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NMF for Big Data with Missing Entries: A Random Projection Based Approach

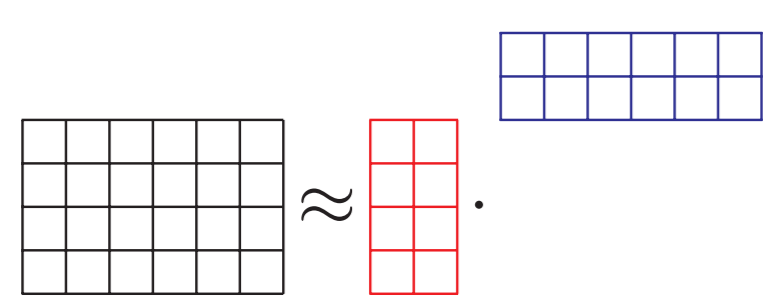
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Context and problem statement

$$X \approx W \cdot H$$



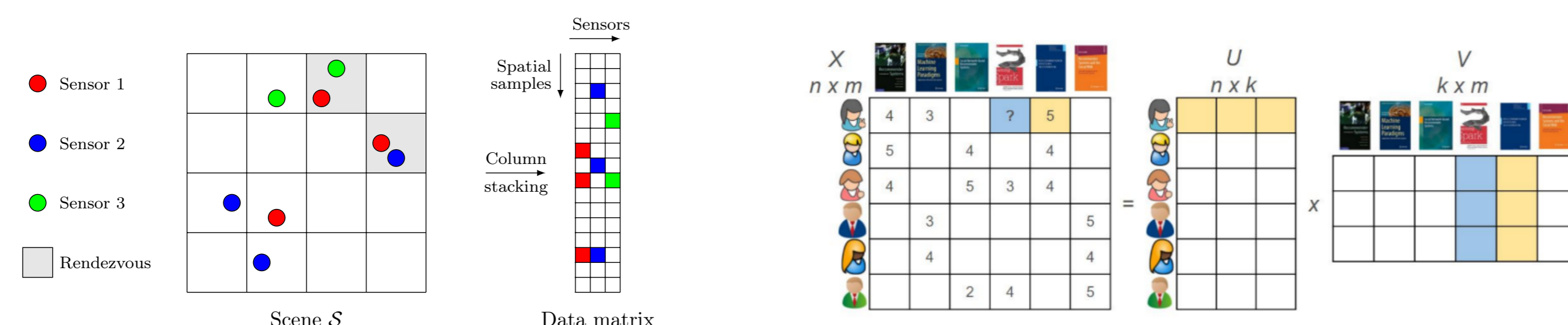
► **Nonnegative Matrix Factorization (NMF)** is a popular tool in Signal/Image Processing and Machine Learning

- Goal: estimate two nonnegative $n \times p$ and $p \times m$ matrices W and H such that an observed low-rank nonnegative $n \times m$ matrix X can be written as $X \approx W \cdot H$
- Some applications:
 - **Source separation**, dictionary learning, graph analysis, topic modelling, **hyperspectral unmixing**...
- Why is NMF so popular? **Better interpretability** than no-sign-constrained approaches



NMF and PCA applied to face dataset (source: [1])

- How is NMF working?
 - Iterative procedure where W and H are alternately updated
 - Historical techniques known to be slow (multiplicative updates, projected gradients, nonnegative least squares, etc)
- NMF and Big Data: How to face the data deluge?
 - Distributed computing (e.g., [2])
 - Online factorization (e.g., [3])
 - Fast solver (e.g., [4])
 - Randomized strategies (e.g., [5, 6, 7])
- In many problems, observed data matrix X with **missing entries or confidence measures** associated to each entry
 - Some applications: collaborative filtering, **source apportionment**, **low-rank nonnegative matrix completion**, **mobile sensor calibration** (aim of the Ph.D. thesis)



mobile sensor calibration (source: [8]).

recommender systems (source: [9])

◊ **Weighted NMF (WNMF):**

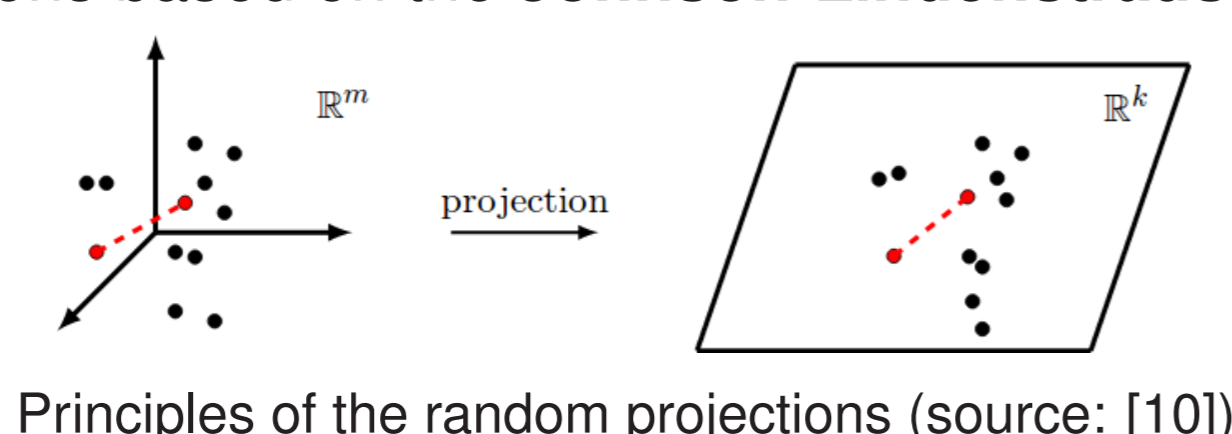
$$\min_{W, H \geq 0} \|Q \circ X - Q \circ (W \cdot H)\|_{\mathcal{F}}$$

- WNMF and Big Data:
 1. Most techniques are based on slow solvers
 2. A few with the fastening strategies for unweighted NMF
 3. **No existing approach with random projections**

◊ **How to use random projections in WNMF?**

Principle of random projections applied to NMF

- **Random projections** is a popular tool in machine learning to speed-up computations while preserving pairwise structure
- Mathematical foundations based on the **Johnson-Lindenstrauss Lemma**



Principles of the random projections (source: [10])

- Gaussian random matrices as projection matrices [5]
- **Structured** random projections [6, 7]
 - ◊ **Randomized Power Iterations** (RPIs) or **Randomized Subspace Iterations** (RSIs)

Require: a target rank ν (with $p \leq \nu \ll \min(n, m)$) and an integer q
 Draw Gaussian random matrices $\Omega_L \in \mathbb{R}^{m \times \nu}$ and $\Omega_R \in \mathbb{R}^{\nu \times n}$
 Define $B_L \triangleq (X X^T)^q \cdot X \cdot \Omega_L$ and $B_R \triangleq \Omega_R \cdot X \cdot (X^T X)^q$
 Derive $L \in \mathbb{R}^{\nu \times n}$ and $R \in \mathbb{R}^{m \times \nu}$ by QR decompositions of B_L and B_R , respectively.

- RSIs are similar but add intermediate QR decompositions in the computations ◊ less sensitive to round-off errors than RPIs

◊ **Why we have designed "compressions matrices" R and L ?**

Dimensionality reduction of X by right (resp. left) multiplication with R (resp. L)

$$\underbrace{X \cdot R}_{X_R} \approx \underbrace{W \cdot H}_{H_R} \cdot R \quad \quad \quad \underbrace{L \cdot X}_{X_L} \approx \underbrace{L \cdot W}_{W_L} \cdot H$$



NMF with weights

Weighted Extensions of NMF aim to solve:

$$\min_{W, H \geq 0} \|Q \circ X - Q \circ (W \cdot H)\|_{\mathcal{F}}$$

1. Direct computations (Ho, 2008):
 - Incorporating the matrix Q in the update rules
2. EM-based strategy (Zhang *et al.*, 2006)
 - E-step: Estimate the unknown entries of X

$$X^{\text{comp}} = Q \circ X + (\mathbb{1}_{n,m} - Q) \circ (W \cdot H), \quad (1)$$

where $\mathbb{1}_{n,m}$ is the $n \times m$ matrix of ones.

- M-step: Apply any standard NMF technique to X^{comp}

◊ **Direct incorporation of random projections is not trivial...**

Proposed Method

Require: initial matrices W and H

repeat

{**E-step**} {Compression makes E-step 3 times slower in our experiments}

Compute X^{comp} as in (1)

Apply RSIs or RPIs to X^{comp} to compute L and R

Define $X_L^{\text{comp}} \triangleq L \cdot X^{\text{comp}}$ and $X_R^{\text{comp}} \triangleq X^{\text{comp}} \cdot R$

{**M-step**} {10-100 times faster than SotA EM-W-NMF methods in our experiments}

for $\text{comp}=1$ **to** MaxOuter **do**

Define $H_R \triangleq H \cdot R$

Solve $\min_{W \geq 0} \|X_R^{\text{comp}} - W H_R\|_{\mathcal{F}}$

Define $W_L \triangleq L \cdot W$

Solve $\min_{H \geq 0} \|X_L^{\text{comp}} - W_L H\|_{\mathcal{F}}$

end for

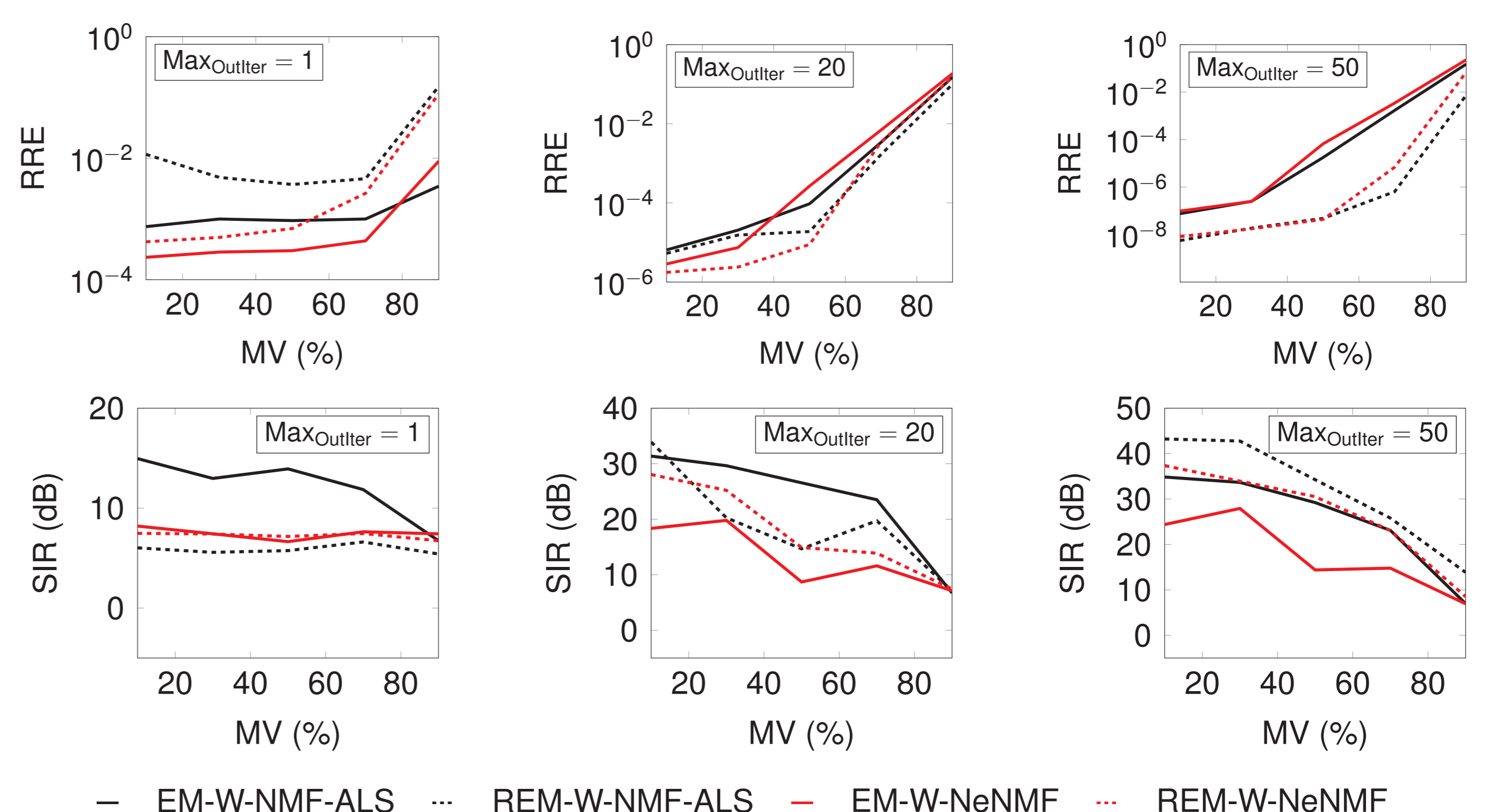
until a stopping criterion

Experiments and Results

To test our method we run each test for 60 seconds and repeat 15 times the following:

- we randomly generate nonnegative factor matrices W^{theo} and H^{theo} , with $n = m = 10000$ and $p = 5$.
- we randomly suppress data to generate X , with a sampling rate varying from 10 to 90% (with a step-size of 20%)
- We compare the proposed REM-W-NMF strategy to the EM-W-NMF with several solvers, i.e., MU, PG, ALS & Nesterov iterations
- measure the accuracy of reconstruction of X Relative Reconstruction Error (RRE) and the accuracy of estimation of H (Signal-to-Interference Ratio—SIR)

$$\text{RRE} \triangleq \frac{\|X^{\text{theo}} - W \cdot H\|_{\mathcal{F}}^2}{\|X^{\text{theo}}\|_{\mathcal{F}}^2} \quad (2) \quad \text{SIR} = \sum_{j=1}^p 10 \log_{10} \left(\frac{\|\hat{h}_j^{\text{coll}}\|^2}{\|\hat{h}_j^{\text{orth}}\|^2} \right) \quad (3)$$



Conclusion and perspectives

- Novel framework to combine random projections and weighted matrix factorization.
- Based on an EM scheme
- The proposed strategy outperforms non-randomized state-of-the-art EM techniques
- In future work, we aim to apply the proposed strategy to informed and structured NMF techniques applied to mobile sensor calibration

References and acknowledgments

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