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# Strain rate sensitivity in superelasticity

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**Abstract.** This paper deals with the influence of martensitic transformation latent heat on the superelastic behavior modelling. Exothermic and endothermic effects are responsible for a strong evolution on the temperature field inside the material that modifies its mechanical response. This is responsible for a strain rate effect that is taken into account by introducing a coupling equation between the production rate of martensite and the temperature change, into a micro-macro modelling of the superthermoelastic behaviour with the assumption that the temperature field remains uniform but different to the test temperature imposed. Numerical simulations so obtained show a good agreement with experimental results performed on Cu-based superelastic alloys. © 2000 Published by Elsevier Science Ltd.

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#### 1. Introduction

From a metallurgical viewpoint, martensitic transformation is a time independent transformation. Individual plate growths or shrinks at the sound velocity (Delaey, 1991). Morphology of martensitic phase is directly related to the nature of control parameters imposed during the transformation. Self accommodated plates growth when the transformation is purely thermally induced and an oriented microstructure is produced when large applied stress is imposed (Roytburd, 1993). Influence of local variations in the stress field is the object of many studies. Several kinds of variations are distinguished. Some of these originate from heterogeneities in the loading conditions. In rod bending, for instance, some parts are stressed in tension while others are stressed in compression. Structural analysis deals with these extrinsic heterogeneities using finite element techniques (Brinson and Lammering, 1993; Leclerc et al., 1996) or formulations derived from the strength of materials as proposed by Tobushi and Tanaka (1991) or Gillet et al. (1995). Other kinds of variation, that can be called intrinsic, originate from incompatibilities in the martensitic transformation strain field. Granular structure in polycrystalline materials and between martensite plates are responsible of these incompatibilities. interactions Thermodynamical analysis and modern scale transition approaches can be successfully applied to derive the overall behaviour of volume element taking these intrinsic heterogeneities into account (Patoor et al., 1988; Sun and Hwang, 1993; Gall and Sehitoglu, 1999).

Less attention was focussed on heterogeneities in the temperature field. Like for the stress field case, they emanate from different phenomena and distinction between extrinsic or intrinsic origin is also valid. Extrinsic variations originate from heterogeneous thermal boundary conditions or come from shape particularities (Fig. 1a). Intrinsic heterogeneities are directly related to the

characteristics of the martensitic transformation, large heat production is associated with transformation from austenite to martensite and an equivalent endothermic effect is associated with reverse transformation (Fig. 1b). This latent heat of transformation strongly affects the temperature field inside the material and it is able to produce very sensitive evolutions in superelastic behaviour (Otsuka et al., 1976) (Fig. 2). Relative importance of temperature evolution generated in that way is directly related to heat exchange conditions existing between the specimen where the transformation takes place and the surrounding. Heat exchange conditions depend on several factors: nature of the surrounding (liquid or gas), natural or forced convection, geometrical shape and aspect ratio of the specimen. All these factors are closely related by the resolution of the heat equation associated with the problem considered. Due to the time dependant character of this equation, the strain rate becomes an important parameter in order to determine the resulting macroscopic behaviour. The diagram drawn in Fig. 2 shows, schematically, the influence of the strain rate on stress-strain curves. In a purely isothermal loading a straightforward relation is observed between the increase of stress which is needed to induce the transformation and material characteristics as the  $M_s - M_f$  temperature interval and the slope  $d\sigma/dT$  of the Clausius-Clapeyron equation. This is no longer the case, when according to strain rate imposed and/or the heat exchange conditions applied, the latent heat of transformation contributes to the influencing of the sample temperature. In these conditions a complex relation relates the end of transformation stress to the material parameters. Qualitatively, it appears from Fig. 2 that the strain rate sensitivity increases with the  $d\sigma/dT$ coefficient or when the  $M_s - M_f$  interval is increased.



Fig. 1. Different origins for thermal heterogeneities observed in SMA element. (a) Extrinsic heterogeneity generates by thermal boundary conditions and geometrical factors. (b) Intrinsic heterogeneity originates from latent heat of transformation.

Since the beginning of the 90's, strain rate effect received attention from several researchers. Federlich and Zanzotto (1991) were the first to model non-isothermal evolution as an extension of Müller isothermal approach (Müller and Xu, 1990). They assumed a random distribution of interfaces between the two phases for one-dimensional (1-D) problems and formulated a temperature-dependent constitutive equation by introducing a non-isothermal hysteresis operator. Leo et al. (1993) proposed to model 1-D heat transfer problem in SMAs considering the quasistatic motion of an unique austenite-martensite interface following the model developed by Abeyaratne and Knowles (1993). This single interface model gives a good account of the stress-

strain behaviour of NiTi wires at different strain rates and it was extended later to describe the constant load behaviour (Shield et al., 1997). In a different way, Brinson et al. (1996) had established a 1-D phenomenological model based on macroscopic thermo-dynamical analysis using the martensite fraction as an internal variable and by giving an uncoupled resolution of the energy balance equation for the heat transfer. This model was applied to resistive heating of NiTi wire under convection in air. Beckker and Brinson (1997) proposed a coupled solution for this problem. A similar problem is addressed by Benzaoui et al. (1997) from a macroscopic model which assumes the existence of a three phase system. Two internal variables: the volume fraction of self-accommodated martensite and the volume fraction of oriented martensite are used to express the Helmholtz free energy of this system. Time effects are well described from integration of the heat equation and a good agreement is obtained with experimental data determined for heating and cooling of a NiTi wire loaded under constant applied stress.



Fig. 2. Exothermic effect for superelastic loading at temperature  $T_{ext}$  increases the temperature and requires a higher stress to go on the transformation that gives a strain rate effect. Isothermal (a) and adiabatic (b) conditions are lower and upper limits for the resulting stress-strain curve.

Heat exchange mechanisms giving low frequency response for actuation Bhattacharyya et al. (1995) proposed to increase it using the SMA actuator as one junction in a thermoelectric element. In that way, the thermoelectric Peltier effect is used as heat sink. They give an analytical solution for the one-dimensional thermoelectric heat transfer problem from a phenomenological model which was developed to describe the thermomechanical response of SMA element. A radically different way is proposed by Lexcellent et al. (1996) to model the effect of the stress rate in tensile test for a single crystal. In their approach, a 3-D micromechanical-based model is used to describe the single crystal behaviour, but influence of the loading rate is reached by integration of the heat equation in the 1-D case considered. A common point between all these different approaches is that a structural point of view is always adopted. Each work describes a well-defined structural problem and solves the heat equation with respect to the particular shape and the particular loading case considered. In that way, extension to other loading conditions is difficult to perform.

Original aspect of the present work is to deal with a fully 3-D framework for the polycrystalline material in such a way any kind of loading conditions can be accounted for. As no geometrical parameters are introduced in this modelling, only intrinsic heterogeneities in the temperature field can be considered. Like for purely mechanical problems a thermodynamical analysis and a scale transition method are used to define the influence of the strain rate on the behaviour of a volume element. Our previous studies have shown that scale transition modelling of superthermoelasticity is able to account for simultaneous variation in the temperature imposed and in the applied stress. The general framework of this analysis and relevant results obtained, like the ability to derive the kinetics of martensitic transformation with respect to evolution in temperature and applied stress, are recalled in Section 2. This section ends with the difficulties added in the superthermoelastic problem by the existence of a non-homogeneous temperature field. Physical origin of the strain rate effect in SMA is discussed in Section 3 and from the consideration of experimental measurement of the temperature field inside a superelastic sample, the assumption of an infinite heat conduction rate is proposed to simplify the time-dependant superthermoelastic problem. Consequences of this assumption on the modelling are detailed in section four where a coupling equation between the temperature change and the martensite fraction produced, or reversed, is derived. Application of the proposed model to copper-based SMA is presented in section five. Numerical simulations are compared with experimental results obtained by Chrysochoos et al. (1993) and by Entemeyer (1996).

#### 2. General framework for scale transition modelling

These last 10 years, several works had established that scale transition frameworks are very suitable to model superelastic behaviour in shape memory alloys (Ono and Sato, 1988; Patoor et al. 1993; Sun and Hwang, 1993; Huang and Brinson, 1998; Gall and Sehitoglu, 1999). A strong predictive capacity is related to these approaches (Patoor et al., 1995, Gall and Schitoglu, 1999) which require no additional assumption to define the kinetics evolution rule for martensitic transformation. These approaches are based upon kinematical description of the transformation strain and thermodynamical analysis at the so-called microscopic level. Assumption of existence of a representative volume element, called RVE, is one of the key points of these approaches. In polycrystalline SMA, in order to consider the two major sources of incompatibilities in the transformation strain, grains are usually chosen as RVE and two different models are used to derive the overall thermomechanical behaviour. One model deals with the existence of the granular structure and it determines the intergranular stress field. The second model, established at the RVE scale, takes into account the growth of several variants of martensite inside each grain. In these descriptions, we assume that the formation of a variant n produces well-defined and uniform transformation strain  $\varepsilon^n$  over volume  $V^n$  (Patoor et al., 1998). Each grain, described as a crystal of the parent austenite phase, is characterised by its crystallographic orientation. Assuming the same uniform elastic behaviour in both phases, the transformation strain,  $\varepsilon^{T}$ , which takes place inside a RVE of volume *V* is defined by:

$$\varepsilon_{ij}^{T} = \frac{1}{V} \int_{V} \sum_{n} \varepsilon_{ij}^{n} \,\theta^{n}(r) \mathrm{d}V = \sum_{n} \varepsilon_{ij}^{n} f^{n} \tag{1}$$

with  $\theta^n(r) = 1$  if  $r \in V^n$  (= 0 if  $r \notin V^n$ )

+ Erratum to "Strain rate sensitivity in superelasticity", International Journal of Plasticity, 17, 2001, pp.273-274.

Volume fractions  $f^n = V^n/V$ , which are defined for each variant, are used to describe the microstructure evolution and are taken as internal variables in the thermodynamical analysis. Evolution in the transformation strain given by Eq. (1) with respect to the loading conditions is obtained from the definition of a thermodynamical potential that is function of the control parameters and internal variables  $f^n$ . Complementary free energy  $\Psi(\sigma, T, f^n)$  defined over RVE of volume V is composed of elastic energy  $W_{elas}$ , chemical energy  $\Delta G_{ch}$  and interfacial energy  $W_{int}$  (Ortin and Planes, 1988; Patoor et al., 1997). Homogeneous loading conditions are applied on the RVE boundary, i.e. the applied stress  $\sigma(r)$  and the temperature T(r) fields are uniforms, we obtain:

$$\Psi(\sigma_{ij}, T, f^n) = -[\Delta G_{ch} + W_{elas} + W_{int} - \sigma_{ij}\varepsilon_{ij}]$$
<sup>(2)</sup>

Chemical energy,  $\Delta G_{ch}$ , depends on the temperature and on the volume  $V_M$  of martensite formed. A linear approximation around  $T_0$ , the thermodynamical equilibrium temperature, is commonly used. Elastic energy,  $W_{elas}$ , is classically determined considering incompatibilities in the transformation strain built an internal stress field  $\tau(r)$  (Mura, 1987; Patoor and Berveiller, 1997).

$$\Delta G_{ch}(T) = B(T - T_0) V_M$$

$$W_{elas} = \frac{1}{2} V \sigma_{ij} M_{ijkl} \sigma_{kl} - \frac{1}{2} \int_V \tau_{ij}(r) \varepsilon_{ij}^T(r) dV$$
(3)

In Eq. (3), the four order tensor M, denotes the elastic stiffness tensor, which is assumed to be uniform in the two phases, and  $\varepsilon^{T}(r)$  is the transformation strain field that is a piecewise uniform function having values  $\varepsilon^{n}$ . According to previous micromechanical analysis (Fassi Fehri et al., 1987; Bensalah, 1992; Siredey et al., 1999), interaction energy,  $E_{int}$ , that is related with the internal stress field, can be expressed as a quadratic function of volume fractions  $f^{n}$  by introducing the interaction matrix  $H^{nm}$ . This matrix takes into account the nature of the interaction between two variants n and m. Two kinds of interaction are considered, selfaccommodation, producing very low internal stress, and incompatibility, building very high internal stress level.

$$E_{int} = -\frac{1}{2} \int_{V} \tau_{ij}(r) \varepsilon_{ij}^{T}(r) \mathrm{d}V = \frac{1}{2} \sum_{n,m} H^{nm} f^{n} f^{m}$$

$$\tag{4}$$

Martensite plates are described using oblate shape inclusions, so interfacial energy,  $W_{int}$ , becomes negligible in regard to the elastic energy. According to Eqs. (2)-(4), the complementary free energy per unit reference volume is expressed by (Patoor et al., 1998):

$$\Psi(\sigma_{ij},T,f^n) = \frac{1}{2}\sigma_{ij}M_{ijkl}\sigma_{kl} + \sigma_{ij}\sum_n \varepsilon_{ij}^n f^n - B(T-T_0)\sum_n f^n - \frac{1}{2}\sum_{n,m} H^{nm}f^n f^m$$
(5)

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Due to physical limitations applied on the  $f^n$ , a Lagrangian functional  $L(\sigma_{ij}, T, f^n)$  is introduced to derive driving forces,  $F^n$ , acting on internal variables (Moumni and Nguyen, 1996).

$$F^n(\sigma_{ij},T,f^n) = \frac{\partial L(\sigma_{ij},T,f^n)}{\partial f^n}$$

with

(6)

$$L(\sigma_{ij},T,f^n) = \Psi(\sigma_{ij},T,f^n) - \lambda_0 \left[\sum_n f^n - 1\right] - \sum_n \lambda_n \left[-f^n - 0\right]$$

Where Lagrange multipliers  $\lambda_0$  and  $\lambda_n$  are positive quantities associated with inequality constraints f = 1 and  $f^n = 0$ . Definition of the driving force becomes:

$$F^{n}(\sigma_{ij},T,f^{n}) = \sigma_{ij}\varepsilon_{ij}^{n} - B(T-T_{0}) - \sum_{m} H^{nm}f^{m} - \lambda_{0} + \lambda_{n}$$
<sup>(7)</sup>

Hysteresis loop observed in superelasticity is described through the definition of a pseudodissipative potential  $W^d$ . This potential is determined considering the driving force  $F^n$  must reach a critical value  $F_c$  to produce evolution in the volume fraction of the variant considered (growing or shrinkage). This resistive dissipative force is assumed to be a positive material constant, applied in the same way on each variant. Second law of the thermodynamics and energy balance require:

$$\dot{\Psi}\big|_{R,T} = \dot{W}^d = \sum_n F^n \dot{f}^n \ge 0 \tag{8}$$

Assuming the dissipation occurs without any coupling effect between the martensite variants, local transformation criteria are defined on each variant for forward and reverse transformations.

$$\sigma_{ij}\varepsilon_{ij}^n - B(T - T_0) - \sum_m H^{nm} f^m - \lambda_0 + \lambda_n = +F_c \quad \text{for austenite to variant } n \tag{9}$$

$$\sigma_{ij}\varepsilon_{ij}^n - B(T - T_0) - \sum_m H^{nm} f^m - \lambda_0 + \lambda_n = -F_c \quad \text{for variant } n \text{ to austenite}$$
(10)

If one of criteria (9) or (10) is satisfied, within the physical limitations imposed on internal variables, the transformation rate  $\dot{f}^n$  on a variant, is determined according to the consistency condition (Rice, 1971; Patoor and Berveiller, 1997).

$$\dot{f}^n = \sum_m (H^{nm})^{-1} \left[ \varepsilon_{kl}^m \dot{\sigma}_{kl} - B\dot{T} \right]$$
(11)

Determination of production rate for all variants activated in this crystal, leads to the following constitutive equation for the transformation strain rate of the RVE.

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$$\dot{\varepsilon}_{ij}^{T} = \sum_{n} \varepsilon_{ij}^{n} \sum_{m} (H^{nm})^{-1} \left[ \varepsilon_{kl}^{m} \dot{\sigma}_{kl} - B\dot{T} \right] = g_{ijkl}(r) \dot{\sigma}_{kl} + n_{ij}(r) \dot{T}$$
(12)

This transformation strain rate is related to local stress and temperature rates applied on the RVE. In this approach, local tangent moduli g(r) and n(r) are defined from material parameters easy to identify. They depend on the number of martensite variants induced inside the RVE, the nature of their interactions and also the crystallographic orientation of the RVE.

$$g_{ijkl}(r) = \sum_{n,m} \varepsilon_{ij}^n (H^{nm})^{-1} \varepsilon_{kl}^m$$

$$n_{ij}(r) = -B\left(\sum_{n,m} \varepsilon_{ij}^n (H^{nm})^{-1}\right)$$
(13)

In polycrystalline material the assumption of uniformity of the loading condition through the entire body considered is not realistic. We have to take into account the intergranular stress field which is built by transformation strain incompatibilities from grain to grain. A second model involving a homogenisation method is needed to determine the overall behaviour taking into account the actual stress applied on each individual grain. Kinematical Eq. (1) is now applied over a polycrystalline body composed by a set of N individual grains. We obtain for the overall transformation strain,  $\varepsilon^{tr}$ , and for the global amount, f, of martensite the following expressions:

$$\varepsilon_{ij}^{T} = \sum_{N} \left\{ \sum_{n} (\varepsilon_{N}^{n})_{ij} f_{N}^{n} \varphi^{N} \right\}$$
  
and (14)  
$$f = \sum_{N} \left\{ \sum_{n} (f_{N}^{n} \varphi^{N}) \right\}$$

Where  $\varepsilon_N^n$  and  $f_N^n$  denote the transformation strain and the volume fraction related to the variant labelled *n* inside the grain labeled *N* with volume fraction  $\varphi^N$ . Considering grains as RVE turns intergranular stress field into piecewise uniform function having constant value inside each grain. For each grain, total strain rate is composed of transformation strain rate expressed using Eq. (12) and elastic strain rate. We obtain:

$$\dot{\varepsilon}_{ij}^{N} = M_{ijkl}\dot{\sigma}_{kl}^{N} + \sum_{n} (\varepsilon_{N}^{n})_{ij} \sum_{m} (H^{nm})^{-1} [(\varepsilon_{N}^{m})_{kl}\dot{\sigma}_{kl}^{N} - B\dot{T}]$$
(15)

From this equation, the following local stress rate applied on grain N is given by:

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$$\dot{\sigma}_{ij}^N = l_{ijkl}^N \dot{\varepsilon}_{kl}^N + m_{ij}^N \dot{T} \tag{16}$$

where local piecewise uniform tangent moduli  $l^N$  and  $m^N$  are defined in each grain from Eqs. (13) and (15).

$$l_{ijkl}^{N} = \left( M_{ijkl} + \sum_{n,m} (\varepsilon_{N}^{n})_{ij} (H^{nm})^{-1} (\varepsilon_{N}^{m})_{kl} \right)^{-1}$$

$$m_{ij}^{N} = B \left( \sum_{n,m} (\varepsilon_{N}^{n})_{kl} (H^{nm})^{-1} \right) \left( M_{ijkl} + \sum_{n,m} (\varepsilon_{N}^{n})_{ij} (H^{nm})^{-1} (\varepsilon_{N}^{m})_{kl} \right)^{-1}$$
(17)

In order to determine the overall behaviour; let us consider a homogeneous reference medium characterised by a set of uniform tangent moduli ( $L^0$  and  $M^0$ ) and subject to uniform strain  $E^0$ . In the actual material, local moduli l(r) and m(r) can be expressed using their deviation parts to these uniform quantities (Patoor et al., 1996).

$$l_{ijkl}(r) = L_{ijkl}^{0} + \delta l_{ijkl}(r)$$
  
and (18)  
$$m_{ij}(r) = M_{ij}^{0} + \delta m_{ij}(r)$$

The stress rate defined by Eq. (16) must be in equilibrium. If there are uniformity of properties inside the homogeneous reference medium and a uniform temperature, the equilibrium condition leads to:

$$L^{0}_{ijkl}(\dot{\varepsilon}_{kl}(r))_{,j} + (\delta l_{ijkl}(r)\dot{\varepsilon}_{kl}(r) - \delta m_{ij}(r)\dot{T})_{,j} = 0$$
(19)

Resolution of Eq. (19) using the Green tensor method, gives the following integral equation:

$$\dot{\varepsilon}_{mn}(r) = \dot{E}_{nm}^{0} + \int_{V} \Gamma_{mnij}^{0}(r - r') \left[ \delta l_{ijkl}(r') \dot{\varepsilon}_{kl}(r') - \delta m_{ij}(r') \dot{T} \right] dV'$$
(20)

Where  $\Gamma^0$  denotes the modified Green tensor. A self-consistent approximation is chosen to solve Eq. (20) (Lipinski and Berveiller, 1989). This framework considers the effective overall medium has the same properties than the homogeneous reference one ( $L^0$  and  $M^0$  taken equal to  $L^{eff}$  and  $M^{eff}$ ) and that local stress and strain rates and local moduli are piecewise uniform functions. In that way, we obtain localization expressions between local and overall strains and we determine the effective behaviour of the polycrystal. This overall behaviour is now completely defined from knowledge of the polycrystalline internal structure evolution. Experimental features are well described by this self-consistent model (Fig. 3). This approach also derives the kinetics rule for the stress-induced transformation, according to Eqs. (11) and (14) we obtain:

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$$\dot{f} = \sum_{N} \left\{ \sum_{n} f_{N}^{n} \varphi^{n} \right\} = \sum_{N} \left\{ \sum_{n,m} (H^{nm})^{-1} \left[ (\varepsilon_{N}^{m})_{kl} \dot{\sigma}_{kl}^{N} - B\dot{T} \right] \varphi^{N} \right\}$$
(21)

From this equation, evolution of volume fraction of martensite with respect to loading parameters is successfully computed (Fig. 4) (Entemoyer et al., 1995; Patoor et al., 1996).



Fig. 3. Comparison between simulation and experimental results performed on a polycrystalline CuAlBe SMA for a superelastic test T = 60 °C.

One drawback in this analysis comes from the assumption of the existence of a uniform temperature field inside the entire body. Due to heat production generated by the martensitic transformation itself, or heat absorption associated to the reverse transformation, this assumption must be carefully checked. For general cases, temperature inside the body is not equal to external temperature,  $T^{ext}$ , imposed on the boundaries of the system (Chrysochoos et al., 1993; McCormick et al., 1993). Both local thermodynamical analysis at the RVE level and homogenisation process must be revisited considering the existence of a non-homogeneous temperature field denoted by  $\theta(r)$ . The local transformation strain rate field defined by Eq. (12) is now expressed as:

$$\dot{\varepsilon}(r) = g(r)\dot{\sigma}(r) + n(r)\dot{\theta}(r) \tag{22}$$

Non-uniformity of the temperature field turns the scale transition framework previously adopted to be no longer valid. As consequence, analysis presented in this section must be restricted to very slow strain rate. In this way it is possible to assume the thermal equilibrium  $\theta(r) = T^{ext}$  is reached at any point r. Nevertheless extension to this scheme to higher strain rates can be performed from the consideration of some experimental observations.



Fig. 4. Numerical determination of the kinetics of a stress-induced transformation for a superelastic uniaxial tensile test with a CuZnAl SMA.

#### 3. Physical origin of the strain rate effect

Martensitic transformation is a first order phase transition. According to this character, direct transformation from austenite to martensite is associated with a heat production (exothermic transformation) and reverse transformation from martensite to austenite produces heat absorption (endothermic transformation). These phenomena strongly modify the temperature field inside the material and have a great influence on the mechanical behaviour observed. On loading, heat production associated with the stress-induced transformation increases the temperature inside the material due to the temperature dependent character of superelasticity; a higher stress level is required to pursue the transformation. This produces an apparent increase of the transformation slope. Similar apparent increase is also observed on unloading. In this case, shrinkage of the martensite plates is associated with heat absorption that produces temperature decrease. A lower stress level is then required to go on with the reverse transformation. A relative amount of these overall evolutions is related to the transformation rate and to the thermal boundary conditions imposed (Fig. 5).

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Fig. 5. Strain rate effect on the superelastic behavior of a CuAlBe SMA (Entemeyer, 1996).

Existence of a non-homogeneous temperature field inside the material means the scale transition model presented in the previous section cannot be applied directly. Fortunately experimental considerations allow us to simplify this problem. In the polycrystalline sample, the transformation progress from many different places is scattered inside the material. Due to this relative uniform distribution of heat sources and high thermal conductivity in metals, a uniform evolution of the temperature can be assumed inside the material. This assumption leads us to consider that the characteristic time for heat propagation is at least one order of magnitude smaller than the characteristic time for mechanical loading. Very high strain rates like those obtained for impact loading are implicitly excluded from this analysis. This uniformity in the temperature field was experimentally established by Chrysochoos et al. (1993) on Cu-based alloys using an infrared camera. They observed a uniform increase of the sample temperature of around six degrees during superelastic tensile test for strain rate not exceeding  $10^{-3}$  s<sup>-1</sup>. McCormick et al. (1993) also report similar observations on NiTi alloys. They reported that the magnitude of this temperature change depends on transformation rate and specimen-environment heat transfer conditions. For mechanical testing in air of NiTi wires of 1-mm diameter and 26 mm gauge length, no temperature change was found for strain rates lower than  $10^{-4}$  s<sup>-1</sup> and near adiabatic condition was reached for  $10^{-2} \text{ s}^{-1}$ . On the same material tested in water, an isothermal behaviour was obtained up to  $10^{-3} \text{ s}^{-1}$  and strain rate of  $1 \text{ s}^{-1}$  was required for adiabatic regime. In the following we assume the existence of a uniform temperature field with a dfferent value of the imposed temperature test  $T^{ext}$ . This is equivalent to assume infinite heat

conduction rate inside the material. This assumption is realistic considering in SMAs the two phases have very close thermal conductivities and heat flux are faster than usual mechanical loading rate. Nevertheless before to introduce this assumption into the modelling we have to discuss lengthscale effects in thermal measurement. Leo et al. (1993) report a non-uniform temperature rise on NiTi wire for strain rate close to  $10^{-3} \text{ s}^{-1}$  in air. They determined the temperature history of the sample by placing several thermocouples along a length of 130 mm of a wire of diameter 0.65-mm. In fact, this observation is not in contradiction with the uniformity of the temperature field assumed in the present paper. Leo's experiment deals with a structural problem where geometrical factors are important. In our case, scale transition determination requires to take into account a maximum of 5000 grains. With an average grain size not exceeding 100 mm, the volume of this material element corresponds to a maximal wire length of 6 mm in Leo's problem. In such a length it is reasonable to consider if the temperature field remains uniform.

#### 4. Micromechanical modeling of the strain rate effect

In order to introduce strain rate effects into the scale transition framework used in this paper, we relate the uniform sample temperature, denoted by T in this section, to the transformation rate of martensite and to the test temperature,  $T^{ext}$ , which is imposed on the boundary of the volume element. Thermodynamical analysis allows to derive this overall relation. Let us consider a crystal of parent phase of volume V. The Helmholtz's free energy of this system is a function of total strain E, temperature T and internal variables  $f^n$ . From Eq. (5) Helmholtz's free energy per unit volume, denoted here by  $\Phi_V$  is expressed by (Patoor and Berveiller, 1997):

$$\Phi_V(E,T,f^n) = \frac{1}{2} \left( E_{ij} - E_{ij}^T \right) C_{ijkl}(E_{kl} - E_{kl}^T) + \frac{1}{2} \sum_n H^{nm} f^n f^m + B(T - T_0) - \sum_n f^n \quad (23)$$

In the following we use the Helmholtz's free energy per unit mass, defined as  $\Phi = \frac{1}{\rho} \Phi_V$  where  $\rho$  denotes the density. Application of the second principle of thermodynamic leads to the Clausius-Duhem inequality:

$$D = \sigma_{ij} \dot{E}_{ij} - \rho \sum_{n} \frac{\partial \Phi}{\partial f^n} \dot{f}^n - \frac{\vec{q}}{T} \operatorname{grad}(T) \ge 0$$
(24)

In this relation *D* is the dissipation and  $\vec{q}$  is the heat flux vector. According to the assumption of uniform temperature field inside the material we have:

$$\operatorname{grad}(T) = 0 \tag{25}$$

Considering the heat conduction satisfied an isotropic linear Fourier equation, Eq. (25) leads to heat flux vector equals to zero.

$$q = -k \operatorname{grad}(T) \tag{26}$$

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Where k denotes the thermal conductivity of the material. Dissipation expresses by Eq. (24) turns to:

$$D_1 = \sigma_{ij} \dot{E}_{ij} - \rho \sum_n \frac{\partial \Phi}{\partial f^n} \dot{f}^n \ge 0$$
(27)

If there is no volume source involved, Chrysochoos and Peyroux (1997) had established that the heat equation can be written as:

$$\rho C_f \dot{T} - k\Delta T = D_1 + \rho T \sum_n \frac{\partial^2 \Phi}{\partial T \partial f^n} \dot{f}^n = 0$$
(28)

Where  $\Delta T$  is the Laplacian of the temperature field. In our case we have  $\Delta T = 0$  and  $C_f$  denotes the specific heat at constant  $f^n$ . The thermomechanical coupling component and the intrinsic dissipation appear on the second part of Eq. (28). As established by Chrysochoos et al. (1993) intrinsic dissipation is very low with regards to the latent heat of transformation and we can neglect its influence on the evolution of the temperature inside the sample. Heat Eq. (28) turns into:

$$\rho C_f \dot{T} = \rho T \sum_n \frac{\partial^2 \Delta}{\partial T \partial f^n} \dot{f}^n \tag{29}$$

so according to expression (23) for the Helmholtz's free energy, the following expression is obtained to relate the variation of the temperature with the production rate of martensite:

$$C_f \dot{T} = \frac{B}{\rho} T \sum_n \dot{f}^n \tag{30}$$

All parameters involved in relation (30) are macroscopic quantities, then Eq. (30) can be easily generalised to polycrystalline material assuming infinite heat flux conduction inside the material and homogeneity of parameters  $C_f$ , B,  $\rho$ . The following overall relation is obtained:

$$\dot{T} = T \frac{B}{\rho C_f} \dot{f}$$
(31)

Eq. (31) relates the evolution of the temperature with respect to the global production rate of martensite. Heat exchange conditions with the surrounding can be taken into consideration introducing a heat exchange parameter denoted by h:

$$\rho C_f \dot{T} + h(T - T^{ext}) = TB\dot{f} \tag{32}$$

For h equals zero adiabatic loading condition is simulated. When h reaches an infinite value transformation occurs under isothermal condition and T equals the imposed test temperature.

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This coupling equation is introduced into the self-consistent modelling of the superelastic behaviour defined in section two of this paper (Entemeyer, 1996).

### 5. Numerical results

The coupling Eq. (32) introduces influence of strain rate effect into the self-consistent modelling of non-isothermal stress-induced martensitic transformation. At each incremental step of the applied loading, the uniform variation of temperature obtained inside the sample is computed according to the global amount of martensite produced, or reversed, during this step and to heat exchange conditions considered. In that way, the purely mechanical conditions imposed on superelasticity turns to a complex non-isothermal loading where both applied stress and temperature changed. This framework is applied to describe superelastic behaviour of copper-based alloys. Two applications are made. The first one consider the CuZnAl alloy that was used by Chrysochoos et al. (1993) to performed simultaneous measurement of mechanical response and temperature evolution. The second application deals with strain rate effects observed on a CuAlBe SMA characterised by Entemeyer (1996).

Table 1.

Material constants given by Chrysochoos et al. (1993) for a CuZnAl superelastic alloy and by Entemeyer (1996) for a CuAlBe SME

		CuZnAl	CuAlBe
Martensite start temperature	M <sub>s</sub>	15°C	-30°C
Austenite finish temperature	$A_f$	19.5°C	-12°C
Stress-temperature coefficient	$d\sigma/dT$	2 MPa/K	2 MPa/K
Young's modulus	Ε	70 GPa	60 GPa
Poisson's ration	ν	0.33	0.3
Density	ρ	7700 kg/m <sup>3</sup>	$7700 \text{ kg/m}^3$
Specific heat	$C_{f}$	393 J/kg/K	400 J/kg/K

### 5.1. Application to CuZnAl alloy

The only data available on simultaneous determination of both temperature evolution and mechanical behaviour in the superelastic alloy are these determined by Chrysochoos et al. (1993) using infrared thermography during a tensile test. Material data corresponding to this experiment are used in that section to validate the model presented in this paper. Chrysochoos et al. (1993) had performed tensile tests on a Cu-25.63 wt.% Zn-4.2-wt.%-Al alloy at constant room temperature of 30 °C with absolute value of the strain rate less than  $10^{-3}$  s<sup>-1</sup>. Material data determined by Chrysochoos et al. (1993) on this alloy are given in Table 1. Fig. 6 recalls the simultaneous experimental measurement obtained.

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Fig. 6. Simultaneous evolution of the temperature (b) and of the superelastic mechanical behavior (a) reported by Chrysochoos et al. (1993) on a CuZnAl polycrystalline alloy. Diagram (a) shows the stress-strain curves obtained and diagram (b) gives the simultaneous time evolution of imposed strain  $\varepsilon$  and measured sample temperature  $\theta$ .

These results are compared with numerical simulation performed using the present approach. Macroscopic material parameters given for this alloy in Table 1 are used to feed this simulation and the microstructural parameters needed by the model are estimated considering usual data for this kind of alloy. Crystallographic data like the habit plane normal **n**, the transformation direction **m** and the amplitude g of the transformation displacement which are required to define the transformation strain associated to each variant (Patoor et al., 1988) are those experimentally determined by DeVos et al. (1978) for CuZnAl alloys. Numerical values for the components of the interaction matrix that appears in constitutive Eq. (12) are chosen according to the results of several micromechanical determinations on this kind of alloy (Fassi Fehri et al., 1987; Bensalah, 1992). These works established this matrix is composed of two kinds of term, weak interaction terms H1 for self-accommodated variants and strong interaction terms H2 for incompatible variants. All these microstructural parameters are listed in Table 2. In order to compute the overall behaviour of a polycrystalline sample from scale transition method with grains as VER, the granular structure is described by 100 spherical grains randomly oriented to give an isotropic crystallographic texture. Numerical results obtained are given in Fig. 7.

Parameter *h*, which appears in Eq. (32), is the only model parameter that needs to be adjusted in order to take into account the particular heat exchange conditions used in the Chrysochoos' experiment, This parameter is fitted according to the mechanical behaviour of Fig. 6a. The choice of a heat exchange parameter *h* equal to  $5 \times 10^{-2}$  W/cm<sup>3</sup>/K gives similar mechanical results for Fig. 6a and 7a. This value is reasonable with regards to the experimental conditions.

From Figs. 6b and 7b we observe that evolutions of the sample temperature computed for the same loading-unloading sequence are in good agreement with the experimental infrared measurement. From this result we may conclude that introduction of Eq. (32) into the self consistent scheme gives a good representation of thermomechanical coupling generated by the latent heat of transformation for superelastic alloys.

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Table 2.

Microstructural parameters required by the scale transition approach used in this paper to simulate the overall response of CuZnAl and CuAlBe SMAs

		CuZnAl	CuAlBe
Habit plane normal	$n_1, n_2, n_3$	0.182; 0.721; 0.669	0.168; 0.705; 0.688
Direction of transformation	$m_1, m_2, m_3$	0.165; 0.655; -0.373	0.151; 0.846; -0.904
Transformation displacement	g	0,23	0,22
Compatible interaction	H1	μ/1000	μ/350
Incompatible interaction	H2	μ/200	μ/70
Crystallographic texture		Isotropic	Isotropic
Number of grains		100	100
Grain shape		Spherical	Spherical



Fig. 7. Numerical determination of the simultaneous evolution of the temperature (b) and of the superelastic mechanical behaviour (b) computed for a CuZnAl polycrystalline alloy at a strain equal to  $5 \times 10^{-2} \text{ s}^{-1}$ .

#### 5.2. Application for CuAlBe alloy

In order to show evidence of the strain rate effect on superelastic alloy, tensile tests were performed at different strain rates on a Cu- 11.6 wt.% Al- 0.5 wt.% Be polycrystalline alloy. Wires diameter 1.5 mm were supplied by Trefimetaux, Serrifontaine, France. These samples were annealed at 690 °C for 5 min and air-cooled. Grain size observed after this treatment is around 300  $\mu$ m and the  $M_s$  temperature is equal to -30 °C. Material data for this alloy are given in Table 1. Wires were tested in displacement control with a screw driven mechanical machine and a 100 kN load cell. A length of 100 mm of wire was considered between the grips. Strain was measured using an extensometer with a gage length of 10 mm. For testing at various temperatures, specimens were kept in a chamber where the temperature was held constant. There was no measurement of the temperature change induced by the mechanical loading. Stress-strain

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curves determined at  $T^{ext} = 60^{\circ}C$  for strain rates equal to 2.3  $10^{-5}s^{-1}$  and 1.4  $10^{-2}s^{-1}$  are presented in Fig. 5.

Numerical simulations for this alloy were performed using material parameters in Table 1 and microstructure data listed in Table 2 for CuAlBe alloy. Numerical values given in Table 2 for this alloy, come from crystallographic measurement performed by Moreau et al. (1995) on the same material and from a micromechanical determination of the interaction matrix for CuAlBe SMA (Siredey et al., 1999). Like for CuZnAl case, the granular structure is described considering 100 spherical grains randomly oriented. For each strain rate, the heat exchange parameter *h* required by Eq. (32) is estimated. We consider isothermal condition at strain rate equal to  $2.3 \ 10^{-5} \text{s}^{-1}$ , so *h* is taken equal to infinite. Adiabatic condition is assumed for strain rate equal to  $1.4 \ 10^{-2} \text{s}^{-1}$  and *h* is taken equal to zero for this second loading condition. Numerical results obtained using these two values of parameter *h* are given in Fig. 8. Comparison with Fig. 5 shows that the increase in the  $d\sigma/d\varepsilon$  coefficient with the strain rate is well described and the 5 °C temperature rise numerically determined for adiabatic condition is in agreement with observations of Chrysochoos et al. (1993). We may conclude that major rate dependant effects observed in superelastic behaviour are well described by the proposed micromechanical approach.



Fig. 8. Numerical simulation of the strain rate sensitivity on the superelastic behaviour for the CuAlBe SMA presented in Fig. 5.

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#### 6. Conclusion

Thermodynamical analysis and modern scale transition approaches are successfully applied to model the superthermoelastic behaviour which deals with simultaneous variation in the imposed temperature and applied stress. We present an extension of this scheme by including the latent heat contribution in order to describe the strain rate sensitivity in superelastic behaviour. From the assumption of the existence of a uniform temperature field different to the imposed test temperature, the heat equation resolution leads to a coupling relation between temperature changes in the sample and global production rate of martensite induced by the loading.

This work constitutes the first study which considers this problem with a fully 3-D framework. A 3-D polycrystalline model is used and, from the uniform temperature assumption and the introduction of a global heat exchange parameter with the surrounding, the heat equation is solved without any structural consideration. In that way extension to different kinds of loading conditions is straightforward.

Major rate dependent effects observed in superelastic behaviour are well described by the proposed micromechanical approach. Temperature variation, computes from this numerical simulation, has the same order of magnitude than that measured by Chrysochoos et al. (1993). Strain rate sensitivity of the stress-strain response for uniaxial tensile test is in agreement with that experimentally observed.

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