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# Quantifying silo flow using MRI velocimetry for testing granular flow models

Luke Fullard\*

*School of Fundamental Sciences,  
Massey University, New Zealand.*

Daniel J. Holland

*Department of Chemical and Process Engineering,  
University of Canterbury, New Zealand.*

Petrik Galvosas

*School of Chemical and Physical Sciences,  
Victoria University, New Zealand*

Clive Davies

*School of Food and Advanced Technology,  
Massey University, New Zealand.*

Pierre-Yves Lagrée, Stéphane Popinet

*Institut Jean le Rond d'Alembert,  
CNRS UMR 7190  
Sorbonne Université,  
Paris, France.*

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In this work we present experimental results of the gravity-driven discharge of poppy seeds from 3D-printed silos. The velocity fields of the flowing poppy seeds are measured using Magnetic Resonance Imaging (MRI) velocimetry techniques. Crucially, this approach allows the velocity field to be determined throughout the flow domain, unlike visual techniques such as Particle Image Velocimetry (PIV) and related methods where only the flow at or near the wall is accessible. We perform the experiment three times; with 3D-printed silos of cone half angles  $30^\circ$  and  $50^\circ$  respectively, and then repeat the  $30^\circ$  silo experiment, but with a layer of poppy seeds glued to the silo wall to create a “rough wall” condition. In our experiments, we observe and quantify velocity fields for three well known granular flow regimes; mass flow, funnel flow, and rat-holing. The results of the experiments are compared to equivalent output of numerical simulations. In this mathematical model, the well-known  $\mu(I)$  friction law is used to define an effective granular viscosity, and the flow is solved using a standard Navier-Stokes type solver. While the results are generally encouraging, it is noted that some aspects of the model are lacking and should be improved; in particular, the rat-holing effect observed in one of the MRI experiments was not predicted by the model, nor was the exact volumetric flow rate from any of the silos. Suggestions for model improvement are discussed.

## I. BACKGROUND AND INTRODUCTION

Granular matter is well known to behave in complex and often unexpected ways. Particles in a granular assembly may act in a solid-like, liquid-like, or gas-like manner, with the transition between these phases often difficult to

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\* L.Fullard@Massey.ac.nz

define and quantify [1]. A commonly studied granular system is gravity-driven silo discharge. In addition to being a system of great practical importance, silo flow can also display a variety of interesting flow dynamics. Depending on the design of the silo (i.e. the silo half angle, the friction between particles, the friction between the silo walls and particles, and the size and shape of the particles), the flow may be either mass-flow, funnel flow, or display rat-holing [2, 3]. In mass flow, all particles in the silo are in motion with no stagnant zones; in funnel-flow there are regions within the silo where particles flow, but there are also stagnant regions (and an interface between flowing/stagnant regions); when a silo displays rat-holing, flow only occurs in a central core approximately the size of the silo opening, with large stagnant regions surrounding this core. Rat-holing can be considered an extreme case of funnel flow, but the flow is often observed to be intermittent and transient, whereas in a general funnel flow the dynamics are much more steady. Due to the variety of flow regimes, the silo provides an excellent test of numerical models of granular dynamics.

Apart from testing numerical codes, quantifying velocity fields in the silo is of great industrial importance, for example, in the study of particle mixing and segregation as particle blends are discharged from a silo. While there have been many Discrete Element Modelling (DEM) [4–6] and continuum models [7–14] developed to study the silo, experimental measurements and validations are still required.

The vast majority of experimental characterisation of the velocity vector field in a discharging silo has been using visual imaging methods in transparent silos (both conical and planar). Techniques such as Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV) have been successfully applied to measure the grain velocity *at the silo walls* [15–20]. On the contrary however, experimental measurements of velocity fields away from silo walls (i.e. in the bulk of the flow) are particularly difficult to obtain. Previous attempts to experimentally quantify 3D velocity fields in silos

have included X-ray CT [21, 22], timing tracer discharge [23], Scanning gamma ray tomography [24, 25], and single profile proton absorptiometry [26], however, all of these methods give limited velocity profile information, and usually provide averaged data, data at discrete points, or data along a line only, rather than on a plane.

Magnetic Resonance Imaging (MRI) is an alternative technique that can study flow in optically opaque systems. MRI has been applied to non-silo granular systems [27–34] to quantify parameters such as velocity fields and packing. Kawaguchi [35] observed the flow type, either mass or funnel flow, in silos using tagged MR imaging. In this approach, bands of particles are tagged at one point in time and then the positions of these tagged particles imaged after a defined delay (in this case 100 ms). The deformation of the tagged layers was observed visually. In theory this technique could be extended to estimate the velocity in a silo using further image processing techniques, but this would give only an indirect measure of the velocity fields. MRI has also been used to obtain the only reported direct, quantitative measurement of the silo velocity data on a plane away from the silo walls that we have found [36], though the range of silo flow conditions studied was limited. The first objective of the current article is to extend the work of Gentzler and Tardos [36] to obtain velocity field data for a wider range of silo flow situations. Firstly, we report on both the vertical and horizontal component of the velocity at the outlet. Secondly, we also measure particles of a large diameter ( $\approx 1\text{ mm}$ ) such that the effect of the surrounding air on the particle dynamics near the orifice is not significant [37]. Thirdly, we consider the effect of changing the hopper geometry. Finally, we consider the effect of rough-walls on the particle dynamics. These last two aspects of the experiment mean that flow is studied across the three major flow regimes observed in silos.

A second objective is to assess the applicability of the so-called  $\mu(I)$  friction law [38] for reproducing the velocity fields which we experimentally measure. Previously, the  $\mu(I)$  friction law has been used to define an effective gran-

139 ular viscosity for use in incompressible contin-  
 140 uum flow models. Such an approach has been  
 141 successfully applied to the granular column col-  
 142 lapse and to some silo flows. [7, 8, 13, 39].  
 143 However, the velocity fields produced by the  
 144 model have not been rigorously tested against  
 145 experimental data. In particular, we examine  
 146 the model applicability to reproduce the three  
 147 silo flow modes, mass flow, funnel flow, and rat-  
 148 holing, which we observe in our experimental  
 149 results.

## 150 II. MATERIALS AND METHODS

### 151 1. Particle properties

152 In this study, poppy seeds were chosen as the  
 153 granular material of interest due to their par-  
 154 ticle size, their price and availability, and the  
 155 fact that they contain abundant free oil which  
 156 allows a strong signal to be detected by the  
 157 MRI equipment. The poppy seeds were non-  
 158 spherical, and were kidney shaped, as seen in  
 159 Figure 1. The poppy long diameter was ap-  
 160 proximately  $1.25\text{ mm}$ , while the short diameter  
 162 was approximately  $0.85\text{ mm}$ . A standard sieve  
 163 experiment was performed and  $\approx 93\%$  of the  
 164 particles were found to be between  $710\ \mu\text{m}$  and  
 165  $1180\ \mu\text{m}$ , with a Sauter mean diameter [40],  $d$ ,  
 166 of  $951\ \mu\text{m}$ .

### 167 2. Silo system design

168 The silo feeding system was designed to the  
 169 specifications of the bore of the MRI apparatus  
 170 in such a way that the poppy seeds were fully  
 171 contained and never came in direct contact with  
 172 the MRI apparatus itself. A system of perspex  
 173 pipes of decreasing diameter was used to feed  
 174 the poppy seeds into the test silo (the region  
 175 to be imaged by the MRI) and then out of the  
 176 bottom of the system. These pipes were con-  
 177 nected using a series of push-fittings with small  
 178 tolerances. Figure 2 A. displays the full system  
 179 of pipes and the test silo, while B. is a close-  
 180 up of the silo itself. The silo was designed in

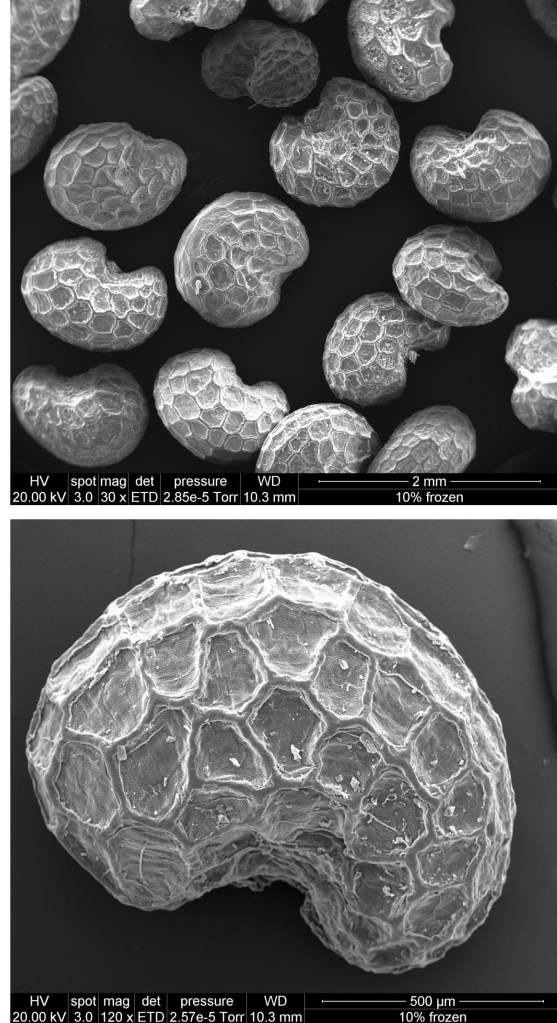


FIG. 1. Scanning Electron Microscope images of a sample of poppy seeds. It is apparent from the image that the seeds are non-spherical with a kidney shape. The surface of the seeds is also seen to be textured. A scale is included at the bottom of each image. **A.** An image of multiple poppy seeds. **B.** A close up of a single poppy seed.

a CAD program, 3D printed from ABS plastic, and the opening at the bottom of the silo,  $D_0$ , was drilled to a diameter of  $6.5\text{ mm}$  (note that this is  $\approx 6.5$  times greater than the Sauter mean diameter,  $d$ , of the particles to avoid jamming

[18, 41]). The inner diameter of the silo,  $W$ , was  $23.5 \text{ mm}$ . Since  $D_0 > 6.5d$ ,  $W > 2.5D_0$ , and the bed height is always deeper than the silo opening diameter, the flow rate from the silo can be expected to be independent of the silo geometry. [42] The silo half angle,  $\phi$ , was changed between each experiment; the first silo had a  $30^\circ$  half angle, the second  $50^\circ$ , and the third was another  $30^\circ$  half angled silo but with rough walls. The rough walled silo was printed in two halves, then poppy seeds were glued onto the inner silo walls in a single layer, and finally, the two halves were glued together to form a full silo. We note that the diameter of the final pipe, labeled pipe 3 in Figure 2, was wider than the silo opening. This design was to avoid the well-known standpipe flow rate effect [43] which does not occur unless the pipe below the silo is full [43]. Since the silo opening diameter was smaller than the exit pipe this was not the case and the standpipe effect was avoided.

### 3. Experimental method

A Bruker Avance I Nuclear Magnetic Resonance spectrometer with a 9.4 T wide bore magnet located at Victoria University of Wellington, New Zealand was used for the experiments. A 30 mm diameter radio-frequency coil was used for excitation and detection. A three-axis shielded Micro2.5 gradient set capable of producing a maximum gradient strength of  $1.51 \text{ T m}^{-1}$  was used for imaging and flow encoding. The pipes and silo were connected together and carefully inserted into the MRI. The silo and upper two pipes were filled from above through a funnel. A bucket was placed under the system to collect the discharged particles. As the particles were discharged the system was periodically refilled from above such that the upper pipe (pipe #1) was never more than half empty. Note that the flow rate from the silo was constant and independent of fill height as is implicit in the Beverloo flow rate equation [44, 45].

The vertical (i.e. in the axial direction) and horizontal (i.e. in the radial direction) components of velocity of the poppy seeds were measured using a phase encoded velocity imaging sequence [46]. The image was obtained using a spin echo acquisition with a slice selective refocussing pulse. To enable accurate measurements of the wide range of velocities present in the system, experiments were repeated with 8 flow encoding gradients. The velocity was calculated from a linear fit to as many of these data points as possible. For the fastest flowing regions, typically only three experiments with the weakest flow encoding gradients were used, while in the slow moving regions all 8 experiments were used. The gradient encoding duration  $\delta$  was set to  $0.7 \text{ ms}$ , the observation time was  $2.5 \text{ ms}$ , and the maximum gradient strength was set to  $0.07 \text{ T m}^{-1}$  in the vertical direction and  $0.14 \text{ T m}^{-1}$  in the horizontal direction. These settings gave a maximum field of flow of approximately  $2 \text{ m s}^{-1}$  with a minimum detectable velocity of  $1 \times 10^{-3} \text{ m s}^{-1}$ ,

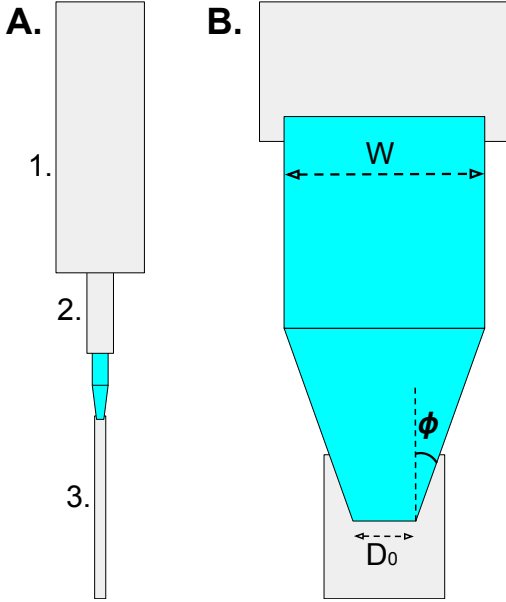


FIG. 2. A sketch of the piping and silo in the experimental set-up (not to scale).

**A.** The system is loaded from above. the seeds flow through the largest pipe #1. into the more narrow pipe #2, through the test silo section, and out through pipe #3. **B.** A close up of the test silo section.

where the minimum detectable velocity corresponds to a signal-to-noise ratio of 2. Images were acquired at a spatial resolution of  $0.45 \text{ mm}$  in the horizontal direction and  $1.18 \text{ mm}$  in the vertical direction with a slice thickness of  $1 \text{ mm}$ . The total acquisition time for the images was approximately 50 minutes. Flow-encoded NMR images can acquire a phase arising from the imaging gradients themselves. It is common practice to correct this phase by acquiring measurements on a static sample. Here images of a static bed were also acquired. The phase change for these was negligible, thus no correction was required.

Three MRI experiments were performed, one with a silo of  $30^\circ$  half angle, one with a silo of  $50^\circ$  half angle, and finally with another silo of  $30^\circ$  half angle, but with rough walls (with particles glued on the silo walls).

#### 4. Numerical model

One goal of this work is to model the silo using a continuum model of granular flow. Recently, the  $\mu(I)$  law for the friction of granular materials has been used to define an effective viscosity in granular flow simulations. This viscosity was successfully implemented into an incompressible Navier-Stokes solver (Gerris Flow Solver [47]) to model dense granular flow in a variety of situations [7, 8, 13, 39]. For our situation, an axisymmetric domain was used so that our 3D silo could be modelled in 2D. The governing equations of incompressible flow were solved in Gerris;

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho \mathbf{g}$$

In the above continuity and momentum equations,  $\mathbf{u}$  is the velocity vector,  $\rho$  the flowing (bulk) density,  $p$  the local isotropic pressure,  $\eta$  the effective (or apparent) granular viscosity, and  $\mathbf{D}$  the rate of strain tensor. The effective viscosity is defined as

$$\eta_{eff} = \frac{\mu(I)p}{D_2}, \quad (3)$$

but in practice a regularised effective viscosity was used to avoid infinite values when the fluid is experiencing small shear;

$$\eta = \min \left( \frac{\mu(I)p}{D_2}, \eta_{max} \right). \quad (4)$$

Here,  $D_2 = \sqrt{\frac{1}{2} D_{ij} D_{ij}}$  is the second invariant of the strain rate tensor, where  $D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$ , and  $\mu(I)$  is the granular friction law;

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}, \quad (5)$$

with  $\mu_1$ ,  $\mu_2$ , and  $I_0$  parameters. The variable  $I$  is the granular inertial number and is defined as

$$I = \frac{d D_2 \sqrt{\rho_p}}{\sqrt{p}}, \quad (6)$$

where  $d$  is the particle diameter and  $\rho_p$  is the solid particle density.

In our axisymmetric numerical model we apply no-slip conditions on both of the velocity components at the silo walls, a symmetry condition along the axis of symmetry, homogeneous Neumann velocity boundary conditions (for each velocity component) at the top and bottom of the silo, and we set  $p = 0$  at the top and bottom of the silo. Note that other boundary conditions could be used at the silo wall (for example, to allow slip at the silo wall [48, 49]), but the effect of more complex boundary conditions is left for future work. For the  $30^\circ$  silo with rough walls, the simulation domain was reduced by a particle diameter in size to account for the reduced dimensions due to the layer of particles glued to the silo walls, but the silo opening was kept at  $6.5 \text{ mm}$ . No other change to the boundary conditions was made.

Parameters used in our simulation are listed in table I. The first friction parameter,  $\mu_1$ , was chosen based on measurements of the angle of repose of the poppy seeds which was found to be approximately  $31^\circ$ , hence,  $\mu_1 = \tan 31 = 0.6$ . The upper limit on the friction angle, defined by parameter  $\tan^{-1}(\mu_2)$ , was expected to be

around  $60^\circ$  since our MRI experimental results for the velocity in the  $30^\circ$  silo (to be presented in Figure 3) showed small slow/stagnant regions at the transition from the conical to cylindrical section. We also noted that larger values of  $I_0$  kept the incompressible  $\mu(I)$  model in the well-posed regime for a wider range of inertial numbers than for low values of  $I_0$  [50]. For this reason, various values of  $\tan^{-1}(\mu_2) \approx 60^\circ$  and  $I_0$  between 0.05 and 1 were tested. It was found that the parameters  $\mu_2 = 1.7$  and  $I_0 = 0.5$  gave a good match to experimental data (to be discussed), gave a wide range of well-posed inertial number values, and, importantly, were physically realistic.

TABLE I: Parameters used in the numerical model.

Name	Symbol	Unit	Value
Bulk density	$\rho$	$kg/m^3$	600
Particle density	$\rho_p$	$kg/m^3$	1000
Particle diameter	$d$	$mm$	0.951
Friction coefficient #1	$\mu_1$	-	0.6
Friction coefficient #2	$\mu_2$	-	1.7
Reference inertial number	$I_0$	-	0.5

### III. RESULTS

#### 1. MRI Experimental Results

The results of the phase encoded velocity imaging sequence experiment were converted into a Matlab data file and plotted as a contour map. In Figure 3 the logarithm of the vertical component of velocity is plotted for each of the three silos, where  $\mathbf{u} = (u, v)$  is the velocity vector with  $u, v$  the horizontal and vertical velocity components respectively. The logarithm of the magnitude of the horizontal component of velocity ( $u$ ) is shown in Figure 4. The lighter (yellow) regions are zones of rapid flow, while the darker (purple/blue) regions indicate slow or stagnant flow. Horizontal velocity measurements were not available for the  $30^\circ$  silo with rough walls because the magnitude of the horizontal component of velocity was very small and

was of the same order as the noise in the experiment.

The most immediate observation from Figure 3 is that for each silo we have a different flow regime. In the  $30^\circ$  silo we observe mass flow. The particles in the silo at every location are in motion, with a possible small exception at the transition from the cone to the cylindrical section. In the  $50^\circ$  silo we observe funnel flow. There is a region of flow in the center of the silo and this region of flowing material widens as we move further up into the silo. There is a clear stagnant region of flow that surrounds the flowing particles. This stagnant region shrinks as we transition higher into the silo. In the  $30^\circ$  silo with rough walls (i.e. with a layer of poppy seeds glued to the wall) we observe the rat-holing effect. There is a fast core (roughly the diameter of the silo opening) of flowing particles surrounded by a region of stagnant material. The size of this stagnant zone does not perceptibly change as we transition higher into the silo. It is also apparent that the velocity field in the flowing zone remains continuous as we move higher in the silo, past the transition from the conical to cylindrical section (i.e. we do not observe velocity discontinuities or shocks). This is in contrast to predictions from Mohr-Coulomb plasticity based models [2, 51].

In order to assess the appropriateness of the incompressible assumption in our numerical model, we quantify the volumetric flow rate as a function of height above the silo opening. For each MRI experiment we use the vertical component of velocity ( $v$ ) to calculate the volumetric flow rate;

$$Q(z) = 2\pi \int_{r(z)=0}^{r(z)=R(z)} v r dr, \quad (7)$$

where  $r(z)$  is the radial coordinate from the axis of the silo, and  $R(z)$  is the radius of the silo at height  $z$  above the opening. The resulting flow rates for each experiment are plotted in Figure 5.

It is apparent from the figure that the volumetric flow rate is approximately constant throughout the silo in the  $30^\circ$  silo, but this is not so for the  $50^\circ$  and  $30^\circ$  silo with roughened

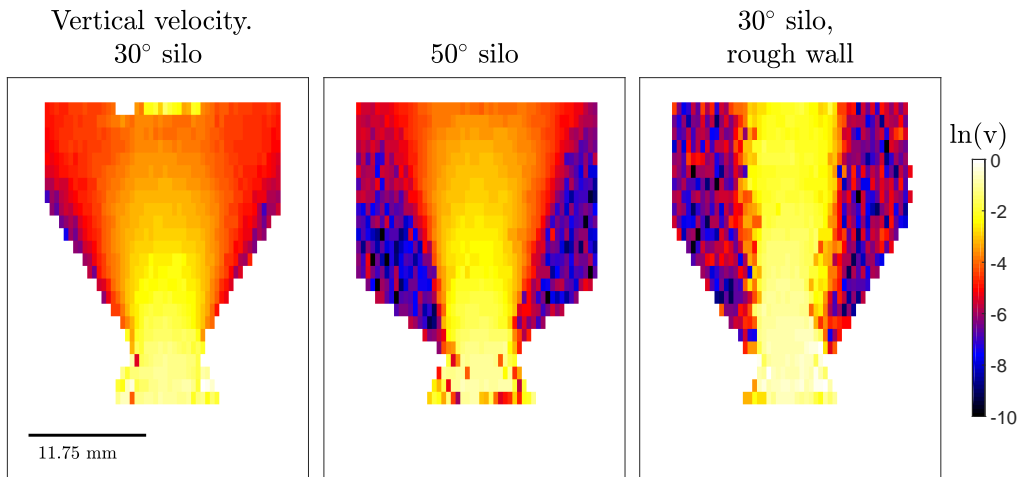


FIG. 3. The log of the magnitude of the vertical component of velocity ( $v$ ) is plotted for each of the three silos. Mass flow is observed in the  $30^\circ$  silo, funnel flow in the  $50^\circ$ , and rat-holing in the  $30^\circ$  silo with rough walls (with particles glued to the silo wall). Yellow regions indicate rapid flow, while purple/blue areas indicate slow to stagnant zones.

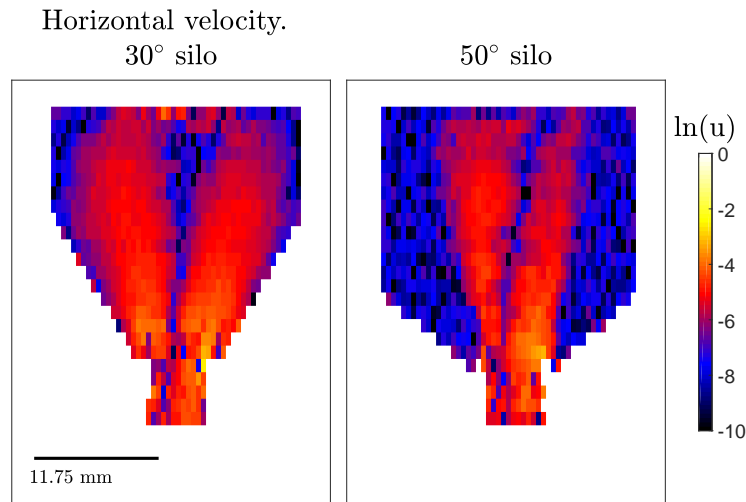


FIG. 4. The log of the magnitude of the horizontal component of velocity ( $u$ ) for the  $30^\circ$  and  $50^\circ$  silos.

414 walls. In these two non-constant flow rate cases,<sup>420</sup>  
 415 the volumetric flow rate  $Q(z)$  is seen to be  $\approx 2 \times$ <sup>421</sup>  
 416 higher near the opening than it is in the bulk of<sup>422</sup>  
 417 the silo. This variation in flow rate could arise<sup>423</sup>  
 418 either from a measurement error or a dilation<sup>424</sup>  
 419 of the flow at the outlet. The signal intensity<sup>425</sup>

at the outlet in all three images is less than half  
 that in the bulk, which would be consistent with  
 a dilation of the flow at the outlet. However,  
 in these measurements there is also significant  
 attenuation of the signal due to the motion of  
 the particles, so the images are not quantitative

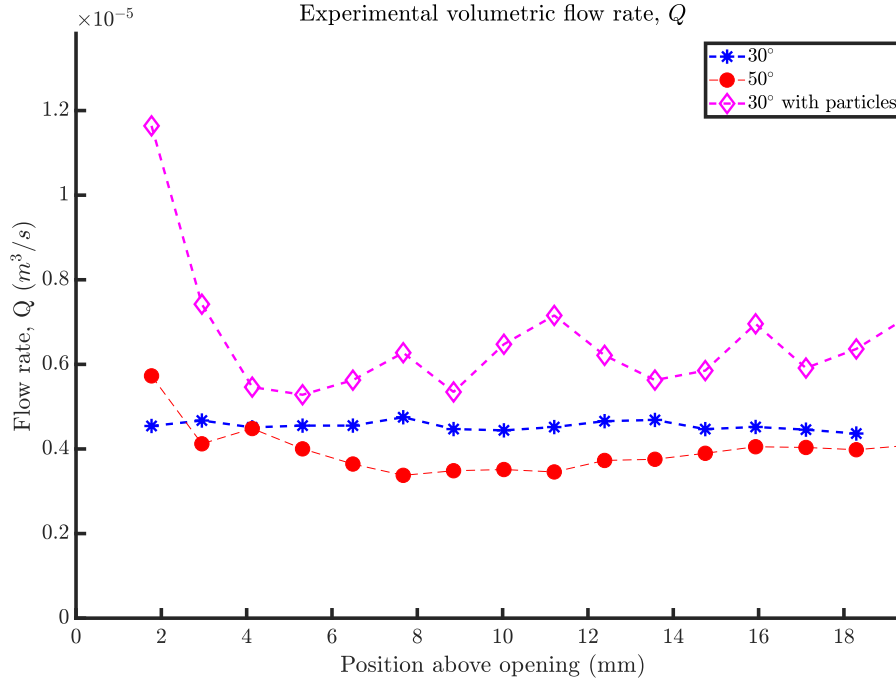


FIG. 5. The volumetric flow rate,  $Q(z)$ , for each of the three silo experiments as a function of height above the silo opening.

in solid fraction. Therefore it is important to consider the errors that arise in measurement of velocity. MRI measurements of the velocity are prone to error in regions of high velocity but this error will tend to cause an underestimation of the velocity as faster moving particles are more heavily attenuated than slower moving particles. The flow rate is seen to increase towards the outlet, hence, it is unlikely that a velocity measurement error could explain the observed flow rate variation. Therefore, it is concluded that, for the funnel flow and rat-holing silos, there is significant dilation of the flow near the opening, and the assumption of incompressibility is likely to be erroneous, at least near the silo opening. In a similar system, a wedge shaped hopper, a significant reduction in bulk density has been observed [52]. As a point of context, in the numerical model the incompressibility condition is enforced (up to a tolerance) and it was found that the change in the volumet-

ric flow rate was less than 1% throughout the silo. Here we assume that the use of an incompressible flow model has only a small effect on the predicted velocity fields, since in the bulk of the silo the flow rate is relatively constant, changing only near the silo opening. However, the dilation near the opening will change the predicted flow rate values. Given this result and model assumption, when comparing experimental and numerical results with an incompressible flow assumption, the velocity should be adjusted to account for the change in volumetric flow rate. In practice this is achieved by normalising the velocity by the volumetric flow rate at each local height above the silo opening. Furthermore, we quantified the mass flow rate,  $\dot{m}$ , from each of the silos by measuring the mass ejected from the system in a given time. For the 30° silo we found  $\dot{m}_{30} = 2.11 \pm 0.07$  g/s, for the 50° silo,  $\dot{m}_{50} = 1.74 \pm 0.09$  g/s, and for the 30° silo with particles on the wall,

468  $\dot{m}_{30}^p = 2.2 \pm 0.1 \text{ g/s}$ . The reduction of the 512  
 469 mass flow rate between the  $30^\circ$  and  $50^\circ$  silos 513  
 470 is compared with corrections made to the Bev-514  
 471 erloo flow rate to account for hopper half angle 515  
 472 [53]. Assuming that the Beverloo parameters 516  
 473 and bulk density is equal between the two silos 517  
 474 of differing half angles, the ratio of the two flow 518  
 475 rates is given as  $M = \frac{f(50^\circ)}{f(30^\circ)}$ , where the func-519  
 476 tion  $f(\alpha) = \sqrt{\frac{1-\cos\alpha}{2\sin^3\alpha}}$ . The theoretical ratio  $M$  520  
 477 is calculated as 0.86, while the experimental ra-522  
 478 tio in our system,  $\frac{\dot{m}_{50}}{\dot{m}_{30}}$  is found to be  $0.82 \pm 0.05$ , 523  
 479 in good agreement with the theoretical value. 524

## 480 2. Numerical Model Results: $30^\circ$ silo 527

481 To directly compare the  $\mu(I)$  numerical re-529  
 482 sults to the MRI experimental results a results 530  
 483 file was imported from Gerris into Matlab which 531  
 484 contained vertical and horizontal components of 532  
 485 velocity. This data was interpolated onto five 533  
 486 horizontal lines which correspond to the loca-534  
 487 tions of measurements taken in the MRI exper-  
 488 iments. Thus, the horizontal and vertical com-  
 489 ponents of velocity predicted in the model could 535  
 490 be directly compared to the experimental data.

491 As previously mentioned, the volumetric flow 536  
 492 rate in the silo experiments was not a constant 537  
 493 near the opening of the silo. Therefore, both 538  
 494 the experimentally measured and numerically 539  
 495 predicted velocity data were normalised by the 540  
 496 volumetric flow rate before being compared. At 541  
 497 each height above the silo opening,  $z$ , the local 542  
 498 volumetric flow rate is calculated using Equa-543  
 499 tion 7. The velocity components are then mul-544  
 500 tiplied by the particle diameter squared and di-545  
 501 vided by the local volumetric flow rate to obtain 546  
 502 the normalised velocity,  $\tilde{\mathbf{u}}$ , where  $\tilde{\mathbf{u}} = \mathbf{u}d^2/Q$ . 547

503 The comparison of the vertical velocity pro-548  
 504 file taken at five heights above the opening for 549  
 505 the  $30^\circ$  silo with smooth walls (i.e. no parti-550  
 506 cles attached to the wall) is shown in Figure 6, 551  
 507 while the horizontal velocity profile is shown in 552  
 508 Figure 7. The distance from the silo opening 553  
 509 to the silo transition (the point where the cone 554  
 510 becomes a cylinder) is  $\approx 14.7 \text{ mm}$ , hence four 555  
 511 of the comparison lines are in the converging 556

conical section of the silo, while one is in the  
 cylindrical section.

It is apparent that the match between the ex-  
 perimentally derived and numerically predicted  
 normalised velocity is good, particularly for the  
 vertical velocity. The normalised velocity pre-  
 dicted by the model has approximately the same  
 maximum and also approximately the same cur-  
 vature and shape as the MRI experimental mea-  
 surements. However, the absolute velocity pre-  
 dicted by the model does not match the exper-  
 iment due to the discrepancy in the volumet-  
 ric flow rate between the two. There is more  
 noise in the horizontal measurements, and the  
 prediction of normalised horizontal velocity is  
 slightly worse near the silo opening, but overall  
 the agreement is satisfying.

As a further test, in Figure 8 we plot the nor-  
 malised vertical component of velocity along the  
 axial centerline of the silo and compare the ex-  
 periment to the model. It is apparent that the  
 model prediction is in very good agreement with  
 the experimental results.

## 503 3. Numerical Model Results: $50^\circ$ silo

In Section III 2, the comparison of numeri-  
 cal and experimental velocity fields for the  $30^\circ$   
 silo with smooth walls, there were no stagnant  
 regions in the flow domain. The transition  
 from flowing to stationary is difficult to capture  
 with simple incompressible Navier-Stokes based  
 models. Figures 9 and 10 show the normalised  
 vertical and horizontal velocity measurements  
 and predictions in the  $50^\circ$  silo. In this silo the  
 distance from the silo opening to the transition  
 point is  $\approx 7.1 \text{ mm}$ , hence in this case two of  
 our velocity contours are in the conical section,  
 while the remaining three are in the cylindrical  
 section.

Remarkably, the match between experimen-  
 tal and numerical model results is quite good.  
 Despite the observed transition from a flowing  
 to a stagnant state in the silo domain, the gran-  
 ular viscosity model is able to capture the (nor-  
 malised) maximum velocity, the curvature and  
 shape of the velocity contours, and the approx-

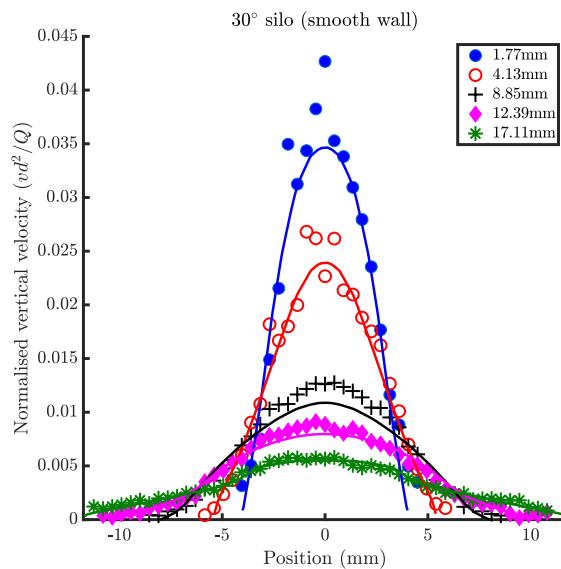


FIG. 6. The vertical velocity MRI measurements (solid circles) compared with those predicted by the numerical model (lines) for the 30° silo.

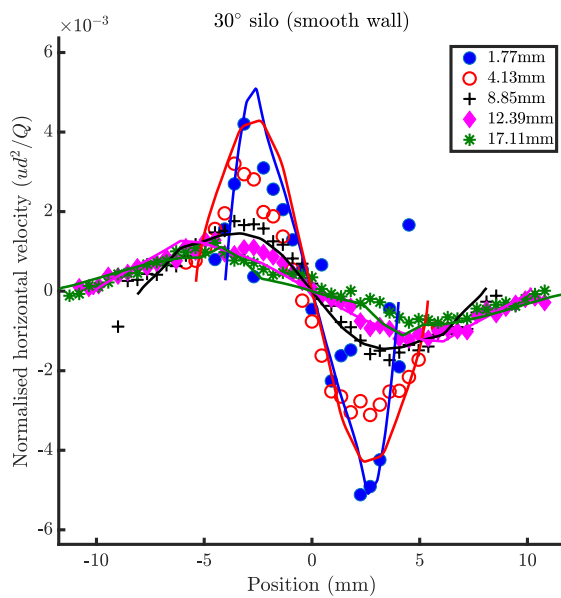


FIG. 7. The horizontal velocity MRI measurements (solid circles) compared with those predicted by the numerical model (lines) for the 30° silo, at the same locations as in the vertical velocity figure.

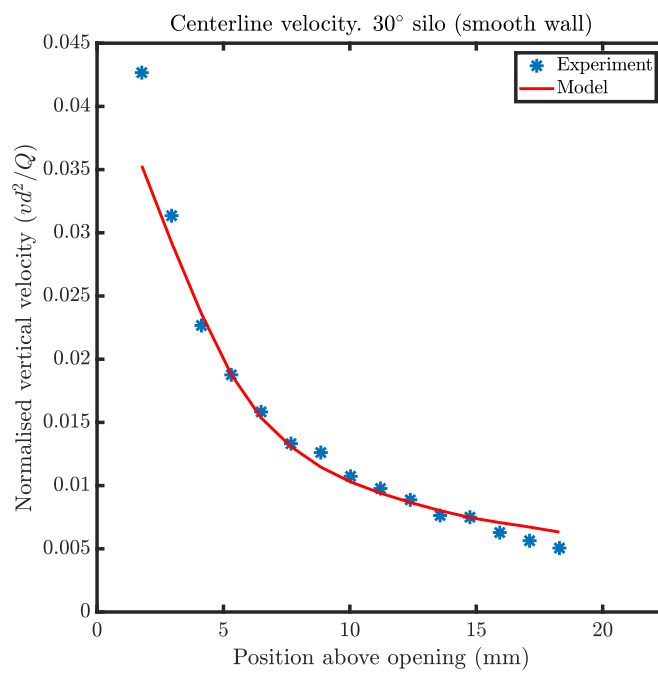


FIG. 8. A comparison of the normalised vertical velocity measured along the axial centerline of the silo compared with that predicted by the model for the 30° silo.

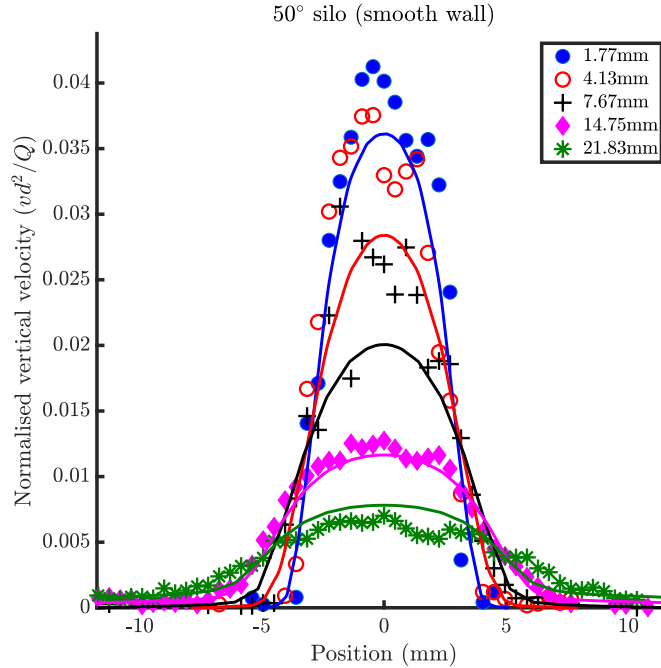


FIG. 9. The vertical velocity MRI measurements (solid circles) compared with those predicted by the numerical model (lines) for the 50° silo.

imate location of the solid/flowing boundary.

Figure 11 compares the model to experimen-  
 tal normalised vertical velocity along the axial  
 centerline of the 50° silo. In this case the ex-  
 perimentally measured velocity contains more  
 noise than in the 30° case, but it is apparent  
 that the model and experiment are of similar  
 and follow a somewhat similar decrease. How-  
 ever, the comparison is not quite as good as in  
 the 30° case.

#### 4. Numerical Model Results: 30° silo with rough walls

The most challenging flow regime to replicate  
 is the rat-holing behaviour observed in the 30°  
 silo with roughened walls. In this case the ob-  
 served magnitudes of horizontal velocity were  
 too small to quantify since they were impercep-  
 tible from the experimental noise. Hence, the

comparison of experimental to numerical pre-  
 dictions was only possible for the vertical ve-  
 locity component. Figure 12 displays the nor-  
 malised vertical velocity profile at five heights  
 above the silo opening, while Fig. 13 is the  
 normalised vertical velocity measured and pre-  
 dicted along the axial centerline of the silo.

It is apparent that the  $\mu(I)$  model predictions  
 completely fail to replicate the measured veloc-  
 ity, particularly far from the silo opening. In  
 the case of rat-holing flow, the  $\mu(I)$  model is  
 unable to capture the observed dynamics.

#### 5. Numerical Model: Sensitivity analysis and flow rates

In order to further compare the experimental  
 and numerical velocity predictions we compare  
 predicted flow rates between the numerical and  
 experimental results, and perform a sensitivity

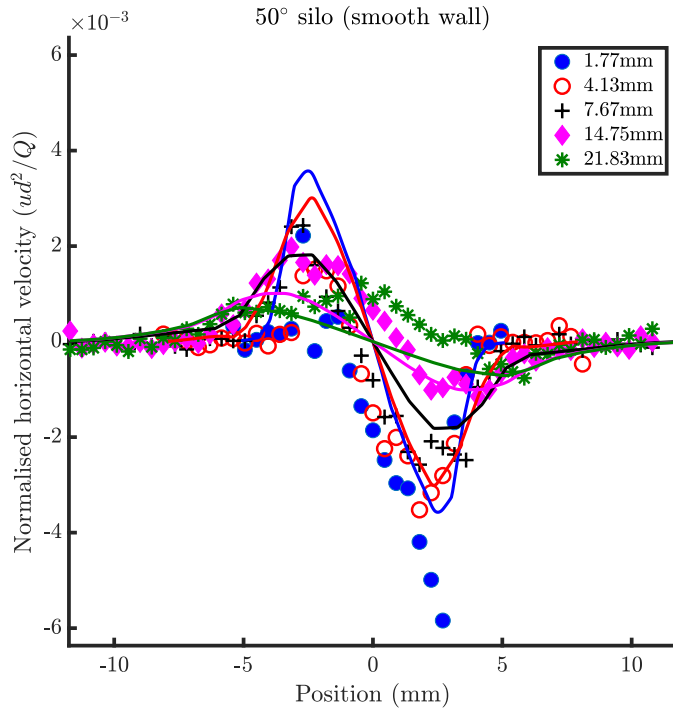


FIG. 10. The horizontal velocity MRI measurements (solid circles) compared with those predicted by the numerical model (lines) for the  $50^\circ$  silo, at the same locations as in the vertical velocity figure.

593 analysis on the numerical model parameters. 613

594 To quantify the “goodness of fit” of the numerical 615  
 595 predictions of velocity to the experimentally measured ones we perform linear least- 616  
 596 squares regression on the normalised vertical velocity data:  $\tilde{v}_{num} = b\tilde{v}_{exp}$  (i.e. we force the 617  
 597 regression to pass through the origin). In the case of a perfect fit between the numerical and ex- 618  
 598 perimental data, the slope of the line,  $b$ , would be unity. The normalised vertical velocity data 619  
 599 at five heights above the silo opening (the same heights as used in Figures 6, 9) are combined 620  
 600 and the regression is performed on the entirety of this data at once. To test the sensitivity 621  
 601 of the model predictions to model parameters this process was repeated 65 times for differ- 622  
 602 ent values of  $I_0$  and  $\mu_2$ . This analysis was performed for both the  $30^\circ$  and  $50^\circ$  silos, resulting 623  
 603 in 130 numerical simulations. In each simulation the value of  $\mu_1$  was kept constant at 0.6, 624  
 604 while the ranges of the other two parameters were  $0.05 < I_0 < 1$ , and  $0.9 < \mu_2 < 2.1$ . In Fig- 625  
 605 ure 14 the slopes resulting from the linear least-squares regression analysis are contoured for the 626  
 606  $30^\circ$  (left) and  $50^\circ$  (right) silo flows respectively. The solid red dot in the contour plots indicates 627  
 607 the values of the parameters used in the current work to produce Figures 6 - 13. The fine 628  
 608 red line in the left plot is the contour of slope = 1 which represents a perfect fit of the numerical 629  
 609 prediction of normalised vertical velocity to its experimental measurement. In general, the 630  
 610  $30^\circ$  silo numerical simulation was better fit for lower  $I_0$  and larger  $\mu_2 - \mu_1$  values, while the  $50^\circ$  631  
 611 simulation had the opposite behaviour. The  $30^\circ$  simulation was always better fit to the experi- 632  
 612 mental data than the  $50^\circ$  one, with reported slopes in the range 0.86 to 1.03 (by compar- 633

613 while the ranges of the other two parameters were  $0.05 < I_0 < 1$ , and  $0.9 < \mu_2 < 2.1$ . In Fig- 615  
 614 ure 14 the slopes resulting from the linear least-squares regression analysis are contoured for the 616  
 615  $30^\circ$  (left) and  $50^\circ$  (right) silo flows respectively. The solid red dot in the contour plots indicates 617  
 616 the values of the parameters used in the current work to produce Figures 6 - 13. The fine 618  
 617 red line in the left plot is the contour of slope = 1 which represents a perfect fit of the numerical 619  
 618 prediction of normalised vertical velocity to its experimental measurement. In general, the 620  
 619  $30^\circ$  silo numerical simulation was better fit for lower  $I_0$  and larger  $\mu_2 - \mu_1$  values, while the  $50^\circ$  621  
 620 simulation had the opposite behaviour. The  $30^\circ$  simulation was always better fit to the experi- 622  
 621 mental data than the  $50^\circ$  one, with reported slopes in the range 0.86 to 1.03 (by compar- 623  
 622 son, the  $50^\circ$  silo slopes were in the range 0.65 to 0.89). For the parameters used in the main 624  
 623

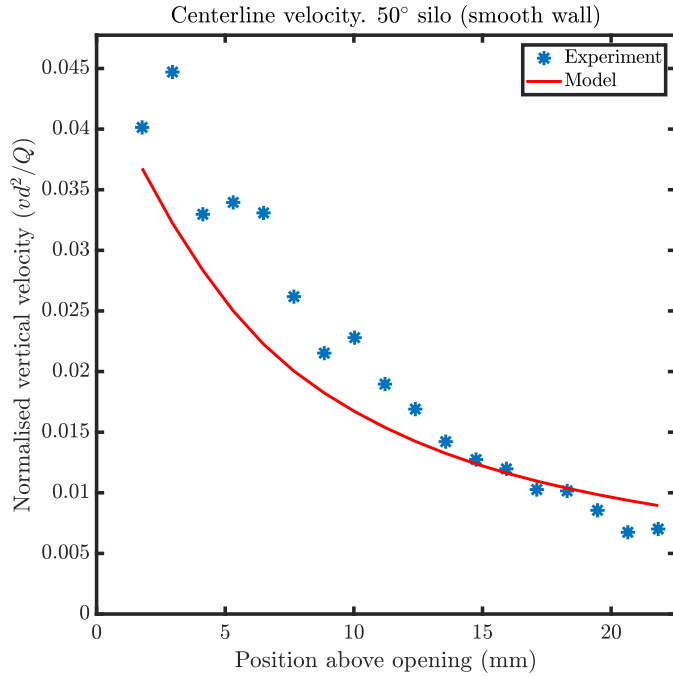


FIG. 11. A comparison of the normalised vertical velocity measured along the axial centerline of the silo compared with that predicted by the model for the  $50^\circ$  silo.

634 text (see Table I) the least squares slopes were 655  
 635 0.94 for the  $30^\circ$  silo, and 0.84 for the  $50^\circ$  one. 656  
 636 Overall, the choice of the parameters  $I_0 = 0.5$  657  
 637 and  $\mu_2 = 1.7$  used in this work is shown to be 658  
 638 a good balance between accuracy for both the 659  
 639  $30^\circ$  and  $50^\circ$  silos. 660

640 Table II presents, for each of the three silos, 661  
 641 the experimentally derived mass and volumetric 662  
 642 flow rates, the numerically predicted volumet- 663  
 643 ric flow rate, and an approximate solids volume 664  
 644 fraction in the bulk of the silo. The solids vol- 665  
 645 ume fraction in the bulk was approximated by 666  
 646 taking the ratio of the experimental mass and 667  
 647 volumetric flow rates (in the bulk of the silo), 668  
 648 then dividing by the particle density ( $\approx 1000$   
 649  $kg/m^3$ ). The predicted flowing solids fraction 669  
 650 in the bulk of the  $30^\circ$  and  $50^\circ$  silos is remark-  
 651 ably similar (0.46 and 0.47 respectively). How-  
 652 ever, the  $30^\circ$  silo with particles glued to the wall  
 653 shows a significantly lower solids volume frac-  
 654 tion of 0.36. As previously noted, the numer-

ical model was of incompressible type, hence  
 was not able to accurately predict the correct  
 flow rate. In the table the predicted volumetric  
 flow rate in the  $30^\circ$  silo simulation was a factor  
 of  $\approx 4.5$  smaller than the experimentally ob-  
 served one. The volumetric flow rate predicted  
 in the  $50^\circ$  silo simulation was a lot closer to  
 the experimentally observed rate, but we cau-  
 tion against interpreting this as a validation of  
 the model. During the sensitivity analysis the  
 predicted flow rate varied by a factor of ten over  
 the ranges of the parameters tested, which in-  
 dicates that it is sensitive to model parameter  
 choice.

#### IV. DISCUSSION AND CONCLUSIONS

In this work we have presented results of ex-  
 perimental and numerical investigation of silo  
 flow in three flow regimes; mass flow, funnel

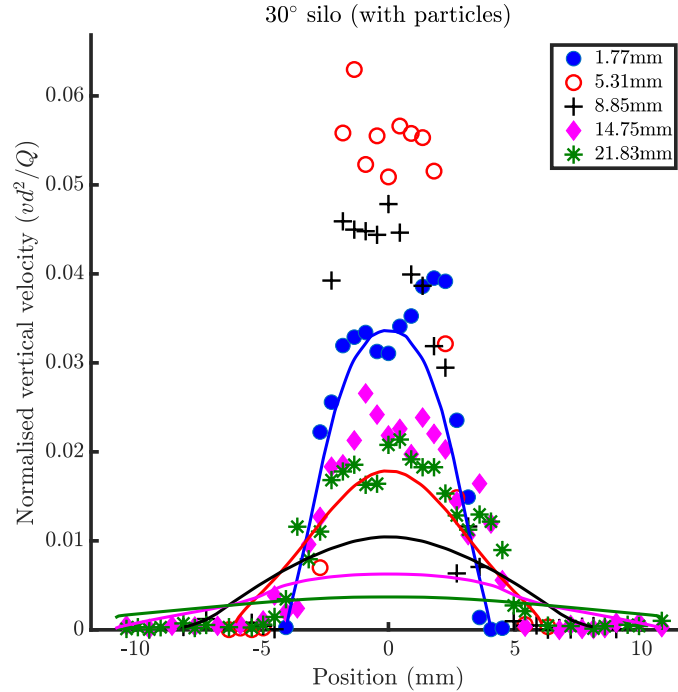


FIG. 12. The vertical velocity MRI measurements (solid circles) compared with those predicted by the numerical model (lines) for the 30° silo with roughened walls.

TABLE II: The experimentally derived and numerically predicted flow rates in the tested silos.

	$\dot{m}$ (g/s)	$Q_{exp}$ (bulk, $cm^3/s$ )	$Q_{num}$ ( $cm^3/s$ )	$\sim\phi_{exp} = (\dot{m}/Q_{exp})/\rho_p$
30°	$2.11 \pm 0.07$	$4.54 \pm 0.05$	0.97	$0.47 \pm 0.02$
50°	$1.74 \pm 0.09$	$3.8 \pm 0.1$	4.2	$0.46 \pm 0.04$
30° (with particles)	$2.2 \pm 0.1$	$6.1 \pm 0.3$	0.81	$0.36 \pm 0.04$

673 flow, and rat-holing. Using MRI velocimetry we687  
 674 measured both the horizontal and vertical com-688  
 675 ponents of velocity throughout the three test689  
 676 silos, including the transition from the converg-690  
 677 ing conical to the cylindrical section. We found691  
 678 that the 30° silo produced a mass flow, the 50°692  
 679 silo produced a funnel flow, and the 30° silo with693  
 680 rough walls produced a rat-holing flow. We also694  
 681 presented results of a numerical model which695  
 682 used the  $\mu(I)$  friction law to define an effec-696  
 683 tive granular viscosity for dense granular flow.697  
 684 This viscosity was used to simulate the silo flows698  
 685 by means of incompressible computational fluid699  
 686 dynamics. 700

It was observed that the apparent volumetric flow rate in the MRI experiments was constant in the 30° silo, but was a function of height above the silo opening for the other two; the flow rate was large near the silo opening but then rapidly fell to a near constant higher in the silo. The flow rate near the opening was roughly  $2\times$  that of the bulk, indicating that there is significant dilation of the flow near the silo exit opening in the 50° and 30° with rough wall cases. This is in contrast to the numerical model which enforced incompressibility of the flow. Recent studies have quantified the effect of solids fraction value at the silo opening on the

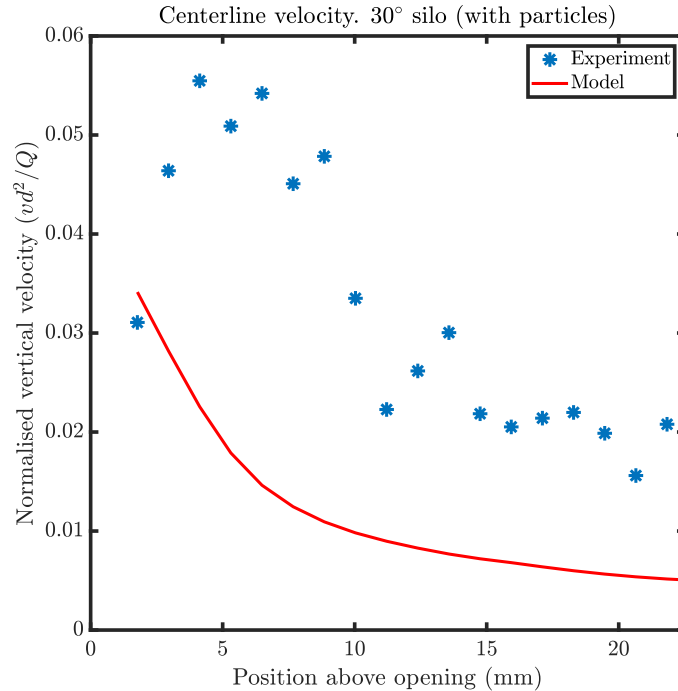


FIG. 13. A comparison of the normalised vertical velocity measured along the axial centerline of the silo compared with that predicted by the model for the 30° silo with roughened walls.

701 flow rate from the silo [54], and reported that 721  
 702 solids fraction in the near opening region could 722  
 703 be as low as half that in the bulk of the silo. We 723  
 704 conclude that to fully capture the experimental 724  
 705 measurement of the flow rate (and hence, the 725  
 706 exact values of velocity) numerical models will 726  
 707 likely need to include dilation effects, particu- 727  
 708 larly for funnel and rat-holing flows. The ef- 728  
 709 fect of dilation for the mass flow silo appeared 729  
 710 negligible, but may be important to accurately 730  
 711 predict the volumetric flow rate from the silo. 731

712 To allow comparison between our experimen- 732  
 713 tal and numerical results, the velocity compo- 733  
 714 nents of each were normalised by the local value 734  
 715 of volumetric flow rate (i.e. the flow rate at 735  
 716 height  $z$  above the silo opening). The resulting 736  
 717 velocity fields derived from the 30° silo simu- 737  
 718 lation showed excellent agreement with the ex- 738  
 719 perimental data. Plots of the vertical and hor- 739  
 720 izontal velocity at a series of heights above the 740

opening showed that both the shape and (nor-  
 malised) maximum of the velocity contours were  
 well matched, as was the vertical velocity compo-  
 nent measured along the center-line of the  
 silo. The comparison in the 50° silo (which  
 operated in the funnel flow regime in the MRI  
 experiment) were surprisingly impressive, with  
 very good agreement between experimental and  
 numerical results. This suggests that for appro-  
 priate values of fitting parameters the  $\mu(I)$  fric-  
 tion law can be used to define an effective gran-  
 ular viscosity for granular dynamics, even in the  
 case where there are transitions from static to  
 flowing regions in the domain of study.

However, for the 30° silo with roughened  
 walls (which displayed rat-holing in the MRI  
 experiment), the simulation results were poorly  
 matched to the experimental data. The grain  
 dynamics in this silo are very complicated and  
 hard to capture with numerical models. Rat-

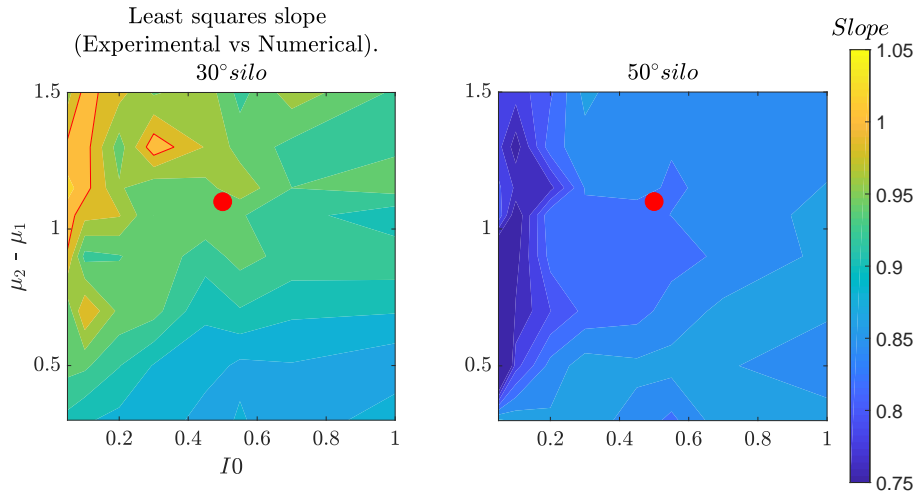


FIG. 14. Sensitivity analysis of the numerical model to parameters  $I_0$  and  $\mu_2 - \mu_1$ . The contour plots display the value of the slope found by performing a least-squares linear regression between the experimental and numerical normalised vertical velocity data. The left graph is the analysis for the  $30^\circ$  silo and the right for the  $50^\circ$  one. The red dot in the plots indicated the value of the parameters used in the current work, while the fine red line in the left plot is the contour of slope = 1 (indicating a perfect fit of the numerical to experimental data).

741 holing flow in a silo is often avoided by smooth-764  
 742 ing the silo walls (thus, changing the stress dis-765  
 743 tribution in the silo) and/or increasing the size766  
 744 of the silo opening. It is a challenge for simple767  
 745 incompressible continuum visco-plastic models768  
 746 of granular flow to capture these “finite particle769  
 747 size” effects. Further work is needed, includ-770  
 748 ing adding the effect of compressibility, to fully771  
 749 capture the observed dynamics in this situation.772

750 It is clear that the  $\mu(I)$  model performs admirably in a silo in the mass and funnel flow773  
 751 regimes for the parameter values chosen, but774  
 752 further model development is needed to fully775  
 753 capture the observed phenomena in rat-holing776  
 754 flow, and to accurately predict the flow rate777  
 755 from the silo. Adding in a degree of compress-778  
 756 ibility into the model and/or accounting for779  
 757 granular non-locality and finite size effects may780  
 758 improve flow rate predictions in the silo and781  
 759 may help to capture more accurately flowing to782  
 760 stagnant phase transitions and potentially the783  
 761 rat-holing phenomenon [55]. Testing these hy-784  
 762 potheses is currently being pursued by the au-785  
 763

thors. Additionally, the  $\mu(I)$  friction law was discovered using experimental data from relatively low friction spherical particles [56, 57]. It is unclear if the  $\mu(I)$  model is the correct friction law to use for natural particles such as the poppy seeds used in this work. Furthermore, particle shape has been shown to be an important factor in the behaviour of general granular systems [58, 59], and silo systems specifically [60, 61]. Using SEM imaging we found that our poppy seeds were kidney bean shaped, and not spherical. Such an effect could be important to include in a numerical model of granular flow, although the factor does not seem critical, since we obtained very good agreement between experimental and numerical results for the  $30^\circ$  and  $50^\circ$  silos. The  $\mu(I)$  parameters in the numerical model were our “best guess”. The first friction coefficient,  $\mu_1$ , was taken as the angle of repose of the poppy seeds, however,  $\mu_2$  and  $I_0$  were chosen to be physically realistic and to try to reduce the ill-posed regions for the  $\mu(I)$  model [50]. To check the dependence of model

787 results on the  $I_0$  and  $\mu_2$  parameters a sensitiv-808  
 788 ity analysis was performed. It was found that809  
 789 the accuracy of the model was retained over a810  
 790 wide range of parameter values, and that our811  
 791 choice of  $I_0$  and  $\mu_2$  was a good balance of ac-812  
 792 curacy for both the 30° and 50° silos. To re-813  
 793 duce model degrees of freedom these parameters814  
 794 should be measured for the specific set of par-815  
 795 ticles [62]. In addition to experimentally quan-816  
 796 tifying model parameters, the development of817  
 797 realistic numerical boundary conditions should818  
 798 be a focus. Developing these boundary condi-  
 799 tions is a significant future research challenge,819  
 800 but recent work has made excellent progress to-820  
 801 wards this goal [48, 49]. The observation in the821  
 802 30° silo that the flow regime changes from mass822  
 803 to rat-holing when the boundary condition is823  
 804 changed exemplifies the necessity of accurat824  
 805 boundary conditions and may indicate some-825  
 806 thing more complex than a simple slip condi-826  
 807 tion is needed. Finally, in recent times it has827

been shown that defining an effective granular  
 viscosity using the  $\mu(I)$  friction model with an  
 incompressible flow assumption can be mathe-  
 matically ill-posed depending on the choice of  
 parameters [63]. Adding the effect of compress-  
 ibility seems to alleviate this issue [50, 64]. Al-  
 though we did not note any issues in our model  
 for our choice of parameters, this fact serves as  
 an additional motivation to transition to a com-  
 pressible flow model of granular drainage from  
 a silo.

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 and MAU1712). Finally, we acknowledge the  
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 tify the mass flow rates in the system.

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