairage.

The spectral distribution of the solar irradiance is actually not constant and varies in time. It has been seen in the previous section that day-to-day changes in total solar irradiance may reach 5 W m^{-2}. Such important changes do not affect the wavelength equally. The visible and infrared parts of the spectrum are the least affected, while the UV (ultra-violet) part is much affected. The amplitude of the variations is 1 to 3 orders of magnitude greater in the UV part of the spectrum than in the visible or infrared. The importance of such changes in UV may be illustrated by the fact that the UV radiation is an important driver of chemical and physical processes in the upper atmosphere of the earth.

5.3 THE IRRADIANCE AND IRRADIATION AT THE TOP OF THE ATMOSPHERE

Let E_0 and H_0 denote the total extra-terrestrial irradiance and irradiation impinging on a horizontal surface located at the top of the atmosphere:

$$E_0 = E_{0N} \cos\theta_S \qquad (5\text{-}9)$$

$$H_0 = E_{0N}\, \Delta t \cos\theta_S \qquad (5\text{-}10)$$

where θ_S is the solar zenithal angle and Δt is a period of time during which θ_S may be considered as a constant, approximately 1 min or less.

These relationships demonstrate that the solar zenithal angle plays a major role on the extra-terrestrial irradiance and irradiation, and subsequently on the solar irradiance and irradiation received at ground level as it will be seen in the next chapter. The greater this angle, the more inclined the sun rays on the horizontal surface and the lower the extra-terrestrial irradiance or irradiation.

These equations also hold for any wavelength and not only for the total irradiance and irradiation. Let $E_0(\lambda)$ and $H_0(\lambda)$ denote the extra-terrestrial spectral irradiance and irradiation impinging on a horizontal surface located at the top of the atmosphere:

$$E_0(\lambda) = E_{0N}(\lambda) \cos\theta_S \qquad (5\text{-}11)$$

$$H_0(\lambda) = E_{0N}(\lambda)\, \Delta t \cos\theta_S \qquad (5\text{-}12)$$

Figure 5.2 displays a typical spectral distribution of the solar irradiance $E_0(\lambda)$ for three θ_S (0°, 30°, 60°) for the wavelength intervals [200, 2000] nm. On may clearly see the influence of the solar zenithal angle. The greater this angle, the lower the spectral extra-terrestrial irradiance received by a horizontal surface.

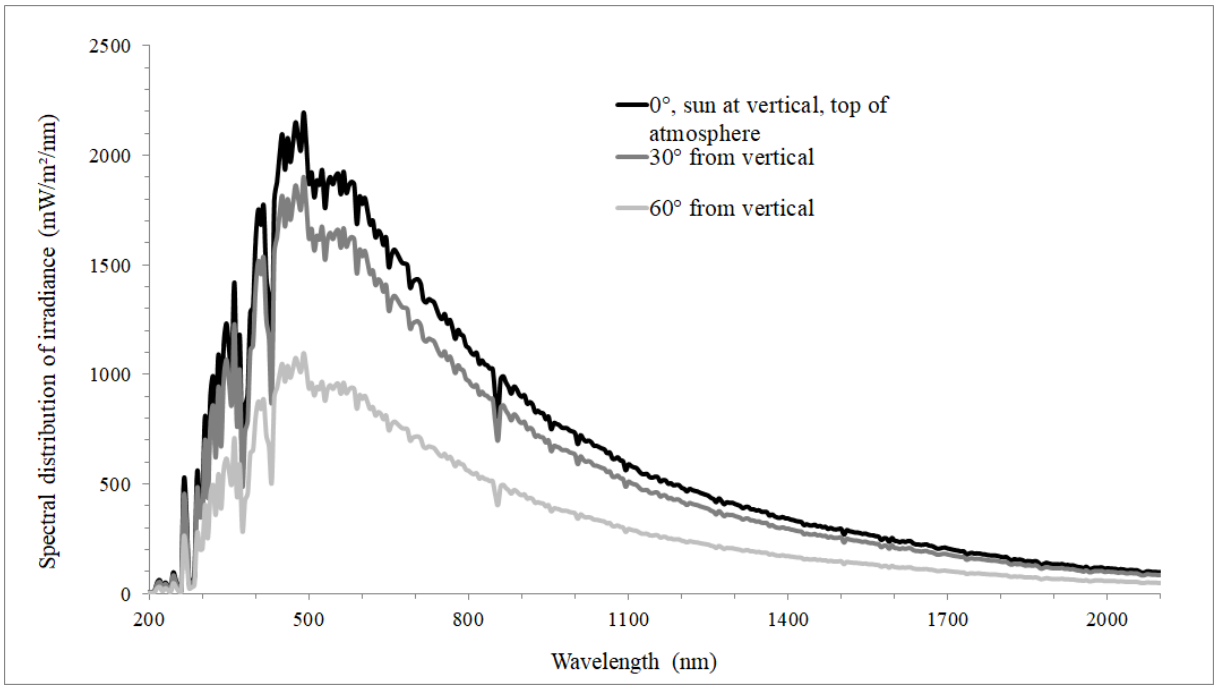

Figure 5.2. Typical spectral distributions of the solar irradiance received by a horizontal surface at the top of the atmosphere for wavelengths from 200 nm to 2000 nm for three solar zenithal angles: 0°, 30°, and 60°.

During a day, E_0 and H_0, as well as $E_0(\lambda)$ and $H_0(\lambda)$, are at their maximum when the solar zenithal angle is at its lowest, i.e. at 12:00 TST. E_0 and H_0 are null when the sun is below the horizon. A seasonal cycle is superimposed on this daily cycle. It is due to the variations of E_{0N} within the year due to the changes in the sun-earth distance, and to changes in solar declination and the astronomical seasons.

At a given instant, E_{0N} does not depend on the latitude while E_0 and H_0 do because θ_S does. Eq. 3-15 provides the variation of θ_S with the latitude Φ and the solar declination δ; ω is the solar angle which is directly related to the true solar time (Eq. 3-14).

$$\cos\theta_S = \sin\Phi\sin\delta + \cos\Phi\cos\delta\cos\omega \qquad (3\text{-}15)$$

The extra-terrestrial daily irradiation H_{0day} and the daily mean of irradiance E_{0day} do not depend on the longitude and depend only on the latitude and the solar declination. It is easier to use the solar angle ω than the time to demonstrate this point. If one denotes t_{SR} and ω_{SR} and t_{SS} and ω_{SS} respectively the times and solar angles for sunrise and sunset, H_{0day} is defined by:

$$H_{0day} = \int_{t_{SR}}^{t_{SS}} E_{0N}\cos\theta_S\, dt \qquad (5\text{-}13)$$

By exploiting Eq 3-14 where the time is expressed in h, it comes:

$$H_{0day} = \left(\frac{12}{\pi}\right)\int_{\omega_{SR}}^{\omega_{SS}} E_{0N}\cos\theta_S\, d\omega \qquad (5\text{-}14)$$

At first approximation, one may assume that E_{0N} does not depend on ω. It follows that:

$$H_{0day} = \left(\frac{12}{\pi}\right) E_{0N}\int_{\omega_{SR}}^{\omega_{SS}} \cos\theta_S\, d\omega \qquad (5\text{-}15)$$

By exploiting Eqs 3-22 to 3-25, and remembering that $\omega_{SS} > 0$ and $\omega_{SS} = -\omega_{SR}$, it comes:

$$H_{0day} = \left(\frac{24}{\pi}\right) E_{0N}\left[\cos\phi\cos\delta\sin\omega_{SS} + \omega_{SS}\sin\phi\sin\delta\right] \qquad (5\text{-}16)$$

or, introducing the daytime S_0 (in h):

$$H_{0day} = E_{0N}\left[\left(\frac{24}{\pi}\right)\cos\phi\cos\delta\sin\omega_{SS} + S_0\sin\phi\sin\delta\right] \qquad (5\text{-}17)$$

Because the time in Eq 3-14 is expressed in h, H_{0day} has a unit of Wh m^{-2}. It is converted in J m^{-2} by multiplying the result of Eq. 5-16 by 3600 s. The daily mean of irradiance E_{0day} is obtained by dividing H_{0day} from Eq. 5-16, which is in Wh m^{-2}, by 24 h.

Figure 5.3 exhibits the daily mean of the irradiance at the top of the atmosphere E_{0day} for various latitudes as a function of the day in the year and Table 5.2 reports the yearly mean of the extra-terrestrial irradiance at these same latitudes.

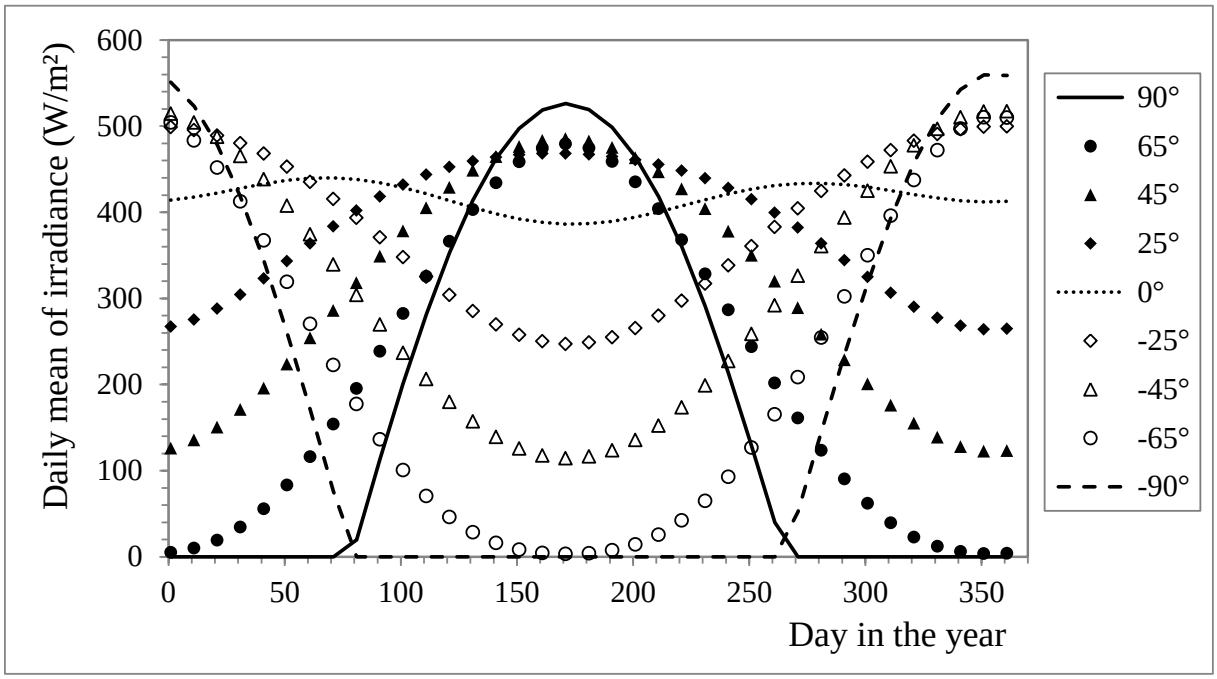

Figure 5.3. Daily mean of irradiance collected on a horizontal plane located at the top of the atmosphere as a function of the day in the year and latitude.

Figure 5.3 shows that the daily mean of irradiance offers a yearly cycle. In the northern hemisphere (full symbols), it exhibits a minimum on approximately 21 December and a maximum on approximately the 21 June, i.e. at the solstices. This is the opposite in the southern atmosphere (empty symbols). For the same latitudes but in opposite hemispheres the irradiances exhibit the same values with a dephasing of half a year.

At the equator, the sun is high at noon all over the year and the solar zenithal angle at noon exhibits low values all over the year. The daytime is 12 h all over the year (cf. Figure 4.2). As a result, the equator exhibits the greatest yearly mean of the irradiance at the top of atmosphere, around 420 W m^{-2} (Table 5.2). The daily mean of irradiance is almost constant at the equator (dashed line in Figure 5.3); it varies at most by 13 % of the yearly mean throughout the year.

The smallest yearly mean of the irradiance at the top of atmosphere is observed at high latitudes. The greater the latitude in absolute value, the smaller the yearly mean of the irradiance at the top of atmosphere (see Table 5.2).

The magnitude of the variation during the year increases as the latitude increases. This is illustrated in Figure 5.3. The changes in irradiance between winter and summer are approximately 200 W m^{-2} at latitude 25° (or -25°), i.e. 54% of the yearly mean; they amount to 480 W m^{-2} at latitude 65° (or -65°), i.e. 220% of the yearly mean, and up to 560 W m^{-2} at latitude 90° (or -90°), i.e. 325% of the yearly mean.

The maximum of irradiance increases as the latitude increases in absolute value: it is for example, approximately 440 W m^{-2} at the equator, 480 W m^{-2} at 35° and 530 W m^{-2} at 90°. However, the maximum is more or less the same from approximately 33° up 67° (respectively -33° to -67°). One may note that at a given latitude in absolute value, the maximum in irradiance is greater in the southern hemisphere than in the northern one. For example, it is 490 W m^{-2} for latitude 45° and 520 W m^{-2} for -45°, or 530 W m^{-2} for 90° and 560 W m^{-2} for -90°.

Latitude	Year - Irradiance (W m^{-2})		
	Mean	Minimum	Maximum
90°	172	0	524
65°	214	3	478
60°	236	24	476
45°	307	120	483
30°	365	227	475
25°	380	261	467
23.45°	384	271	463
11.5°	408	345	439
0°	416	384	438

Latitude	Year - Irradiance (W m^{-2})		
	Mean	Minimum	Maximum
-90°	172	0	559
-65°	214	3	510
-60°	236	23	509
-45°	307	113	516
-30°	365	213	506
-25°	380	245	498
-23.45°	384	255	495
-11.5°	408	326	461

Table 5.2. The yearly mean of total irradiance received by a horizontal plane located at the top of the atmosphere at various latitudes

The equations above have been written for the total irradiance and irradiation. As there is no dependence of the solar angle and the solar zenithal angle with the wavelength, these equations are also valid for any wavelength and any spectral interval.

5.4 THE IRRADIANCE AND IRRADIATION AT THE TOP OF THE ATMOSPHERE ON AN INCLINED SURFACE

Let β and α be the inclination and azimuth angles of an inclined plane located at the top of the atmosphere at a given geographical site (Figure 3.6). The irradiance E_{0incl} on that plane is given by:

$$E_{0incl}(\beta, \alpha) = E_{0N} \cos\theta \qquad (5\text{-}18)$$

where the angle of incidence θ of the solar rays is formed by the normal to the plane and the rays and is given by Eq 3-19.

The solar irradiation $H_0(\beta, \alpha)$ received during a period of time $[t_1, t_2]$, expressed in TST, is computed by integrating Eq. 5-18 on the corresponding interval $[\omega_1, \omega_2]$:

$$H_0(\beta, \alpha) = E_{0N} \int_{t_1}^{t_2} \cos\theta \; dt = \left(\frac{12}{\pi}\right) E_{0N} \int_{\omega_1}^{\omega_2} \cos\theta \; d\omega \qquad (5\text{-}19)$$

By exploiting Eqs 3-21 to 3-23, it comes:

$$H_0(\beta, \alpha) = (12/\pi)\, E_{0N} [A(\sin\omega_2 - \sin\omega_1) - B(\cos\omega_2 - \cos\omega_1) + C(\omega_2 - \omega_1)] \qquad (5\text{-}20)$$

Interesting cases are those of the daily irradiation $H_{0day}(\beta, \alpha)$ received by the inclined plane*:*

$$H_{0day}(\beta, \alpha) = \left(\frac{12}{\pi}\right) E_{0N} \int_{\omega_1}^{\omega_2} \cos\theta \; d\omega \qquad (5\text{-}21)$$

where the integration limits, ω_1 and ω_2, are the solutions of

$$\cos\theta = 0 \qquad (5\text{-}22)$$

which is solved by the means of Eqs 3-21 and 3-22.

In the case of a vertical plane orientated to the east (β=π/2; α=π/2), these limits ω_1 and ω_2 are

$$\omega_1 = \omega_{SR} \; ; \; \omega_2 = 0 \qquad (5\text{-}23)$$

The sunrise angle ω_{SR} is given by Eqs. 4-4 and 4-5. It follows:

$$A = 0 \; ; \; B = -\cos\delta \; ; \; C = 0$$

$$H_{0day}(\pi/2, \pi/2) = (12/\pi)\, E_{0N} \cos\delta\, (1 - \cos\omega_{SR}) \qquad (5\text{-}24)$$

It may be observed that a vertical plane orientated to the west (α=3π/2) receives a similar quantity, though the limits are now: $\omega_1 = 0$; $\omega_2 = \omega_{SS}$:

$$H_{0day}(\pi/2, 3\pi/2) = H_{0day}(\pi/2, \pi/2) \quad (5\text{-}25)$$

For planes orientated to the north or south, the first step is to compute the solar hour angle ω_{EW} corresponding to the instant when the sun passes in the east-west plane containing the unit surface [This happens for $\Psi_S = \pi/2$ and the solution is found by exploiting Eq. 3-17 and 3-18 for tangent]:

$$\cos\omega_{EW} = \tan\delta / \tan\Phi \text{ if not at pole} \quad (5\text{-}26)$$

$$\text{if } \omega_{SS} > \pi/2, \text{ i.e., } \delta \text{ and } \Phi \text{ are of opposite signs, } \omega_{EW} = \omega_{SS}$$

For a vertical plane orientated to the south and located in the northern hemisphere ($\beta=\pi/2$; $\alpha=\pi$)

$$\omega_1 = -\omega_{EW} \; ; \; \omega_2 = \omega_{EW}$$

$$A = \cos\delta \sin\Phi \; ; \; B = 0 \; ; \; C = \sin\delta \cos\Phi$$

$$H_{0day}(\pi/2, \pi) = (24/\pi)\, E_{0N} [\cos\delta \sin\Phi \sin\omega_{EW} + \omega_{EW} \sin\delta \cos\Phi] \quad (5\text{-}27)$$

For a plane located in the northern hemisphere, orientated to the south and inclined by a tilt β equal to the latitude Φ ($\beta=\Phi$; $\alpha=\pi$):

$$\omega_1 = -\omega_{EW} \; ; \; \omega_2 = \omega_{EW}$$

$$A = \cos\delta \; ; \; B = 0 \; ; \; C = 0$$

$$H_{0day}(\pi/2, \Phi) = (24/\pi)\, E_{0N} \cos\delta \sin\omega_{EW} \quad (5\text{-}28)$$

The surface may change orientation during the day as is the case for tracking solar collectors. Such surfaces are concentrating collectors, with their axis aligned east-west or north-south with continuous adjustment to minimize the incidence angle θ, or two-axes trackers continuously facing the sun. In these cases, the incidence angle is given by:

horizontal east-west axis	$\cos\theta = (1 - \cos^2\delta \sin^2\omega)1/2$	(5-29)
polar mounting (north-south axis, $\beta=\Phi$)	$\cos\theta = \cos\delta$	
two-axes trackers	$\cos\theta = 1$	

Because the time in Eq 3-14 is expressed in h, $H_{0day}(\beta, \alpha)$ has a unit of Wh m^{-2}. It is converted in J m^{-2} by multiplying the result of one of the above equations by 3600 s. The daily mean of irradiance $E_{0day}(\beta, \alpha)$ is obtained by dividing $H_{0day}(\beta, \alpha)$ obtained by one of the above equations by 24 h.

To illustrate this section, Figure 5.4 exhibits the daily mean of the irradiance at the top of the atmosphere $E_{0day}(\beta, \alpha)$ for a plate located at latitude 45° orientated towards the south and whose tilt in radian is equal to respectively 0 (horizontal), $\pi/4$ (45°) and $\pi/2$ (90°).

The daily mean irradiance received by a vertical plate ($\beta=\pi/2$) is the same than that received on a horizontal surface during boreal winter, but is much lower during boreal summertime. The minimum irradiance is respectively 120 and 110 W m^{-2} for horizontal and vertical plates; the maximum is 490 W m^{-2}, respectively 410 W m^{-2}. Though the maxima differ notably, the yearly means are fairly similar: 310 W m^{-2} for horizontal and respectively 290 W m^{-2} for the vertical plate.

The same plate, but now inclined at $\pi/4$, receives more radiation as a whole than the horizontal surface: the yearly mean irradiance is approximately 400 W m^{-2} for the inclined surface and 310 W m^{-2} when horizontal. The variation in irradiance during the year is also much less for the inclined surface: it has two maxima at 440 W m^{-2} and a minimum at 350 W m^{-2}, while the minimum and maximum are respectively 120 W m^{-2} and 490 W m^{-2} for the horizontal plate.

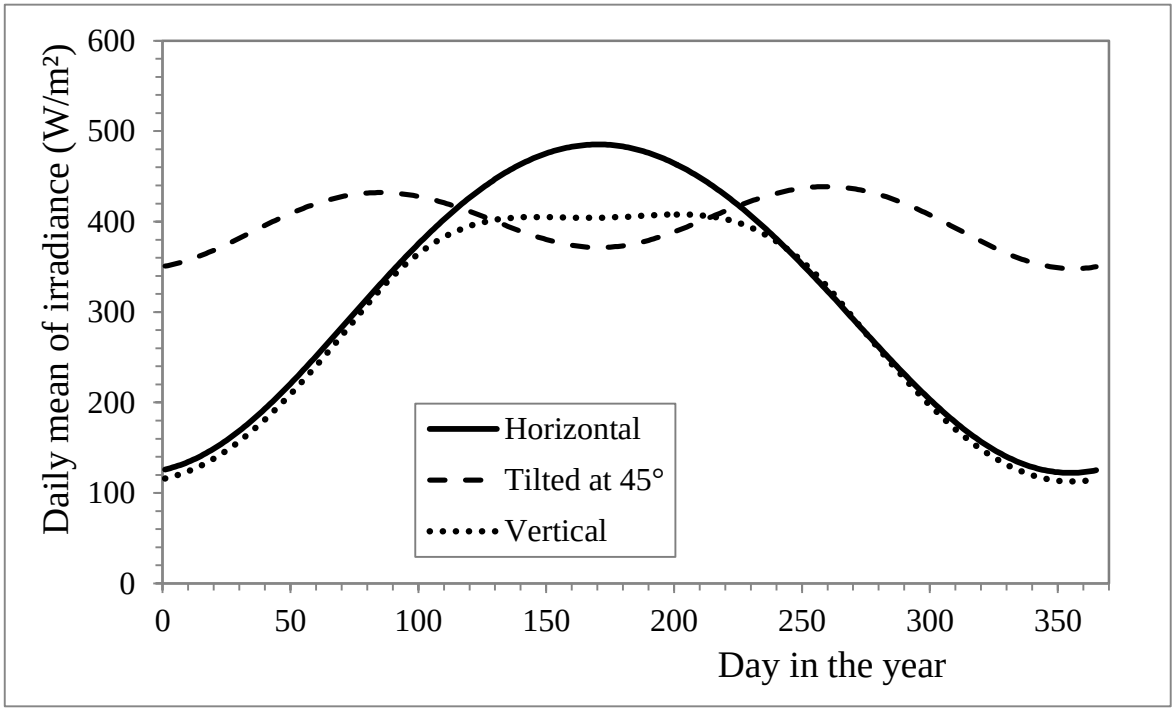

Figure 5.4. Daily mean of irradiance collected on a plane located at latitude 45° at the top of the atmosphere as a function of the day in the year, when the plane is horizontal, tilted at 45° and vertical facing south.

6 THE SOLAR RADIATION AT GROUND LEVEL

In summary

As the solar radiation makes its way from the top of the atmosphere downwards the ground, it is depleted when passing through the atmosphere due to interactions with the constituents of the atmosphere. On average, less than half of extra-terrestrial radiation reaches ground level.

The description and modelling of the optical processes affecting the solar radiation within the atmosphere is called radiative transfer. Absorption is a process present in the atmosphere whereby the energy absorbed by a constituent at a given wavelength is converted into another form and is no longer present in the light. Absorption may occur at very specific wavelengths, called absorption lines or may occur over a wide continuum of wavelengths. Scattering is a physical process associated with light and its interaction with matter occurring at all wavelengths. Particles and molecules deflect the incident wave and re-radiate that energy in all directions, thus abstracting energy from the incident wave. The scattering pattern indicates the relative probability of a photon to be scattered in a given direction; it depends on the size of the particle or molecule, its shape and other properties, and on the incident wavelength.

Whatever the sky conditions, cloud-free or cloudy, the solar zenithal angle plays a major role in the radiative transfer as it influences the optical path of the radiation. The smaller the solar zenithal angle, the smaller the optical path, and the smaller the extinction of the radiation.

The clearness index is the ratio of the radiation received at ground level to that received at the top of the atmosphere. It is a dimensionless quantity that may be defined for any wavelength or a spectral interval and for any time duration. Typical values are 0.1 for very cloudy conditions and 0.7-0.8 for cloud-free conditions.

In clear skies, i.e. cloud-free skies, aerosols and water vapour are the main contributors to depletion. In such conditions, approximately 20 % to 30 % of the total extra-terrestrial radiation is lost during its downwelling path by scattering and absorption phenomena by aerosols and molecules. This amount differs with wavelength and the spectral distribution of the solar radiation is modified as the radiation crosses the atmosphere downwards.

Clouds have a major importance as a whole; they are the major depleting constituents in the atmosphere. Clouds are diverse: optically thin clouds allow a noticeable proportion of radiation to reach the ground whereas optically thick clouds create obscurity by stopping the radiation downwards. In a first approximation, clouds may be considered as spectrally neutral, i.e. their presence does not alter the shape of the spectral distribution of the radiation.

The direct radiation is the radiation coming from the direction of the sun. Only direct radiation is present at the top of the atmosphere. A horizontal surface at ground level receives a depleted part of this direct radiation. It also receives the radiation that has been scattered by the constituents of the atmosphere and that originates from the sky vault in all directions, except that of the sun which is already accounted for. This multi-source radiation is called the diffuse radiation. The global radiation is the sum of the direct and diffuse radiation.

If the receiving plane is inclined, it may receive the direct radiation only partly and the fraction of sky viewed by the plane must be considered for computing the diffuse part impinging on the plane. The plane may also receive a part of the radiation that is reflected by the surrounding landscape towards the plane.

A good knowledge of the optical properties of the atmosphere is necessary to understand and further model the depletion of the radiation. The description and modelling of the optical processes affecting the solar radiation within the atmosphere is called radiative transfer. The main processes in radiative transfer are discussed, and especially the effects of molecules, aerosols, gases and clouds that are depicted by absorption and scattering phenomena.

6.1 THE ABSORPTION AND SCATTERING PHENOMENA

Absorption is a process present in the atmosphere. The energy absorbed by a constituent at a given wavelength is converted into another form and is no longer present in the light. Absorption may occur at very specific wavelengths, which may be denoted absorption lines or may occur over a wide continuum of wavelengths.

Scattering is a physical process associated with light and its interaction with matter. It occurs at all wavelengths. In this process, a particle or a molecule is in the path of an incoming light; it deflects the incident wave and re-radiates that energy in all directions. In that way, the scattering process continuously abstracts energy from the incident wave.

If the scattering is isotropic, then the energy is radiated equally in all directions. Usually, the scattering is not isotropic and how it radiates the energy in a given direction is described by the scattering pattern. The pattern indicates the relative probability of a photon to be scattered in a given direction. The pattern depends on the size of the particle or molecule, its shape and other properties, and on the incident wavelength.

Figure 6.2 exhibits the schematic scattering pattern in the case where particles or molecules are much smaller than the incident wavelength. Given an incident light (black arrow), the radiation is scattered in all possible directions, indicated by the several arrows. The further the limit of the pattern from the particle or molecule (in grey), the greater the probability the light may be scattered in that direction. Here, the scattering pattern is almost an ellipse. One notes that the probability of the light to be scattered forward, i.e. in the direction of the incident light, or to be reflected (backward scattering) is greater than the probability of being scattered at 90° or -90°. Such a scattering pattern is the case of the scattering of the solar radiation by molecules in the atmosphere.

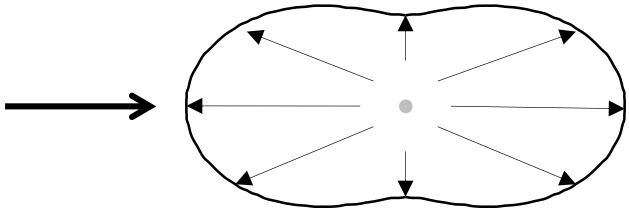

Figure 6.1. Schematic scattering patterns in the case where particles or molecules are much smaller than the incident wavelength. The thick black arrow denotes the incident light. The further the limit of the pattern from the particle or molecule (in grey), the greater the probability the light may be scattered in that direction.

Other schematic scattering patterns are shown in Figure 6.2 according to the size of the particle. Given an incident light (black arrow), the radiation is scattered in several possible directions. In many cases, the pattern is elongated along the direction of the incoming light, which means that the probability of finding photons in directions perpendicular to that of the incident beam is small. When the particle size increases, the amount of scattered energy is greater in the forward directions with possible complex patterns like the lowermost pattern. The patterns given in this Figure 6.2 are schematic; the actual scattering patterns are much more complex without symmetry and depend on the nature, shape and size of the particle.

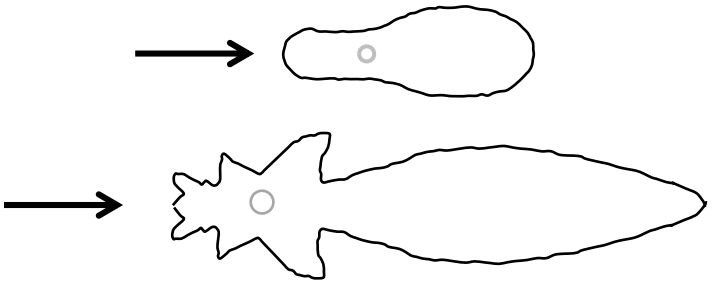

Figure 6.2. Schematic scattering patterns according to the size of the anisotropic scattering particle. The black arrow denotes the incident light. The further the limit of the pattern from the particle or molecule (in grey), the greater the probability the light may be scattered in that direction.

The atmosphere contains many particles and molecules and it occurs that one particle or molecule scatters the light that has been scattered by another particle or molecule and so on. This is called multiple scattering and is illustrated in Figure 6.3. A photon is diverted from the incident beam and is scattered in another direction. It may be further scattered and so on. For a beam light such as in Figure 6.3, it happens that a ray that has been scattered

in a direction away from that of the beam may be scattered by other particles and may reappear in the direction of the beam.

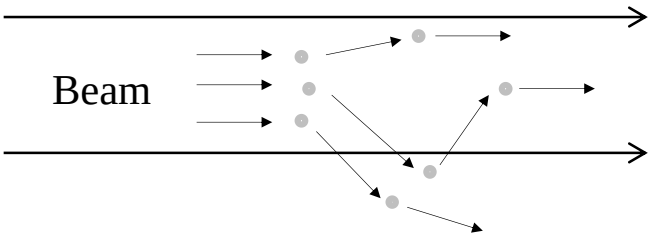

Figure 6.3. Illustration of multiple scattering

When particles are much smaller than the incident wavelength, the scattering may be described by Rayleigh's law. Under this law, the amount of scattering is inversely proportional to the fourth power of the wavelength. It follows that the smallest wavelengths are much more scattered by the particle than the longest ones. This is the case for the air molecules whose sizes are much smaller than the wavelengths of the incident light. In the visible range, the blue wavelengths are more scattered than the red ones.

For particles whose sizes are comparable to or slightly greater than the wavelength, the Mie's law may apply. It describes the scattering by a homogeneous sphere. Actual particles are not spheroids and the Mie's law offers a first approximation. It may depict the scattering effects due to the so-called aerosols, i.e. particles that are in suspension in the atmosphere, usually in the lower part, i.e. within a few km above the ground.

Both scattering and absorption remove energy from the light crossing the atmosphere. The depletion of the radiation is called extinction or attenuation. Absorption, scattering and extinction are wavelength-dependent. The optical depth is a measure of the extinction of the radiation during its vertical travel through a layer of molecules or particles or clouds, with a solar zenithal angle equal to 0. The optical depth is the natural logarithm (ln) of the ratio of the incident power to the transmitted one through a layer. The greater the optical depth, the greater the extinction. The optical depth is positive and unitless. It ranges from 0 to 5 for the aerosols while the cloud optical depth of clouds may be greater than 100. The optical depth may be called optical thickness.

The extinction of a beam depends also upon the length of its travel, i.e. its optical path within the atmosphere. The length of the optical path is called the air mass. The lower the sun above the horizon or equivalently the greater the solar zenithal angle, the greater the air mass and the greater the extinction by the atmosphere.

Absorption of energy by particles and molecules leads to emission because the temperature of these constituents is not null. This is the same physical process than that occurring in the sun leading to the emission of the solar radiation. The ground and the ocean emit energy at longer wavelengths than solar radiation. This longwave energy is also absorbed and scattered by the atmosphere. The atmosphere itself emits energy because its temperature is not null.

A surface at ground level receives solar radiation in short wavelengths (0.3 to 4) μm, either in a direct form, i.e., as a beam from the direction of the sun, or in a diffuse form. The sum of these direct and diffuse components forms the global irradiance. The term global may be omitted if no confusion is possible.

Because their surface temperature is not null, the various constituents of the earth: solids, liquids and gases, emit radiation with longer wavelengths (4 to 80) μm. The terrestrial radiation is always in diffuse form. The surface receives part of this the terrestrial radiation which should be taken into account if an energy budget is made on this surface.

6.2 The Depletion of the Solar Radiation by the Atmosphere

When the solar radiation penetrates into the atmosphere, whether cloud-free or cloudy, it is absorbed and scattered. Only the longer wavelengths reach the lower layers of the atmosphere. The downwelling solar energy at ground level is mostly comprised in the range [0.3, 4] μm. Most of the ultraviolet rays whose wavelengths are

less than 300 nm are absorbed by ozone. Radiation with wavelengths greater than 4 μm is largely absorbed by water vapour.

All the atmospheric components contribute to a greater or lesser degree to the extinction of solar radiation on its downwelling path towards the ground. The extinction takes place as a result of the mechanisms of absorption and scattering that affect the whole solar spectrum. In the upper layers of the atmosphere, the main processes are the absorption of the X-ray and ultraviolet regions of the solar spectrum, and scattering in the violet and blue ranges. As the radiation penetrates further downwards, the extinction affects the longer wavelength portions of solar radiation. The variations in the concentrations of water vapour and ozone, and in the concentrations and optical characteristics of aerosols under the influence of various processes of interaction between the solar radiation field, the atmosphere and the underlying surface, result in constant fluctuations with time in spectral extinction in the atmosphere. This is why it is so difficult to assess precisely the effect of atmosphere on radiation.

In the upper layers of the atmosphere, the absorption of solar radiation is caused by oxygen, ozone and nitrogen oxides and, in the lower stratosphere and troposphere, by water vapour, carbon dioxide, aerosols and other minor components.

The basic atmospheric gases –nitrogen and oxygen– mainly attenuate radiation in ultraviolet and visible regions of the spectrum through molecular scattering. Solar radiation is also scattered by gas molecules and water vapour and by solid aerosol particles, liquid drop components (cloud particles, fog) and ice crystals. The scattering process in the atmosphere results in the production of scattered radiation, part of which goes back into space.

Absorption is mostly caused by ozone, which depletes radiation strongly at wavelengths of less than 0.3 μm and weakly between (0.5 and 0.7) μm, and by water vapour, which exhibits numerous and wide absorption lines above 0.65 μm.

The length of the geometrical path from the top of the atmosphere down to the ground is equal to the inverse of the cosine of solar zenithal angle θ_S. The greater θ_S, the greater the length that the light should travel. The optical path is the optical equivalent but taking into account the optical effects of the atmosphere on the radiation. The more turbid or cloudy the atmosphere, the longer the optical path for the same θ_S. The longer the optical path, the greater the depletion. The optical depth is a function of the wavelength.

The relative air mass, often abbreviated in air mass, is the ratio of the optical path for a given θ_S to that for θ_S =0. It may be approximated by the inverse of the cosine of θ_S which is fairly accurate when θ_S <60°; otherwise, correction must be brought[10]. The air mass is a function of the wavelength.

The clearness index, often noted *KT*, is defined as the ratio of the solar radiation received at ground level and that received at the top of the atmosphere for the same geographical location, same instant and on a similar surface. It is the fraction of the radiation received at the top of the atmosphere passing through the atmosphere. In the case of a horizontal surface receiving the irradiance *E* at ground level, *KT* is defined as:

$$KT = E / E_{0N} \cos\theta_S \qquad (6\text{-}1)$$

where θ_S is the solar zenithal angle and $E_{0N} \cos\theta_S$ is the irradiance received on a horizontal surface at the top of atmosphere. The clearness index is a dimensionless quantity. It can be computed as a ratio of irradiances or irradiations, and will yield the same value in both cases. It may be defined for any wavelength or a spectral interval and for any time duration: $(KT)_h$, or day $(KT)_{day}$ etc.

As it will be seen in the following section, the clearness index may greatly vary during a day with the solar zenithal angle, and may also greatly vary with the wavelength. However, when used for duration greater than the day, and for the total radiation, and more generally for any spectral interval that contributes significantly to the total radiation, it is a convenient quantity to express the overall transmittance of the atmosphere and to compare different situations, e.g. at different latitudes as it removes the effect of the changes in cosine of the solar zenithal angle. Typical values are 0.1 for very cloudy conditions and greater than 0.7 for cloud-free conditions.

[10] Greif et al., op. cited

6.3 THE RADIATIVE TRANSFER IN THE CLOUD-FREE ATMOSPHERE

This section deals with the clear skies, i.e. cloud-free skies, also said cloud-free atmosphere, or cloud-free conditions, or clear-sky conditions. The effects of clouds are treated in a further section. A cloud-free sky may be said pure or turbid, depending on its load in atmospheric constituents, such as water vapour or aerosols.

As seen above, gas molecules scatter the radiation and this process may be described by Rayleigh's law. It follows that the smaller the wavelength, the greater the optical thickness of the atmosphere containing only these molecules. This atmosphere is said to be dry and clean and is sometimes called a Rayleigh atmosphere. For visible light, blue photons are more scattered than the others, the least scattered being the red ones. In addition, the probability of any scattering direction is noticeable as shown in Figure 6.2. This is why the sky is blue: the blue photons are scattered several times in all directions, and consequently form an important fraction of the diffuse radiation which originates from all areas of the sky vault. When the water vapour content increases, this process is less visible: wavelengths are scattered more uniformly and the sky vault appears white. On the contrary, in the case of a dry atmosphere, the sky vault appears deep blue.

Aerosols play an important role in the scattering of radiation and to absorption in a lesser extent. Their exact role depends upon the chemical composition, concentration and size distribution. Nowadays, Ångström's law is often used to describe the optical depth or thickness of the aerosols, $\delta_{aer}(\lambda)$ as a function of the wavelength λ:

$$\tau_{aer}(\lambda) = \beta \left(\lambda/\lambda_1\right)^{-\alpha} \qquad (6\text{-}2)$$

where α and β are two coefficients, and λ_1 and λ two wavelengths. β characterizes the magnitude of the aerosol optical thickness; it is equal to δ_{aer} for $\lambda=\lambda_1$. If λ_1 is the 1 μm wavelength, then β is called the Ångström turbidity coefficient and it comes:

$$\tau_{aer}(\lambda) = \beta \lambda^{-\alpha} \qquad (6\text{-}3)$$

where λ is in μm. Typically, the aerosol optical depth ranges from 0 (no aerosol) up to 5.

α is called the Ångström exponent. It characterizes the spectral behaviour of the optical thickness and further, the influence of the aerosols on the radiation. It is related to the dimensions of the particles and the nature of the statistical distribution of their sizes: the smaller the particles, the larger the exponent. Observations show a considerable range of variations for α. Negative values are possible; α ranges typically from -1 to 4 and. If the Ångström exponent is equal to 0, there is no dependence of the optical depth with the wavelength. If α is positive, the optical depth decreases with the wavelength. Inversely if negative, the aerosol optical depth increases with the wavelength.

If one has a measure of the aerosol optical depth at one wavelength and knows the Ångström exponent, then one may compute the aerosol optical depth for any wavelength using Eq 6-2. Reciprocally, one may compute the Ångström exponent knowing the aerosol optical depth at two wavelengths.

$$\alpha = -log\left(\tau_{aer}(\lambda_1)/\tau_{aer}(\lambda_2)\right) \Big/ log\left(\lambda_1/\lambda_2\right) \qquad (6\text{-}4)$$

Usually, the optical effect of the aerosols only is called turbidity. If the aerosol load in the atmosphere in cloud-free conditions is small, then the extinction effect is small and the sky is said very clear. When the aerosol load is noticeable, the sky is said turbid.

One may report here on another quantity called the Linke turbidity factor that summarizes the atmospheric extinction in cloud-free conditions. The Linke turbidity factor is a very convenient approximation to model the atmospheric absorption and scattering of solar radiation. It describes the optical thickness of the atmosphere due to both absorption by the water vapour and absorption and scattering by the aerosol particles relative to a dry and clean atmosphere. The greater the Linke turbidity factor, the greater the attenuation of the radiation by the clear sky atmosphere. This factor is often given for an air mass equal to 2, i.e. approximately for a solar zenithal angle of 30°, or equivalently a solar elevation angle of 60°. If the atmosphere were dry and clean, the Linke turbidity factor would be equal to 1. A value of 3 is typical for Africa and Europe; it can amount to 7 or more in the case of a polluted area, e.g., a city with heavy traffic. It is a convenient quantity to summarize the turbidity of the

atmosphere and is often used by engineers and other practitioners. It is a key input to several models that assess the downwelling solar radiation received at ground under clear skies.

The atmospheric extinction is sometimes conveniently expressed by the visibility, which is the maximum distance at which a black object, of sufficient angular dimensions, can be distinguished by an observer at ground level against the luminous background of the sky on the horizon. The visibility is expressed in km. It is clearly linked to the content of particles in the atmosphere: dust, droplets etc. In foggy weather the visibility is less than 1 km and increases as the purity of the atmosphere increases. For example, from the authors' office on the French Riviera, high mountains located in the Corsica island 200 km away can be distinguished when the atmosphere is pure and dry, usually in early morning in winter. The visibility is often reported in meteorological reports, especially at airports. Note that it is a measure of the horizontal extinction at surface level, whereas the extinction of the solar radiation due to the atmosphere occurs along a vertical or oblique path.

Table 6.1 summarizes the contribution of the various atmospheric constituents to the extinction of the solar radiation impinging at the top of the atmosphere during its downward path towards the ground under cloud-free conditions.

Type of constituent	Scattering	Absorption
Ozone	Can be neglected	Strong for λ less than 300 nm
Gases (other than ozone)	Strong (λ^{-4}). Increases as the wavelength decreases	Small
Water vapour	Can be neglected	Noticeable for λ greater than 650 nm
Aerosols	$\lambda^{-\alpha}$ with $-1 \leq \alpha \leq 4$	Small

Table 6.1. Summary of the contribution of atmospheric constituents to the extinction of the solar radiation during its downward path under cloud-free conditions (λ is the wavelength)

6.4 THE SPECTRAL DISTRIBUTION AT GROUND LEVEL IN THE CLOUD-FREE ATMOSPHERE

Several graphs of typical spectral distributions of the downwelling solar irradiance received by a horizontal plane at mean sea level are now presented to illustrate the spectral effects of the cloud-free atmosphere on the radiation. They result from a numerical model for radiative transfer in the atmosphere. The simulations were made for the day June 19 –day number 170 in the year– and for a typical atmosphere encountered in summer at mid-latitudes. The receiving surface is horizontal.

Figure 6.4 exhibits a typical spectral distribution for three conditions: *i)* no atmosphere, i.e. at top of the atmosphere (in black), *ii)* clear sky (visibility of 50 km, dark grey), and *iii)* turbid cloud-free conditions (visibility of 10 km, light grey). The solar zenithal angle is 30°. The spectral distribution shows how much power is available for each wavelength. The surface under the curve is the irradiance integrated over the spectral range under concern.

As a whole, one may note a tendency of the spectral irradiance to decrease as the turbidity increases: the irradiance at the top of the atmosphere is greater than that of the very clear sky which in turn is greater than that in turbid conditions, at any wavelength. In this example, the total irradiance is 1174, 885, and 803 W m^{-2} at respectively the top of the atmosphere, the surface in clear sky and the surface in cloud-free but turbid conditions. Respectively 75 % and 68 % of the solar radiation impinging the horizontal surface at the top of the atmosphere went through the atmosphere, i.e. the total clearness index is respectively 0.75 and 0.68.

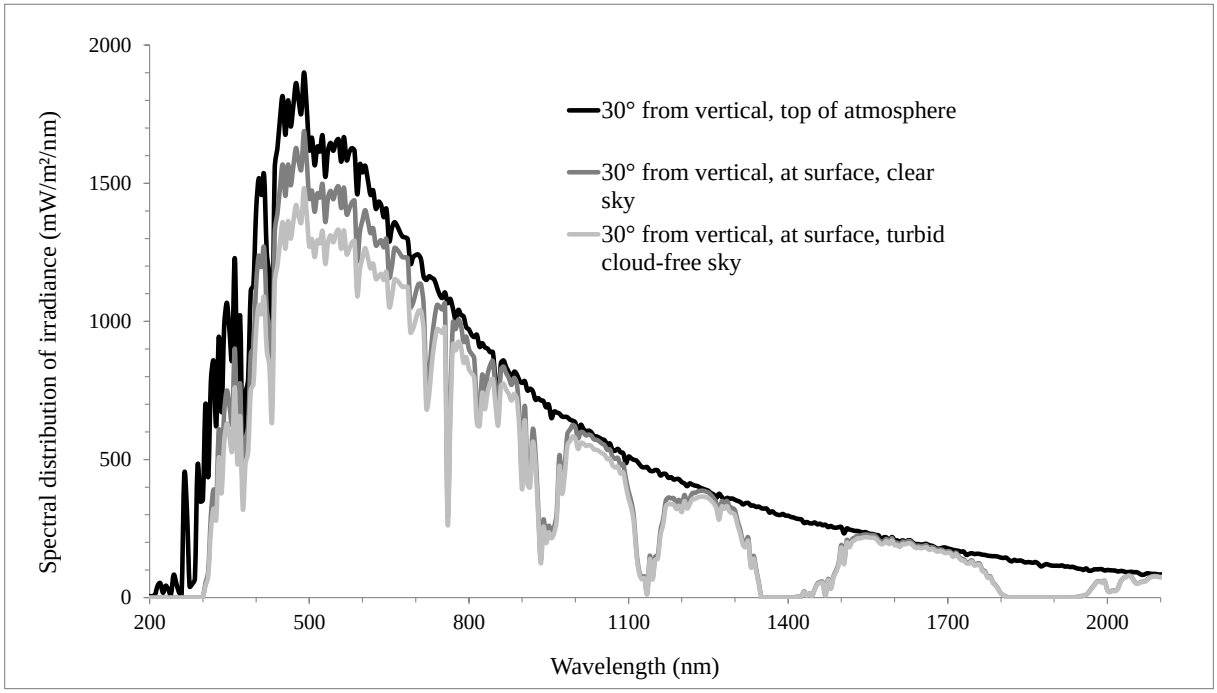

Figure 6.4. Typical spectral distribution of irradiance received by a horizontal surface at the top of the atmosphere and at ground level for very clear sky conditions and turbid cloud-free conditions. The solar zenithal angle is 30°.

The shape of the spectral distribution at sea level is fairly similar to that at the top of the atmosphere. The spectral distribution is well-peaked around 550 nm and about half of the radiation is found within the visible part [380, 780] nm.

However, several changes are noticeable as one may observe spikes and gaps compared to the spectral distribution at the top of the atmosphere. One notes the extinction of the radiation at the shortest wavelengths: the radiation impinging on the ground is small. This is due to absorption by ozone and to scattering by air molecules backwards outside the atmosphere. The irradiance for the shortest wavelengths is more depleted by the atmosphere than for wavelengths greater than 600 nm.

Between 700 and 800 nm are located absorption lines by oxygen and water vapour. The absorption is very large compared to the other wavelengths, yielding spikes in the distribution. At 900 nm and greater wavelengths, the radiation is absorbed by water vapour. This is not done uniformly but by absorption ranges as seen by the departures from the spectrum at the top of the atmosphere. Apart these large gaps due to absorption, the irradiance is the same in the three conditions at wavelengths greater than 1200 nm. The black and grey lines are superimposed. The scattering effects are very limited and the difference is due to the absorption effect.

Table 6.2 reports the contribution of the irradiance observed for various wavelengths to the total irradiance, i.e. integrated over the spectral range [250, 20000] nm for the three spectral distributions in Figure 6.4. One may note that the contributions are very similar for all intervals for the two cases at surface: clear conditions, and cloud-free but turbid conditions. One may see that the contribution is the same for the three distributions for the UV-B, UV-A, or the [280, 2800] nm interval which the typical range covered by a pyranometer. It greatly differs between the top of the atmosphere and the surface in the range [400, 1100] nm, [1100, 4000] nm, or the visible or PAR intervals. The visible band contributes to almost 60 % of the total irradiance at surface compared to 50 % at the top of the atmosphere.

Spectral range, in nm	TOA	Clear	Turbid	Spectral range, in nm	TOA	Clear	Turbid
250 to 20000	100	100	100	280 to 315 (UV-B)	1	<1	<1
400 to 1100	67	77	76	315 to 400 (UV-A)	7	7	6
1100 to 4000	25	18	19	380 to 780 (visible CIE)	49	57	56
280 to 2800 (pyranometer)	97	99	99	400 to 700 (PAR)	40	46	45

Table 6.2. Fraction of the total irradiance in percent, for various intervals of wavelength for the spectral distribution shown in Figure 6.4. PAR stands for photosynthetically available radiation. CIE stands for Commission internationale de l'éclairage.

The following example illustrates the influence of the solar zenithal angle θ_S on the total irradiance and on the spectral distribution. The case of a horizontal surface located at ground level for clear sky conditions (visibility of 50 km) is simulated for three solar zenithal angles: 0°, 30°, and 60°. Table 6.3 reports the total irradiance and the clearness index for this specific example, at respectively the top of the atmosphere and at the surface in clear sky conditions for each solar zenithal angle.

	0°	30°	60°
Irradiance at the top of atmosphere (W m^{-2})	1354	1174	673
Irradiance at ground level (W m^{-2})	1045	885	464
Clearness index	0.77	0.76	0.69

Table 6.3. Example of the total irradiance received by a horizontal surface at the top of the atmosphere and at ground level in clear sky conditions, and clearness index for three solar zenithal angles: 0°, 30° and 60°.

From this Table, one observes that the total irradiance decreases as θ_S increases as expected. One may observe that the decrease is greater at surface than at the top of the atmosphere; this is shown by the fact that the clearness index decreases as θ_S increases. Actually, the optical path increases with θ_S thus increasing the magnitude of the absorption and scattering phenomena. A crude approximation of the total irradiance E in clear sky conditions is given by[11]:

$$E = E_{0N} \cos\theta_S^{1.15} \qquad (6\text{-}5)$$

or equivalently:

$$KT = \cos\theta_S^{0.15} \qquad (6\text{-}6)$$

where there is an additional influence of θ_S other than $\cos\theta_S$ on the modelling of the depletion via the term $\cos\theta_S^{0.15}$. Such an approximation may be useful as a rule of thumb. There are several other models and several web services that provide accurate estimates of the irradiance or irradiation in cloud-free conditions; see e.g. www.soda-pro.com.

The following Figure 6.5 exhibits the spectral distribution of the irradiance for this example for the three solar zenithal angles: 0° (black line), 30° (dark grey line) and 60° (light grey line). This Figure is similar to Figure 5.2 that deals with the spectral distribution but at the top of the atmosphere.

[11] Perrin de Brichambaut and Vauge (op. cited)

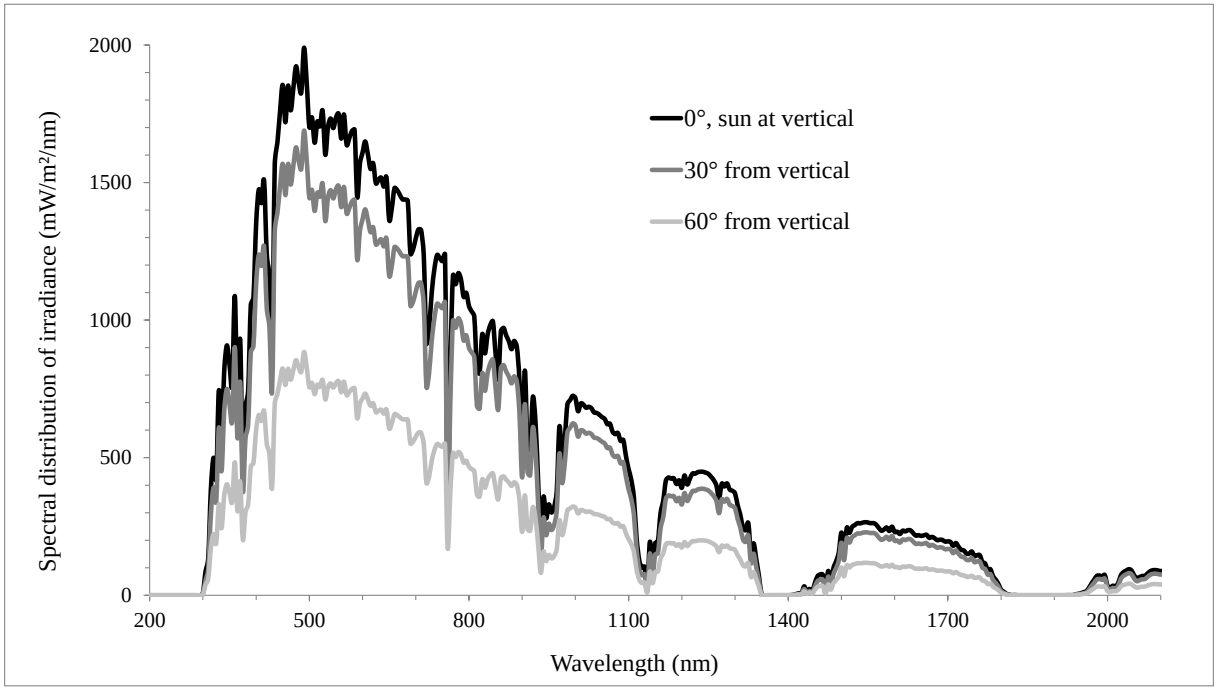

Figure 6.5. Typical spectral distribution of solar irradiance received by a horizontal surface at ground level for wavelengths from 200 nm to 2000 nm in clear sky conditions for three solar zenithal angles: 0°, 30° and 60°.

The influence of θ_S on the depletion of the solar radiation by the atmosphere can be seen at all wavelengths. As discussed above, the influence is more pronounced at the shorter wavelengths. The irradiance for the shortest wavelengths is more depleted as θ_S increases than for wavelengths greater than 600 nm.

To illustrate the extinction of radiation by the atmosphere, one may look at the changes of the clearness index *KT* with the wavelength. Figure 6.6 exhibits the spectral distribution of *KT*. If the atmosphere were completely transparent, *KT* would be equal to 1 for all wavelengths and any θ_S. *KT* is small for the shortest wavelengths because of absorption by ozone and scattering by air molecules. *KT* then increases as the wavelength increases and reaches a significant value for wavelengths greater than 350 nm. *KT* decreases as θ_S increases; the effects are more pronounced for greater solar zenithal angles.

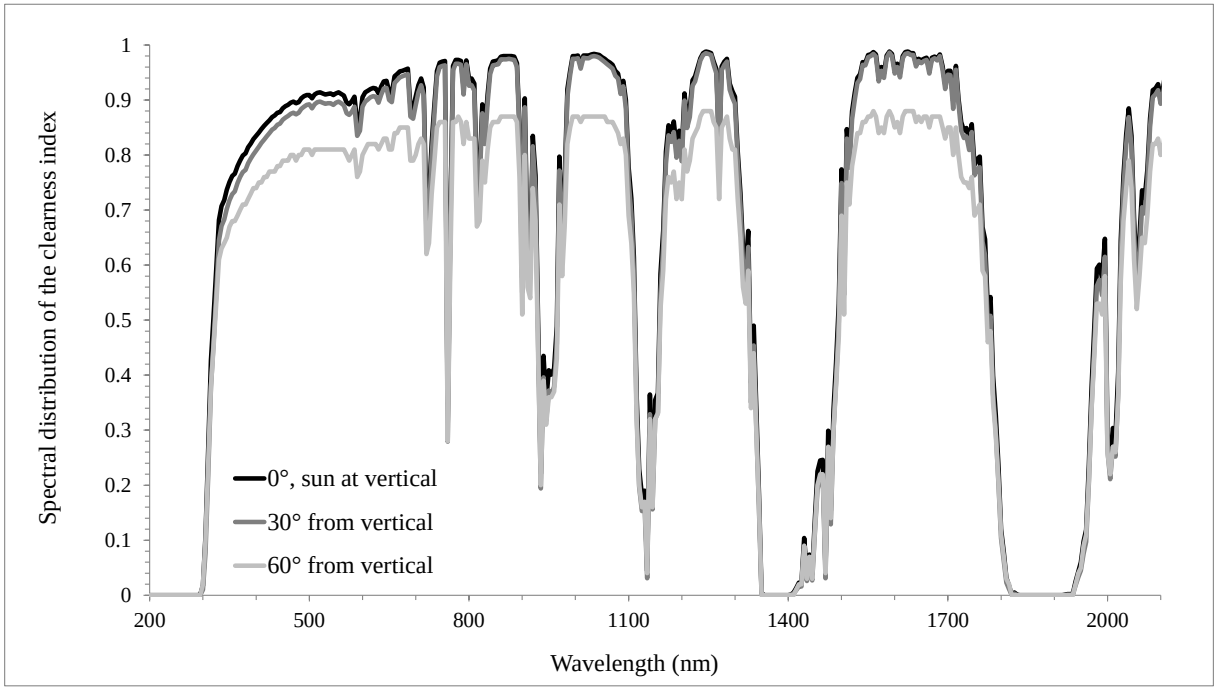

Figure 6.6. Clearness index KT as a function of the wavelength for the example shown in Figure 6.5.

6.5 THE ROLE OF CLOUDS

Clouds consist of a large number of water drops or ice crystals. Consequently, they produce intensive scattering of radiation passing through them and create a strong extinction of radiation in its way towards the ground. The amount of radiation passing downwards through a cloud decreases rapidly as the geometrical thickness of the cloud increases. The optical depth of the cloud characterizes the strength of the depletion. It is a unitless quantity. Value 0 means no extinction. A value of 3-4 is usually enough to make it hard to see the sun clearly from the ground through the cloud. The optical depth is related to the geometrical thickness of the cloud but depends on the type of cloud and the size of the water drops or ice crystals. There are various types of clouds. Consequently, their effects on the radiation vary greatly, from low extinction by thin cirrus to complete extinction by thick cumulus. The extinction by clouds is also a function of the wavelength: the spectral distribution observed in cloud-free conditions is changed by the presence of clouds.

If clouds deplete radiation on the one hand, on the other hand they reflect the radiation because their albedo is large. Two effects are of particular interest here because they increase the amount of solar radiation received at ground level. The first effect is that part of the radiation reflected by the ground is reflected back to the ground by the bottom of the clouds. The second effect is that clouds of finite size may concentrate light because the radiation may reflect on their sides towards the surface of interest. This latter phenomenon may explain why it is possible to observe hourly irradiation, or intra-hourly irradiation, greater than the corresponding extra-terrestrial irradiation.

The following example illustrates the depletion of the radiation by a cloud as a function of the wavelength. The solar zenithal angle is 30°. Two optical depths are given: a thin cloud of optical depth set to 5 and a thick cloud whose optical depth is 15. Also drawn is the spectral distribution for a clear sky (visibility of 50 km). Figure 6.7 exhibits the spectral distribution of irradiance and Figure 6.8 exhibits that for the clearness index.

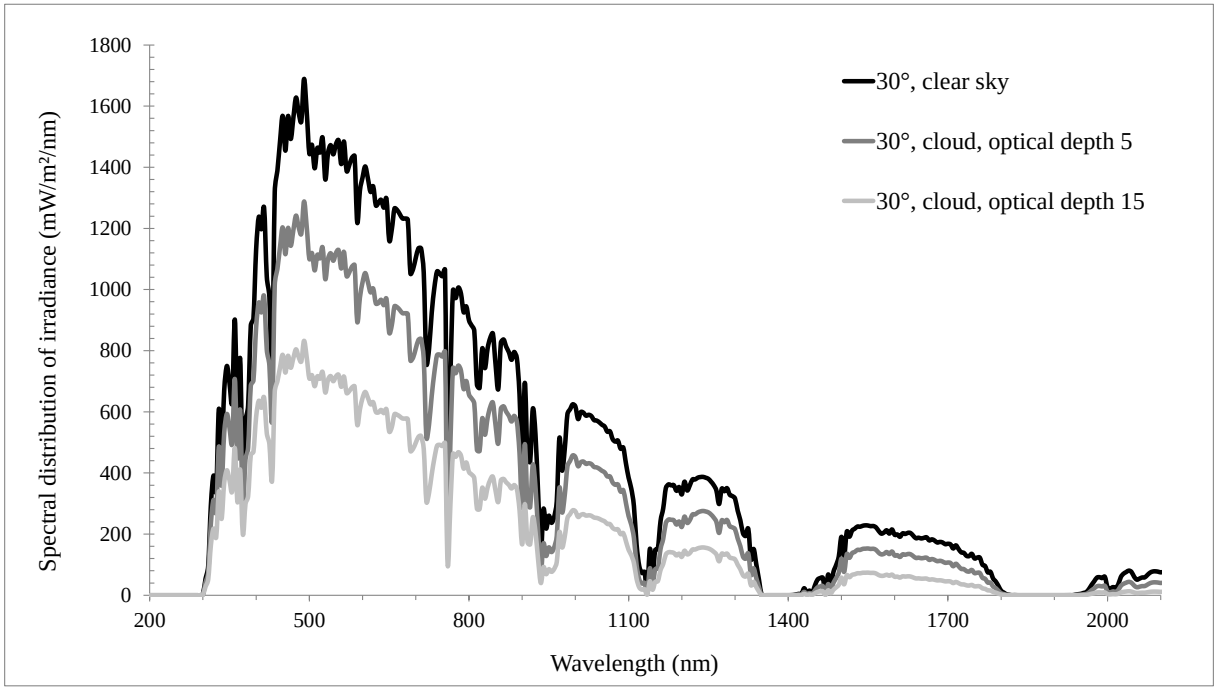

Figure 6.7. Typical spectral distribution of solar irradiance for wavelengths from 200 nm to 2000 nm for a solar zenithal angle of 30°. The surface is horizontal at ground level. Clear sky conditions (visibility of 50 km, black line) and cloudy conditions with two optical depths: 5 (dark grey line) and 15 (light grey line).

One observes that the irradiance in cloudy conditions (grey lines) is much less than that for clear skies (black line). As expected, the irradiance decreases as the optical depth of the cloud increases at all wavelengths. The clearness index *KT* is less than 0.7 at all wavelengths for the cloud optical depth 5 and less than 0.4 for the cloud optical depth 15.

The total irradiance is 1174 W m^{-2} at the top of the atmosphere. It amounts to 885, 637 and 387 W m^{-2} respectively for clear sky, and cloud optical depth 5 and 15 at ground level. The total clearness index is 0.76, 0.55, and 0.33 respectively for clear sky, and cloud optical depth 5 and 15. These numbers illustrate the major influence of the clouds on the solar radiation. The presence of a thin cloud of optical depth 5 is much more important than a low visibility of 10 km: the total irradiance is 637 for the cloud and 803 W m^{-2} for the visibility 10 km, and the clearness index is 0.55 for the cloud and 0.69 for the visibility 10 km.

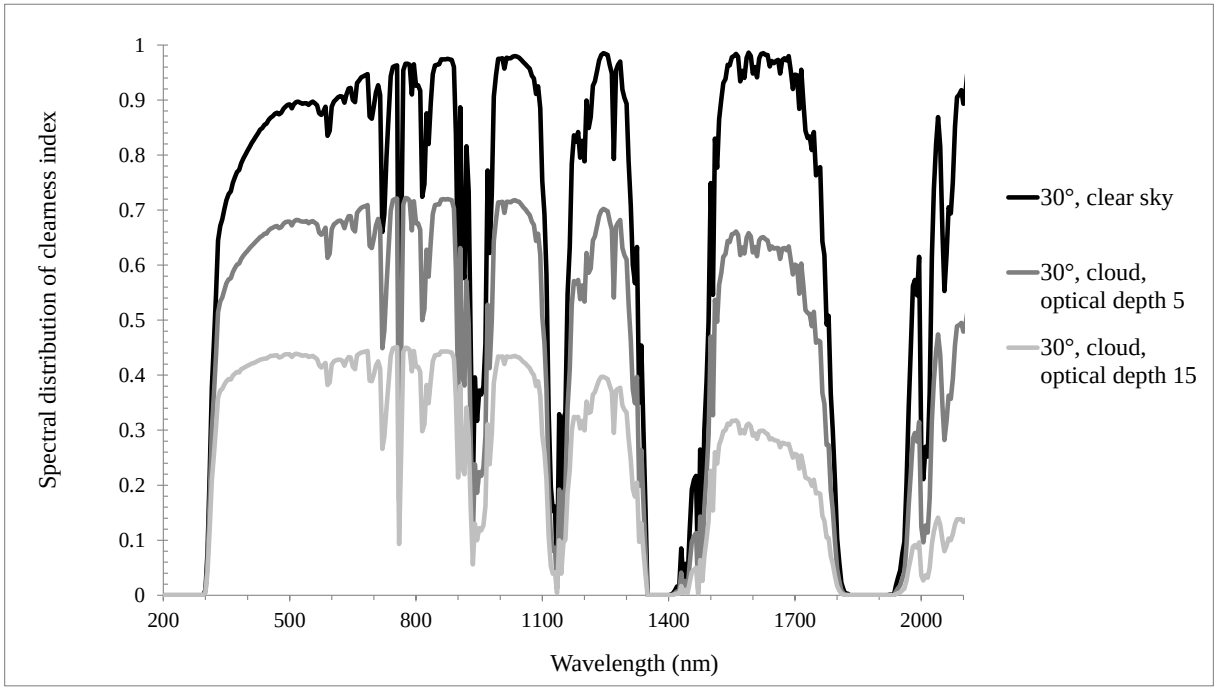

Figure 6.8. Typical spectral distribution of clearness index for wavelengths from 200 nm to 2000 nm for a solar zenithal angle of 30°. The surface is horizontal at ground level. Clear sky conditions (visibility of 50 km, black line) and cloudy conditions with two optical depths: 5 (dark grey line) and 15 (light grey line).

6.6 THE RADIATIVE COMPONENTS AT GROUND LEVEL

As already mentioned a horizontal plane at ground level receives radiation in direct -also called beam- or diffuse form. The global radiation is the sum of the direct and diffuse radiation.

The direct radiation is the radiation coming from the direction of the sun. Only direct radiation is present at the top of the atmosphere. A horizontal surface at ground level receives a depleted part of this direct radiation. The direct radiation may comprise photons that have been scattered once or more and that have re-joined the beam. Depending on the exact definition used for the direct component or of the system used to measure the direct radiation[12], it may include part of the radiation originating from the circumsolar region.

The diffuse radiation is the radiation that has been scattered by the constituents of the atmosphere and that originates from the sky vault in all directions, except that of the sun which is already accounted for through the direct component. It also comprises the part of the radiation that has been reflected by the ground and which is scattered by the atmosphere back towards the horizontal plane.

If the receiving plane at ground level is inclined, it may receive the direct radiation only partly and the fraction of sky viewed by the plane must be considered for computing the diffuse part impinging on the plane. The plane may also receive a part of the radiation that is reflected by the surrounding landscape towards the plane. This component is called the reflected component. The three components are illustrated in Figure 6.9.

[12] Blanc P., Espinar B., Geuder N., Gueymard C., Meyer R., Pitz-Paal R., Reinhardt B., Renne D., Sengupta M., Wald L., Wilbert S., Direct normal irradiance related definitions and applications: the circumsolar issue. Solar Energy, 110, 561-577, 2014. doi: 10.1016/j.solener.2014.10.001.

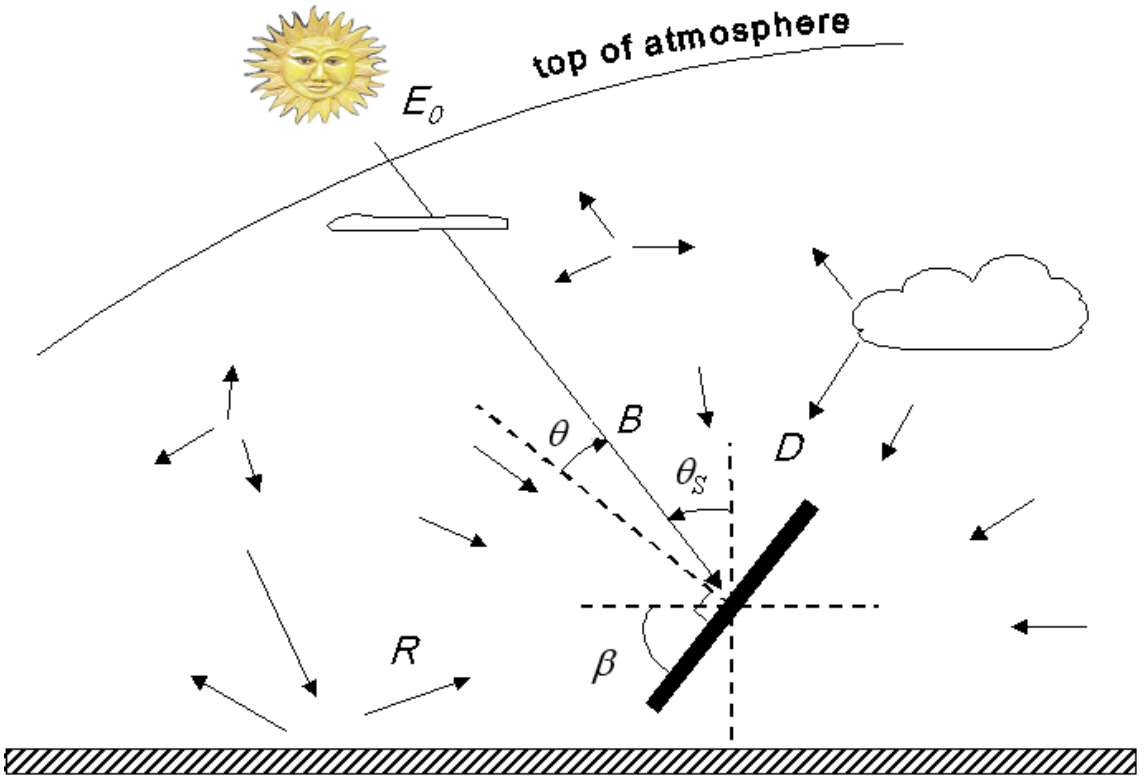

Figure 6.9. Scheme showing the various components composing global irradiance received by an inclined plane.

The global irradiance is the sum of these three components: direct *B*, diffuse *D* and reflected *R*:

$$E(\beta, \alpha, t) = B(\beta, \alpha, t) + D(\beta, \alpha, t) + R(\beta, \alpha, t) \qquad (6\text{-}7)$$

In the case of a horizontal surface, the reflected component is null. Similarly, the global irradiation is the sum of three components: direct, diffuse and reflected irradiations. These relationships hold for any wavelength or ranges of wavelengths.

Figure 6.10 exhibits the global, direct and diffuse irradiances at ground level as a function of wavelength for a solar zenithal angle of 30° and two cases of cloud-free conditions: clear sky (visibility of 50 km) and turbid (visibility of 10 km). As already seen, the global irradiance is greater for the clear sky case than for the turbid case as a whole and at all wavelengths. This is different for the direct and diffuse irradiances.

For the clear sky case, the scattering by the atmospheric constituents is weak: the direct irradiance is much larger than the diffuse one. However, this is not true for shorter wavelengths where molecular scattering occurs. For wavelengths shorter than 350 nm, the two components are equal. As the wavelength increases, the influence of the molecular scattering decreases and the diffuse irradiance decreases after a peak at approximately 450 nm. There is no scattering by aerosols to contribute significantly to the diffuse irradiance. The diffuse irradiance peaks at a shorter wavelength than the global irradiance; it shows low values for wavelengths greater than 800 nm and is close to 0 for wavelengths greater than 1100 nm. Accordingly, the direct irradiance is the major contributor to the global irradiance for wavelengths greater than 350 nm and the sole contributor for wavelengths greater than 1100 nm.

The turbid case shows a fairly similar behaviour (grey lines) though the magnitude of the effects may be different. As there are more scatterers than in the clear sky case, the magnitude of the scattering effect increases and the diffuse irradiance is by far the major contributor to the global irradiance for the shorter wavelengths. The diffuse irradiance is greater than that for the clear sky case as a whole and at all wavelengths. It peaks at 450 nm and then decreases to reach values close to 0 for wavelengths greater than 1300 nm. The diffuse and direct irradiances are equal at approximately 600 nm. The direct irradiance is less than that for the clear sky case as a whole and at all wavelengths. It peaks between 600 and 700 nm and then decreases. It remains the major

contributor to the global irradiance for wavelengths greater than 700 nm and the sole contributor for wavelengths greater than 1400 nm.

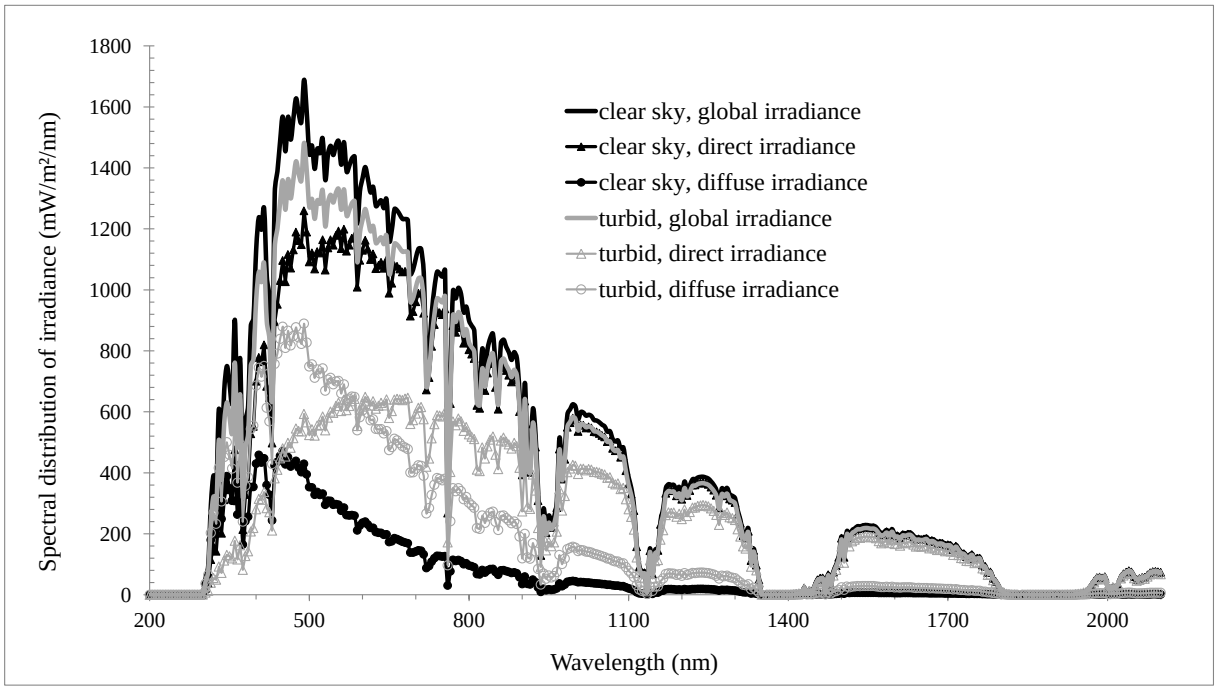

Figure 6.10. Global, direct and diffuse irradiances at ground as a function of wavelength for a solar zenithal angle of 30° and two cases of cloud-free conditions: clear sky (visibility of 50 km) and turbid (visibility of 10 km).

The spectral behaviour of the relative contribution of the diffuse irradiance to the global irradiance is illustrated in Figure 6.11. It reports the ratio of the diffuse irradiance to the global irradiance for the two cases. An additional case of a cloudy situation with a cloud optical depth of 5 is also drawn. As a whole and at each wavelength, the ratio is greater for the turbid case than for the clear sky case: the contribution of the diffuse is more pronounced when the optical turbidity increases. As already discussed, the ratio is the greatest for the shorter wavelengths. It amounts to 0.6 for the clear sky case and up to 0.85 for the turbid case.

In each case, the ratio decreases as the wavelength increases. The decrease is more pronounced when the sky is clear. As the turbidity increases or as the cloud optical depth increases in the case of cloudy atmosphere, the ratio tends to be constant with the wavelength. For the cloud case, the ratio is 1, meaning that the direct irradiance is null at all wavelengths and that the diffuse irradiance is the sole contributor to the global irradiance.

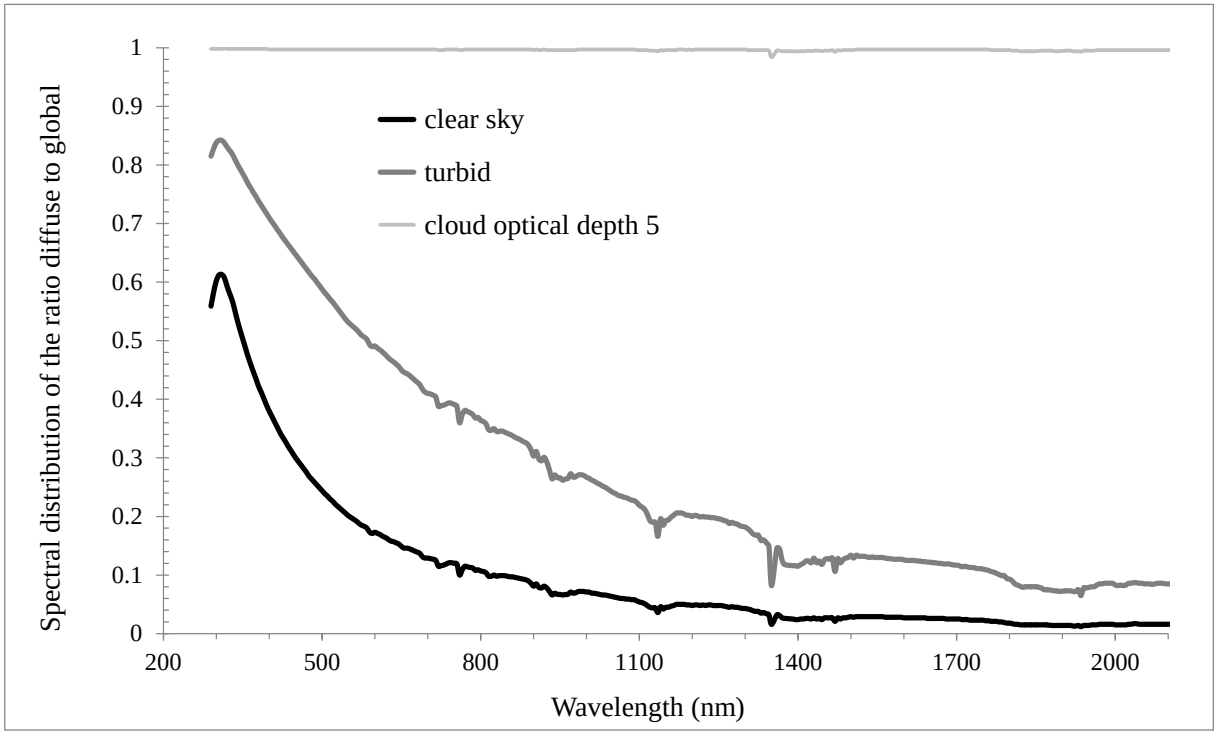

Figure 6.11. Ratio of the diffuse irradiance to the global irradiance as a function of wavelength for a solar zenithal angle of 30° and two cases of cloud-free conditions: clear sky (visibility of 50 km) and turbid (visibility of 10 km) as well as a cloudy situation with an optical depth of 5.

Several models or web services (see e.g. www.soda-pro.com) are proposed to estimate the global, direct and diffuse irradiances, or irradiations, for any type of sky. Typical inputs are the instant (day, hour), geographical location (latitude, longitude, elevation) and one or more variables describing the optical state of the atmosphere.

Most often, measurements are made of global irradiation on the horizontal plane. Measurements of direct and diffuse irradiations are less frequent. Consequently, models should be exploited for estimating the direct and diffuse irradiations for a horizontal surface from the global irradiation measured on this surface. Many publications are devoted to such problems. It is beyond the scope of this textbook to deal with such models. They are numerous and depend also on the data available. Nevertheless, a first insight is provided. The typical approach to obtain the irradiation on an inclined surface is to firstly split the horizontal global irradiation $H(0, 0)$ into direct $B(0, 0)$ and diffuse $D(0, 0)$ irradiations on a horizontal plane. Computation is then made on each irradiation separately to obtain direct irradiation $B(\beta, \alpha)$ and diffuse irradiation $D(\beta, \alpha)$ on an inclined surface. The reflected irradiation $R(\beta, \alpha)$ is added and the sum provides the global irradiation $H(\beta, \alpha)$.

Once the direct irradiance $B(0, 0, t)$ is known on a horizontal surface, the irradiance on an inclined plane $B(\beta, \alpha, t)$ is given by:

$$B(\beta, \alpha, t) = B(0, 0, t) \cos\theta(t) / \cos\theta_S(t) \qquad (6\text{-}8)$$

Assuming there is no shade during the period of integration, the direct irradiation $B(\beta, \alpha)$ is easily computed for any inclined surface by exploiting Eq. 5-19:

$$B(\beta, \alpha) = B(0, 0) \, [\, H_0(\beta, \alpha) / H_0(0, 0) \,] \qquad (6\text{-}9)$$

A simple model for reflected irradiation $R(\beta, \alpha)$ is based on the assumption that the reflection from the ground is isotropic, i.e., the irradiation is the same whatever the direction from which the reflected rays are coming. In this case, the fraction of ground viewed by this plane is:

$$r_g(\beta) = (1 - \cos\beta) / 2 \qquad (6\text{-}10)$$

and the reflected irradiation is given by:

$$R(\beta, \alpha) = \rho_g H(0, 0) r_g(\beta) \quad (6\text{-}11)$$

where ρ_g is the ground albedo, i.e., the fraction of irradiation that is reflected by the ground.

The major difficulty when assessing irradiation on inclined surfaces lies in the estimation of the diffuse irradiation. The major cause of difficulty arises from the fact that the distribution of radiances on the sky vault is not simple. It is anisotropic, with a marked circumsolar effect around the direction of the sun, and is closer to isotropy when the sky becomes completely overcast. Clouds scattered in the sky or grouped in a particular direction increase the anisotropy. It is also evident from previous figures that this distribution depends upon the spectral distribution of the irradiation and therefore on the optical properties of the sky, including cloud effects.

A very simple model consists in considering the sky vault as a source with uniform radiance. This model is inaccurate and is not recommended at all, but its presentation may improve understanding of diffuse irradiation. In this case, as for reflected irradiation, the fraction of sky viewed by the plane may be considered as:

$$r_i(\beta) = (1 + \cos\beta) / 2 \quad (6\text{-}12)$$

and the diffuse irradiation is given by:

$$D(\beta, \alpha) = D(0, 0) r_i(\beta) \quad (6\text{-}13)$$

where $D(0, 0)$ is the diffuse irradiation received on the horizontal surface.

Authors' biography

Lucien Wald was born in 1953. He graduated in theoretical physics from universities in Marseille and Paris, France, in 1976 and 1977. He specialised himself in fluid dynamics in geophysics and atmosphere optics. He obtained the Ph.D. degree in 1980 from the University Pierre and Marie Curie in Paris, and the *Doctorat d'Etat ès Sciences* in 1985 from the University of Toulon, both on the applications of remote sensing to oceanography. He joined MINES ParisTech in 1980 and was appointed Professor in 1991. He has focused his own research on geophysics (meteorology, solar radiation), remote sensing and image processing. He has written more than one hundred articles in peer-reviewed journals, books or chapters in the field of Earth observation and environment, namely marine environment, terrestrial environment, urban environment, meteorology, solar radiation, effects of solar radiation on human health. He published well-received articles in data fusion, including a patent on sensor fusion. He obtained the Autometrics Award in 1998 and the ERDAS Award in 2001 for articles on data fusion. His activities in information technologies have been rewarded by the famous French Blondel Medal in 1996.

He served the European Association of Remote Sensing Laboratories (EARSeL) as secretary-general from 1997 to 2001. He has co-organised several international conferences and working groups, including the series of conferences "Fusion of Earth data" (1996-2000), the session "data fusion" at the EARSeL Annual Assembly (1996-2009) and the session "energy meteorology" at the European Meteorological Society (EMS) Annual Assembly (2009-2017). He was the scientific coordinator of several projects funded by the European Union.