

```

> des := {delta(t) = 0.01*t, delta(t0) = delta0, diff(phi(t), t) =
tan(delta(t))/l, diff(x(t), t) = V*cos(phi(t)), diff(y(t), t) =
V*sin(phi(t)), phi(t0) = phi0, x(t0) = x0, y(t0) = y0};
> desSol := dsolve(des, numeric, parameters = [V, l, t0, x0, y0,
phi0, delta0]);
> desSol(parameters = [109, 5, 0, 0, 0, 0, 0]);
> desSol(90);
> with(plots);
> odeplot(desSol, [[t, x(t)], [t, y(t)], [t, phi(t)]], 0 .. 300,
labels = ["t", "Trajectory"], legend = ["x(t) - Position in x",
"y(t) - Position in y", "phi(t) - Heading Angle"]);
> odeplot(desSol, [t, x(t), y(t)], 0 .. 300);
> odeplot(desSol, [x(t), y(t), phi(t)], 0 .. 150, labels = ["x -
Position", "y - Position", "phi - Heading Angle"]);

```

$$des := \left\{ \delta(t) = 0.01 t, \delta(t0) = \delta0, \frac{d}{dt} \phi(t) = \frac{\tan(\delta(t))}{l}, \frac{d}{dt} x(t) = V \cos(\phi(t)), \frac{d}{dt} y(t) = V \sin(\phi(t)), \phi(t0) = \phi0, x(t0) = x0, y(t0) = y0 \right\}$$

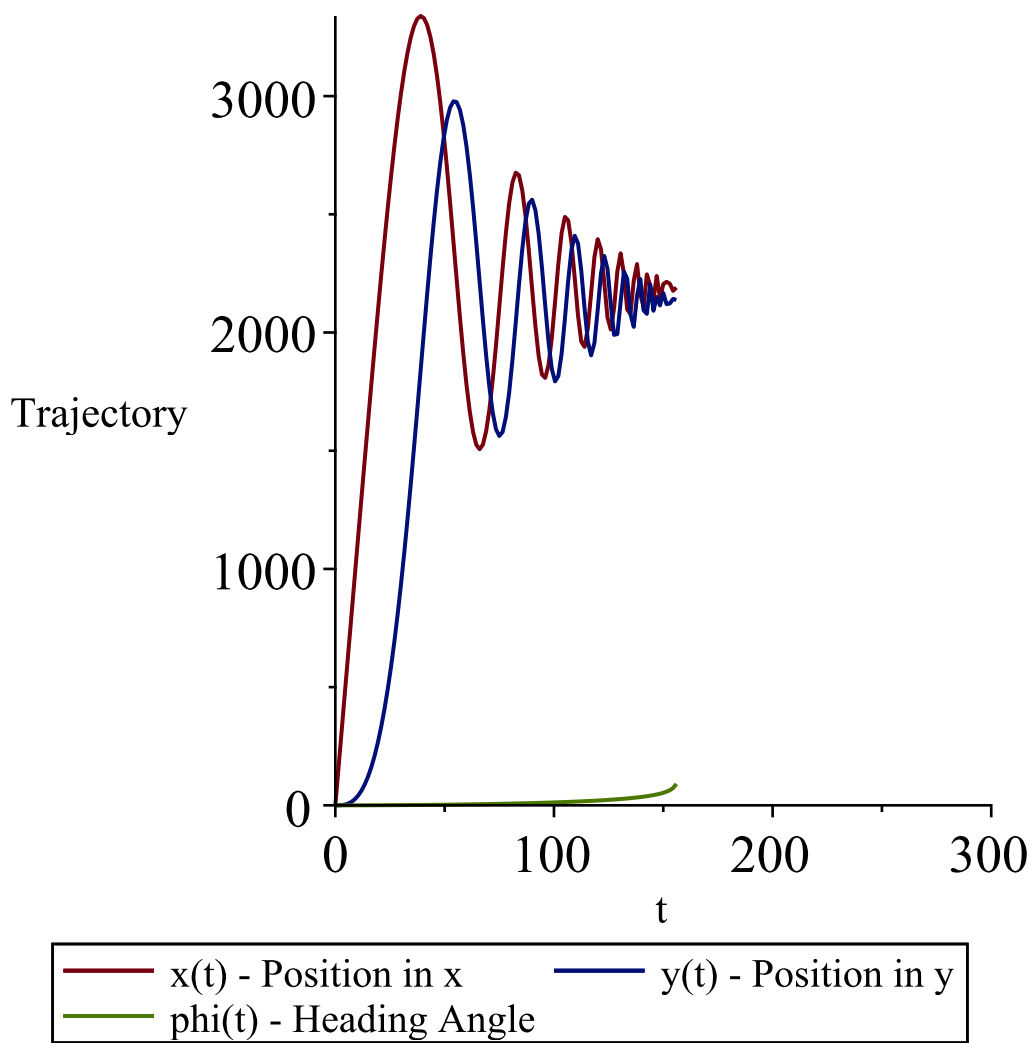
*desSol* := **proc**(*x\_rkf45\_dae*) ... **end proc**

[*V* = 109., *l* = 5., *t0* = 0., *x0* = 0., *y0* = 0.,  $\phi0$  = 0.,  $\delta0$  = 0.]

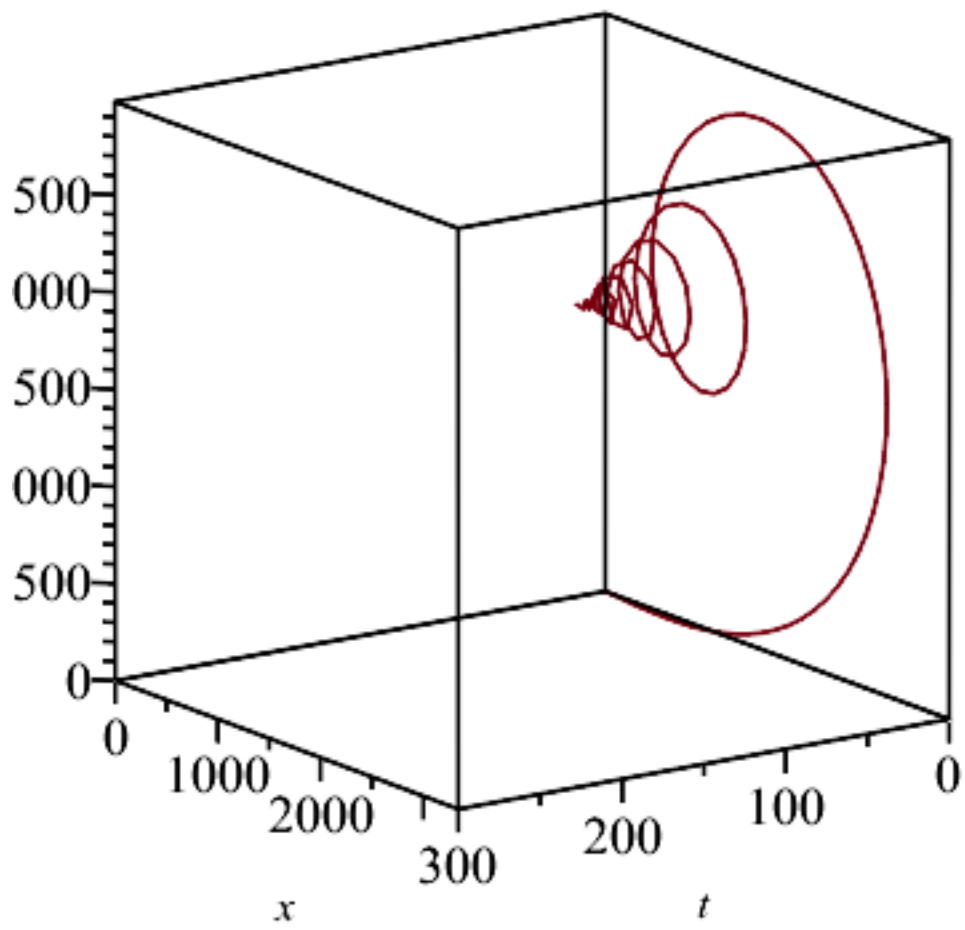
[*t* = 90.,  $\delta(t)$  = 0.9000000000000000,  $\phi(t)$  = 9.50884887136091, *x(t)* = 2184.94156683081, *y(t)* = 2561.96008935270]

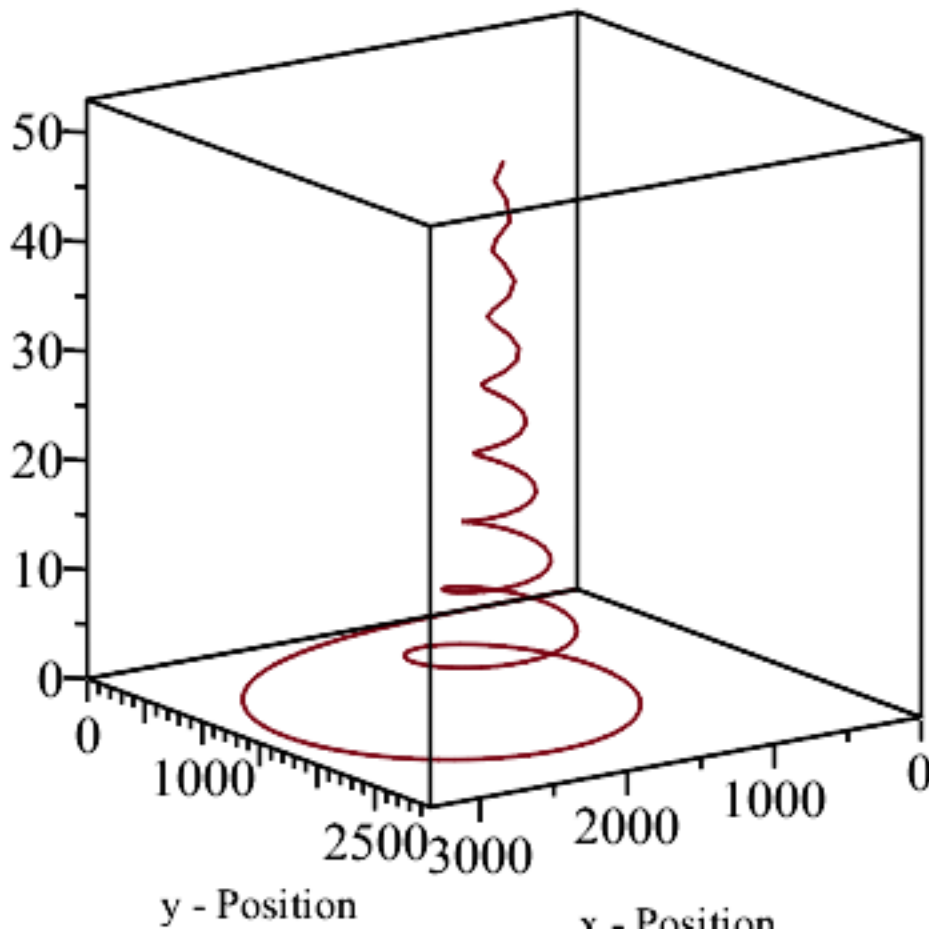
[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra\_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

Warning, cannot evaluate the solution further right of 157.07963, probably a singularity



[Warning, cannot evaluate the solution further right of 157.07963, probably a singularity](#)





> eval(des, t=10);

$$\left\{ \begin{aligned} \delta(10) = 0.10, \delta(t0) = \delta 0, \left( \frac{d}{dt} \phi(t) \right) \Big|_{\{t=10\}} &= \frac{\tan(\delta(10))}{l}, \left( \frac{d}{dt} x(t) \right) \Big|_{\{t=10\}} \\ &= V \cos(\phi(10)), \left( \frac{d}{dt} y(t) \right) \Big|_{\{t=10\}} = V \sin(\phi(10)), \phi(t0) = \phi 0, x(t0) = x0, y(t0) = y0 \end{aligned} \right. \quad (1)$$

Ici je redéfinis l'équation ordinaire différentielle du modèle odométrique.

> fsolve(des);  
 A(k) = <<1 | 0 | -v\*sin(phi(t))\*dt>, <0 | 1 | v\*cos(phi(t))\*dt> ,

```

<0 | 0 | 1>>;
B(k) = <<cos(phi(t))*dt | 0>, <sin(phi(t))*dt | 0>, <tan(0.01*t)*
dt/l | V*dt/(l*cos(0.01*t)^2)>>;
C = <<1 | 0 | 0>, <0 | 1 | 0>, <0 | 0 | 1>>;
X(t, k) = <<x(t)>, <y(t)>, <phi(t)>>;
U(t) = <<V>, <0.01*t>>;
X(t, k + 1) = A(k)*X(t, k) + B(k)*U(t);
Y(t) = C*X(t, k);
dt = 1;

```

$$\begin{aligned}
 & 0. \\
 A(11) &= \begin{bmatrix} 1 & 0 & -V \sin(\phi(6)) dt \\ 0 & 1 & V \cos(\phi(6)) dt \\ 0 & 0 & 1 \end{bmatrix} \\
 B(11) &= \begin{bmatrix} \cos(\phi(6)) dt & 0 \\ \sin(\phi(6)) dt & 0 \\ \frac{0.06007210383 dt}{l} & \frac{1.003608658 V dt}{l} \end{bmatrix} \\
 C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 X(6, 11) &= \begin{bmatrix} x(6) \\ y(6) \\ \phi(6) \end{bmatrix} \\
 U(6) &= \begin{bmatrix} V \\ 0.06 \end{bmatrix} \\
 X(6, 12) &= A(11) X(6, 11) + B(11) U(6) \\
 Y(6) &= C X(6, 11) \\
 dt &= 1
 \end{aligned}$$

(2)

Ici je reprends le schéma numérique de Parent-Ollivier.

```

> for t from 0 by 1 to 5 do
  for k from 0 by 1 to 10 do
    V=109;
    l=5;
    A(k) = <<1 | 0 | -V* sin(phi(t))>, <0 | 1 | V* cos(phi(t))>, <0 | 0 | 1>;
    print( <<1 | 0 | -109 * sin(phi(t))>, <0 | 1 | 109 * cos(phi(t))>, <0 | 0 | 1> )
    B(k) = <<cos(phi(t)) | 0>, <sin(phi(t)) | 0>, <tan(0.01 * t) / l | V / (l * cos(0.01 * t)^2)>);
    print( <<cos(phi(t)) | 0>, <sin(phi(t)) | 0>, <tan(0.01 * t) / 5 | 109 / (5 * cos(0.01 * t)^2)> )
    C = <<1 | 0 | 0>, <0 | 1 | 0>, <0 | 0 | 1>;
    X(t, k) = <<x(t)>, <y(t)>, <phi(t)>;
    print( <<x(t)>, <y(t)>, <phi(t)> )
    U(t) = <<V>, <0.01 * t>;
    print(A(k)·X(t, k) + B(k)·U(t))
  end do;
end do;

```

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(0) X(0, 0) + B(0) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(1) X(0, 1) + B(1) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(2) X(0, 2) + B(2) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(3) X(0, 3) + B(3) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(4) X(0, 4) + B(4) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(5) X(0, 5) + B(5) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(6) X(0, 6) + B(6) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(7) X(0, 7) + B(7) U(0)$$



$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(8) X(0, 8) + B(8) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(9) X(0, 9) + B(9) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(0)) \\ 0 & 1 & 109 \cos(\phi(0)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(0)) & 0 \\ \sin(\phi(0)) & 0 \\ 0. & 21.80000000 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \\ \phi(0) \end{bmatrix}$$

$$A(10) X(0, 10) + B(10) U(0)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(0) X(1, 0) + B(0) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(1) X(1, 1) + B(1) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(2) X(1, 2) + B(2) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(3) X(1, 3) + B(3) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(4) X(1, 4) + B(4) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(5) X(1, 5) + B(5) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(6) X(1, 6) + B(6) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(7) X(1, 7) + B(7) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(8) X(1, 8) + B(8) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(9) X(1, 9) + B(9) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(1)) \\ 0 & 1 & 109 \cos(\phi(1)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(1)) & 0 \\ \sin(\phi(1)) & 0 \\ 0.002000066670 & 21.80218015 \end{bmatrix}$$

$$\begin{bmatrix} x(1) \\ y(1) \\ \phi(1) \end{bmatrix}$$

$$A(10) X(1, 10) + B(10) U(1)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(0) X(2, 0) + B(0) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(1) X(2, 1) + B(1) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(2) X(2, 2) + B(2) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(3) X(2, 3) + B(3) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(4) X(2, 4) + B(4) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(5) X(2, 5) + B(5) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(6) X(2, 6) + B(6) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(7) X(2, 7) + B(7) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(8) X(2, 8) + B(8) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(9) X(2, 9) + B(9) U(2)$$



$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(2)) \\ 0 & 1 & 109 \cos(\phi(2)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(2)) & 0 \\ \sin(\phi(2)) & 0 \\ 0.004000533418 & 21.80872233 \end{bmatrix}$$

$$\begin{bmatrix} x(2) \\ y(2) \\ \phi(2) \end{bmatrix}$$

$$A(10) X(2, 10) + B(10) U(2)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(0) X(3, 0) + B(0) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(1) X(3, 1) + B(1) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(2) X(3, 2) + B(2) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(3) X(3, 3) + B(3) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(4) X(3, 4) + B(4) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(5) X(3, 5) + B(5) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(6) X(3, 6) + B(6) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(7) X(3, 7) + B(7) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(8) X(3, 8) + B(8) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(9) X(3, 9) + B(9) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(3)) \\ 0 & 1 & 109 \cos(\phi(3)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(3)) & 0 \\ \sin(\phi(3)) & 0 \\ 0.006001800648 & 21.81963177 \end{bmatrix}$$

$$\begin{bmatrix} x(3) \\ y(3) \\ \phi(3) \end{bmatrix}$$

$$A(10) X(3, 10) + B(10) U(3)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(0) X(4, 0) + B(0) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(1) X(4, 1) + B(1) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(2) X(4, 2) + B(2) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(3) X(4, 3) + B(3) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(4) X(4, 4) + B(4) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(5) X(4, 5) + B(5) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(6) X(4, 6) + B(6) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(7) X(4, 7) + B(7) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(8) X(4, 8) + B(8) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(9) X(4, 9) + B(9) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(4)) \\ 0 & 1 & 109 \cos(\phi(4)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(4)) & 0 \\ \sin(\phi(4)) & 0 \\ 0.008004269400 & 21.83491723 \end{bmatrix}$$

$$\begin{bmatrix} x(4) \\ y(4) \\ \phi(4) \end{bmatrix}$$

$$A(10) X(4, 10) + B(10) U(4)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(0) X(5, 0) + B(0) U(5)$$



$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(1) X(5, 1) + B(1) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(2) X(5, 2) + B(2) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(3) X(5, 3) + B(3) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(4) X(5, 4) + B(4) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(5) X(5, 5) + B(5) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(6) X(5, 6) + B(6) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(7) X(5, 7) + B(7) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(8) X(5, 8) + B(8) U(5)$$

$$\begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix}$$

$$\begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix}$$

$$A(9) X(5, 9) + B(9) U(5)$$

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & -109 \sin(\phi(5)) \\ 0 & 1 & 109 \cos(\phi(5)) \\ 0 & 0 & 1 \end{bmatrix} \\
 & \begin{bmatrix} \cos(\phi(5)) & 0 \\ \sin(\phi(5)) & 0 \\ 0.01000834168 & 21.85459097 \end{bmatrix} \\
 & \begin{bmatrix} x(5) \\ y(5) \\ \phi(5) \end{bmatrix} \\
 & A(10) X(5, 10) + B(10) U(5)
 \end{aligned}$$

**(1.1)**