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Birgitta Dresp, Chiara Silvestri, René Motro. Which geometric model for the curvature of 2-D shape contours?. *Spatial Vision*, 2007, 20 (3), pp.219-64. 10.1163/156856807780421165 . hal-02168826

**HAL Id: hal-02168826**

**<https://hal.science/hal-02168826v1>**

Submitted on 25 Sep 2024

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# Which geometric model for the curvature of 2-D shape contours?

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**Abstract**—We investigated the geometric representations underlying the perception of 2-D contour curvature. 88 arcs representing lower and upper halves of concentric circles, or halves of ellipses derived mathematically through planar projection by affinity with the circles, a special case of Newton's transform, were generated to produce curved line segments with negative and positive curvature and varying sagitta (sag) and/or aspect ratio. Aspect ratio is defined here as the ratio between the sagitta and the chord-length of a given arc. The geometric properties of the arcs suggest a regrouping into four structural models. The 88 stimuli were presented in random order to 16 observers eight of whom were experienced in the mathematical and visual analysis of 2-D curvature ('expert observers'), and eight of whom were not ('non-expert observers'). Observers had to give a number, on a psychophysical scale from 0 to 10, that was to reflect the magnitude of curvature they perceived in a given arc. The results show that the subjective magnitude of curvature increases exponentially with the aspect ratio and linearly with the sagitta of the arcs for both experts and non-experts. Statistical analysis of the correlation coefficients of linear fits to individual data represented on a logarithmic scale reveals significantly higher correlation coefficients for aspect ratio than for sagitta. The difference is not significant when curves with the longest chords only ( $7^\circ$ – $10^\circ$ ) are considered. The geometric model that produces the best psychometric functions is described by a combination of arcs of vertically and horizontally oriented ellipses, indicating that perceptual sensations of 2-D contour curvature are based on geometric representations that suggest properties of 3-D structures. A 'buckled bar model' is shown to optimally account for the perceptual data of all observers with the exception of one expert. His perceptual data can be linked to a more analytical, less 'naturalistic' representation originating from a specific perceptual experience, which is discussed. It is concluded that the structural properties of 'real' objects are likely to determine even the most basic geometric representations underlying the perception of curvature in 2-D images. A specific perceptual learning experience may engender changes in such representations.

*Keywords:* 2-D contour curvature; geometric cues; psychometric function; laws of sensation; shape perception; perceptual learning.

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## INTRODUCTION

Recent studies on the possible link between 2-D shape perception and the known properties of 3-D objects in the ‘real’ world have led to suggest that the perception of 2-D shape properties is largely determined by statistical relationships between geometrical characteristics of image representations and their physical sources in ‘natural’ visual environments (Howe and Purves, 2005). As discussed earlier by others (e.g. Attneave, 1954; Ogilvie and Daicar, 1967; Wilson and Richards, 1989; Morgan, 2005), the visual perception of 2-D curvature plays an important role in form perception and object recognition. Curvature may be particularly useful to the processes that allow the human perceptual system to generate 3-D representations of complex objects and scenes on the basis of simple 2-D contour images. Also, 2-D curvature has been shown to contribute to the apparent non-rigidity of objects and parts (e.g. Cavanagh and von Grünau, 1989), thereby providing ecologically significant cues for the perception of facial expressions (Lyon *et al.*, 2000). 3-D shape representation may be enabled by local shape biases favouring symmetry or other structural regularities in the 2-D image, or by combinations of pictorial image cues (e.g. Biederman, 1987; Koenderink and van Doorn, 2003). Visual experience and learned knowledge about object properties, i.e. learned associations between specific two-dimensional projections and their correlated three-dimensional structures, may be another important factor (e.g. Biederman, 1987; Sinha and Poggio, 1996; Howe and Purves, 2005).

The 2-D geometry of curves is abundantly exploited by design engineers and architects in the conceptual design of complex spatial structures (see Fig. 1) such as spherical domes or modern free-form-design structures, so-called ‘blob designs’, in general. The use of curvilinear spaces in architecture dates back to the dawn of building shelter. Vernacular architecture is, by the nature of the materials and construction techniques used, based on curved shapes even though descriptive 2-D geometry was not necessarily referred to then for the planning and execution of building projects. It has been suggested that evolution may have produced specialized brain mechanisms that efficiently exploit the geometry of visual perceptual space (Heeley and Buchanan-Smith, 1996; Foley *et al.*, 2004). However, neither the brain processes through which 2-D curve geometry would enable 3-D shape representation, nor which of the geometrical cues available in a simple 2-D curve would best account for its perceived quality in terms of a shape, have been hitherto identified.

The early visual processing of curved line segments has been amply investigated in psychophysical studies using two-alternative forced-choice (2 AFC) procedures for measuring visual discrimination thresholds. In such experiments, observers have to compare briefly presented curve segments and make a rapid decision regarding local differences in curvature, or curve orientation.



**Figure 1.** In the ‘real’ world, geometric 2-D representations of spatial structures provide design models widely exploited by visual 3-D rendering techniques in modern architecture. Some of the geometric models commonly used for visual rendering inspired the 2-D geometric models tested in our experiment here. Certain structural properties of our models bear a relationship with structural properties of curved objects occurring in the visual world that surrounds us. Such structural properties may have primed man’s visual perceptual experience of 3-D curved space since the dawn of building shelter and may help us gain insight into the perceptual processes through which shape representations are generated on the basis of 2-D geometric regularities or cues.

### *Local visual processing of 2-D curvature*

The ability of human observers to detect curvature in 2-D line stimuli is highly developed and falls within the domain of so-called ‘visual hyperacuity’. This is reflected by the fact that the spatial precision with which man can distinguish a curve from a straight line represents only one fifth of the physical spacing between two neighbouring visual receptors (cones), and one tenth of the smallest receptive field centre of ganglion cells found in the primate retina (Watt and Andrews, 1982; Watt *et al.*, 1987). This precision with which we are capable of telling a curve from a perfectly straight line seems to rely on geometric cues two of which have been identified in psychophysical experiments using methods of relative perceptual judgement (e.g. Foster *et al.*, 1993; Whitaker and McGraw, 1998). One such cue is the distance between the line or chord that joins the two ends of a curve and the parallel line tangent to that curve. In terms of geometry, this cue is referred to as the sagitta, or sag (Foster *et al.*, 1993), of the arc formed by a curved line. The other one is the sagitta divided by the length of the chord joining the two ends of a curve, i.e. the ratio between sagitta and chordlength of the arc formed by a curved line. This geometric cue has been referred to previously as aspect ratio by others (Whitaker and McGraw, 1998). Its effect on visual performance in curve discrimination tasks was found to be scale-invariant since images of curves with identical aspect ratios but of different sizes produced identical detection thresholds.

Scale-invariance of visual processing represents a certain functional or ecological advantage (e.g. Whitaker and McGraw, 1998) and, among the different geometric cues available in an image, the visual system may prefer cues providing such an advantage. While sagitta gives a strictly local cue of relative height in the visual field, or position of the point of maximum curvature in respect to other points of the curve, aspect ratio conveys more global information about the area covered by the curve. The efficient use of either cue by the visual system might depend on stimulus attributes such as the length or presentation duration of curve segments. Curvature discrimination was, for example, found to be optimal for stimuli with a length of at least 30 min of visual arc (e.g. Watt, 1984; Kramer and Fahle, 1995). Also, certain characteristics of the discrimination performance were reported to change with stimulus duration (Ferraro and Foster, 1986; Watt, 1987). Such changes may be reflected by an increase in visual sensitivity to curvature for durations up to two seconds, and by a change from discrete to continuous performance for increasing stimulus durations, between 100 milliseconds and longer. Rapid curvature detection with curved line targets presented for 100 milliseconds within arrays of multiple curves (so-called ‘distractors’) was found to produce variations in visual performance as a function of the sagitta and number of ‘distractors’, interpreted in terms of a discrete processing model based on parallel category coding of small and briefly presented curve segments (Foster and Savage, 2002). The temporal properties and length tuning of rapid curvature discrimination in 2 AFC tasks suggest that the local visual processing of curvature information may rely on orientation selective detectors (Koenderink and Richards, 1988; Zetzsche and Barth, 1990) optimally sensitive to a narrow window of variations in size and time course of the stimuli. Some evidence that orientation selective cortical neurons with so-called end-stopped receptive field properties (e.g. Hubel and Wiesel, 1959) may serve this purpose has been made available through single-cell recordings from the visual brains of mammals (e.g. Dobbins *et al.*, 1987; see also Dobbins *et al.*, 1989).

Visual curvature discrimination measured in 2 AFC tasks may provide an account for the brain mechanisms through which the visual system extracts strictly local information of 2-D curvature. How these mechanisms are used in perception to generate the global 2-D shape interpretation of a curve in terms of its principal object property ‘curvature’ remains unanswered at the level of rapid visual discrimination.

### *Measuring sensation to reveal the structural laws of 2-D contour curvature*

In classical psychophysics, visual sensations are considered to reflect the expression of brain mechanisms involved in the processing of spatial or temporal structural information carried by stimuli. Thurstone (1927) and subsequently Stevens (1956, 1975) introduced psychophysical scales, so-called sensation scales, as measures for variations in the response of the perceptual system to variations in a given property such as loudness, brightness, or heat for a given class of stimuli such as sounds, lights or temperatures. The psychometric functions which describe the scaled

responses of the perceptual system to critical dimensions of a given stimulation are referred to as psychophysical laws, or laws of sensation.

In this study, we employ a psychophysical scale to measure variations in the response of the visual perceptual system to variations in cues for 2-D contour curvature provided by curve segments. While threshold-based approaches to the rapid visual discrimination of curves have permitted determination of individual just noticeable differences between two briefly presented curves, our psychophysical approach measures the sensation of curvature that is elicited by a given curved stimulus. Such a procedure probes a higher level of shape processing compared with the threshold-based approach.

Curves were generated using descriptive geometry for the production of four distinct structural models, which will be described later herein. It has been pointed out earlier that the curvature of 2-D line segments is a potent cue to shape (e.g. Attneave, 1954; Morgan, 2005). If it is true that the perception of 2-D shape properties is largely determined by statistical relationships between geometrical or structural characteristics of 2-D stimuli and their physical sources in 'natural' visual environments, then the potential of 2-D curves for activating shape representations should be optimal with stimuli that can be related to such physical sources. The capacity of the visual brain to rely on parts for the generation of representations of wholes (e.g. Biederman, 1987), and on segments for the perceptual completion of object contours, describes one of the most striking abilities visual perception has evolved (see Pessoa *et al.*, 1998, for a review). Statistically likely structural relationships between 2-D geometry and 'natural' environments may involve internal representations, or sensation scales, which exploit this perceptual ability.

*Do expert brains optimally use the geometry of visual space for perception?*

To optimize form and design, engineers and architects use descriptive geometry through computerized tools that exploit shape grammars such as Formian for the 2-D visualization of 3-D shapes. This creative process is referred to as 'digital rendering' by experts in the field. Digital rendering puts into practice the repeated visual and mental manipulation of 2-D curves, which are progressively transformed into increasingly complex 2-D shapes to create images representing a 3-D 'natural' world. Does such a repeated visual and mental manipulation of curve geometry generate a learning process that will optimize, or change the use of geometric cues for perception? In other words, when engineers or architects learn to optimize the use of the 2-D geometric properties of curves in the design process, do they implicitly learn to optimally exploit these properties for the processing of statistical relationships between geometrical or structural characteristics of 2-D shapes and their physical sources in 'natural' visual environments?

### The spatial geometry of 2-D curvature

Contour curvature of ‘natural’ physical objects reveals a wide range of round or oblong shapes that relate to two geometric 2-D shape descriptions: circles and ellipses. In terms of pure geometry, curves or arcs derived from circles and ellipses share certain properties, the circle being a particular case of the ellipse. Ellipses may be described as hyperbolic images of points falling on straight lines or circles. The Cartesian description of circles and ellipses provides the two principal geometric models for the possible generation of 2-D contour curvature by the human perceptual system. From these two principal geometric models, four major structural models for 2-D contour curvature, with comparable mathematical properties, can be obtained. Each of these structural models is mathematically derived from the general model represented by that of concentric circles. This allows the generation of a wide range of possible curves with variable aspect ratio, variable sagitta, or both. Here, we define aspect ratio as the ratio between sagitta and chord-length of the arc formed by a curve, as in Whitaker and McGraw (1998).

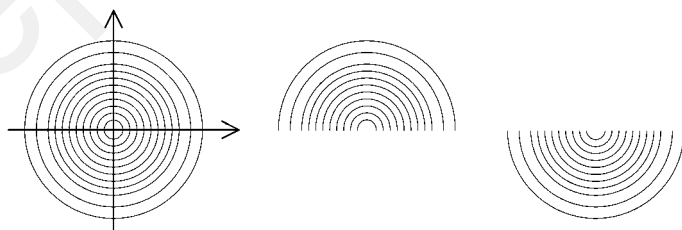
*Arcs of concentric circles.* The Cartesian description of a circle with centre 0 is

$$R_{(1)}^2 = x^2 + y^2,$$

where  $R$  is the radius of the circle, and  $x$  and  $y$  the two-dimensional spatial coordinates of the points falling on its perimeter. Concentric circles share the same centre (here 0). A second concentric circle may be obtained from a first one by

$$R_{(2)}^2 = (R_{(1)} \pm \delta R)^2.$$

The halves of several concentric circles can be defined as arcs or curves with varying sagitta and constant aspect ratio (see Fig. 2). The sagitta of the arc or curve describing one half of a circle is identical to the radius ( $R$ ) of the full circle, the length of the chord of the arc is twice the radius ( $R \times 2$ ) of the full circle. Such a combination of curves, or arcs, does not suggest a 3-D structure (see again Fig. 2). This structural model will be, as of now, referred to as model 1.



**Figure 2.** Arcs defined by the halves of several concentric circles (*left*) produce curves with ‘positive’ (*middle*) or ‘negative’ curvature (*right*) and identical aspect ratios. Aspect ratio is here defined as the ratio between the sagitta of a given arc and the length of its chord. The sagitta of an arc described by one half of a circle is identical to the radius ( $R$ ) of the full circle; the chord-length of an arc is identical to twice the radius ( $R \times 2$ ) of the full circle.

*Arcs of vertical ellipses derived from concentric circles.* For straightforward comparison, in terms of pure geometry, between the four structural models referred to here, we derived ellipses directly from the model based on the concentric circles. In Cartesian space, an ellipse can be defined as the projected image of two concentric circles. The image  $(x, y) = (b^x, a^y)$  of a principal circle  $C_{(0,a)}$  and the image  $(x, y) = ((a/b)x, y)$  of a secondary circle  $C_{(0,b)}$  describe what is referred to as the planar projection of ellipses by affinity with circles (see Fig. 2), or sometimes also as a particular case of Newton's transform (e.g. Lockwood, 1967; Yates, 1974). Planar projections by affinity are widely used in digital rendering to create 3-D effects in images.

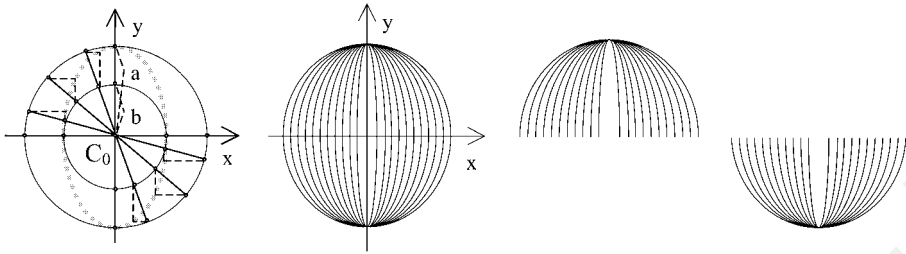
The halves of vertical ellipses thus obtained from several concentric circles can be defined as arcs or curves with constant sagitta and varying chord-length and aspect ratio (see Fig. 2). The sagitta of each such arc is identical to the radius ( $R$ ) of the principal circle  $C_{(0,a)}$  the full ellipse is derived from. The chord-length of a given arc is identical to twice the radius ( $R \times 2$ ) of the secondary circle  $C_{(0,b)}$  the full ellipse is derived from. The combination of curves in this model suggests 3-D properties of a dome-like spatial structure that may occur in the 'natural world' (see Fig. 3). This structure will be, as of now, referred to as model 2.

*Arcs of horizontal ellipses derived from concentric circles.* Planar projection of ellipses by affinity with the concentric circles of model 1 also permits the generation of horizontal ellipses. The halves of these horizontal ellipses are defined as arcs or curves with varying sagitta, constant chord-length, and varying aspect ratio (see Fig. 4). The sagitta of each of these arcs is identical to the radius ( $R$ ) of the secondary circle  $C_{(0,b)}$  the full ellipse is derived from. The chord-length of the arcs is identical to twice the radius ( $R \times 2$ ) of the principal circle  $C_{(0,a)}$  from which the full ellipses are derived. Again, such a combination of elliptic arcs suggests a dome-like spatial structure with apparent 3-D properties that may represent a 'natural' structure. This structure will be, as of now, referred to as structural model 3.

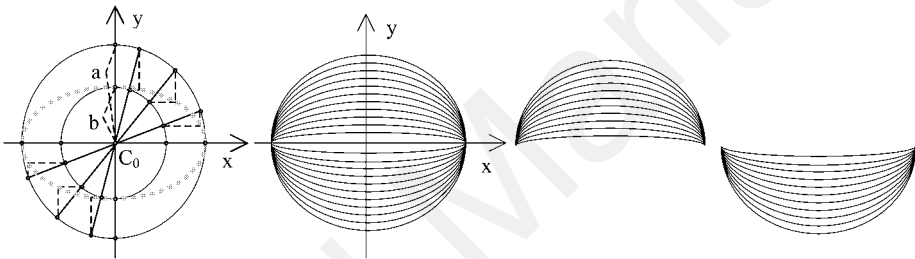
*Combining arcs of vertical and horizontal ellipses.* Combinations of vertical and horizontal ellipses derived from one and the same set of concentric circles permits the generation of arcs with varying sagitta, chord-length and aspect ratio. These geometric parameters can be straightforwardly derived from the radii of the concentric circles, as already described above with regard to Figs 3 and 4. The combination of curves in this model suggests a spatial structure consisting of a mesh of nodes and lines with apparent 3-D properties (see Fig. 5). This structure will be, as of now, referred to as structural model 4.

In the 'real' world, such mesh-like structures, with nodes and connecting elements, provide design models widely used in modern free-form architecture (see again Fig. 1). As illustrated in Figs 2–5, using one and the same 2-D model of concentric circles and then deriving vertical and horizontal ellipses through planar projection by affinity permits the generation of four spatial models for 2-D contour

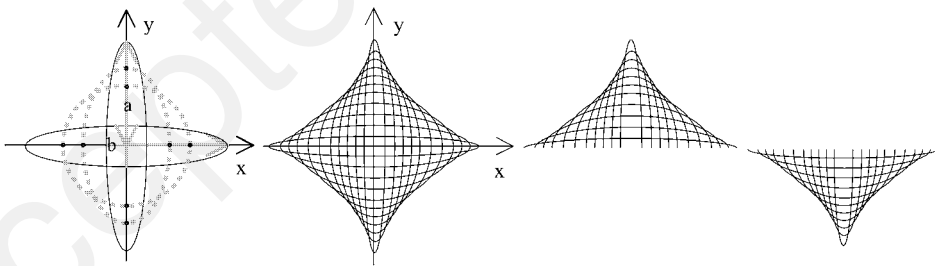




**Figure 3.** Arcs derived from vertical ellipses (*middle-left*) were obtained from concentric circles through planar projection by affinity, a special case of Newton's transform. This produces another set of curves with 'positive' (*middle-right*) or 'negative' curvature (*right*). The sagittae of these curves are identical while their aspect ratios vary. The sagitta of an arc described by one half of a given ellipse is identical to the radius ( $R$ ) of the principal circle from which the full ellipse is derived; the chord-length of an arc is identical to twice the radius ( $R \times 2$ ) of the secondary circle from which the full ellipse is derived.



**Figure 4.** Arcs derived from horizontal ellipses (*middle-left*) obtained by the same kind of transform (*left*) lead to a third model of curves with 'positive' (*middle-right*) or 'negative' curvature (*right*). The chord-lengths of these curves are identical, their sagittae and aspect ratios vary.



**Figure 5.** Combinations of arcs from vertical and horizontal ellipses (*left and middle-left*) produce curves (*middle-right and right*) with varying sagittae, chord-lengths and aspect ratios. This fourth curve model generates a curved, mesh-like spatial structure with apparent 3-D properties.

curvature each of which consists of a specific combination of circular or elliptic curves producing some kind of spatial structure. As can be seen in the examples, only the 2-D structures obtained from ellipses (models 2, 3 and 4) reveal apparent properties of 3-D shape. In architecture and design, such digital effects are gen-

erated through specific shape grammars with the aim to produce 2-D images that faithfully represent 3-D properties of 'natural' objects.

## EXPERIMENTAL

Which of the four geometric models considered herein provides the best account for the perception of the curvature of contour segments? Model 1 is based on a combination of concentric circles which does not seem to produce a structure suggesting 3-D properties, whereas models 2, 3 and 4 show structural models derived from ellipses the combination of which suggests spatial structures with apparent 3-D properties, such as dome-like structures (see again Figs 2 and 3) or a bulging mesh surface that looks like that of a tent (see again Fig. 5). Considering that the perception of 2-D shape properties is predicted to depend upon some statistical relationship between structural properties of a 2-D image representation and the properties of a 'natural' object (Howe and Purves, 2005), geometric models suggesting properties of 'natural' objects should generate stronger shape percepts. If this is so, then curved contour segments derived from planar projection of ellipses (structural models 2, 3 and 4) would be expected to produce stronger sensations of curvature than segments of concentric circles.

In addition, perceptual learning may be important for shape perception in 2-D images (e.g. Sinha and Poggio, 1996). The four structural models tested here may therefore produce different shape sensations when comparing perceptual judgements of laypeople to perceptual judgements of experts, such as design engineers or architects. If experts use the geometric cues contained in the stimuli more optimally, or differently, for shape representation compared with non-experts, then significant differences in sensation would be expected between the two groups.

### *Subjects*

Sixteen psychophysically untrained observers with normal or corrected-to-normal vision participated in the experiments. Eight (four women and four men) were students or professionals in biology or psychology and considered non-experts in the domain of geometry and spatial structures. The other eight observers (three women and five men) were students or professionals in architecture or civil engineering and all experienced users of spatial design tools such as AUTOCAD or CATIA. These observers were considered experts in the domain of descriptive geometry and spatial structures.

### *Stimuli*

The stimuli for the experiments were generated by a computer graphic design software tool (AUTOCAD) on an IBM computer (Pentium III) equipped with a standard colour screen with a display resolution of 1024 × 768 pixels. All arcs,

or stimuli, were derived, as explained in detail in the introduction, from eleven concentric circles the diameter of which varied between 1.2 and 10 degrees of visual angle. Lower and upper halves of these concentric circles and all the vertical and horizontal ellipses derived from them produced a total of 88 arcs with 'positive' (upward) and 'negative' (downward) curvature in the two-dimensional plane. 22 of these arcs were circular in shape, 66 were elliptic. In a given experimental session, these arcs were presented in random order on the computer screen to individual observers placed at a distance of approximately one meter from the screen. The 2-D geometry of these arcs, and how they generate four structural models to be tested in our experiment, are given in the Introduction and illustrated by Figs 2–5. The two individual geometric parameters of each arc that were put to the test, sagitta (1) and aspect ratio (2), are defined as (1) the distance between the chord of a given arc and the parallel line tangent to it, and as (2) the ratio between sagitta and chord-length. Arcs with 'positive' and 'negative' curvature derived from concentric circles (structural model 1) all had constant aspect ratio (0.5), their sagitta varied between 0.6 and 5 degrees of visual angle and their chord-lengths between 1.2 and 10 degrees. Arcs derived from the ellipses of structural model 2 had constant sagitta (5 degrees of visual angle), their aspect ratios varied between 0.5 and 4.2 and their chord-lengths between 1.2 and 9 degrees of visual angle. The sagitta of arcs of ellipses of structural model 3 varied between 0.6 and 4.5 degrees of visual angle with a constant chord-length of 10 degrees. Aspect ratio varied between 0.06 and 0.5. The sagitta of arcs of structural model 4 varied between 0.6 and 5 degrees of visual angle, the chord-lengths between 1.2 and 10 degrees and the aspect ratio between 0.06 and 4.2. All arcs were defined by a contour of the width of a single pixel. Their luminance (held constant at  $40 \text{ cd/m}^2$ ) was determined by means of a photometer (Minolta LS 110) used for calibrating grey levels (R–G–B combinations) of a square of 1 degree of visual angle, switched on in the centre of the dark screen. The luminance of the dark background of the screen on which the arcs were presented was held constant at  $2 \text{ cd/m}^2$ .

### *Procedure*

Observers were seated comfortably in a semi-dark room in front of the computer screen. They were told that they were going to view a series of arcs, one at a time, and were asked to produce a number between 0 and 10 that was to reflect the magnitude of the curvature they spontaneously perceived when a given arc came up on the screen. A straight line was shown at the beginning of the trials to make sure that each of the observers spontaneously replied 0 to rate its curvature, indicating that they had understood the principle of the psychophysical scaling procedure, although none of the 16 observers knew what a psychophysical scale actually was. No modulus for a curvature to be associated with the upper limit of the psychophysical scale (here 10) was given to avoid influencing the subjects' internal scales in favour of any one of the structural models tested. The 88 stimulus arcs and 12 straight lines of varying length were presented in random order. For each

observer, a different random sequence of stimuli was generated. The duration of presentation was not limited, but observers were encouraged to make their decisions as rapidly as possible. Typing the 'enter' key triggered the presentation of the next arc. Each of the 88 arcs was shown only once to a given observer within a single series of random presentations.

## RESULTS

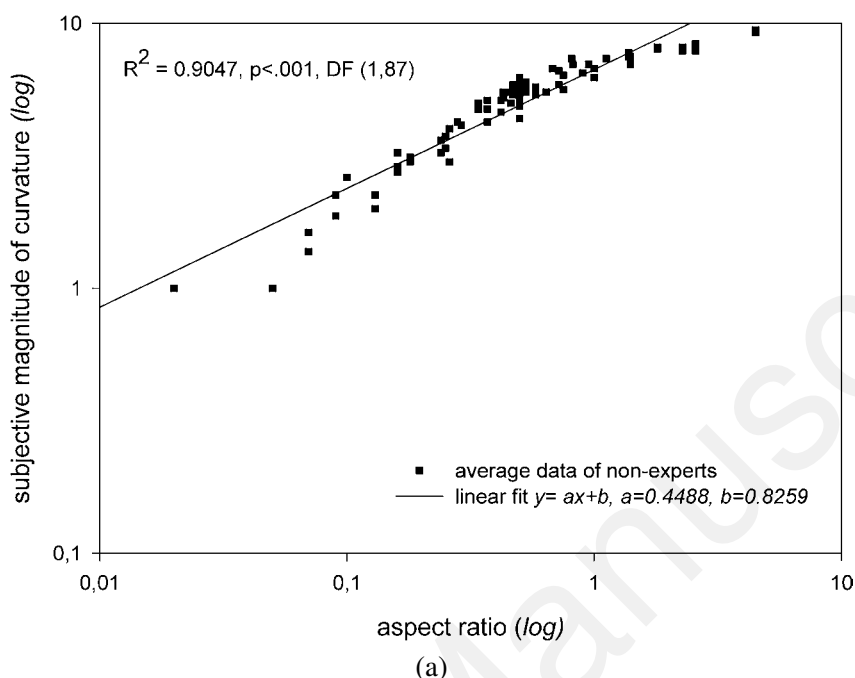
In previous, threshold-based studies investigating the rapid visual processing of curvature (see the Introduction), population statistics based on comparisons between means were applied to determine which of the independent variables tested would account best for the variance in curve discrimination thresholds (e.g. Foster *et al.*, 1993; Foster and Savage, 2002). Here, regression analysis was used to determine the strength of the relationship between physical magnitudes of sagitta and subjective magnitudes of curvature, and between physical magnitudes of aspect ratio and subjective magnitudes of curvature for each of the four structural models tested. Such an analysis allows us to determine the predictive power of a given independent variable, associated with a given structural model, based on the regression model that provides the best statistical fit for individual and/or average data of perception.

Separate analyses for data of non-experts and experts were performed. All data were analyzed individually before proceeding to analyses of average data. For each group of observers, the average magnitude of curvature was first analyzed as a function of the aspect ratio and the sagitta of the 88 arcs irrespective of the structural geometric model the arcs were derived from. Subsequently, separate analyses were conducted for each of the four structural geometric models.

### *Non-expert observers*

The data of the eight non-experts are shown in Figs 6 and 7. There were no noticeable differences between individuals of this group and, therefore, only average data are shown here.

*Subjective magnitude of curvature as a function of aspect ratio and sagitta.* Subjective magnitudes of curvature were analyzed as a function of the sagitta and the aspect ratio of the 88 arcs presented in the experiment. These data are shown in Fig. 6. Regression analyses of the average data plotted on a logarithmic scale were performed. These analyses reveal a good linear relationship between subjective magnitude of curvature and aspect ratio (Fig. 6(a)), and a somewhat poorer linear relationship between subjective magnitude of curvature and sagitta (Fig. 6(b)). The average linear function relating subjective magnitude to aspect ratio produced a noticeably higher correlation coefficient than the average function obtained for sagitta. The correlation coefficients of linear fits to the individual data (not shown



**Figure 6.** This figure shows average data of non-expert observers. Subjective magnitudes of curvature are plotted as a function of the sagitta (Figs 6(b), (d) and (f)) and the aspect ratio (Figs. 6(a) and (e)) of the arcs presented, with different symbols for the different ranges of chord-lengths. Regression analysis of the data plotted on a logarithmic scale (Figs 6(a) and (b)) reveals a good linear relationship between subjective magnitude of curvature and aspect ratio (Fig. 6(a)), and a somewhat poorer linear relationship between subjective magnitude of curvature and sagitta (Fig. 6(b)). The linear function relating subjective magnitude to aspect ratio is found to produce a noticeably higher correlation coefficient than the function obtained for sagitta. Statistical analysis ( $t$ -test) of the correlation coefficients of linear fits to individual data reveals a statistically significant difference ( $t(1, 14) = 7.9254, p < 0.001$ ) between correlation coefficients of functions fitted for aspect ratio and functions fitted for sagitta (Fig. 6(c)), with aspect ratio producing a statistically better linear fit to individual sensations of curvature than sagitta. The difference in correlation coefficients is no longer statistically significant when stimuli with the longest chords only are taken into account for data analysis.

here) revealed a systematic difference in correlation coefficients for observers from this group.

Statistical analysis ( $t$ -test) of the correlation coefficients of linear fits to the individual data reveals a statistically significant difference ( $t(1, 14) = 7.9254, p < 0.001$ ) between correlation coefficients of functions fitted for aspect ratio and functions fitted for sagitta. Correlation coefficients of linear fits to individual data as a function of observers' initials are shown in Fig. 6(c). The statistical difference between correlation coefficients reveals that aspect ratio produces a better linear fit to individual sensations of curvature than sagitta. However, if only arcs with the longest chord-lengths are taken into account for the analysis, the

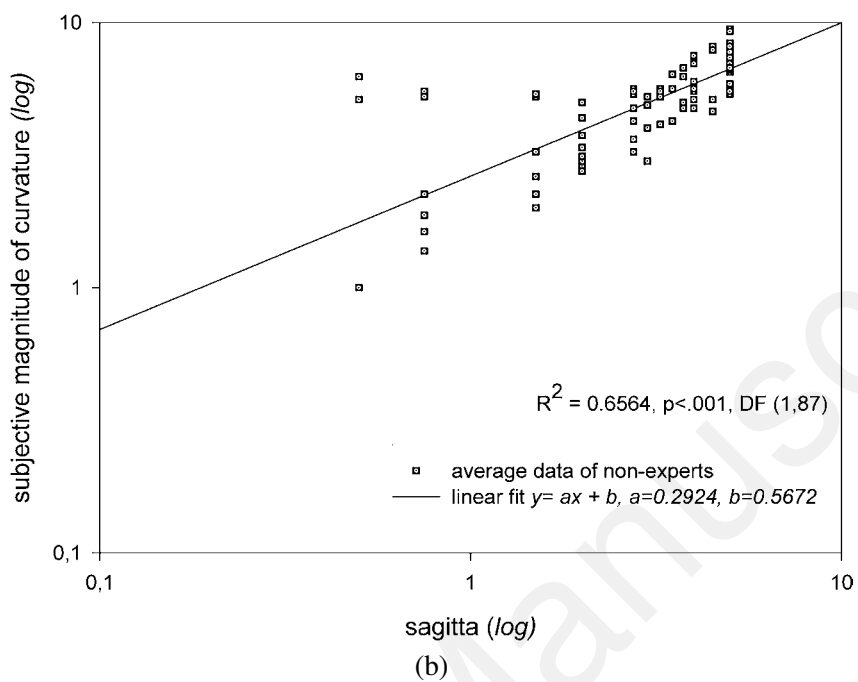


Figure 6. (Continued.)

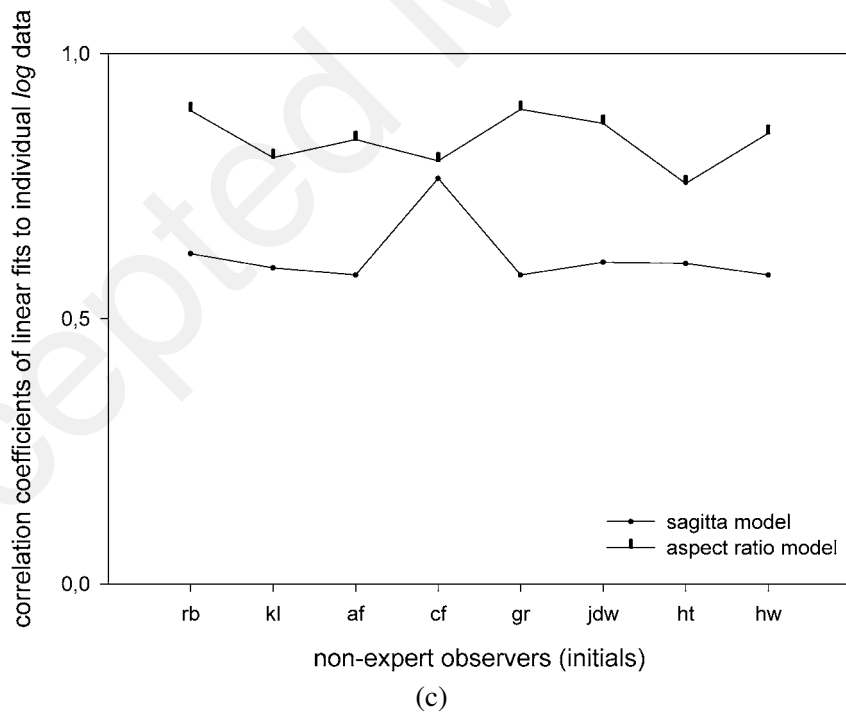


Figure 6. (Continued.)

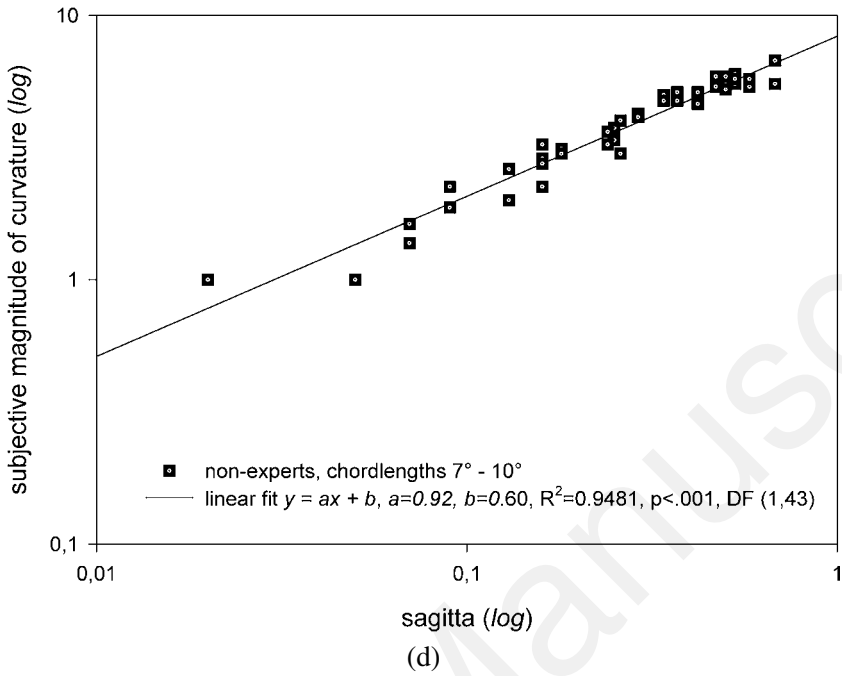


Figure 6. (Continued.)

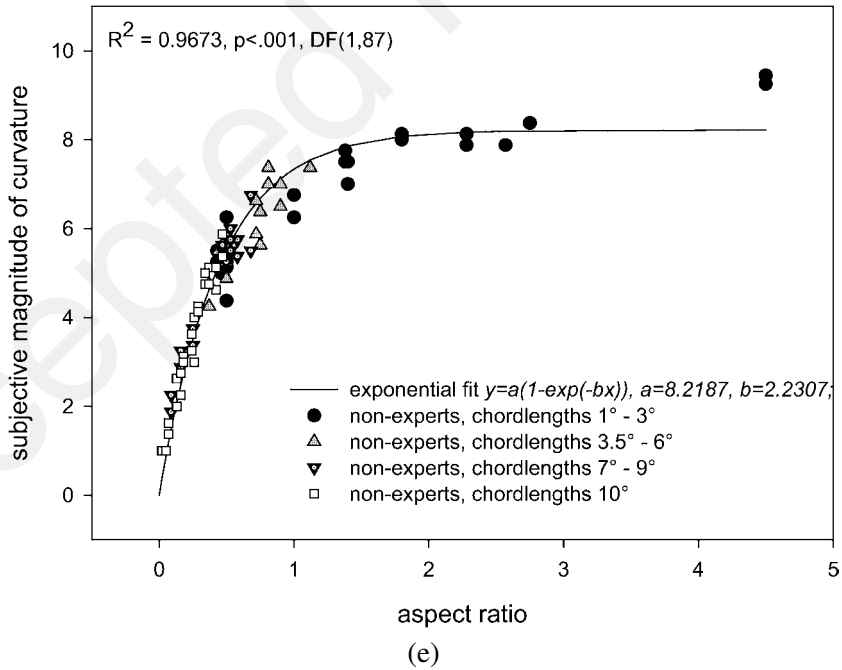
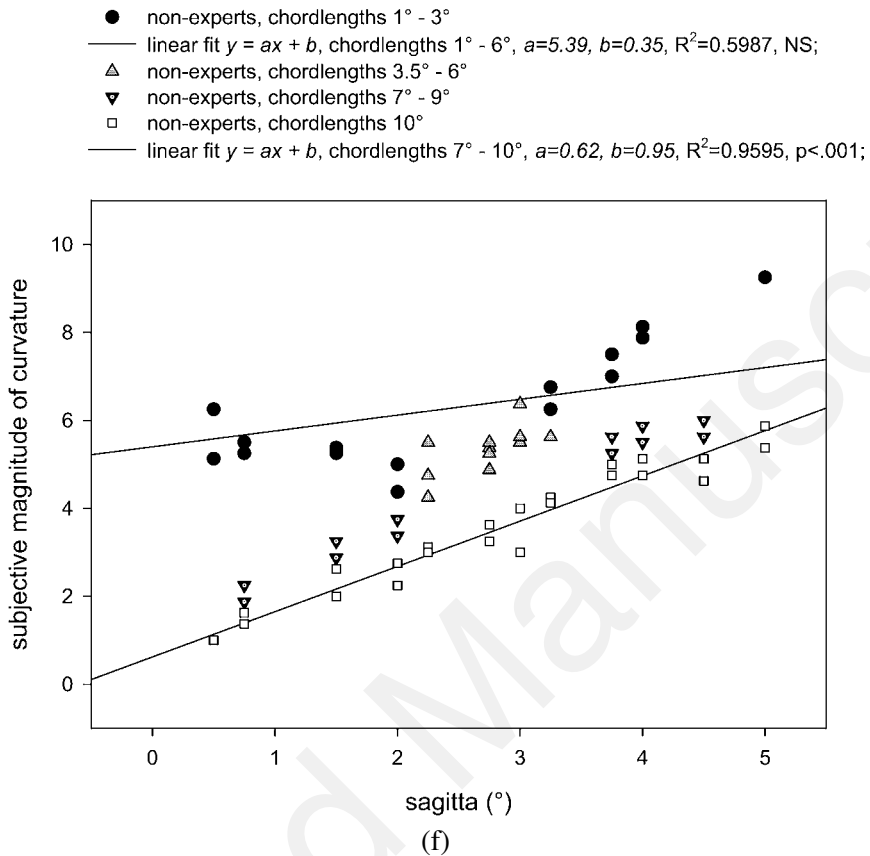


Figure 6. (Continued.)



**Figure 6.** (Continued.)

difference in effects of aspect ratio and sagitta is no longer statistically significant. This is explained by the interdependency between sagitta and chord-length. This interdependency is revealed by the fact that sagitta is a more reliable predictor of subjective magnitudes of curvature for stimuli with chord-lengths of 7–10 degrees of visual angle (Figs 6(d) and 6(f)).

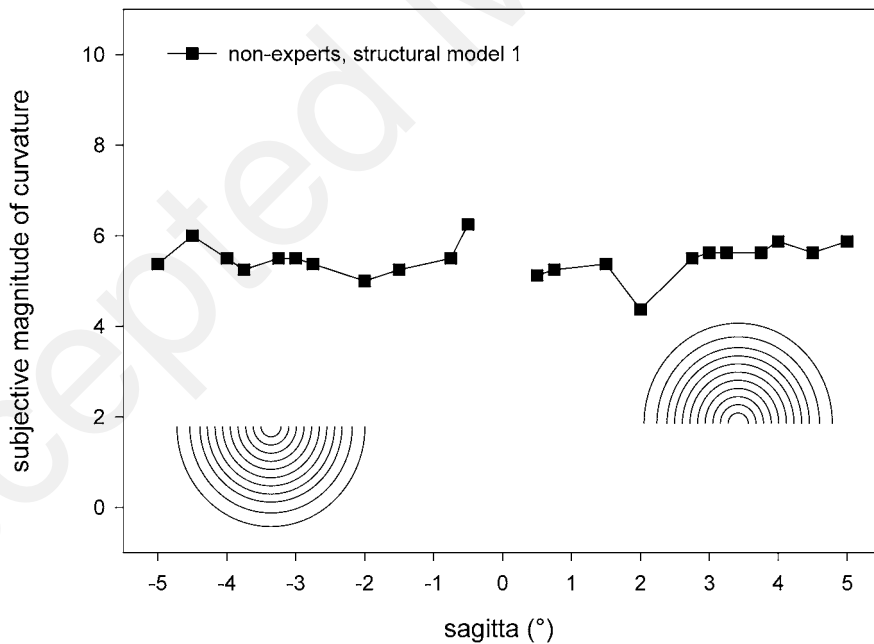
Psychometric functions were also fitted to the average data plotted on a linear scale. These analyses reveal that an exponential function provides the best fit for subjective magnitude of curvature as a function of aspect ratio (see Fig. 6(e)). A linear function was found to provide a reasonably satisfactory fit for subjective magnitude as a function of sagitta regardless of chord-length ( $R^2 = 0.7433$ ;  $p < 0.001$ ,  $DF(1, 87)$ ), with a noticeably better fit when chord-lengths between 7 and 10 degrees were considered only (Figs 6(d) and 6(f)). Comparing the global data for sagitta and aspect ratio, however, shows that subjective magnitude of curvature plotted as a function of sagitta (Fig. 6(b)) presents noticeably more scatter than subjective magnitude of curvature plotted as a function of aspect ratio (Fig. 6(a)).



*Curvature representation as a function of the four structural models.* The average data of non-expert observers were analyzed as a function of the four structural geometric models that served as a basis for generating the 88 arcs presented as stimuli. These data are shown in Fig. 7. The arcs of the concentric circles generated for the experiment provided model 1. These arcs have constant aspect ratio and varying sagitta. Analysis of subjective magnitude of curvature as a function of sagitta of arcs from this structural model shows that arcs from concentric circles produce constant subjective magnitude in the middle region of the psychophysical scale (Fig. 7(a)). Arcs with ‘negative’ and ‘positive’ curvature produce symmetric curvature.

The arcs of the vertical ellipses providing model 2, derived from the concentric circles of model 1, have constant sagitta and varying aspect ratio. For this model, subjective magnitude of curvature increases symmetrically with increasing aspect ratio of arcs with ‘negative’ and ‘positive’ curvature (see Fig. 7(b)). This increase covers only the upper half of the psychophysical scale. The function that best describes the relationship between subjective magnitude of curvature and the aspect ratio of the arcs, irrespective of the polarity of their curvature, is found to be an exponential function (see Fig. 7(c)).

The arcs of the horizontal ellipses providing model 3, also derived from the concentric circles of model 1, have varying sagitta and varying aspect ratio.



(a)

**Figure 7.** This figure shows average data of non-expert observers as a function of the four geometric models that served as a basis for generating the 88 arcs presented in the experiments.

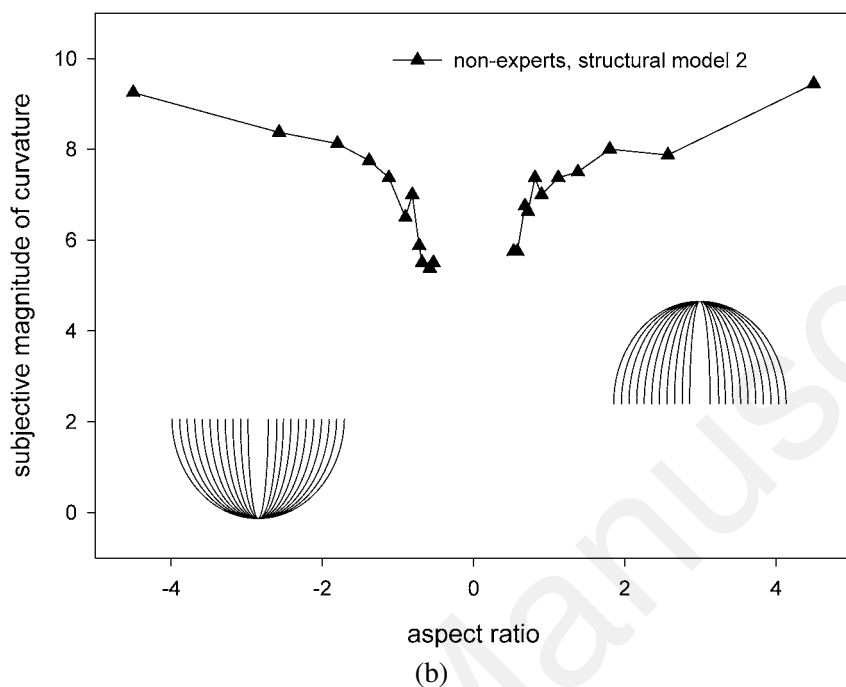


Figure 7. (Continued.)

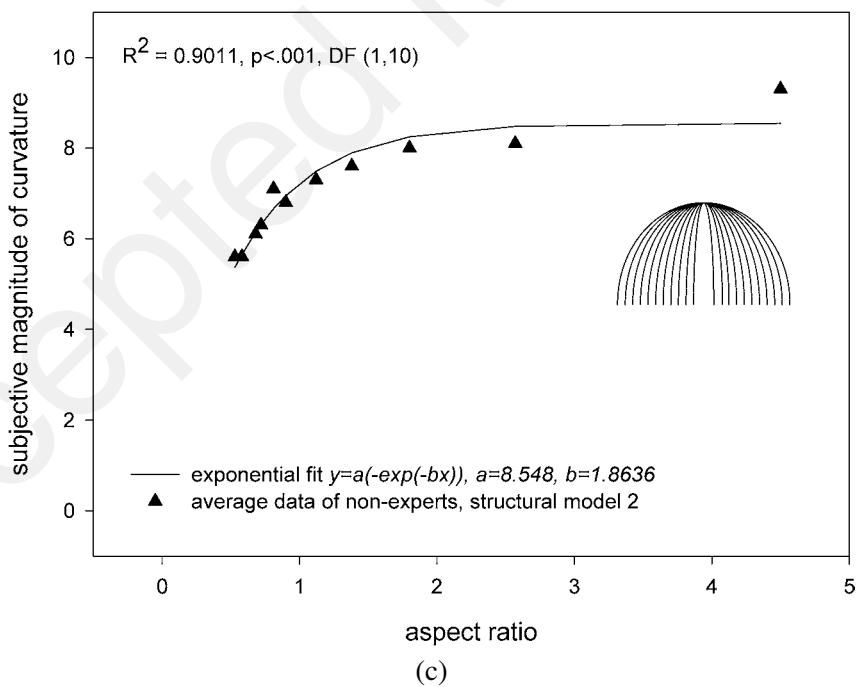


Figure 7. (Continued.)

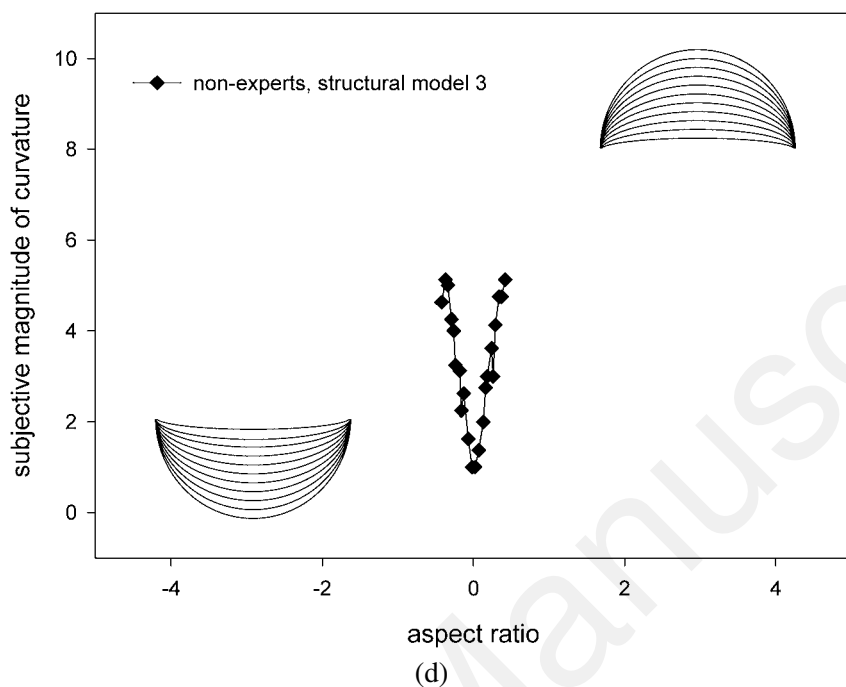


Figure 7. (Continued.)

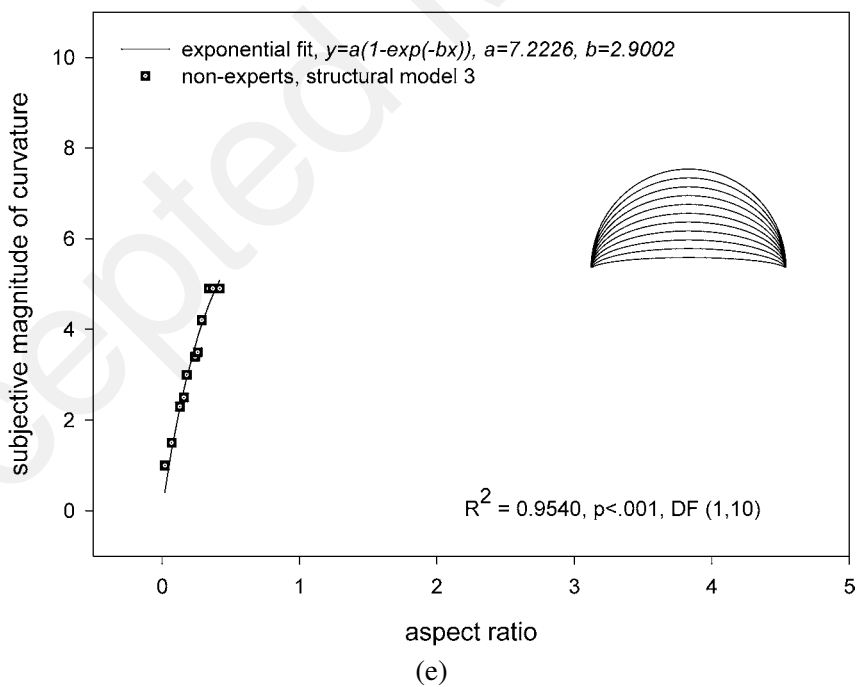


Figure 7. (Continued.)

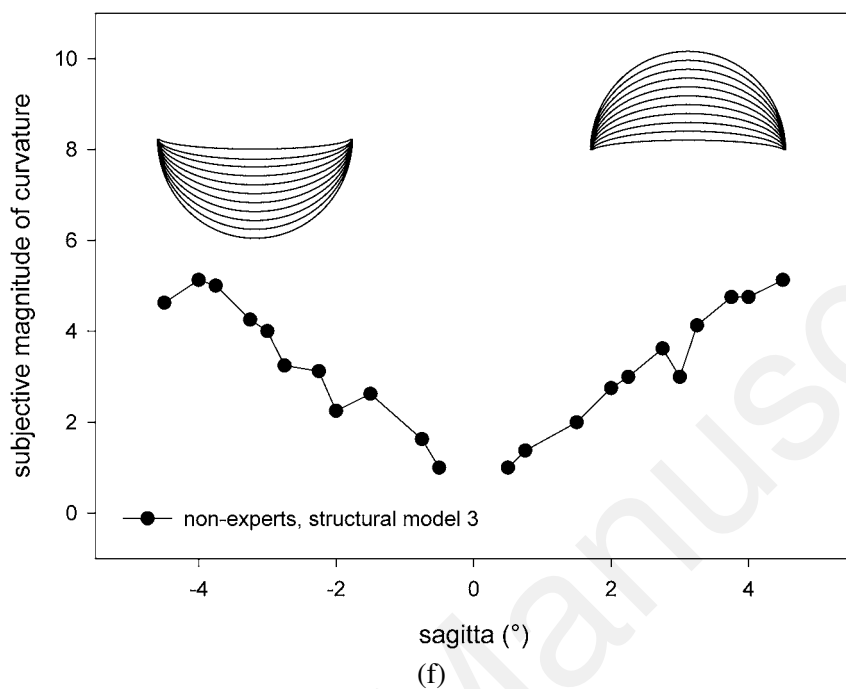


Figure 7. (Continued.)

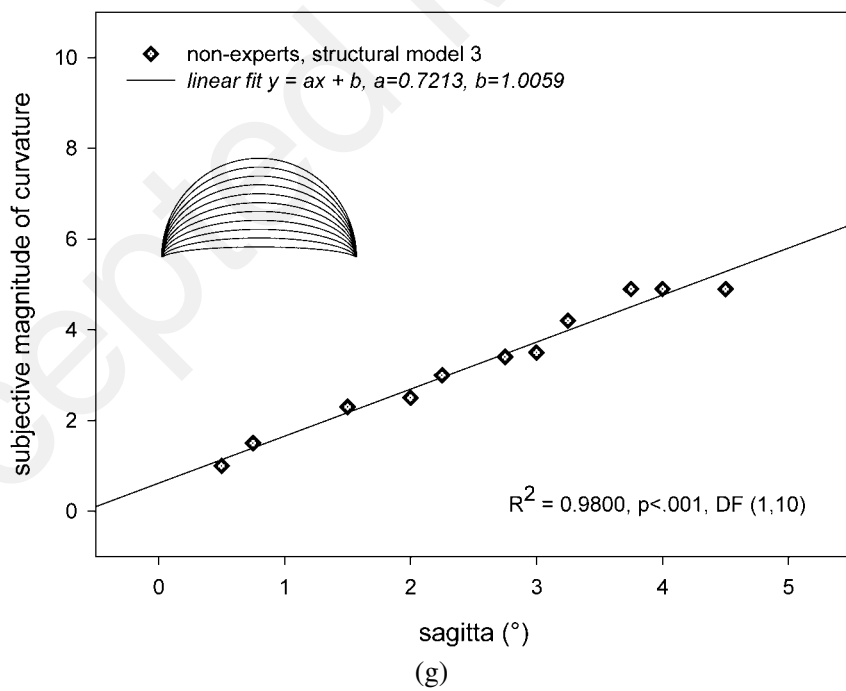


Figure 7. (Continued.)

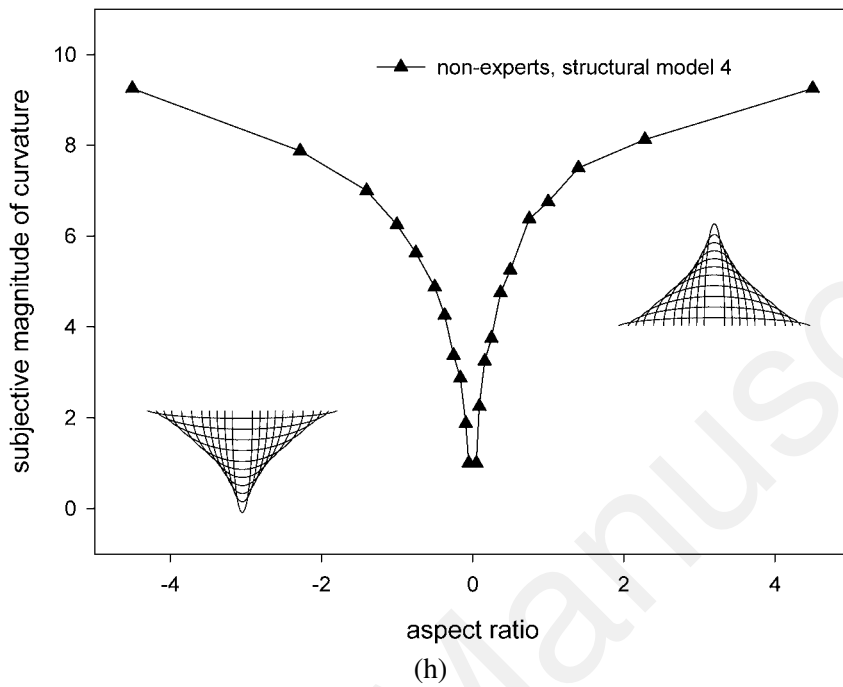


Figure 7. (Continued.)

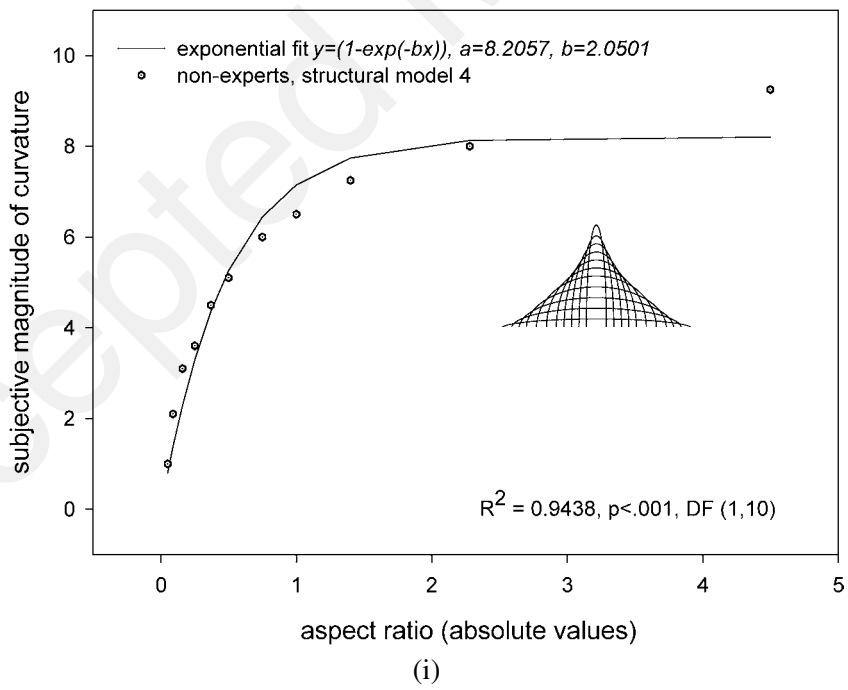


Figure 7. (Continued.)

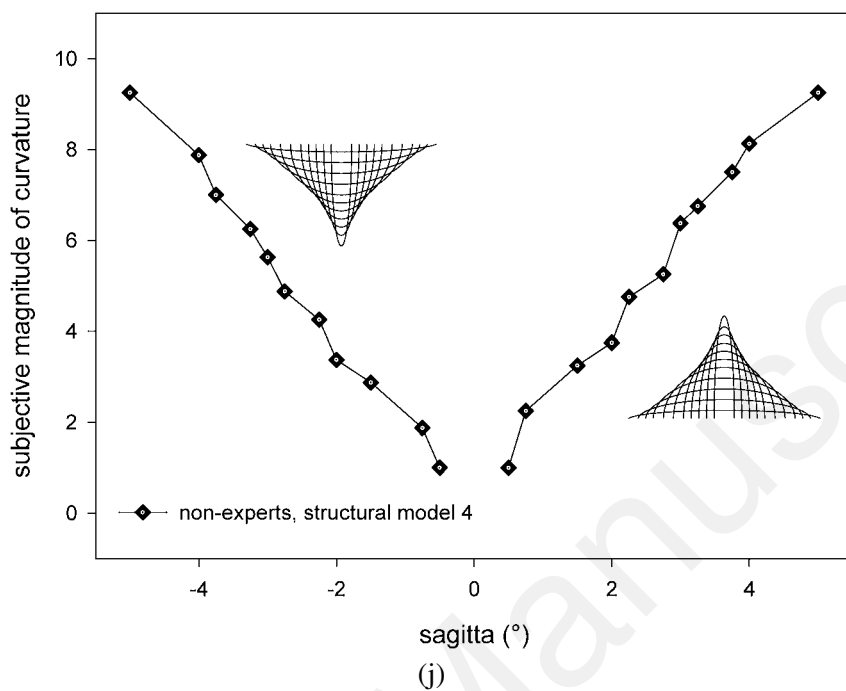


Figure 7. (Continued.)

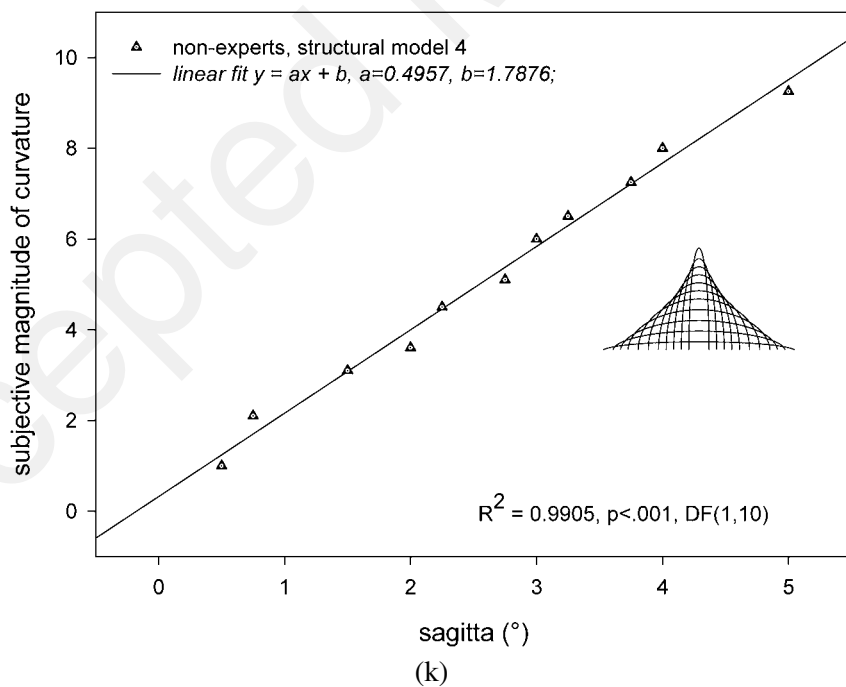


Figure 7. (Continued.)

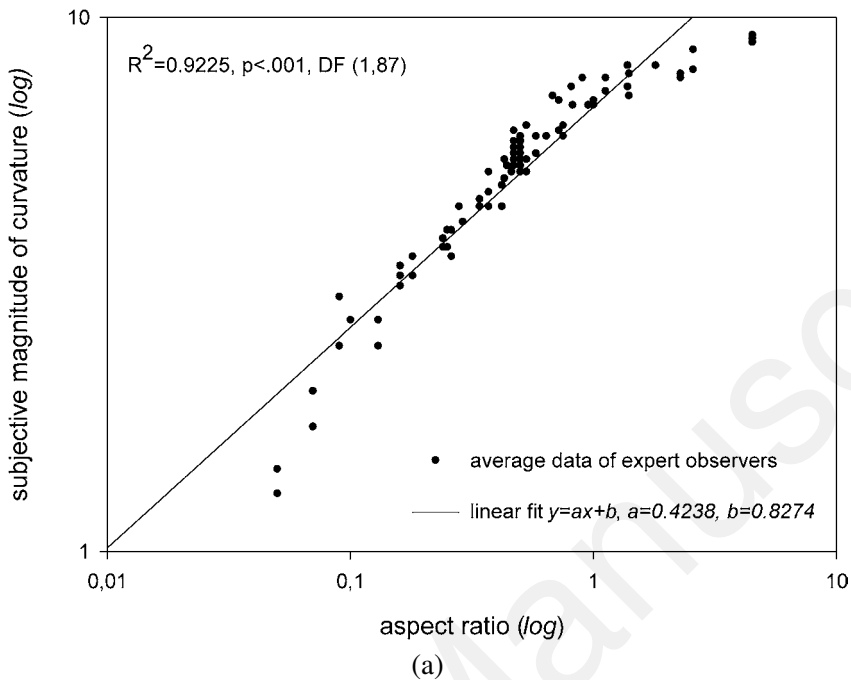
For model 3, the subjective magnitude of curvature increases symmetrically with increasing aspect ratio (Fig. 7(d)) or sagitta (Fig. 7(f)) of arcs with ‘negative’ and ‘positive’ curvature. This increase covers only the lower half of the psychophysical scale. The function that best describes the relationship between the subjective magnitude of curvature and the aspect ratio of the arcs of model 3, irrespective of the polarity of their curvature, is found to be an exponential function (Fig. 7(e)). The function that best describes the relationship between the subjective magnitude of curvature and the sagitta of the arcs, irrespective of the polarity of their curvature, is a linear function (Fig. 7(g)).

The combination of arcs of vertical and horizontal ellipses, all derived from the concentric circles of model 1, provided structural model 4. Arcs of model 4 have varying sagitta and varying aspect ratio. These variations cover the full range of values generated for the experiment. For arcs of model 4, subjective magnitude of curvature increases symmetrically with increasing aspect ratio (Fig. 7(h)) or sagitta (Fig. 7(j)) of arcs with ‘negative’ and ‘positive’ curvature. This increase covers the full psychophysical scale. The function that best describes the relationship between subjective magnitude of curvature and the aspect ratio of the arcs of model 4, irrespective of the polarity of their curvature, is found to be an exponential function (Fig. 7(i)). The function that best describes the relationship between subjective magnitude of curvature and the sagitta of the arcs, irrespective of the polarity of their curvature, is a linear function (see Fig. 7(k)).

### *Expert observers*

The data of the eight expert observers were analyzed in the same way as the data of the non-experts. Psychometric functions of individuals were compared and there were no noticeable differences between seven of the individuals of this group. Their average data are shown in Figs 8 and 9. The data of the expert observer who produced results that were noticeably different from those of the others are dealt with later in a separate section.

*Subjective magnitude of curvature as a function of sagitta and aspect ratio.* The subjective magnitude of curvature of seven of the eight experts shows variations as a function of sagitta and aspect ratio that are in every aspect similar to those obtained with non-experts. Regression analysis of the experts’ data plotted on a logarithmic scale reveals a good linear relationship between subjective magnitude of curvature and aspect ratio (Fig. 8(a)), and a somewhat poorer linear relationship between subjective magnitude of curvature and sagitta (Fig. 8(b)), as was found with the non-experts. The linear function relating subjective magnitude to aspect ratio is found to produce a noticeably higher correlation coefficient than the function obtained for sagitta, as in the data of non-experts. The statistical analysis ( $t$ -test) of the correlation coefficients of linear fits to the individual data of the seven experts (Fig. 8(c)) reveals a statistically significant difference ( $t(1, 12) = 6.0556$ ,  $p < 0.001$ ) between correlation coefficients of functions fitted for aspect ratio



**Figure 8.** This figure shows average data of expert observers. Subjective magnitudes of curvature are plotted as a function of the sagitta (Figs 8(b), (d) and (f)) and the aspect ratio (Figs 8(a) and (e)) of the arcs presented in the experiment. Regression analysis of the data plotted on a logarithmic scale (Figs 8(a) and (b)) reveals a good linear relationship between subjective magnitude of curvature and aspect ratio (Fig. 8(a)), and a somewhat poorer linear relationship between subjective magnitude of curvature and sagitta (Fig. 8(b)), which becomes better when stimuli with the longest chords only are considered (Fig. 8(d)). The linear function relating subjective magnitude to aspect ratio is, as in the data of non-experts, found to produce a noticeably higher correlation coefficient than the function obtained for sagitta. Statistical analysis (*t*-test) of the correlation coefficients of linear fits to individual data reveals, as with the experts, a statistically significant difference ( $t(1, 12) = 6.0556$ ,  $p < 0.001$ ) between correlation coefficients of functions fitted for aspect ratio and functions fitted for sagitta (Fig. 8(c)), with aspect ratio producing a statistically better linear fit to individual sensations of curvature than sagitta. Again, the difference in correlation coefficients is not significant when individual data for stimuli with the longest chords only are fitted.

and functions fitted for sagitta. As with the non-experts, aspect ratio produces a statistically better linear fit to individual sensations of curvature than sagitta when all the chord-lengths are taken into account. Again, the difference is no longer significant when the stimuli with the longest chords only are taken into account. This is, as in the data for the non-experts, explained by the interdependency of sagitta and chord-length (Figs 8(d) and 8(f)). Psychometric functions fitted to average data represented on a linear scale (Figs 8(e) and 8(f)) reveal that an exponential function provides the best fit for subjective magnitude of curvature as a function of aspect ratio (Fig. 8(e)), and a linear function a reasonably satisfactory



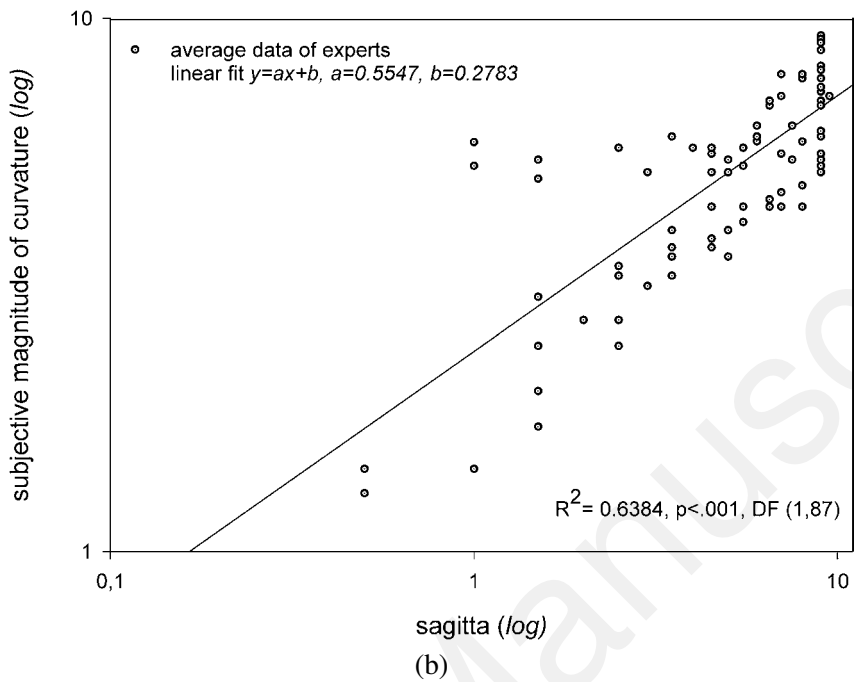


Figure 8. (Continued.)

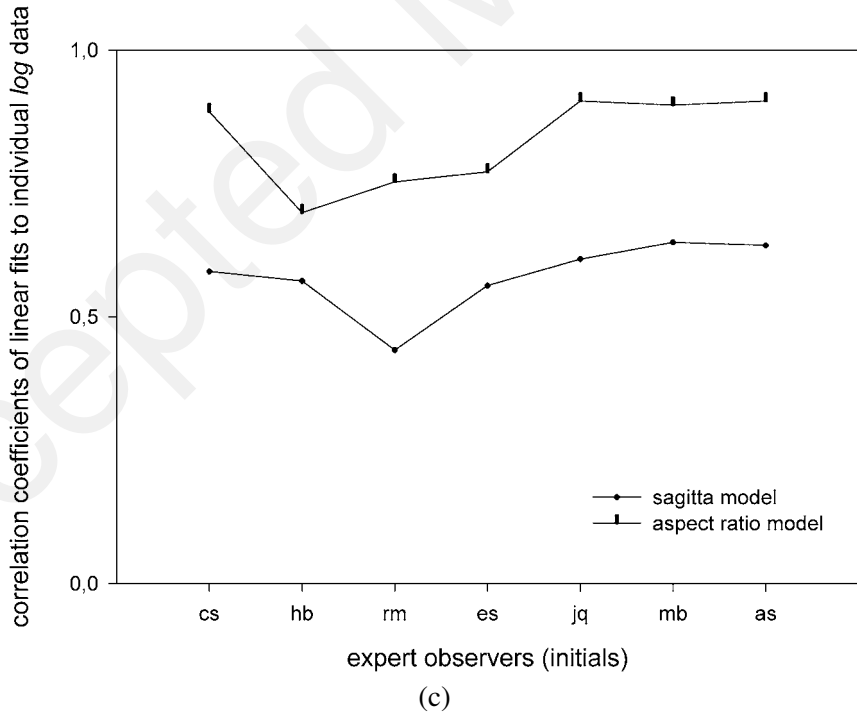


Figure 8. (Continued.)

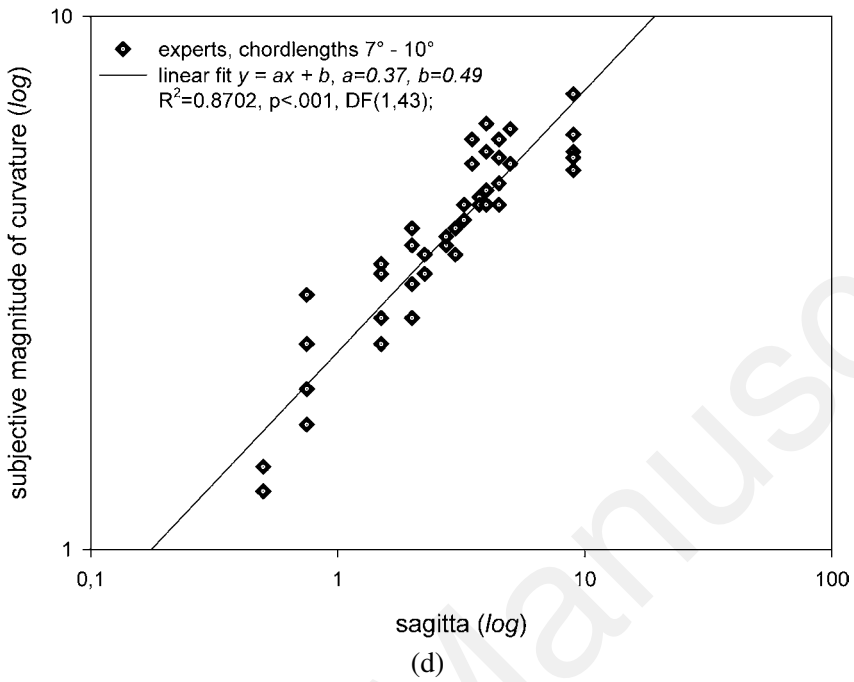


Figure 8. (Continued.)

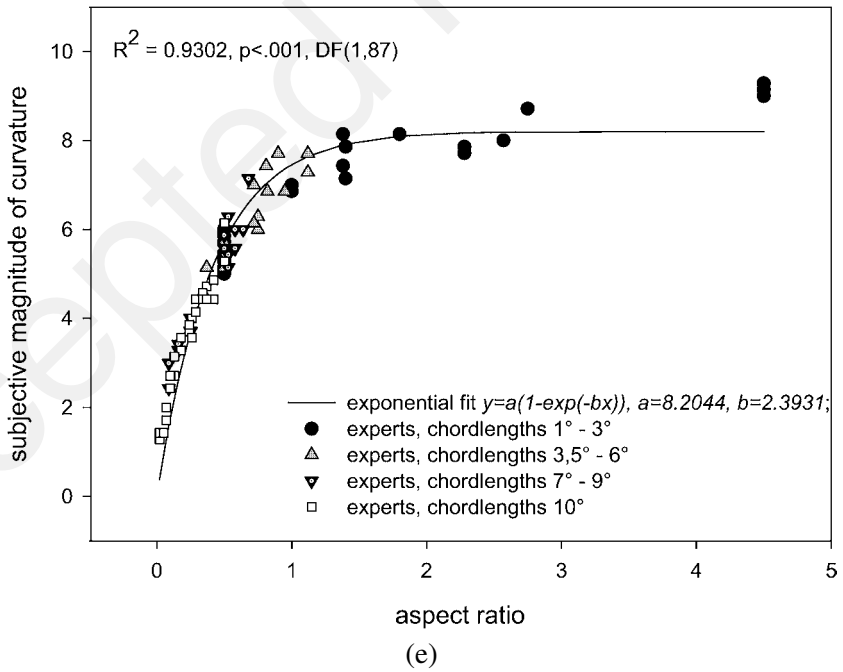
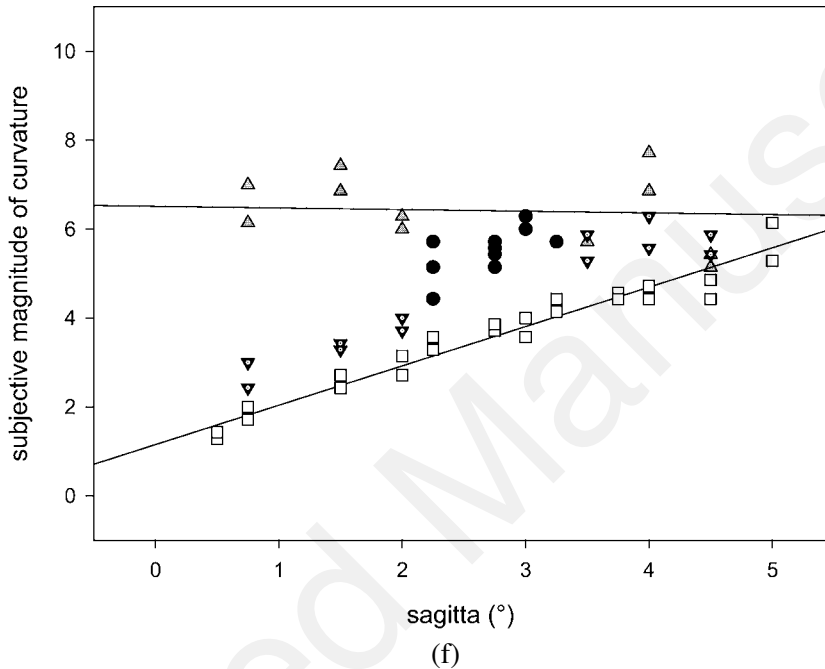


Figure 8. (Continued.)

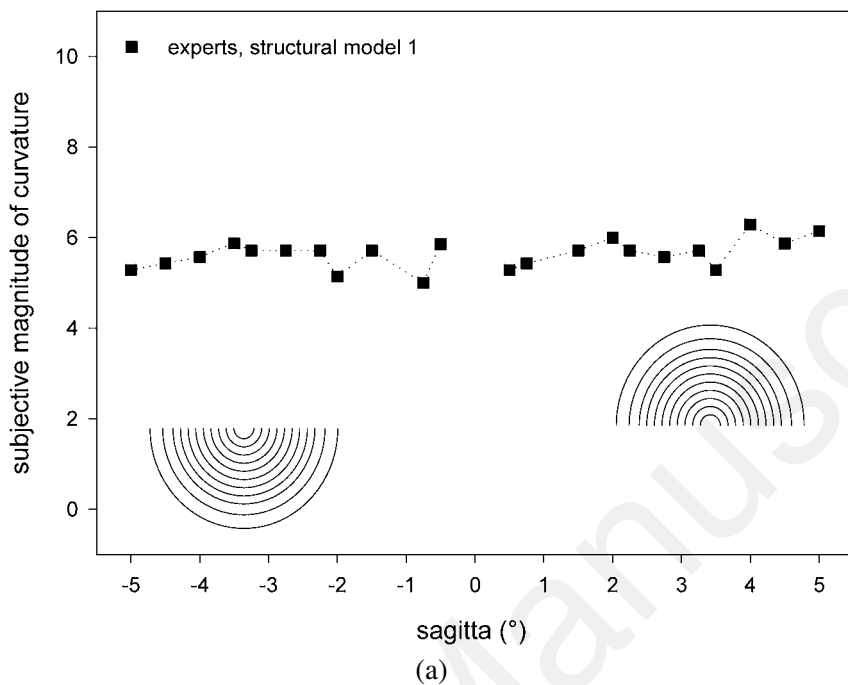
- experts, chordlengths 1° - 3°
- △ experts, chordlengths 3,5° - 6°
- linear fit  $y = ax + b$ , chordlengths 1° - 6°:  $a=6.51$ ,  $b=-0.03$ ,  $R^2=0.0209$ , NS;
- ▼ experts, chordlengths 7° - 9°
- experts, chordlengths 10°
- linear fit  $y = ax + b$ , chordlengths 7° - 10°:  $a=1.15$ ,  $b=0.88$ ,  $R^2= 0.9553$ ,  $p<.001$ ;



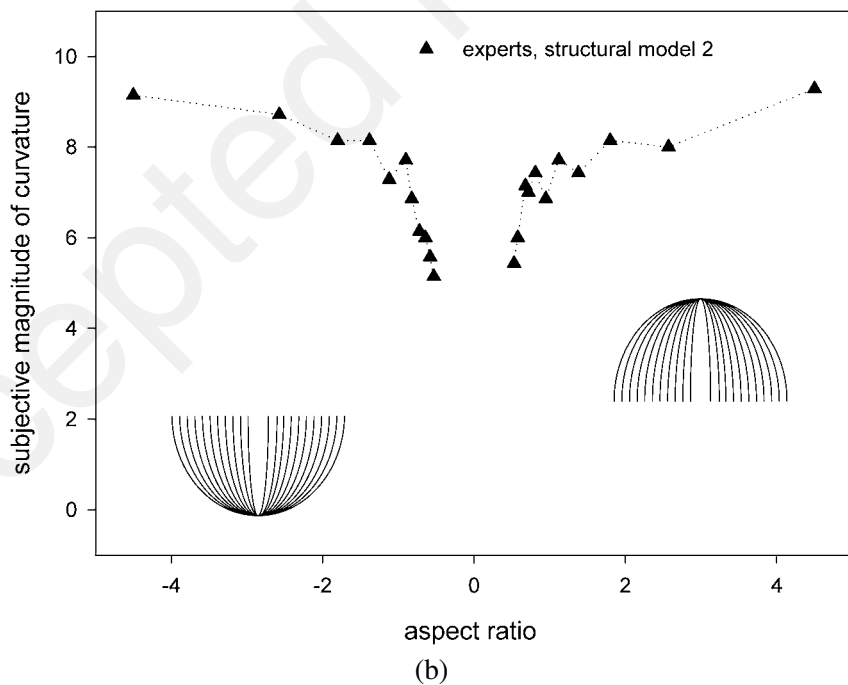
**Figure 8.** (Continued.)

fit for subjective magnitude as a function of sagitta regardless of chord-length ( $R^2 = 0.6149$ ;  $p < 0.001$ ,  $DF(1, 87)$ ), with a noticeably better result when stimuli with the longest chords only are taken into account (Fig. 8(f)). Again, when comparing the data for aspect ratio (Fig. 8(a)) and sagitta (Fig. 8(b)), we find more scattered data for sagitta than for aspect ratio.

*Curvature representation as a function of the four structural models.* The average data of the expert observers analyzed as a function of the four geometric models show that the arcs of concentric circles (model 1) produce constant subjective magnitude in the middle region of the psychophysical scale (Fig. 9(a)). The same result was obtained with the non-experts. Again, arcs with ‘negative’ and ‘positive’ curvature produce symmetric data. For arcs of vertical ellipses (model 2), subjective magnitude of curvature increases symmetrically with increasing aspect ratio of arcs with ‘negative’ and ‘positive’ curvature (Fig. 9(b)). This increase concerns the upper half of the psychophysical scale only, as already found with the non-experts.



**Figure 9.** This figure shows average data of the expert observers as a function of the four geometric models.



**Figure 9.** (Continued.)

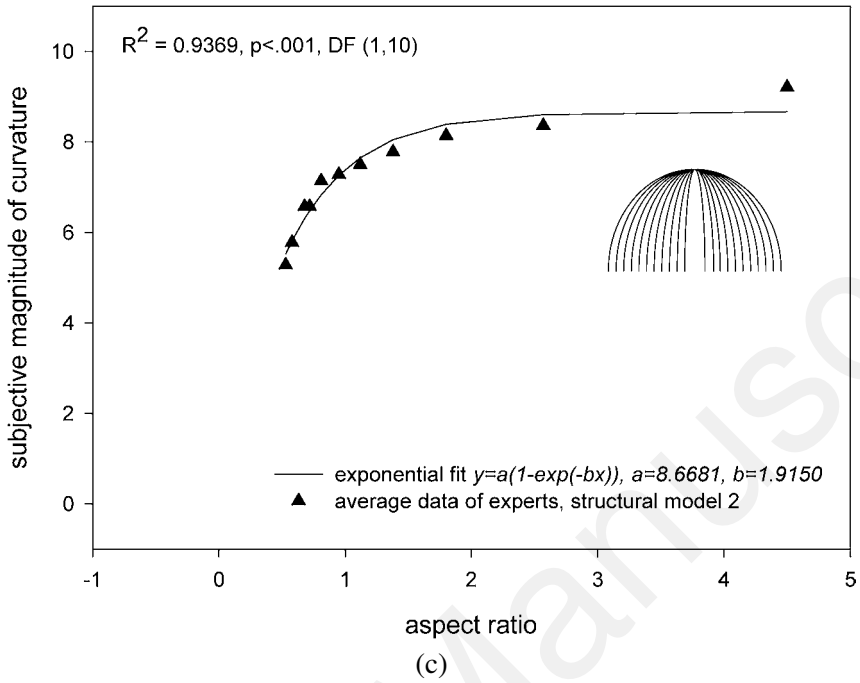


Figure 9. (Continued.)

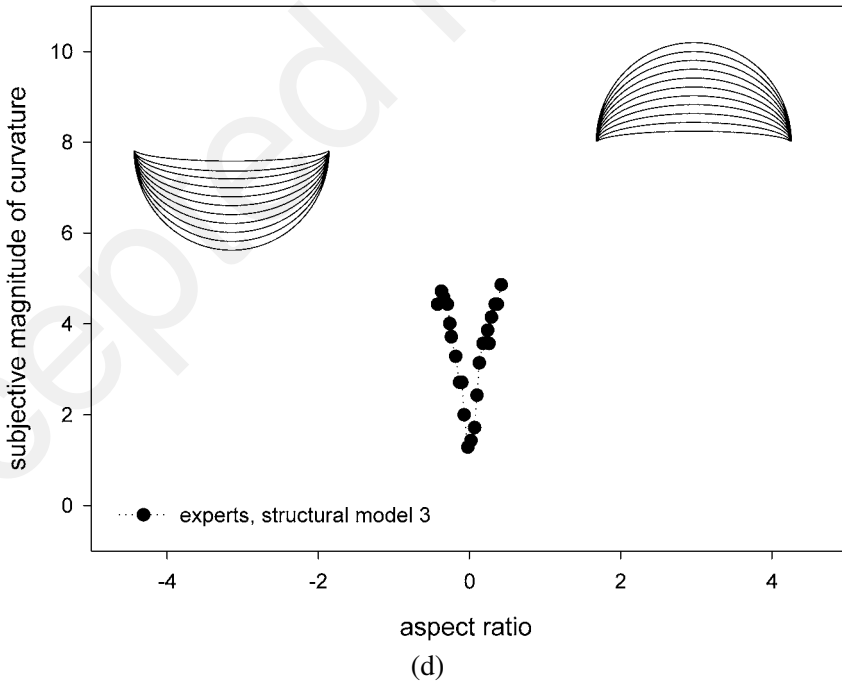


Figure 9. (Continued.)

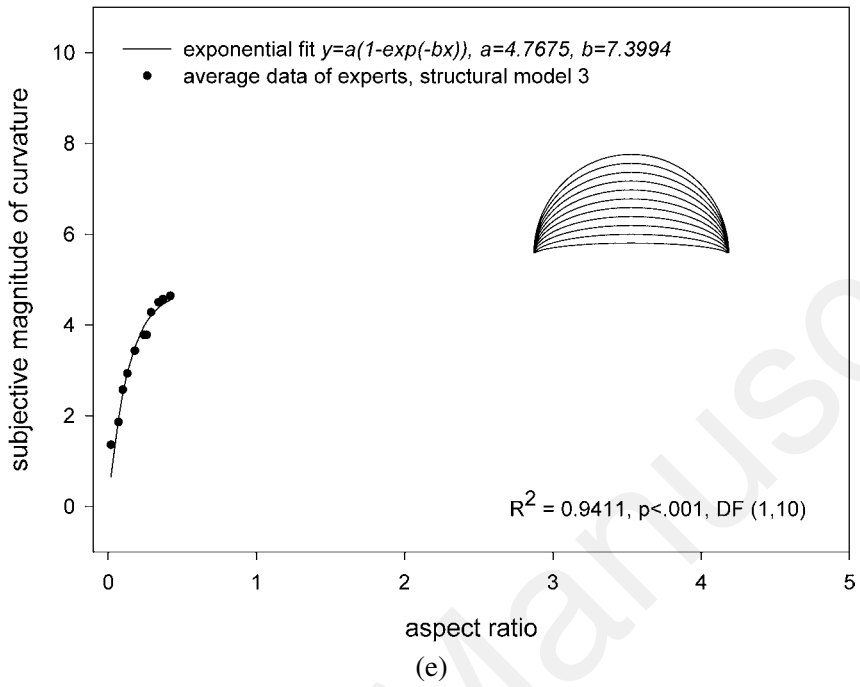


Figure 9. (Continued.)

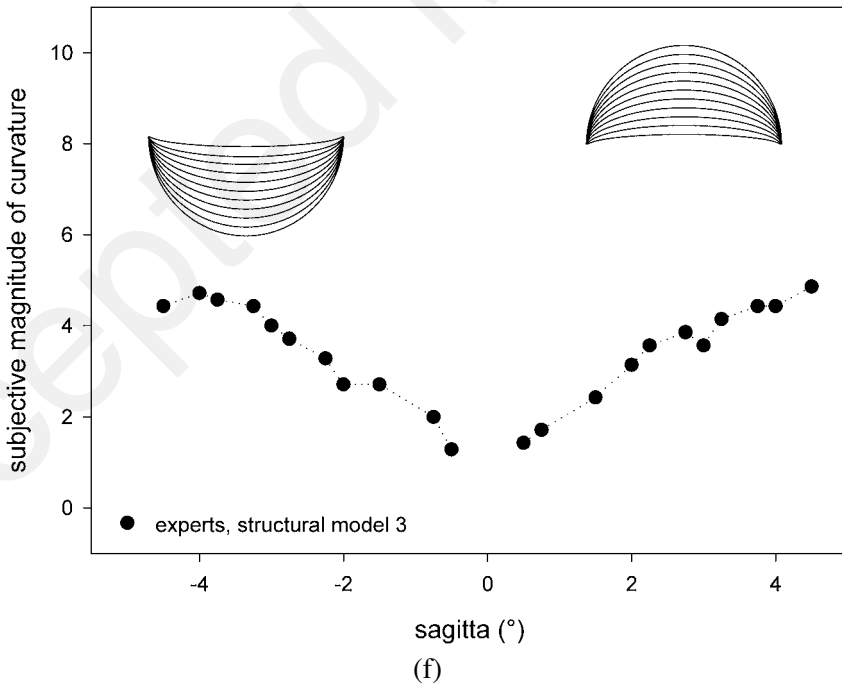


Figure 9. (Continued.)

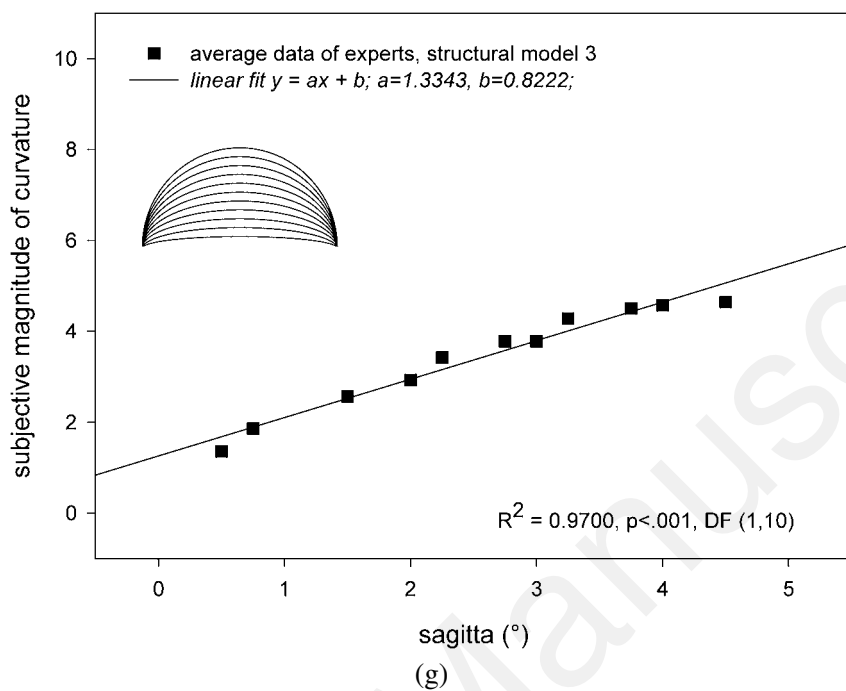


Figure 9. (Continued.)

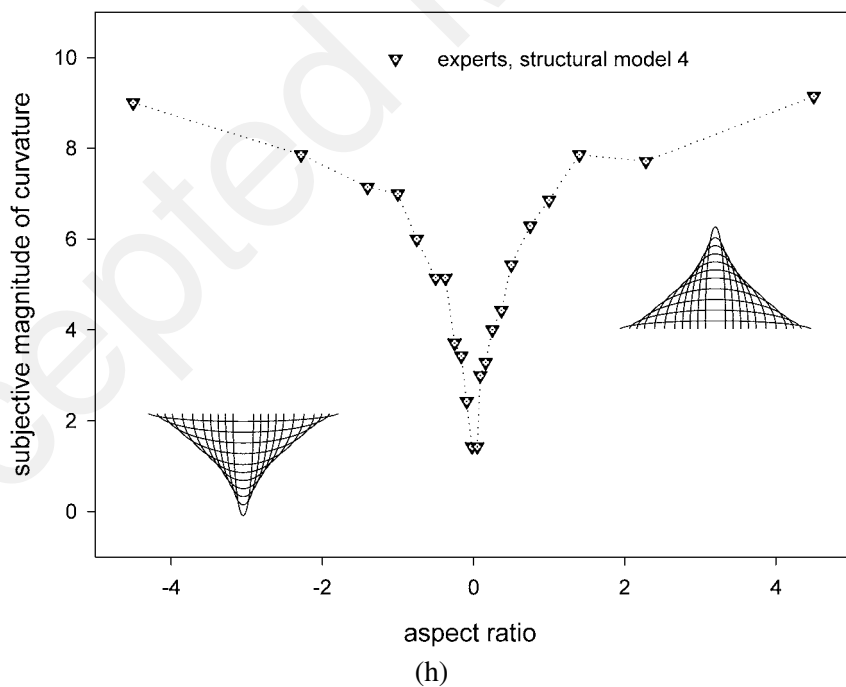


Figure 9. (Continued.)

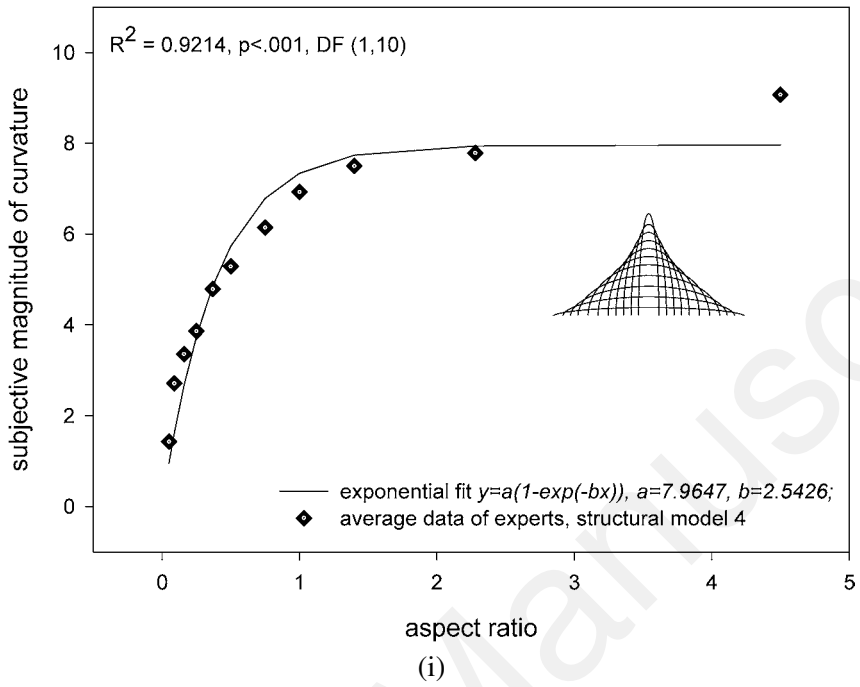


Figure 9. (Continued.)

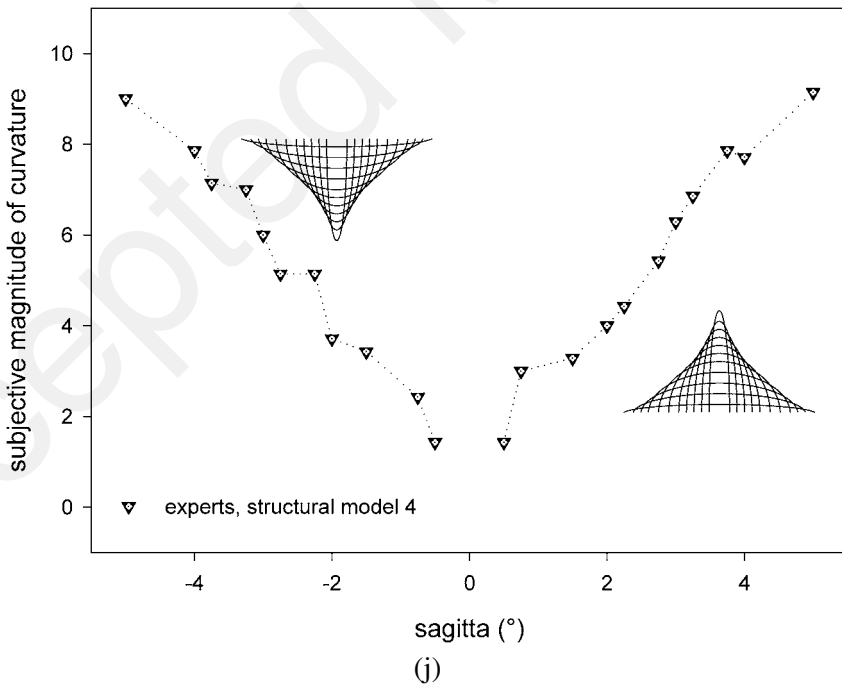
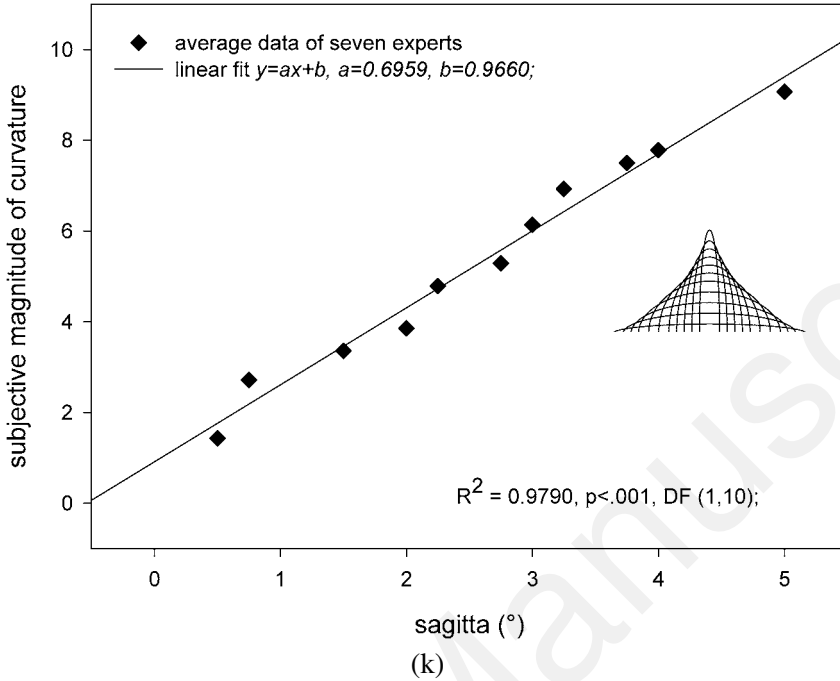


Figure 9. (Continued.)





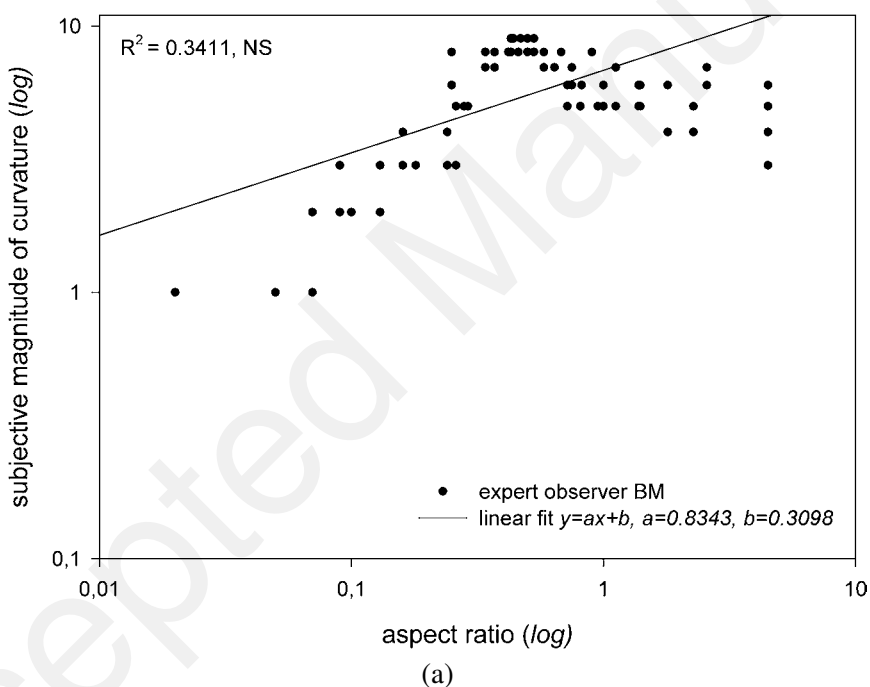
**Figure 9.** (Continued.)

The function that best describes the relationship between subjective magnitude of curvature and the aspect ratio of the arcs is, again, found to be an exponential function (Fig. 9(c)). For arcs of horizontal ellipses (model 3), subjective magnitude of curvature increases symmetrically with increasing aspect ratio (Fig. 9(d)) or sagitta (Fig. 9(f)) of arcs with ‘negative’ and ‘positive’ curvature. Again, this increase concerns the lower half of the psychophysical scale only. The function that best describes the relationship between subjective magnitude of curvature and the aspect ratio of the arcs, irrespective of the polarity of their curvature, is an exponential function (Fig. 9(e)), as previously shown in the data of the non-experts. The function that best describes the relationship between subjective magnitude of curvature and the sagitta of the arcs is, again, a linear function (Fig. 9(g)). For the combination of arcs of vertical and horizontal ellipses (model 4), subjective magnitude of curvature increases symmetrically with increasing aspect ratio (Fig. 9(h)) or sagitta (Fig. 9(j)). As for the non-experts, this increase covers the full psychophysical scale. The function that best describes the relationship between subjective magnitude of curvature and the aspect ratio of the arcs is, again, an exponential function (Fig. 9(i)). The function that best describes the relationship between subjective magnitude of curvature and sagitta is, again, a linear function (Fig. 9(k)).

*Expert observer BM: an exceptional case*

Expert observer BM, an experienced civil engineer and expert in the theoretical analysis of form-force coupling in architectural design, produced data that were different from those of all the other observers. His data are shown in Figs 10 and 11.

*BM: subjective magnitude of curvature as a function of sagitta and aspect ratio.* At a first glance, BM's visual sensations of curvature, reflected by subjective magnitudes of curvature plotted as a function of the sagitta and the aspect ratio of the arcs presented in the experiment do not reveal any systematic variation of sensation as a function of either of the two geometric variables tested. Regression analysis of BM's data plotted on a logarithmic scale (Figs 10(a) and 10(b)) reveals no significant linear relationship between either subjective magnitude of curvature



**Figure 10.** This figure shows results of expert observer BM, a particular case whose data required a separate analysis. Subjective magnitudes of curvature are plotted as a function of the sagitta (Figs 10(b), (d) and (f)) and the aspect ratio (Figs 10(a), (c) and (e)) of the arcs presented in the experiment. Regression analysis of the data plotted on a logarithmic scale reveals no significant linear relationship between either subjective magnitude of curvature and aspect ratio, or subjective magnitude of curvature and sagitta. No suitable psychometric function could be fitted to BM's global average data represented on a linear scale. When stimuli with only the longest chord-lengths were considered, however, we obtain satisfactory linear fits for both aspect ratio and sagitta plotted on a logarithmic scale (Figs 10(c) and (d)) or on a linear scale (Figs 10(e) and (f)). BM's data thus reveal an unusual interdependency of aspect ratio and chord-length not observed with any other of the subjects, non-experts or experts.

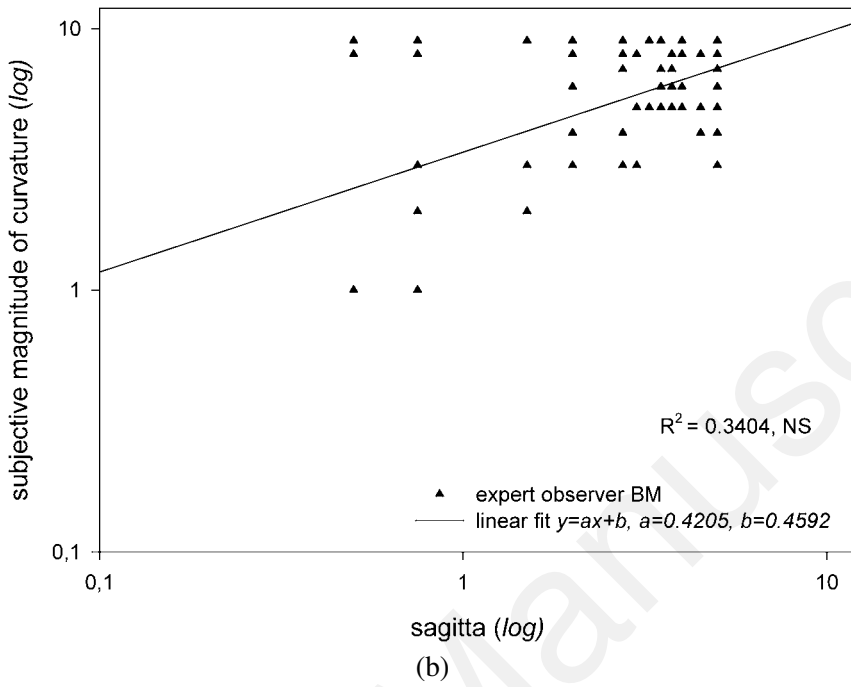


Figure 10. (Continued.)

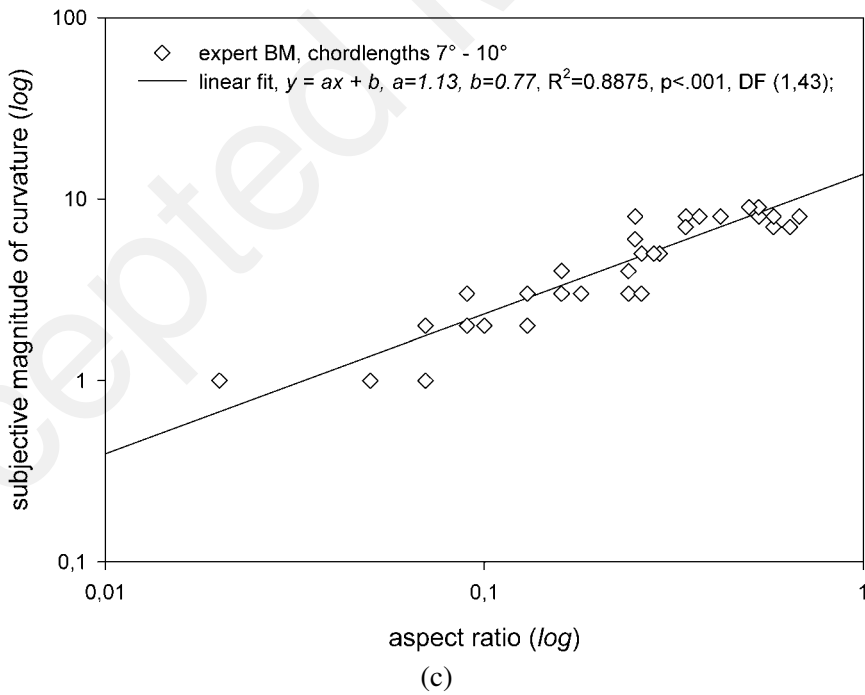


Figure 10. (Continued.)

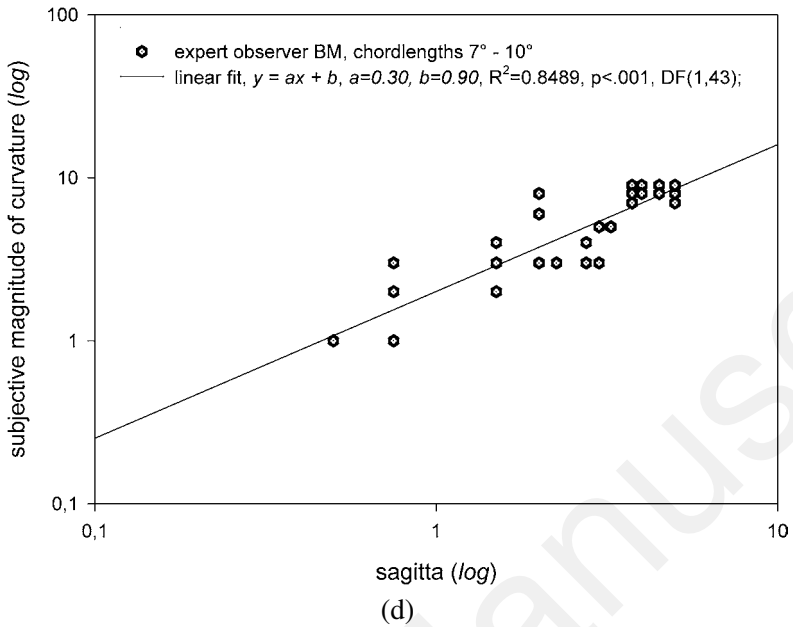


Figure 10. (Continued.)

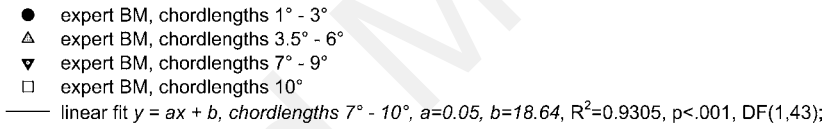
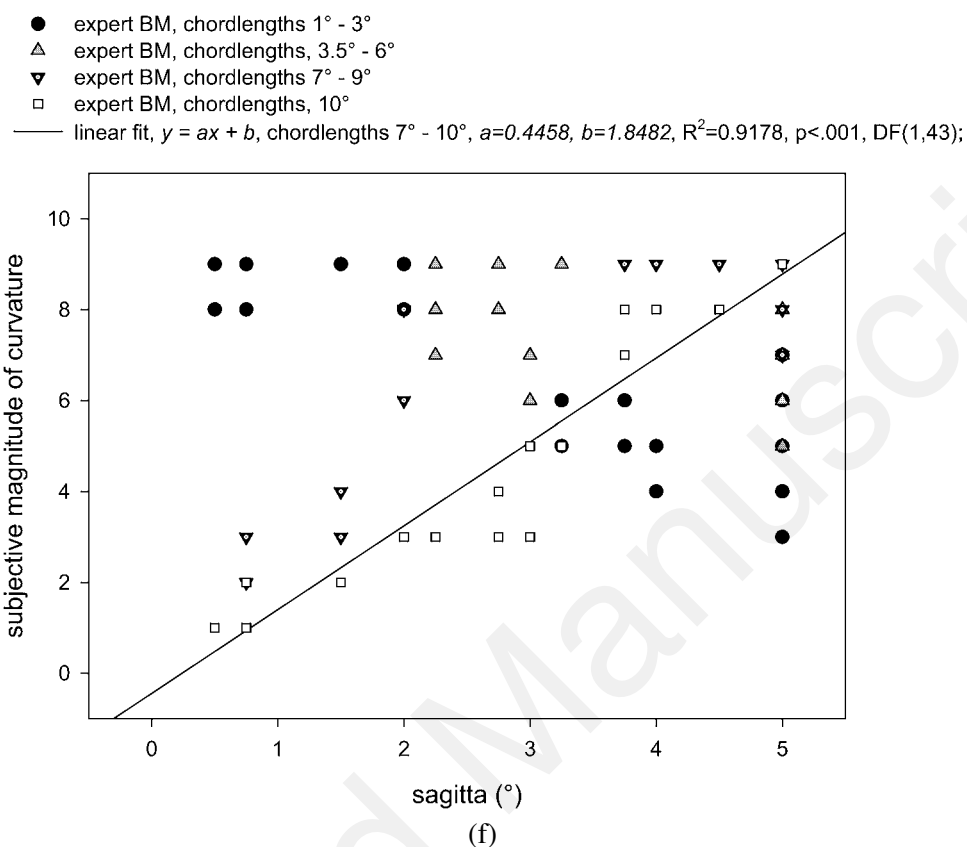


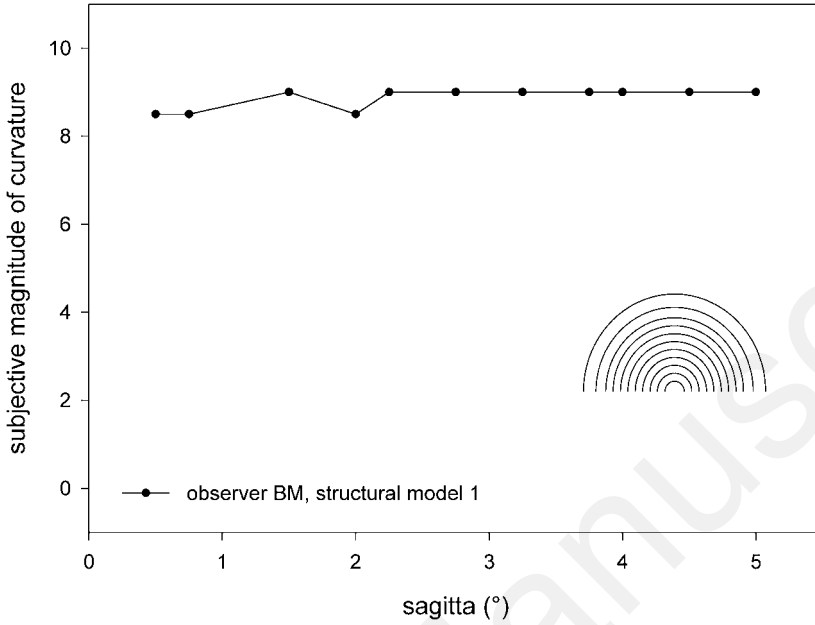
Figure 10. (Continued.)



**Figure 10.** (Continued.)

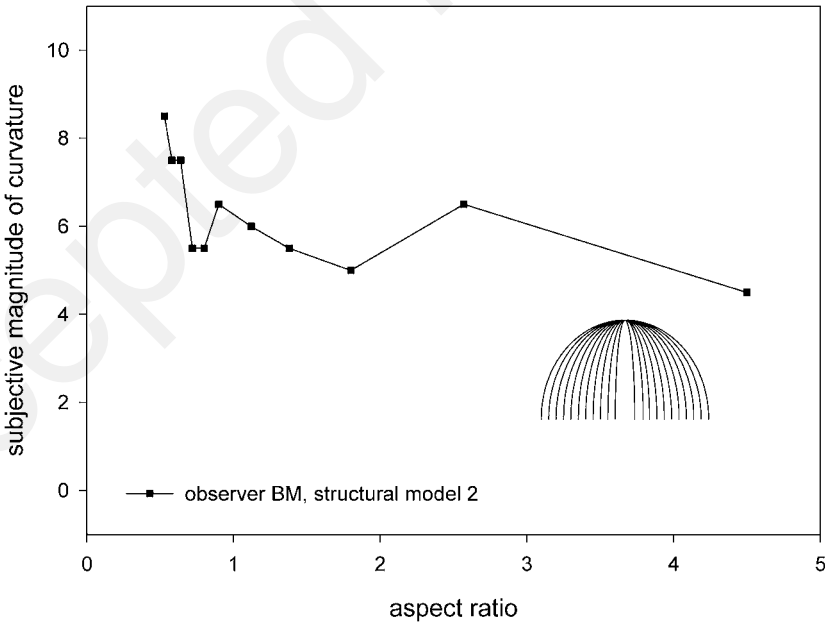
and aspect ratio (Fig. 10(a)), or subjective magnitude of curvature and sagitta (Fig. 10(b)). No suitable psychometric function could be fitted to BM's global data represented on a linear scale (Figs 10(e) and 10(f)). However, further analyses as a function of the chord-lengths of the stimuli revealed significant linear relationships between BM's subjective magnitude of curvature and the sagitta of arcs with chords of 7–10 degrees of visual angle (Figs 10(d) and 10(f)) and between curvature and the aspect ratio of arcs with chords of at least 7 degrees (Figs 10(c) and 10(e)). Thus, BM's data reveal a dependency between sagitta and chord-length similar to that described for the other experts. In contrast, BM's results exhibit such a dependency also between aspect ratio and chord-length, which is a very unusual observation given that aspect ratio proves to be a solidly scale-invariant predictor for all the other observers, experts or not.

*BM: curvature as a function of the four structural models tested.* As with all other observers, BM's data revealed symmetric sensations for arcs with 'negative' and 'positive' curvature and only average subjective magnitudes for the two polarities of curvature are therefore shown here. Arcs of concentric circles (model 1)



(a)

**Figure 11.** This figure shows BM's results as a function of the four geometric models. Like with all other observers, BM's data revealed symmetric sensations for arcs with negative and positive curvature and, therefore, average subjective magnitudes for the two polarities of curvature are shown here only.



(b)

**Figure 11.** (Continued.)

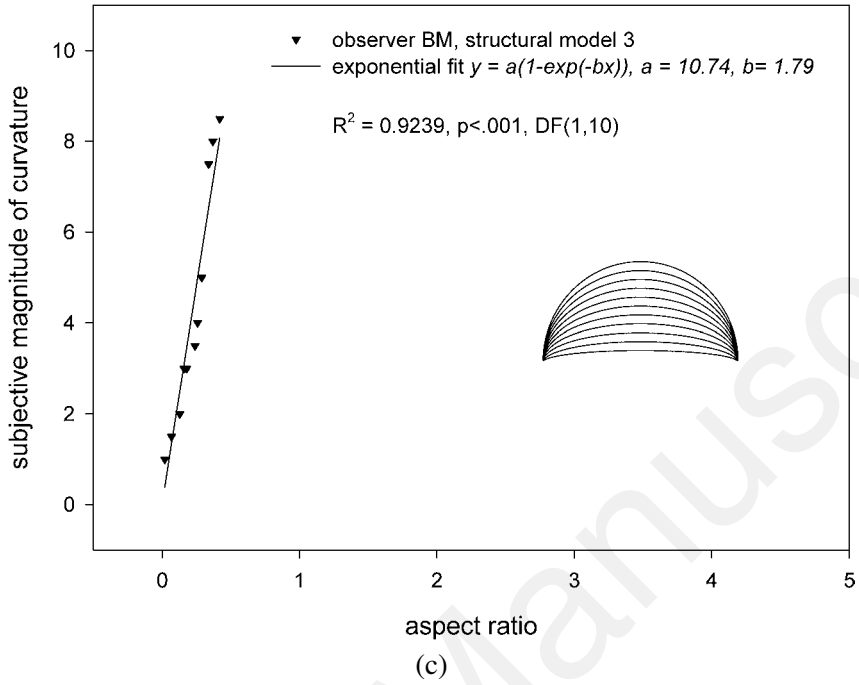


Figure 11. (Continued.)

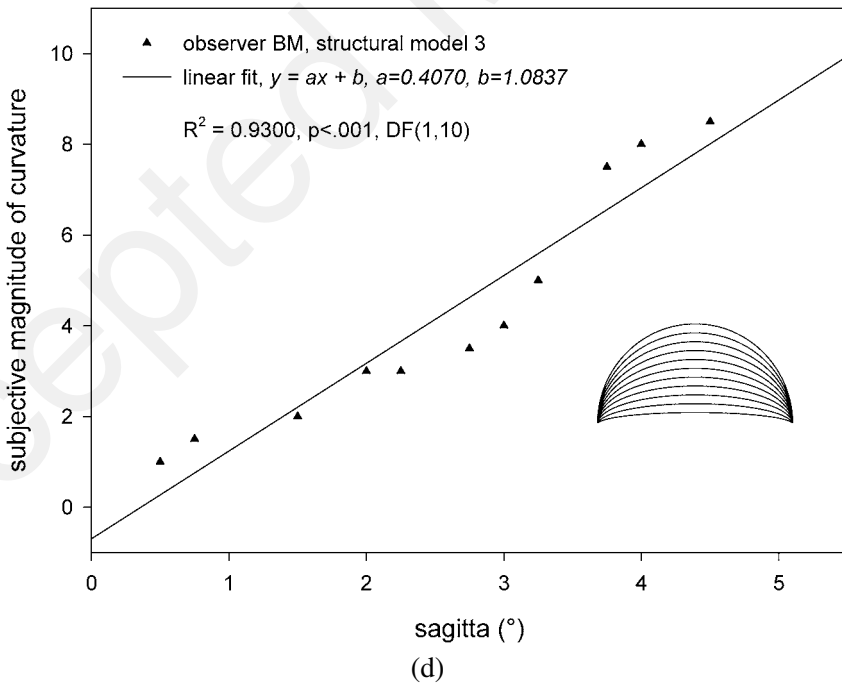


Figure 11. (Continued.)

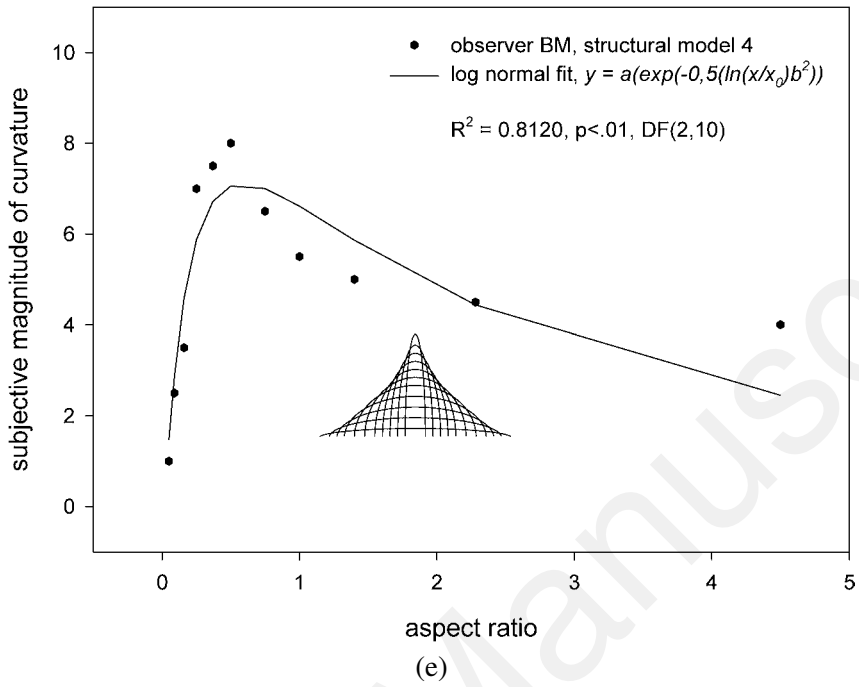


Figure 11. (Continued.)

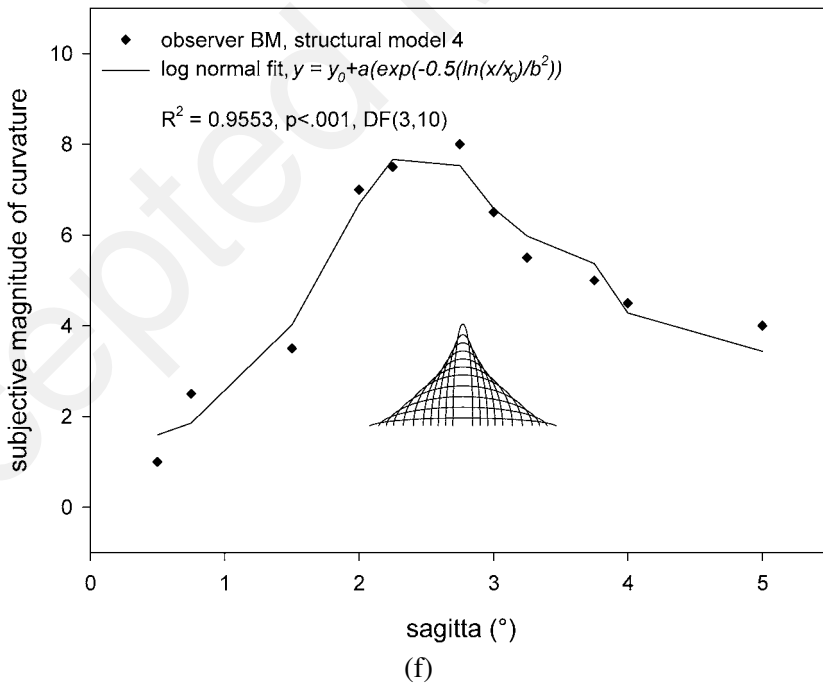


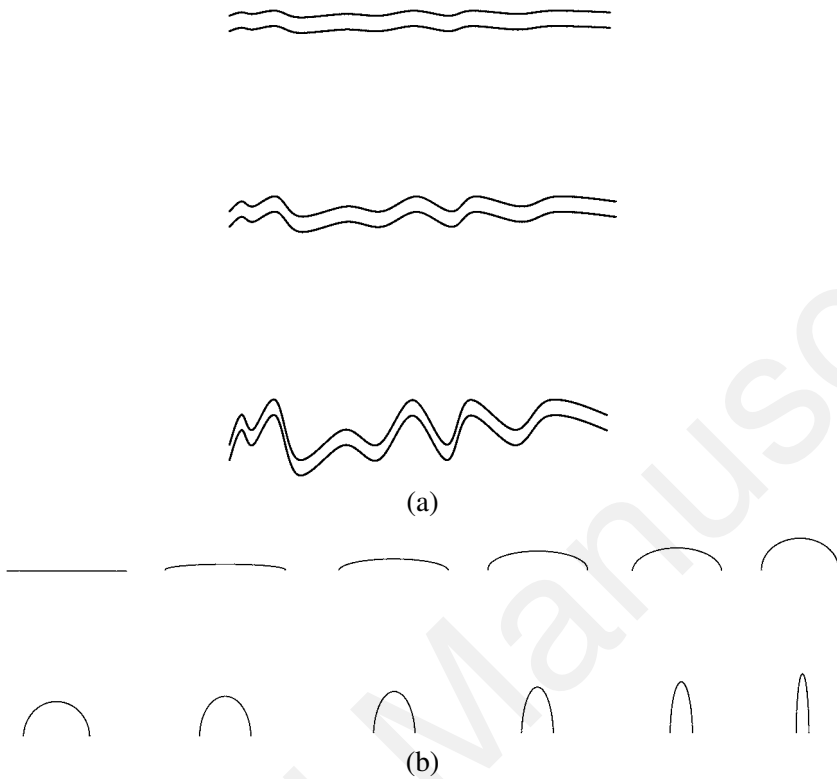
Figure 11. (Continued.)



produce, like with all other observers, constant subjective magnitude as a function of variations in sagitta, with the major difference that BM's sensation levels out in the upper end of the psychophysical scale (Fig. 11(a)), whereas the other observers showed constant magnitudes of sensation in the middle region of the scale. For arcs of vertical ellipses (model 2), BM's subjective magnitude of curvature decreases with increasing aspect ratio of arcs with negative or positive curvature (Fig. 11(b)). The individual data of all other observers had shown exponential increases in sensation here. For arcs of horizontal ellipses (model 3), subjective magnitude of curvature increases with increasing aspect ratio (Fig. 11(c)) or sagitta (Fig. 11(d)). This increase covers the full psychophysical scale for observer BM. For all the others, the increase in sensation was found to cover only the lower half of the psychophysical scale. However, the function that best describes the relationship between BM's subjective magnitude of curvature and the aspect ratio of the arcs of model 3 is, as with all the other observers, found to be an exponential function (Fig. 11(c)). The function that best describes the relationship between subjective magnitude of curvature and the sagitta of the arcs of model 3, is found to be a linear function (Fig. 11(d)), as with the other observers. For BM, the combination of arcs of vertical and horizontal ellipses (model 4) generates subjective magnitudes of curvature which produce a U-shaped function for increasing aspect ratio (Fig. 11(e)) or sagitta (Fig. 11(f)). This result is very different from that obtained with any of the other observers for this geometric model (model 4). The function that best describes the relationship between BM's subjective magnitude of curvature and the aspect ratio of the arcs of model 4 is found to be a three-parameter log-normal fit (Fig. 11(e)). The function that best describes the relationship between BM's subjective magnitude of curvature and the sagitta of the arcs of model 4 is a two-parameter log-normal fit (Fig. 11(f)). In the light of these data it appears quite clearly that the 'best' curve in terms of perceived curvature is, for observer BM, one that has the shape of half a circle, regardless of its sagitta or chord-length. For all the other observers, the 'best' curve corresponds to the shape of a narrow, vertical ellipse.

## DISCUSSION

The results of this study provide new insights into the perceptual processes that generate shape representations on the basis of structural properties of 2-D curvature. First, they clarify the relative importance of local geometric cues in the perceptual representation of global shape attributes such as perceived curvature. From the simple 2-D view of a single curve, the important role of local geometric cues in shape perception may not be too obvious. However, when two parallel curves with multiple double curvature are combined, it becomes quite clear, as in Fig. 12(a), which illustrates how the strength of the global shape percept, or shape sensation, increases with the local sagittae or aspect ratios.



**Figure 12.** (a) To produce visible 3-D effects through 2-D curvature, at least two parallel curves are required. The strength of the resulting shape percept, or shape sensation, then increases with increasing local sagitta or aspect ratio, also sometimes referred to as the ‘local height-to-width ratio’ (e.g. Stevens, 1981a and 1981b). (b) A ‘buckled bar model’ optimally accounts for variations in the subjective magnitude of arcs describing halves of ellipses derived from concentric circles (our structural model 4). When the arcs of structural model 4 are presented as an ordered series, beginning with the arc that has the smallest aspect ratio and sagitta and ending with the arc that has the highest aspect ratio and sagitta, we can figure a horizontal bar that is progressively bent, or buckled.

Here in our data, both the aspect ratio and the sagitta of 2-D curves account for the generation of global shape sensations in terms of curvature. The aspect ratio, however, is found to provide the more reliable predictor. This can be explained in the light of earlier data from threshold-based studies, tailored to investigate the rapid visual discrimination of curves.

#### *Local geometric cues and scale-invariance*

The rapid visual analysis of small curve segments, presented for comparatively short durations, involves a discrete, or coarse, processing mode (Ferraro and Foster, 1986; Watt, 1987) where visual sensitivity increases with exposure duration. Such a processing mode privileges the use of the most readily available local cue, the

sagitta. Sagitta provides a strictly local measure of relative height in the visual field at a precise location of a given curve: the point of maximum height. Thus, under conditions where stimuli are smaller than 1 degree of visual angle and exposure durations not longer than a second, sagitta has, indeed, proven to provide a better account for the variance observed in curvature discrimination thresholds than aspect ratio (Foster *et al.*, 1993; Foster and Savage, 2002). However, other threshold-based studies using curves with considerably larger sizes (up to 4.2 degrees of visual angle) have produced results showing that aspect ratio seemed to provide the best cue (Whitaker and McGraw, 1998). It appears that larger stimuli and/or longer exposure durations privilege a finer, more global and continuous perceptual analysis of curvature (Ferraro and Foster, 1986; Watt, 1987). Such an analysis may involve processing of shape properties based on a more integrated, presumably scale-invariant cue, the aspect ratio. Aspect ratio conveys information about the total area covered by a given curve and therefore seems the ideal predictor for the perception of global shape properties. The magnitude estimates from our study here, with comparatively large stimuli and unrestricted exposure duration, point toward such an interpretation.

While aspect ratio is shown to provide a highly reliable, scale-invariant predictor for perceptual representations of curvature, the predictive power of sagitta is, in contrast, found to depend on the chord-length of the curves. For chord-lengths between 1 and 6 degrees of visual angle, sagitta does not produce reliable predictions, whereas for chord-lengths between 7 and 10 degrees, it becomes as good a predictor as aspect ratio. This could be so because the perceptual system has an inbuilt prototype representation of the 'ideal curve' with the 'ideal size'. Thus, for curved shapes closely matching this 'ideal' representation, the perceptual system may effectively combine the use of sagitta and aspect ratio, whereas with shapes smaller than the 'ideal curve', it may be constrained to privilege either of the two cues. A bias in favour of sagitta for the smallest, briefly exposed shapes (Foster *et al.*, 1993), and a bias in favour of aspect ratio for shapes of intermediate sizes (Whitaker and McGraw, 1998, our present study) would then, again, make perfect sense.

The magnitude estimates from this study reveal an important functional aspect of the perceptual processes involved in the generation of shape representations such as curvature: scale invariance. For example, although the circular arcs of our structural model 1 have different sizes and varying sagitta, they all produce identical perceptual magnitudes of curvature. Moreover, the predictive power of aspect ratio is found to be independent of the chord-lengths of the curves, except in the case of observer BM. The perceptual system of experts as well as of that of most non-experts may have a general tendency to privilege the use of scale invariant representations, and such a preference may already be present at early levels of visual processing, as threshold measurements from certain curve discrimination experiments would, indeed, suggest (Whitaker and McGraw, 1998).

*Which structural model accounts best for the perceived curvature of 2-D shape contours?*

While our structural model 1, which produces no variations in curvature for any of the observers tested here, does not suggest any 3-D shape properties, the three other structural models (referred to as models 2, 3 and 4) do suggest shape properties that may be found in 'real' 3-D objects. Only these models are found to produce variations in subjective magnitude of curvature, consistent with an optimal use of aspect ratio by all observers, experts or non-experts, except BM. Arcs from structural model 2, which were all halves of vertical ellipses derived from the concentric circles of model 1, produce increasing subjective magnitudes of curvature with increasing aspect ratio. This increase covers only the upper half of the psychophysical scale only, which is consistent with increases in the aspect ratio of the arcs of this model, covering only values in the upper range of those used. Arcs from structural model 3, which were all halves of horizontal ellipses derived from the concentric circles of model 1, produce increasing subjective magnitudes of curvature only in the lower half of the psychophysical scale, which is consistent with the increase in aspect ratios, covering only values in the lower range of those used. The 'best' psychometric functions, with variations in sensation covering the whole psychophysical scale used, are obtained with our structural model 4. The elliptic arcs of model 4, when represented all together on a Cartesian plan, form a mesh-like spatial structure which strongly suggests properties of 'real' 3-D objects (see again Figs 1 and 5). We believe it likely that the perceptual representations of 2-D shape, reflected here through measures of sensations of curvature, originate from a statistical relationship between the structural properties of the 2-D images of the arcs of model 4 and their 3-D counterparts in 'real' objects.

*A 'buckled bar model' for internal representations of curved objects?*

How a 'natural' model in terms of a statistical relationship between geometrical 2-D characteristics of our stimuli and a possible physical source in the 'real' 3-D world (e.g. Howe and Purves, 2005) could explain our psychophysical data may be illustrated by linking the internal curvature representations revealed by the data of our experiment to an external physical source in terms of a learned action on a real object: the buckling of a bar. In our experiment, the data obtained with the curves of our structural model 4 produce the best psychometric functions. Now, when the arcs of structural model 4 are presented as an ordered series (Fig. 12(b)), beginning with the one that has the smallest aspect ratio and sagitta and ending with the one that has the highest aspect ratio and sagitta, we can immediately figure a horizontal bar that is progressively bent, or buckled. Although observers never saw this sequence in the experiment, the internal representations it refers to have been learnt and re-activated over and over again in multiple active or passive situations since we were children. It may therefore not be too surprising that this 'natural' model of 2-D curvature appears to provide a suitable model for the structural or morphological

representations at the origin of the sensations of 15 out of the 16 observers tested here. The straight line at the beginning of the series shown in Fig. 12 produces no sensation of curvature, while the last arc of the series shown produces a maximal subjective magnitude of curvature in most observers.

*2-D shape attributes and perceptual experience.* While 15 out of the 16 observers tested here seem to straightforwardly use an internal representation that bears certain structural and statistical relationships with the active transformation of a straight object into a curved one, an operation we spontaneously learn to execute playfully when we are children, expert observer BM's shape sensations seem to originate from a rather different kind of structural representation. In his data, we find, for example, that model 3 produces an increase in subjective magnitude with increasing aspect ratio and sagitta covering the full psychophysical scale. This increase in sensation with increasing aspect ratio is predicted by an exponential function, increasing sensation with increasing sagitta is predicted by a linear function. When considering observer BM's global data, however, neither the aspect ratio, nor the sagitta of the curves seem to prove a valid cue to perception, given the fact that no suitable psychometric function can be fitted where all other observers produced perfectly satisfactory predictive models.

However, when analyses are performed as a function of the chord-length of the curves, we discover that aspect ratio as well as sagitta reliably predict BM's perceptual data when curves with the longest chords only are considered. Thus his results reveal a dependency between sagitta and chord-length similar to that described for the other experts but, in addition, exhibit such a dependency also between aspect ratio and chord-length. This is a very unusual observation given that aspect ratio proves to be a solidly scale-invariant predictor for all the other observers, be they experts or not. Clearly, BM uses a structural model of curve representation that is radically different from the model used by all the others. In the light of the data it becomes clear that the 'best' curve, in terms of perceived curvature is, for BM, one that has the shape of half a circle, regardless of the sagitta or chord-length. For all the other observers, the 'best' curve corresponds to the shape of a narrow, vertical ellipse and perception of curvature decreases when the sagitta or the aspect ratio of that ellipse decreases, as in our structural models 2 and 4. In contrast, BM's sensations of curvature tend to diminish as the curves approach the shape of the narrowest, vertical ellipse (models 2 and 4). These findings suggest that a specific perceptual expertise, or learning process, can transform 'natural' structural representations of curved shapes into specific representations that exploit the geometry of 2-D space in a different, possibly more 'analytical' way. Sinha and Poggio (1996) have shown that the adult human visual system is capable of learning new associations between two-dimensional pictures and projectionally coherent 3-D structures. The data produced by BM, one out of eight experts tested here, may possibly be explained by such a kind of specific, associative learning.

## Acknowledgements

We are grateful to Professor Enzo Siviero for discussions related to this work. The helpful comments and suggestions of the anonymous reviewers were greatly appreciated. Financial support of the Dipartimento Costruzione dell'Architettura (DCA) at Istituto Universitario Architettura Venezia (IUAV) to C. Silvestri is gratefully acknowledged. In addition, we should like to express our gratitude to the sixteen expert and non-expert observers who agreed to participate in this study.

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