



HAL
open science

Performance evaluation of solar energy predictor for wireless sensors

Bingying Li, Yuehua Ding, Yide Wang

► **To cite this version:**

Bingying Li, Yuehua Ding, Yide Wang. Performance evaluation of solar energy predictor for wireless sensors. Fifth Sino-French Workshop on Information and Communication Technologies (SIFWICT 2019), Jun 2019, Nantes, France. hal-02167956

HAL Id: hal-02167956

<https://hal.science/hal-02167956>

Submitted on 28 Jun 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Direction-of-Arrival Estimation Using Co-prime Array in the Context of Both Circular and Non-circular Sources

Bingying LI, Yuehua DING
School of Electronic and Information Engineering
South China University of Technology
Guangzhou, China
eeyhdng@scut.edu.cn

Yide WANG
IETR, UMR 6164, School of Polytech' Nantes
University of Nantes
Nantes, France
Yide.wang@univ-nantes.fr

Abstract—Recently, coprime array has been proposed to estimate the direction of arrival (DoA) of $O(MN)$ sources using only $O(M + N)$ physical sensors. In this paper, we investigate a new method to achieve higher degrees of freedom for the coprime array under the coexistence of K_1 non-circular and K_2 circular sources. By exploiting the properties of the signal sources, a new model is constructed based on a longer virtual uniform linear array (ULA). Simulation results show that the proposed method can identify more sources once the condition $K_1 + 2K_2 < 2MN$ is satisfied.

Index Terms—DoA estimation, coprime array, MUSIC, non circularity and circularity.

I. INTRODUCTION

Direction of arrival (DoA) estimation plays a significant role in array signal processing. It has wide application in many fields such as radar, sonar and wireless communication systems [1] - [3]. During the past several decades, research in DoA estimation of narrowband signals has achieved a huge progress since the multiple signal classification (MUSIC) [4] was proposed. Its mainstream is focused on half wave length spaced uniform linear arrays (ULA). It is well known that the resolution capability of a ULA is directly limited by the number of its physical sensors. For example, with a ULA of N sensors, the number of resolvable uncorrelated sources by MUSIC-like algorithms is $N - 1$.

To overcome the above mentioned limitation, coprime array has been recently proposed, which is a non-uniform linear array constituted of two uniform linear subarrays with large inter-element distances [5]. This new geometry shape can increase the degrees of freedom of the array by exploiting the covariance matrix of the received signals. A new system model can be obtained by vectorizing the covariance matrix of the received signals. It can be viewed as a virtual ULA with a much wider aperture, which is able to detect $O(MN)$ sources using only $O(M + N)$ physical sensors. The spatial smoothing method is then used to overcome the correlation effect [6]. Actually, this method has been widely applied in many systems to get further improvements in sources' DoA

finding [7] - [9].

Moreover, researchers are also interested in the exploitation of the properties of incoming signals which can bring further estimation improvement. When the incoming signals are from non-circular sources, the DoA estimation accuracy and the maximum number of detectable signals can be greatly increased [10]. [11] developed an algorithm to cope with more general scenarios, where the non-circular and circular sources are mixed together. It can make use of the existing non-circular signals to increase the maximum number of detectable signals. It combines the received signal vector and its complex conjugate counterpart to form a new signal vector and exploits the covariance matrix of the new signal vector. It also proposes different estimators for non-circular sources and for circular sources to achieve higher accuracy.

For the coprime arrays, the property of non-circular signals is exploited in [12] to enhance the DoA finding performance, while the case of mixed signals including non-circular and circular signals is still not investigated. Motivated by this observation, in this paper, we make use of coprime array and exploit the properties of real and complex circular sources to increase the degrees of freedom in DoA estimation.

The rest of this paper is organized as follows: Section II presents the MUSIC-based DoA estimation for a classical ULA, then for a coprime array. The proposed method is presented in Section III. Simulation results are provided in section IV. Section V concludes the paper.

Notation: Capital letters of boldface are used for matrices. Lowercase letters of boldface denote column vectors. $(\cdot)^H$ and $(\cdot)^T$ denote the operations of Hermitian (complex conjugate transpose) and transpose, respectively. The operation of complex conjugate is denoted by $(\cdot)^*$. $E[\cdot]$ denotes the expectation. $\|\cdot\|$ represents the Frobenius norm. \otimes denotes the Kronecker product. Diagonal matrices are denoted by $diag(\cdot)$. j is used to represent the imaginary part of a complex number. \mathbf{I}_M is

the $M \times M$ identity matrix. $\mathbf{0}$ represents a zero matrix or vector.

II. PROBLEM FORMULATION

A. Classical model

Consider a ULA of M sensors with d the distance between two adjacent sensors. Suppose K uncorrelated narrowband sources impinging on this array from directions θ_k , $k = 1, \dots, K$. The used snapshots number is denoted by N . The received signal at the array sensors is the superposition of these signals, which can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]$; $\mathbf{s}(t) = [s_1(t), \dots, s_K(t)]^T$; $\mathbf{n}(t)$ represents the spatially and temporally white noise; $s_k(t)$ denotes the k -th signal received at the first sensor; $\mathbf{n}(t)$ and $\mathbf{s}(t)$ are assumed to be uncorrelated and $\mathbf{a}(\theta_k)$ is the steering vector corresponding to $s_k(t)$, given by

$$\mathbf{a}(\theta_k) = [1, e^{j\frac{2\pi}{\lambda} d \sin \theta_k}, \dots, e^{j(M-1)\frac{2\pi}{\lambda} d \sin \theta_k}]^T$$

λ is the wavelength of the impinging narrowband signals, and d is usually set to be $\lambda/2$.

B. MUSIC algorithm

The correlation matrix of the received signal in (1) can be expressed as

$$\begin{aligned} \mathbf{R}_{\mathbf{xx}} &= E[\mathbf{x}(t)\mathbf{x}^H(t)] \\ &= \mathbf{A}E[\mathbf{s}(t)\mathbf{s}^H(t)]\mathbf{A}^H + \sigma_n^2 \mathbf{I}_M \\ &= \mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma_n^2 \mathbf{I}_M \\ &= \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H + \mathbf{G}\mathbf{\Gamma}\mathbf{G}^H \end{aligned} \quad (2)$$

where the columns of \mathbf{U} generate the signal-subspace, formed by the eigenvectors corresponding to the K largest eigenvalues and the columns of \mathbf{G} span the noise-subspace, formed by the eigenvectors corresponding to the $M-K$ smallest eigenvalues. Since the signal-subspace and noise-subspace are orthogonal, the actual source steering vector $\mathbf{a}(\theta_k)$ and the noise-subspace are orthogonal.

$$\mathbf{a}(\theta_k)^H \mathbf{G} = 0, \text{ for } k = 1, 2, \dots, K \quad (3)$$

Therefore, the spatial pseudo-spectrum can be expressed as

$$p(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{G}\mathbf{G}^H \mathbf{a}(\theta)} \quad (4)$$

C. Coprime array

Assume N and M are two coprime numbers. Consider a linear array with $N + 2M - 1$ non-uniformly located sensors along the X-axis, whose positions are given by the following set in terms of X-coordinate:

$$S = \{Mnd, 0 \leq n \leq N-1\} \cup \{Nmd, 1 \leq m \leq 2M-1\}$$

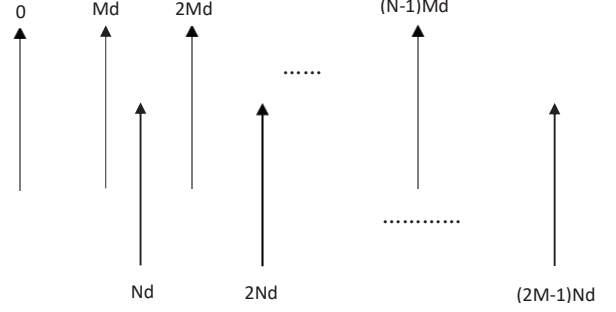


Fig. 1. Coprime array.

where, d is the fundamental spacing.

Similar to model (1), the received signal is denoted by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (5)$$

where $\mathbf{a}(\theta)$ is the $(N + 2M - 1) \times 1$ steering vector whose elements are given by

$$e^{j\frac{2\pi}{\lambda} l \sin \theta}, l \in S$$

The correlation matrix of the received signal is given by:

$$\begin{aligned} \mathbf{R}_{\mathbf{xx}} &= E[\mathbf{x}(t)\mathbf{x}^H(t)] \\ &= \mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma_n^2 \mathbf{I}_{N+2M-1} \\ &= \sum_{i=1}^K \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I}_{N+2M-1} \end{aligned} \quad (6)$$

Since the sources are temporally uncorrelated, $\mathbf{R}_{\mathbf{ss}}$ is a diagonal matrix with diagonal elements $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$ which are equal to the actual sources powers. By vectorizing the correlation matrix $\mathbf{R}_{\mathbf{xx}}$. The following vector is obtained:

$$\begin{aligned} \mathbf{z} &= \text{vec}(\mathbf{R}_{\mathbf{xx}}) \\ &= \text{vec}\left[\sum_{i=1}^K \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i)\right] + \sigma_n^2 \tilde{\mathbf{1}}_n \\ &= \mathbf{B}(\theta_1, \dots, \theta_K) \mathbf{p} + \sigma_n^2 \tilde{\mathbf{1}}_n \end{aligned} \quad (7)$$

where $\mathbf{B}(\theta_1, \dots, \theta_K) = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \dots, \mathbf{a}^*(\theta_K) \otimes \mathbf{a}(\theta_K)]$, $\mathbf{p} = [\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2]^T$, and $\tilde{\mathbf{1}}_n = [\mathbf{e}_1^T, \mathbf{e}_2^T, \dots, \mathbf{e}_{N+2M-1}^T]^T$ with \mathbf{e}_i a column vector of all zeros, except the one at the i -th position, which is equal to one. Comparing (7) with (1), \mathbf{z} behaves like a signal vector received by a longer virtual array and \mathbf{B} acts as the manifold matrix. The equivalent source signal vector is represented by \mathbf{p} and the noise becomes a deterministic vector given by $\sigma_n^2 \tilde{\mathbf{1}}_n$. According to \mathbf{B} , the locations of the virtual sensor array are given by the values in the following set including the subset of cross differences $\{\pm(Mn - Nm)d, 0 \leq n \leq N-1, 1 \leq m \leq 2M-1\}$ and the subset of self-differences $\{(Mn_1 - Mn_2)d, 0 \leq n_1, n_2 \leq N-1\}, \{(Nm_1 - Nm_2)d, 1 \leq m_1, m_2 \leq 2M-1\}$. It includes $2MN + 1$ consecutive differences from $-MNd$ to MNd [5]. By extracting and sorting those rows corresponding to the $2MN + 1$ consecutive differences, a new model is obtained as follows:

$$\tilde{\mathbf{z}} = \tilde{\mathbf{B}}\mathbf{p} + \sigma_n^2 \tilde{\mathbf{e}}' \quad (8)$$

where $\tilde{\mathbf{e}}'$ is a vector of all zeros except a "1" at the $(MN+1)$ -th position. (8) is equivalent to a ULA receiving K coherent sources \mathbf{p} with deterministic noise term $\sigma_n^2 \tilde{\mathbf{e}}'$. By applying MUSIC with spatial smoothing, it can identify MN sources.

III. PROPOSED METHOD FOR MIXED SIGNALS

Recently, [12] proposed a method to enhance the DoA finding performance for non-circular signals by using coprime array. In this paper, we extend the discussion to the mixed signals including non-circular and circular signals.

A. Data model

Consider the model in (5), assume K temporally uncorrelated narrowband signals are impinging on this array from directions $\{\theta_i, i = 1, 2, \dots, K\}$ with powers $\{\sigma_i^2, i = 1, 2, \dots, K\}$, in which there are K_1 non-circular signals and K_2 circular signals, respectively, $K = K_1 + K_2$. Equation (5) can be rewritten as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{A}_1\mathbf{s}_{nc}(t) + \mathbf{A}_2\mathbf{s}_c(t) + \mathbf{n}(t) \quad (9)$$

where $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2]$ with $\mathbf{A}_1 = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_{K_1})]$ and $\mathbf{A}_2 = [\mathbf{a}(\theta_{K_1+1}), \mathbf{a}(\theta_{K_1+2}), \dots, \mathbf{a}(\theta_{K_1+K_2})]$ denoting the array manifold matrices, and $\mathbf{s}_{nc}(t)$ and $\mathbf{s}_c(t)$ denoting the non-circular real signals and complex circular signals respectively. $\mathbf{n}(t)$ represents the spatially and temporally white noise, which is uncorrelated with the sources. By combining the original signal vector and its complex conjugate counterpart, a new extended signal model can be formed:

$$\begin{aligned} \tilde{\mathbf{x}} &= \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}^*(t) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_1^* \end{bmatrix} \mathbf{s}_{nc} + \begin{bmatrix} \mathbf{A}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2^* \end{bmatrix} \begin{bmatrix} \mathbf{s}_c(t) \\ \mathbf{s}_c^*(t) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}^*(t) \end{bmatrix} \end{aligned} \quad (10)$$

B. Co-array model

Based on model (10), we get the correlation matrix of the extended received signal:

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} &= E[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^H] \\ &= \begin{bmatrix} E[\mathbf{x}(t)\mathbf{x}^H(t)] & E[\mathbf{x}(t)\mathbf{x}^T(t)] \\ E[\mathbf{x}^*(t)\mathbf{x}^H(t)] & E[\mathbf{x}^*(t)\mathbf{x}^T(t)] \end{bmatrix} \end{aligned} \quad (11)$$

Considering the fact that the signals are uncorrelated with each other and with the noise, and that both the mean and the elliptic covariance of a circular signal are equal to zero, we have $E[s_i s_i^T] = E[s_i^* s_i^H] = 0$, for $K_1 + 1 \leq i \leq K_1 + K_2$, and the following equations hold:

$$E[\mathbf{x}(t)\mathbf{x}^H(t)] = \sum_{i=1}^K \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I}_{N+2M-1} \quad (12)$$

$$E[\mathbf{x}(t)\mathbf{x}^T(t)] = \sum_{i=1}^{K_1} \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^T(\theta_i) \quad (13)$$

$$E[\mathbf{x}^*(t)\mathbf{x}^H(t)] = \sum_{i=1}^{K_1} \sigma_i^2 \mathbf{a}^*(\theta_i) \mathbf{a}^H(\theta_i) \quad (14)$$

$$E[\mathbf{x}^*(t)\mathbf{x}^T(t)] = \sum_{i=1}^{K_1} \sigma_i^2 \mathbf{a}^*(\theta_i) \mathbf{a}^T(\theta_i) + \sigma_n^2 \mathbf{I}_{N+2M-1} \quad (15)$$

Next, we vectorize these four parts of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ to get the following vectors:

$$\begin{aligned} \mathbf{z}_1 &= \text{vec}(E[\mathbf{x}(t)\mathbf{x}^H(t)]) \\ &= \mathbf{B}_1(\theta_1, \dots, \theta_{K_1+K_2}) \mathbf{p}_1 + \sigma_n^2 \tilde{\mathbf{I}}_n \end{aligned} \quad (16)$$

$$\begin{aligned} \mathbf{z}_2 &= \text{vec}(E[\mathbf{x}(t)\mathbf{x}^T(t)]) \\ &= \mathbf{B}_2(\theta_1, \dots, \theta_{K_1+K_2}) \mathbf{p}_2 \end{aligned} \quad (17)$$

$$\begin{aligned} \mathbf{z}_3 &= \text{vec}(E[\mathbf{x}^*(t)\mathbf{x}^H(t)]) \\ &= \mathbf{B}_3(\theta_1, \dots, \theta_{K_1+K_2}) \mathbf{p}_3 \end{aligned} \quad (18)$$

$$\begin{aligned} \mathbf{z}_4 &= \text{vec}(E[\mathbf{x}^*(t)\mathbf{x}^T(t)]) \\ &= \mathbf{B}_4(\theta_1, \dots, \theta_{K_1+K_2}) \mathbf{p}_4 + \sigma_n^2 \tilde{\mathbf{I}}_n \end{aligned} \quad (19)$$

The following extended model can be obtained by stacking $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3$ and \mathbf{z}_4 :

$$\begin{aligned} \mathbf{z} &= [\mathbf{z}_1^T, \mathbf{z}_2^T, \mathbf{z}_3^T, \mathbf{z}_4^T]^T \\ &= \mathbf{B} \mathbf{p}_* + \sigma_n^2 \tilde{\zeta}_n \end{aligned} \quad (20)$$

where $\mathbf{B} = [\mathbf{B}_1^T, \mathbf{B}_2^T, \mathbf{B}_3^T, \mathbf{B}_4^T]^T$, with $\mathbf{B}_1(\theta_1, \dots, \theta_{K_1+K_2}) = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \dots, \mathbf{a}^*(\theta_{K_1+K_2}) \otimes \mathbf{a}(\theta_{K_1+K_2})]$, $\mathbf{B}_2(\theta_1, \dots, \theta_{K_1+K_2}) = [\mathbf{a}(\theta_1) \otimes \mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_{K_1+K_2}) \otimes \mathbf{a}(\theta_{K_1+K_2})]$, $\mathbf{B}_3(\theta_1, \dots, \theta_{K_1+K_2}) = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}^*(\theta_1), \dots, \mathbf{a}^*(\theta_{K_1+K_2}) \otimes \mathbf{a}^*(\theta_{K_1+K_2})]$, $\mathbf{B}_4(\theta_1, \dots, \theta_{K_1+K_2}) = [\mathbf{a}(\theta_1) \otimes \mathbf{a}^*(\theta_1), \dots, \mathbf{a}(\theta_{K_1+K_2}) \otimes \mathbf{a}^*(\theta_{K_1+K_2})]$; $\mathbf{p}_* = [\mathbf{p}_1^T, \mathbf{p}_2^T, \mathbf{p}_3^T, \mathbf{p}_4^T]^T$, with $\mathbf{p}_1 = \mathbf{p}_4 = [\sigma_1^2, \sigma_2^2, \dots, \sigma_{K_1+K_2}^2]^T$, $\mathbf{p}_2 = \mathbf{p}_3 = [\sigma_1^2, \sigma_2^2, \dots, \sigma_{K_1}^2, 0, \dots, 0]^T$ in which there are zeros for the last K_2 elements; and $\tilde{\zeta}_n = [\tilde{\mathbf{I}}_n^T, \mathbf{0}^T, \mathbf{0}^T, \tilde{\mathbf{I}}_n^T]^T$, $\tilde{\mathbf{I}}_n = [\mathbf{e}_1^T, \mathbf{e}_2^T, \dots, \mathbf{e}_{N+2M-1}^T]^T$ with \mathbf{e}_i denoting column vector of all zeros, except the one at the i -th position, which is equal to one, and $\mathbf{0}$ represents a zero column vector with the same size as $\tilde{\mathbf{I}}_n$. Similar to Section II-C, \mathbf{B}_i can be regarded as a manifold matrix and the corresponding equivalent source signal vector is represented by \mathbf{p}_i , while the noise becomes a deterministic vector.

The distinct rows of \mathbf{B}_1 and \mathbf{B}_4 behave like the manifold of a longer array whose sensor locations are given by the values in the set including the subset of cross differences $\{\pm(Mn - Nm)d, 0 \leq n \leq N-1, 1 \leq m \leq 2M-1\}$ and that of self-differences $\{(Mn_1 - Mn_2)d, 0 \leq n_1, n_2 \leq N-1\}$, $\{(Nm_1 -$

$Nm_2)d, 1 \leq m_1, m_2 \leq 2M - 1\}$. This set includes $2MN + 1$ consecutive differences from $-MN$ to MN . The sensor positions represented by the rows of \mathbf{B}_2 and \mathbf{B}_3 are given by the set including subset $\{\pm(Mn + Nm)d, 0 \leq n \leq N - 1, 1 \leq m \leq 2M - 1\}$, $\{(Mn_1 \pm Mn_2)d, 0 \leq n_1, n_2 \leq N - 1\}$ and subset $\{(Nm_1 \pm Nm_2)d, 1 \leq m_1, m_2 \leq 2M - 1\}$.

The positions of the virtual sensors include more than $4MN + 1$ consecutive differences. Hence from \mathbf{B}_i , for non-circular signals, we can extract a manifold matrix of a much longer ULA with $2M_{max} + 1$ sensors located at $nd, -M_{max} \leq n \leq M_{max}$, where $M_{max} > 2MN$.

By extracting and sorting those rows from \mathbf{B} which correspond to the $2M_{max} + 1$ consecutive sequence, we construct a new matrix \mathbf{M} of size $(2M_{max} + 1) \times K$. The (n, i) -th element of \mathbf{M} is given by $e^{j2n\pi d \sin\theta_i/\lambda}$, $i = 1, 2, \dots, K, n = -M_{max}, -M_{max} + 1, \dots, -1, 0, 1, \dots, M_{max} - 1, M_{max}$. This is equivalent to extracting and sorting the corresponding rows from the observation vector \mathbf{z} to form a new vector $\tilde{\mathbf{z}}$:

$$\tilde{\mathbf{z}} = \mathbf{M}\mathbf{p}_* + \sigma_n^2 \tilde{\mathbf{e}}'$$

where $\tilde{\mathbf{e}}' \in \mathbf{R}^{(2M_{max}+1) \times 1}$ is a vector of all zeros except a 1 at the $(M_{max} + 1)$ -th position. The vector $\tilde{\zeta}_n$ consists of 0s and 1s with the 1s occurring exactly at those rows where the phase terms in \mathbf{B} are equal to 0. Therefore, corresponding to the center row, the $(M_{max} + 1)$ th element of $\tilde{\mathbf{e}}'$ is equal to 1. \mathbf{M} is exactly identical to the manifold matrix of an ULA with $2M_{max} + 1$ sensors located from $-M_{max}d$ to $M_{max}d$.

C. Spatial smoothing

For $\tilde{\mathbf{z}}$, it consists of the powers of the actual sources behaving like fully coherent sources, spatial smoothing is needed. We divide the virtual ULA with $2M_{max} + 1$ sensors into $M_{max} + 1$ overlapping subarrays, each with $M_{max} + 1$ elements, where the i -th subarray corresponding to the i -th to the $(M_{max} + i)$ -th rows of $\tilde{\mathbf{z}}$ has sensors located at $\{(n + i - M_{max})d, n = 0, 1, \dots, M_{max}\}$. We denote the signal received by the i -th subarray as

$$\tilde{\mathbf{z}}_i = \mathbf{M}_i \mathbf{p}_* + \sigma_n^2 \mathbf{e}'_i \quad (21)$$

where \mathbf{M}_i is a $(M_{max} + 1) \times K$ matrix taking the elements from the i -th to the $(M_{max} + i)$ -th rows of \mathbf{M} and \mathbf{e}'_i is a vector of all zeros except a "1" at the $(M_{max} + 2 - i)$ -th position. Define $\mathbf{R}_i = \tilde{\mathbf{z}}_i \tilde{\mathbf{z}}_i^H$. Taking the average of \mathbf{R}_i over all i , we get

$$\mathbf{R}_s = \frac{1}{M_{max} + 1} \sum_{i=1}^{M_{max}+1} \mathbf{R}_i \quad (22)$$

Since $M_{max} > 2MN$, \mathbf{R}_s is the spatially smoothed matrix with which we are able to estimate the DoA of up to $2MN$ non-circular sources or MN circular sources. As for mixed source signals, it can identify $K_1 + K_2$ sources once $K_1 + 2K_2 < 2MN$ is satisfied.

D. Estimating method

\mathbf{R}_s is a full rank matrix and the eigendecomposition can be applied as

$$\mathbf{R}_s = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H + \mathbf{G}\mathbf{\Gamma}\mathbf{G}^H \quad (23)$$

where the diagonal matrix $\mathbf{\Lambda}$ of $(K_1 + 2K_2) \times (K_1 + 2K_2)$ contains the $K_1 + 2K_2$ eigenvalues of \mathbf{R}_s and the diagonal matrix $\mathbf{\Gamma}$ contains the $M_{max} - K_1 - 2K_2$ eigenvalues of \mathbf{R}_s . According to [11], we partition \mathbf{G} as $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$ where \mathbf{G}_1 and \mathbf{G}_2 are two matrices with equal dimension. In general, we can use (24) to estimate the DoA of each signal as follows:

$$f(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{G}\mathbf{G}^H \mathbf{a}(\theta)} \quad (24)$$

To improve the estimation performance for circular signals, (25) is employed as follows:

$$f_c(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{G}_1 \mathbf{G}_1^H \mathbf{a}(\theta)} \quad (25)$$

IV. SIMULATION RESULTS

In this section, simulation is performed to examine the performance of the proposed method in 3 different cases. In the simulation, we assumed $M = 5$, $N = 7$, the fundamental spacing $d = \lambda/2$.

A. Case I: circular signals

In this case, we consider $D = 35$ circular signals with DoAs θ uniformly distributed from $\sin(\theta) = -1$ to $\sin(\theta) = 1$. The SNR is set to be 0 dB using 3000 snapshots. With $M = 5$ and $N = 7$, we can estimate up to $MN = 35$ sources from distinct directions. Fig.2 shows the MUSIC spectrum. Apparently, it can identify all the DoAs clearly.

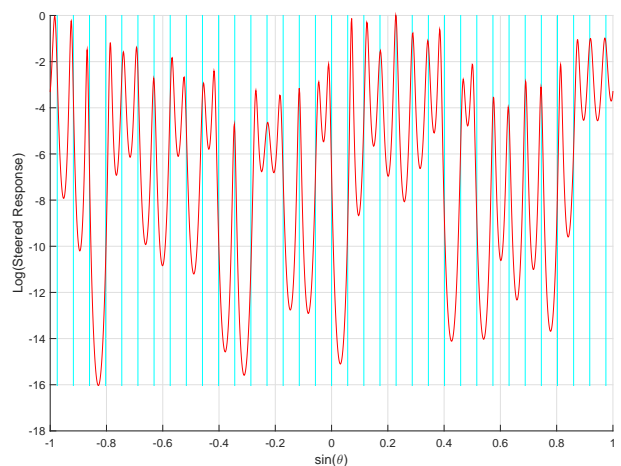


Fig. 2. Spectrum for DOA estimation of circular signals using proposed method, $M=5, N=7, \text{SNR}=0\text{dB}$.

B. Case II: real signals

For the real signal case, we use $D = 70$ real signals with DoAs θ uniformly distributed from $\sin(\theta) = -1$ to $\sin(\theta) = 1$. The SNR is set to be 0 dB . 1000 snapshots is used, $M = 5$ and $N = 7$. Fig.3 shows that the proposed method can identify 70 sources clearly, which is 2 times more than the traditional MUSIC based on coprime array.

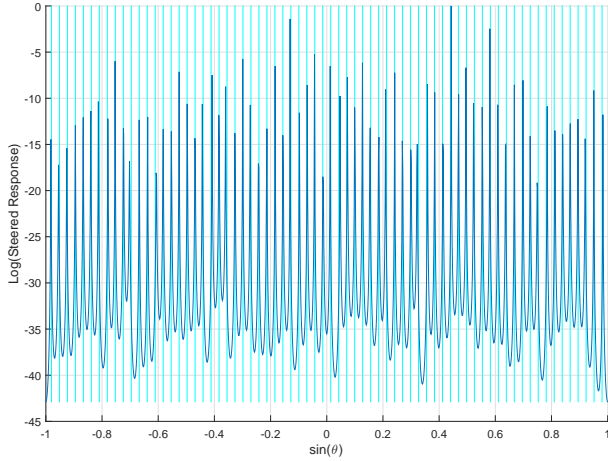


Fig. 3. Spectrum for DOA estimation of real signals using proposed method, $M=5, N=7, SNR=0\text{dB}$.

C. Case III: co-existence of circular and non-circular signals

As for the mixed signals, we consider $K_1 = 30$ non-circular signals and $K_2 = 10$ circular signals with $M = 5$ and $N = 7$. The SNR is set to be 0 dB . 3000 snapshots are used in simulation. Fig.4 shows that the proposed estimator is able

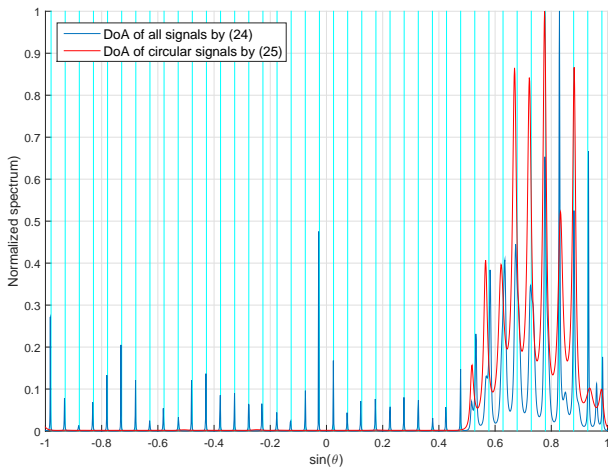


Fig. 4. Spectrum for mixed signals using proposed method.

to identify both non-circular signals and circular signals with good accuracy.

V. CONCLUSIONS

In this paper, a new DOA estimation method is proposed for the co-prime array in the context of K_1 non-circular and K_2 circular sources. Simulation results shows that the proposed method increases degrees of freedom greatly and distinguish the circular signal and the non-circular signal clearly once the condition $K_1 + 2K_2 < 2MN$ is satisfied.

REFERENCES

- [1] G. Zheng, B. Chen, and M. Yang, "Unitary ESPRIT algorithm for bistatic MIMO radar," *Electronics Letters*, vol. 48, no. 3, pp. 179-181, Feb. 2012.
- [2] R. Cao, B. Liu, F. Gao, and X. Zhang, "A low-complex one-snapshot DOA estimation algorithm with massive ULA," *IEEE Communications Letters*, vol. 21, no. 5, pp. 1071-1074, May 2017.
- [3] M. Jiang, J. Huang, W. Han, F. Chu. "Research on target DOA estimation method using MIMO sonar," *IEEE Conference on Industrial Electronics & Applications*. IEEE, pp. 1982-1984, Jun. 2009.
- [4] R.O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, Mar. 1986.
- [5] P. Pal and P. P. Vaidyanathan, "Coprime sampling and the music algorithm," *Digital Signal Processing and Signal Processing Education Meeting (DSP/SPE)*, pp. 289-294, Mar. 2011.
- [6] T. J. Shan "On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-33, NO. 4, pp. 806-811, Aug. 1985
- [7] J. Li, M. Shen, D. Jiang, "DOA estimation based on combined ESPRIT for co-prime array," *IEEE Conference on Antennas & Propagation*. IEEE, pp. 117-118, Feb. 2017.
- [8] A. Nguyen, T. Matsubara, and T. Kurokawa, "High-Performance DOA Estimation for Coprime Arrays with Unknown Number of Sources," *Asia-Pacific International Symposium on Electromagnetic Compatibility (APEMC)*, pp. 369-371, Jun. 2017.
- [9] J. Li, D. Li, D. Jiang, and X. Zhang, "Extended-Aperture Unitary Root MUSIC-Based DOA Estimation for Coprime Array," *IEEE Communications Letters*, Vol. 22, no. 4 pp. 752-755, Apr. 2018
- [10] P. Charge, Y. Wang, and J.Saillard, "A root-music algorithm for non circular sources," *2001 IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, pp. 7-11, May 2001.
- [11] F. Gao, A. Nallanathan, Y. Wang, Improved MUSIC Under the Coexistence of Both Circular and Noncircular Sources, *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3033-3038, Jul. 2008.
- [12] H. Zhai, X. Zhang, W. Zheng, and P. GONG, "DOA Estimation of Noncircular Signals for Unfolded Coprime Linear Array: Identifiability, DOF and Algorithm(May 2018)," *IEEE Access*, Vol. 6, pp. 29382-29390, May 2018.