A Benders Approach for the Two-echelon Stochastic Multi-period Capacitated Location-Routing Problem

Imen Ben Mohamed, Walid Klibi, Ruslan Sadykov, Halil Sen, François Vanderbeck

A Benders Approach for the Two-echelon Stochastic Multi-period Capacitated Location-Routing Problem

Imen Ben Mohamed[1,2,3], Walid Klibi[1], Ruslan Sadykov[3], Halil Sen[3,2], François Vanderbeck[2,3]

[1] The Centre of Excellence in Supply Chain (CESIT), Kedge Business School, France
imen.benmohamed@kedgebs.com

[2] Mathematics Institute of Bordeaux (IMB),University of Bordeaux, France

[3] RealOpt, Inria-Bordeaux-Sud-Ouest, Bordeaux, France

Mots-clés : *2E-CLRP, uncertainty, multi-period, Benders approach.*

1 Introduction and Problem context

The continuing increase in world population is causing considerable progress in global urbanization. This continuous growth is impacting the traffic movement in cities, causing rising levels of congestion and pollutant emissions. Many practitioners and academics consider the single echelon capacitated location-routing problem, where the aim is to find an optimal number of warehouses and their locations, while building routes around them to serve the customers [3]. In the novel context, distribution networks need to be beyond one-echelon to offer more dynamic adjustment to the need of the business over time and to cope with all random factors [1]. In the two-echelon capacitated location-routing problem (2E-CLRP), the network topology includes an intermediate echelon of capacitated distribution platforms (DPs) standing between the capacitated warehouse platforms (WPs) and the customers [5]. These DPs are devoted to consolidation and transshipment operations and no storing activity is performed. Therefore, two-echelon models help to reduce urban congestion and nuisance, and increase mobility.

Location decisions have a long-term lasting effect. Such decisions should be planned as a set of sequential actions to be implemented at different periods of a given time horizon. In addition, distribution practices have got more complex over the years, and have higher uncertainty. Thus, a more realistic stochastic and multi-period characterization of the planning horizon is considered in the current work. More specifically, the horizon is partitioned into a set of planning periods shaping the uncertainty of time-varying demands and promoting the structural adaptability of the network. As far as we know, stochastic and multi-period settings are not tackled yet in 2E-CLRPs literature ([3, 2]).

In this work, we focus on the stochastic multi-period variant of the 2E-CLRP (2E-SM-CLRP), a hierarchical decision problem where the design (i.e., location and capacity) and routing decisions are decided with respect to their temporal hierarchy [1]. Location and capacity decisions are made on a yearly basis, whereas routing decisions are made on a daily basis in a response to the customer orders received. More precisely, our model aims to decide at each period on opening, operating and closing of WPs and DPs, as well as the capacity allocated to the links between platforms to efficiently distribute the goods to the customers. In the second level, the goal is to construct vehicle routes that visit customers using a vehicle routed from an operating DP while minimizing the total expected cost. The model is a two-stage stochastic program with recourse that reflects the temporal hierarchy of decision-making.

2 Benders approach for 2E-SM-CLRP

In this paper, we develop a Benders decomposition approach to solve the 2E-SM-CLRP. The algorithm separates the problem into a Benders master problem (MP) and a number of Benders subproblems, which are easier to solve than the original problem. By using linear programming duality, all subproblems variables are projected out and the relaxed (MP) contains only the remaining master variables and artificial variables representing the lower bounds on the cost of each subproblem. In the first-stage, location (WPs and DPs) and capacity assignment decisions are taken by solving the Benders master problem. When these first-stage decisions are fixed in the original problem, the resulting subproblem is a multi-depot vehicle-routing problem with limited capacities (CVRP-CMD) that can be decomposed by period and by scenario. The CVRP-CMD is formulated by a set-partitioning formulation. However, solving these subproblems as an integer program does not produce effective dual values to generate Benders cuts. In order to overcome this difficulty, we iteratively tackle the second-stage integer program to get valid and useful Benders cuts. As a preprocessing step, we solve the linear relaxation of a bin packing problem (BPP) defined by customers demand to pack into a minimum number of equal-sized vehicles. This provides a lower bound on the number of vehicles required for each period and each scenario. Then, at each iteration of the Benders approach, a relaxed integer MP, including only a small subset of Benders cuts, is optimally solved to obtain a valid dual bound and first-stage solutions. Using such fixed first-stage decisions (operating DPs and capacity assignment), we first relax the integrality restrictions in the Benders subproblems (i.e., CVRP-CMD). For each period and scenario, we solve the LP of the set partitioning formulation of the CVRP-CMD through a column generation approach to generate a Benders cut from the dual solutions. These cuts are then added to the MP. If there is no new Benders cuts, the integer subproblems are then solved through the branch-cut-and-price algorithm from Sadykov et al. [4]. This algorithm is demonstrated to be the best performing exact approach for many classical variants of the vehicle-routing. The expected cost from integer feasible solutions of subproblems yields a primal bound for the 2E-SM-CLRP. If the gap is still large, combinatorial Benders cuts are added to MP to eliminate that such super-optimal solutions from the search tree. This process is repeated without the BPP step recursively until an optimal solution is found or the relative gap is smaller than a given threshold ϵ.

3 Computational experiments

In order to achieve comprehensive conclusions, we generated several sets of realistic instances by varying the problem size, the network characteristics, the demand process, the cost structure and the capacities. Sampling methods are used to calibrate the number of scenarios to be included in the optimization phase. The results show that our algorithm provides the means to solve optimally a large set of instances, and to get good lower bounds on large-scale instances up to 50 customers and 25 demand scenarios under a 5-year planning horizon. Relevant managerial insights are also derived on the behavior of the location decisions under the stochastic multi-period characterization of the planning horizon.

Références

[1] I. Ben Mohamed, W. Klibi and F. Vanderbeck. Designing a two-echelon distribution network under demand uncertainty. *Submitted to European Journal of Operational Research*, 2018.

[2] R. Cuda, G. Guastaroba and M. G. Speranza. A survey on two-echelon routing problems. *Computers & Operations Research*, 55 :185–199, 2015.

[3] C. Prodhon and C. Prins. A survey of recent research on location-routing problems. *European Journal of Operational Research*, 238(1) :1–17, 2014.

[4] R. Sadykov, E. Uchoa and A. Pessoa. A Bucket Graph Based Labeling Algorithm with Application to Vehicle Routing. *Cadernos do LOGIS*, Niterói, Brazil, 7,2017.

[5] C. Sterle. *Location-Routing models and methods for Freight Distribution and Infomobility in City Logistics*. Università degli Studi di Napoli "Federico II". Phd thesis, 2010.