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# Unified modelling of adiabatic shear banding and subsequent micro-voiding driven dynamic failure of viscoplastic solids

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## ABSTRACT

Adiabatic shear bands (ASBs) are intense shear localisation zones resulting from thermomechanical instability in dynamically loaded, high strength structural materials. In the wake of the ASB, Micro-voids (MVs) have been observed which coalesce and ultimately rupture the material. ASBs thus favour premature MV induced damage and further material failure. In the present work, the coupled effects of ASB+MV on the representative volume element (RVE) are reproduced in a unified model within irreversible thermodynamics framework considering a large scale postulate. The influence of the softening mechanisms such as thermal softening and/or DRX induced softening on the ASB onset is accounted for. The model is numerically implemented as a user material subroutine into the engineering finite element (FE) computation code LS-DYNA and its feasibility and performances are assessed on both RVE and structural scales. The dynamic shear compression loading of a hat shaped structure is simulated for that purpose. The respective contributions of ASB and MV on the progressive material deterioration can be clearly identified.

## Keywords:

Adiabatic shear banding, Ductile damage, Dynamic plasticity, Numerical simulation

## 1 INTRODUCTION

The present study aims at modelling the adiabatic shear banding and the consecutive micro-voiding induced material deterioration. Adiabatic shear banding is a distinct deformation mechanism which occurs in viscoplastic solids during high strain rate and impact loading acting as a precursor to failure such as plugging failure, ductile fracture, explosive failure, etc., see [1],[2],[3]. The Adiabatic Shear Bands (ASBs) are seen typically in high strength metals and alloys such as steels and titanium and aluminium alloys which are often used as component materials for aircraft and other structures. The ASBs are undesirable when they cause catastrophic failures of structures such as

34 protection plates for ballistic applications [4] involving military or civil threats (missile,  
35 bird strike, etc). On the other hand, they are desirable in military application such as  
36 kinetic energy penetrators in which the formation of ASBs on the side ends of the  
37 penetrator helps in a self-sharpening-like mechanism ([5],[6],[7]). They also find  
38 industrial application in high speed machining in which they have mitigated effect in the  
39 sense that they help to reduce the cutting force but at the expense of cutting force  
40 oscillation and rougher surface finish [8],[9]. In the context of engineering design, it is  
41 thus crucial to be able to numerically reproduce the consequences of this phenomenon.

42 The ASBs are narrow regions of intense plastic shear deformation and strength  
43 softening resulting from thermo-mechanical instability induced by a competition  
44 between hardening and softening mechanisms in the regions of low stress triaxiality  
45 during high strain rate loading. The mechanisms causing the softening which triggers  
46 the ASB formation are mostly plastic dissipation-induced temperature rise and possibly  
47 dynamic recrystallization (DRX) in certain materials. The shear bands are distinctly  
48 oriented causing local degradation of the material properties and a form of induced  
49 anisotropy at the scale of the specimen. Once the ASB is formed, the softer material  
50 within the band serves as a site for the nucleation and growth of micro-voids (MV).  
51 These micro-voids additionally contribute to softening mechanism and accelerate the  
52 post-localization drop in strength of the RVE in the large scale postulate framework. The  
53 micro-voids coalesce and lead to the formation of cracks which ultimately fracture the  
54 material. The evidence of the presence of micro-voids within the ASB was shown by  
55 various experimental observations ([10],[2],[11]). The experimental observation  
56 reported in on the fracture surface of impacted shear compression specimens showed  
57 the evidence of micro-voids in the ASB wake which proved that the fracture occurred  
58 due to two successive mechanisms: adiabatic shear banding and then micro-voiding.

59 The ASB in itself is a weak discontinuity, causing shape change, as there is cohesion of  
60 matter within the band; the MV is a strong discontinuity with the absence of matter  
61 within, creating irreversible damage in the material and imposing volume change and  
62 related hydrostatic components to the induced strain rate because of the void growth.  
63 Depending on the loading, the ASB does not necessarily lead to micro-voiding and  
64 further failure; and in certain cases, upon unloading, the ASB is quenched by the cooler  
65 surrounding material, undergoes phase transformation and finally becomes harder at  
66 room temperature than the surrounding matrix material [12]. Therefore considering a  
67 unique variable accounting indistinctly for ASB and MV does not allow for distinguishing  
68 the zones in the structure where ASB takes place, with potential initial properties  
69 recovering after unloading and cooling as explained above, and where irreversible void-  
70 growth induced irreversible damage as a precursor to crack formation takes place. Being  
71 able to distinguish the respective contributions of ASB and MV on the current (during

72 loading) and residual (post loading) material deterioration is accordingly of major  
73 interest.

74 Many approaches describing the ASB in the literature are developed at a scale lower  
75 than the bandwidth ([13], [14]and [15]). However, as the ASB width in itself is of the  
76 order of micro-meters, this postulate requires the use of very fine meshing in the finite  
77 element (FE) simulations. Whereas in the present study, a large scale postulate is  
78 adopted, in which the representative volume element (RVE) has a length scale greater  
79 than the ASB width; in other words, the shear band is embedded within the RVE. This  
80 approach facilitates the use of coarser meshes in FE simulations and thus makes the  
81 model feasible to implement on engineering structural applications. Large scale  
82 approaches based on variational methods and enriched finite elements can be found in  
83 [16], [17], [18] and [19], and a review of ASB-oriented modelling approaches in [20].  
84 While modelling of ASB can be widely found in the literature, the subsequent mechanism  
85 of micro-voiding and its modelling as a distinct contribution to the failure process is  
86 largely overlooked. The present work aims to model in a unified approach the ASB and  
87 subsequent MV induced degradation.

88 A physics motivated phenomenological three-dimensional constitutive model was  
89 initially developed by Longère et al. [21] in an irreversible thermodynamics framework  
90 which described the material degradation induced predominantly by the ASB. The aim  
91 here is to use an enlarged version of the aforementioned model based on the model  
92 developed recently by Longère and Dragon [22] in order to explicitly reproduce the  
93 consequences of micro-voiding in the band wake. The view is to clearly distinguish in  
94 the post-critical regime the respective contributions to the material softening coming  
95 from the two mechanisms at stake, namely adiabatic shear banding on one hand and  
96 micro-voiding on the other hand. The ASB+MV model is here implemented as user  
97 material into the engineering finite element (FE) computation code LS-DYNA in the  
98 context of standard FE kinematic formulation. The numerical feasibility of the enriched  
99 model and the results showing the distinct ASB and MV effects are then demonstrated  
100 on the RVE and structural scales.

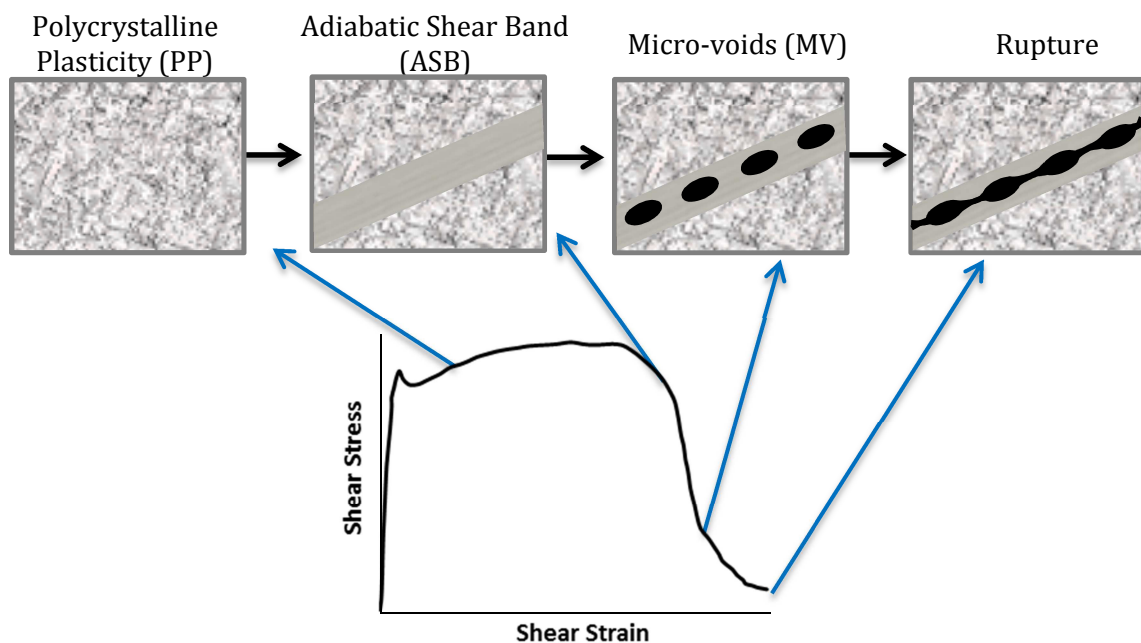
101 Section 2 presents the development of the constitutive model. The numerical issues  
102 encountered in the finite element implementation and performance of enriched ASB+MV  
103 model is assessed by numerical implementation on RVE scale and initial boundary value  
104 problems (IBVP) in Section 3. Finally the conclusions are drawn in Section 4 and some  
105 prospective work is suggested.

## 106 2 BASIC CONCEPTS AND CONSTITUTIVE MODELLING

107 The physical aspects and mechanical consequences of ASB and MV are presented below  
108 followed by the development of the constitutive model in an irreversible  
109 thermodynamics framework.

### 110 2.1 PHYSICAL AND MECHANICAL ASPECTS

111 The high strength metals and alloys under high strain rate loading first exhibit elasto-  
112 plasticity. During this stage, the self-heating caused by plastic dissipation and/or  
113 softening due to dynamic recrystallization may lead to the formation of a localized shear  
114 deformation (ASB) which causes material strength deterioration and alters the  
115 kinematics of the plastic flow. Further, damage is encountered within the shear bands  
116 causing the material failure. The scenario for ASB and MV assisted failure which is  
117 considered for modelling in the present study and their effects on the shear stress-shear  
118 strain curve are depicted in Fig. 1. The regular mechanism of polycrystalline plasticity is  
119 in action during the initial stage of hardening. Then the formation of the ASB reduces the  
120 material strength. The drop in the material strength in the post critical stage is  
121 potentially accelerated by the formation of micro-voids in the wake of the ASB. The  
122 micro-voids coalesce and ultimately rupture the material.



123 **Fig. 1** Successive mechanisms acting during ASB induced deterioration. Schematic drawings of  
124 microstructures are inspired from [2], see Fig.8 therein, and of shear stress-shear strain curve from [23],  
125 see Fig.12 therein.

126 In the following subsections, the salient physical and mechanical features involved are  
127 presented.

## 128 **2.1.1 Physical considerations**

129 The physical features and the formation process of the adiabatic shear bands and micro-  
130 voids are presented in this section.

### 131 *2.1.1.1 Adiabatic shear banding*

132 As mentioned above, the ASBs are distinctly oriented shear localizations resulting from a  
133 thermo-mechanical instability phenomenon. The heat produced from the plastic  
134 deformation during high strain rate loading, when not having enough time to be  
135 conducted away leads to local increase in temperature giving rise to a ‘quasi-adiabatic’  
136 condition. This causes a local thermal softening of the material. Thermo-mechanical  
137 instability comes into play as a competition arises between the thermal softening and  
138 strain or/and strain-rate hardening. When the rate of this thermal softening surpasses  
139 the strain/strain rate hardening, the material deformation becomes unstable and  
140 concentrates in narrow softened bands which are termed as ASBs. This widely accepted  
141 mechanism of ASB inception and growth was first proposed by Zener and Hollomon  
142 [24]. For certain alloys such as those of titanium, DRX is shown to contribute  
143 significantly to the softening leading to the formation of ASB (see [25],[26])

### 144 *2.1.1.2 Micro-voiding in the ASB wake*

145 Although the ASBs occur in conditions of negative or nil stress triaxiality, micro-voids  
146 have been observed to form and grow within the bands. The micro-voiding could be  
147 initiated in the band wake by the hydrostatic tension when there is a change in the  
148 loading path. Or as stated in [20], when the loading is interrupted after the formation of  
149 the ASB, the quenching of the hot band by the surrounding bulk material may lead to  
150 phase transformation involving possible volume increase (as for martensitic  
151 transformation) and strain incompatibility-induced crack formation at the band/matrix  
152 interface. However, the ASB induced failure is generally observed under monotonic  
153 loading.

154 Various perspectives on the stress triaxiality condition for the formation of micro-voids  
155 have been proposed which is discussed in section 3.1.1.3. Under a global shear pressure  
156 loading, Longère et al. [27] demonstrated that micro-voids can initiate and grow around  
157 inclusions in the material. Another theory cited by Odeshi et al. [11] is that the opening  
158 of the voids could be because of the high singular temperatures within the ASB causing

159 atomic mobility or creating a lower flow stress in the ASB than the surrounding regular  
160 material thereby generating a local tensile stress.

### 161 *2.1.1.3 Ultimate rupture*

162 The ASB by itself is not a failure mode i.e. a strong discontinuity but it acts as a precursor  
163 to premature material failure. The micro-voids which germinate within the ASB coalesce  
164 to the formation of cracks leading to the ultimate rupture of the material. Liu et al. [28]  
165 performed dynamic compression on Ti6Al4V alloy cylinders leading to ASB induced  
166 failure. The microscopic observation of the crushed specimens showed ASBs containing  
167 voids and coalesced voids (cracks).

## 168 **2.1.2 Mechanical consequences**

169 The mechanical consequences of ASB+MV initiation and evolution on the RVE (material  
170 point) response are double: material and kinematic.

### 171 *2.1.2.1 Preliminaries*

172 ASBs are not a type of crack or a conventional ‘damage’ as there is cohesion of matter  
173 within the bands and without any surface separation. Neither are they slip bands which  
174 occur under quasi static conditions as the physics of the phenomenon is different. What  
175 is observed is a large gradient of the shear strain between the matrix and the band. It is  
176 considered here as a ‘degradation’ mechanism as it degrades the mechanical properties  
177 of the overall material/structure in which it is present.

178 ASBs are therefore classified as ‘weak’ discontinuity which involves a discontinuity of  
179 the gradient of the displacement/velocity i.e. of the strain/strain rate field. A ‘strong’  
180 discontinuity such as a crack on the other hand involves a discontinuity of the  
181 displacement/velocity field. The weak discontinuity (ASB) serves as the path for the  
182 initiation of micro-voids which coalesce to form a strong discontinuity (crack) and the  
183 material fails. Like the ASB, the micro-voids formed in the wake of the ASB also have an  
184 effect on the deterioration and kinematic response of the material.

185 In the present study, a large scale postulate is adopted, in which the representative  
186 volume element (RVE) has a length scale  $W$  greater than the ASB width  $\lambda$ , i.e.  $W > \lambda$ ; in  
187 other words, the shear band is embedded within the RVE. In the presence of ASB coexist  
188 two different deformation patterns inside the medium: a ‘regular’ one relative to the  
189 material outside the bands, and a ‘singular’ one relative to the material inside the bands,  
190 see [21], [29]. Let suppose that the evolution of ‘regular’ and ‘singular’ processes can be

191 described via the evolution of state variables such like relevant measures of elastic  
 192 strain  $\mathbf{e}^e$ , temperature  $T$ , strain hardening  $\kappa$ , damage  $\delta$ , metallurgical state  $\xi$ , and so on :

193 
$$V_{\text{regular}} = (\mathbf{e}^e, T, \kappa, \delta, \xi, \dots) \text{ and } V_{\text{singular}} = (\mathbf{e}^{e*}, T^*, \kappa^*, \delta^*, \xi^*, \dots)$$

194 where  $V_{\text{regular}}$  and  $V_{\text{singular}}$  represent respectively the sets of ‘regular’ and ‘singular’ state  
 195 variables. At an advanced stage of deformation, ‘singular’ elastic strain can be neglected,  
 196 while a specific ‘singular’ variable describing intense shearing will be introduced  
 197 further. We then have tentatively:

198 
$$V_{\text{regular}} = (\mathbf{e}^e, T, \kappa, \delta, \xi, \dots) \text{ and } V_{\text{singular}} = (T^*, \kappa^*, \delta^*, \xi^*, \dots)$$

199 Before the ASB onset, solely ‘regular’ processes run:

200 
$$V_{\text{regular}} = (\mathbf{e}^e, T, \kappa, \delta, \xi, \dots) \text{ and } V_{\text{singular}} = (0, 0, 0, 0, \dots)$$

201 After the ASB onset, evolutions of ‘regular’ and ‘singular’ processes, associated to RVE  
 202 whose length scale  $W$  is greater than the band width  $\lambda$ , i.e.  $W > \lambda$ , and which are  
 203 respectively located outside and inside the band, become significantly distinct:

204 
$$V_{\text{regular}} = (\mathbf{e}^e, T, \kappa, \delta, \xi, \dots) \text{ and } V_{\text{singular}} = (T^*, \kappa^*, \delta^*, \xi^*, \dots)$$

205 In our approach, the evolution of the ‘singular’ dissipative processes (intervening inside  
 206 the band) contributing to this macromechanical softening is described via the evolution  
 207 of a single internal variable called  $\mathcal{D}$ . The softening of the RVE behaviour being  
 208 considered as a form of mechanical degradation, the variable  $\mathcal{D}$  must then be able to  
 209 describe the deterioration state of the RVE induced by the presence of the bands (ASB  
 210 related feedback deterioration).

211 The variable  $\mathcal{D}$  is then a function of the ‘singular’ state variables and of the  
 212 characteristic length  $\lambda$  of the band:

213 
$$\mathcal{D} = \mathcal{D}(V_{\text{singular}}) = \mathcal{D}(\lambda, T^*, \kappa^*, \delta^*, \xi^*, \dots)$$

214 A micromechanical analysis followed by any averaging procedure could have lead to an  
 215 explicit form for  $\mathcal{D}$ . This is not the purpose of the present work as it is hardly feasible  
 216 regarding strongly non-linear phenomena involved and high strain rate context. What  
 217 we are putting forward is a phenomenological description involving ‘singular’

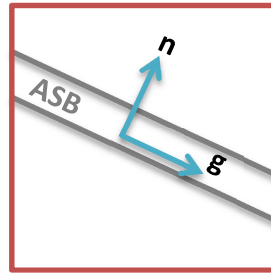


218 dissipative evolution due to ASB and further micro-voiding entirely incorporated in the  
 219 evolution of  $\mathcal{D}$ . In our phenomenological approach, evolution law of  $\mathcal{D}$  is derived from  
 220 the normality rule under the assumption of the existence of a corresponding dissipative  
 221 potential.

### 222 2.1.2.2 Orientation

223 According to the large scale postulate, the shear band or the shear band cluster is  
 224 considered as an entity embedded within the RVE, see Fig. 2. The vector  $\mathbf{g}$  is collinear to  
 225 the slip direction in the band plane and  $\mathbf{n}$  represents the normal to the band plane.  
 226 Accordingly, the structural tensors used in the following are defined in (1) where  $\mathcal{O}^S$   
 227 represents the symmetric and  $\mathcal{O}^{AS}$  the skew-symmetric parts of the tensor. According to  
 228 experimental evidences, the ASBs develop along the planes of maximum shear stress  
 229 within the RVE and thus their orientations are calculated.

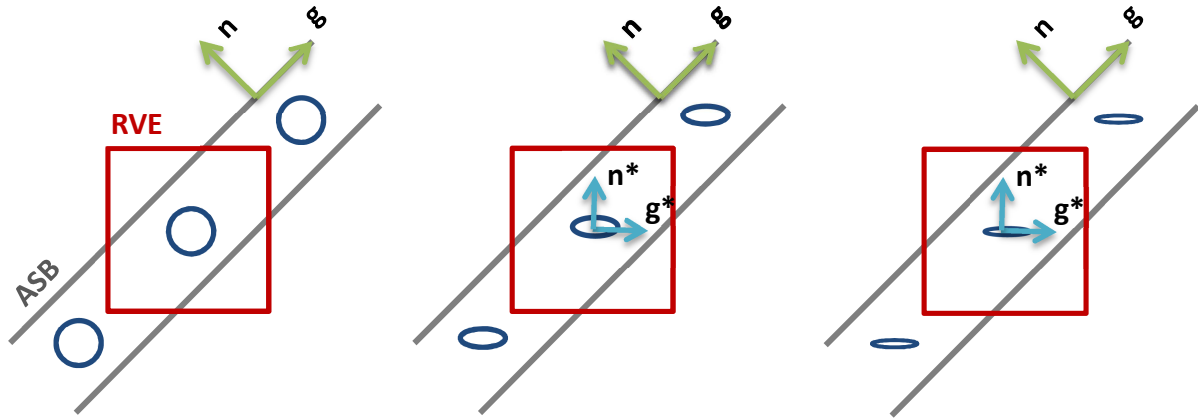
$$230 \quad \mathbf{N} = \mathbf{n} \otimes \mathbf{n} ; \mathbf{M} = (\mathbf{g} \otimes \mathbf{n})^S ; \mathbf{T} = (\mathbf{g} \otimes \mathbf{n})^{AS} \quad (1)$$



231  
 232  
 233

**Fig. 2** ASB embedded within the RVE and orientation vectors

234 Different configurations of micro-voids developing in the ASB wake can be  
 235 distinguished. A thick wall cylinder implosion technique was carried out by Xue et al. [2]  
 236 to produce shear bands on Ti-6Al-4V alloys. The microscopic observation of the ASB  
 237 showed spherical void nucleation. The spherical voids were later seen to elongate to  
 238 ellipse shape rotate along the direction of the maximum shear stress. Impact tests on hat  
 239 shaped specimen by Peirs. et al. [1] showed micro-cracks in the wake of the ASB  
 240 oriented along the highly deformed grains which is different from the ASB orientation.  
 241 Accordingly, Fig. 3 shows an illustration of the possible micro-void shape and  
 242 orientation vectors  $\mathbf{n}^*$ ,  $\mathbf{g}^*$  for micro-voiding occurring inside the band. Corresponding  
 243 MV structural tensors are denoted with an asterisk as  $\mathbf{N}^*$ ,  $\mathbf{M}^*$ ,  $\mathbf{T}^*$ .



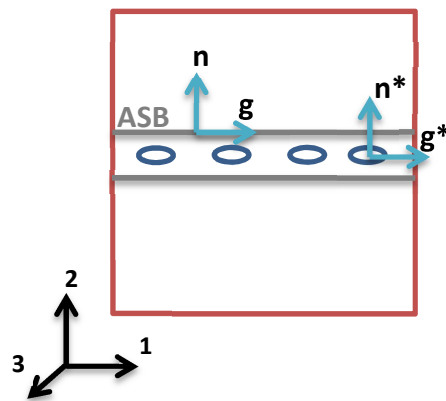
244 **Fig. 3** Illustration of micro-void configurations. a) Spherical void configuration b) Ellipsoidal void  
 245 configuration c) Micro-crack configuration.

246 **2.1.2.3 Material consequences**

247 As the ASB and MV are strongly oriented in space, they anisotropically degrade the  
 248 elastic and plastic material properties of the RVE containing the band based on their  
 249 orientation. The induced degradation may therefore be described by a 2<sup>nd</sup> order tensor  
 250 variable  $\mathbf{D}$  defined in (2). The components of  $\mathbf{D}$  are: the scalar degradation intensities  
 251 denoted by  $\mathcal{D}^{ASB}$ ,  $\mathcal{D}^{MV}$  and the orientation tensors  $\mathbf{N}, \mathbf{N}^*$  of ASB and MV respectively.

252 
$$D_{ij} = \mathcal{D}_{ASB} N_{ij} + \mathcal{D}_{MV} N_{ij}^* \quad (2)$$

253 Consider the illustration in Fig. 4, the normal to the ASB and to the MV is in direction 2,  
 254 in which case, the deterioration tensor is calculated below:



255

256

**Fig. 4** Illustration ASB orientation normal to the 2 direction.

$$257 \quad \mathbf{n} = \mathbf{n}^* = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \mathbf{N} = \mathbf{N}^* = \begin{bmatrix} 0 & 0 & 0 \\ & 1 & 0 \\ sym & & 0 \end{bmatrix} \rightarrow \mathbf{D} = (\mathcal{D}^{ASB} + \mathcal{D}^{MV}) \begin{bmatrix} 0 & 0 & 0 \\ & 1 & 0 \\ sym & & 0 \end{bmatrix} \quad (3)$$

258 According to Fig. 4, the RVE can be viewed as a 3-layer material or sandwich material  
 259 with a thin soft layer inserted between two thick hard layers, inducing a form of  
 260 orthotropy. The resulting elastic shear modulus  $\mu_{12}$  and elastic axial (Young's) modulus  
 261  $E_{22}$  are accordingly expected to progressively vanish while the other elastic moduli are  
 262 not or less affected.

$$263 \quad \begin{cases} \tau_{12} = 2\mu_{12}(\mathcal{D}_{ASB})e_{12}^e \\ \tau_{22} = E_{22}(\mathcal{D}_{ASB})e_{22}^e \end{cases} \text{ with } \begin{cases} \begin{cases} \mu_{12}(\mathcal{D}_{ASB}) \rightarrow \mu_c \square \mu_0 \\ \mathcal{D}_{ASB} \rightarrow \mathcal{D}_{ASBc} \end{cases} \\ \begin{cases} E_{22}(\mathcal{D}_{ASB}) \rightarrow E_c \square E_0 \\ \mathcal{D}_{ASB} \rightarrow \mathcal{D}_{ASBc} \end{cases} \end{cases} \rightarrow \begin{cases} \begin{cases} \tau_{12}(\mathcal{D}_{ASB}) \rightarrow \tau_{12c} \\ \mathcal{D}_{ASB} \rightarrow \mathcal{D}_{ASBc} \end{cases} \\ \begin{cases} \tau_{22}(\mathcal{D}_{ASB}) \rightarrow \tau_{22c} \\ \mathcal{D}_{ASB} \rightarrow \mathcal{D}_{ASBc} \end{cases} \end{cases} \quad (4)$$

$$\begin{cases} \begin{cases} \mu_{12}(\mathcal{D}_{MV}) \rightarrow 0 \\ \mathcal{D}_{MV} \rightarrow \mathcal{D}_{MVc} \end{cases} \\ \begin{cases} E_{22}(\mathcal{D}_{MV}) \rightarrow 0 \\ \mathcal{D}_{MV} \rightarrow \mathcal{D}_{MVc} \end{cases} \end{cases} \rightarrow \begin{cases} \begin{cases} \tau_{12}(\mathcal{D}_{MV}) \rightarrow 0 \\ \mathcal{D}_{MV} \rightarrow \mathcal{D}_{MVc} \end{cases} \\ \begin{cases} \tau_{22}(\mathcal{D}_{MV}) \rightarrow 0 \\ \mathcal{D}_{MV} \rightarrow \mathcal{D}_{MVc} \end{cases} \end{cases}$$

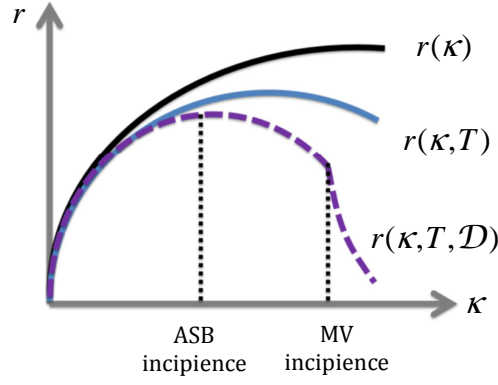
264 where  $\boldsymbol{\tau}$  is the elastic Kirchhoff stress tensor and  $\mathbf{e}^e$  the elastic strain tensor,  $(\mu_0, E_0)$   
 265 and  $(\mu_c, E_c)$  are the initial and critical, elastic shear and axial (Young's) moduli. Note  
 266 that for the case of ASB,  $(\mu_c, E_c)$  have non zero values since ASB preserves matter  
 267 cohesion.

268 Furthermore, the effect of the combined deterioration of ASB and MV can be seen on the  
 269 strain hardening force  $r(\kappa)$  as presented in Fig. 5. The black curve represents the strain  
 270 hardening solely as a function of the isotropic hardening variable  $\kappa$ . The blue curve  
 271 carries an additional influence of the temperature i.e. thermal softening. The purple  
 272 curve is subjected to the ASB and MV induced deterioration and hence the reduction in  
 273 hardening or conversely softening behaviour.

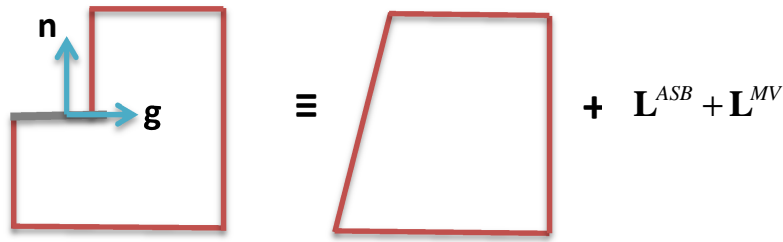
#### 274 2.1.2.4 Kinematic consequences

275 The kinematic consequences of the presence of the ASB and hence MV are viewed as  
 276 those of a 'macro-dislocation' considering an RVE as illustrated in Fig. 6.

277



278 **Fig. 5** Strain hardening force influenced by thermal softening and ASB+MV induced deterioration



279

280 **Fig. 6** ASB viewed as a macro dislocation and the equivalent RVE with kinematic consequences. The  
 281 kinematic variables  $L^{ASB}$  and  $L^{MV}$  allow to smooth the boundary discontinuity caused by the ASB and  
 282 MV, as in crystalline plasticity and continuum damage mechanics.

283 We are designating the total velocity gradient  $\mathbf{L}$  as the sum of a symmetric part  $\mathbf{d}$ , the  
 284 strain rate, and an antisymmetric part  $\boldsymbol{\omega}$ , the spin rate, i.e.  
 285  $\mathbf{L} = \mathbf{d} + \boldsymbol{\omega}$  ;  $\mathbf{d} = \text{sym}[\mathbf{L}]$  ;  $\boldsymbol{\omega} = \text{antisym}[\mathbf{L}]$ . The velocity gradient created by the regular  
 286 plastic deformation outside the band is designated as  $\mathbf{L}^p$ . The ASB induces a  
 287 supplementary velocity gradient  $\mathbf{L}^{ASB}$  as a result of glide velocity  $\dot{\gamma}^{ASB}$  in the direction of  
 288 the unit vector  $\mathbf{g}$  ; likewise the MV induced velocity gradient is termed as  $\mathbf{L}^{MV}$ . The  
 289 introduction of  $\mathbf{L}^{ASB}$  and  $\mathbf{L}^{MV}$  facilitates the smoothing of the boundary discontinuity  
 290 caused by ASB as it is done in crystalline plasticity, see Fig. 6.

291 
$$\mathbf{L}^{in} = \mathbf{L}^p + \mathbf{L}^{ASB} + \mathbf{L}^{MV} \quad (5)$$

292 The deterioration-induced velocity gradient for a given band pattern  $\alpha$  is partitioned  
 293 into symmetric and skew-symmetric parts leading to the corresponding strain rate  $\mathbf{d}$   
 294 and spin  $\boldsymbol{\omega}$  respectively as given below.

$$\begin{aligned}
& L_{ij}^{ASB} \propto \sum_{\alpha} \dot{\gamma}^{ASB,\alpha} g_i^{\alpha} n_j^{\alpha} \\
& L_{ij}^{MV} \propto \sum_{\alpha} \dot{\gamma}^{MV,\alpha} g_i^{\alpha} n_j^{\alpha} + \frac{\dot{\epsilon}_m^{MV}}{3} I_{ij}
\end{aligned}
\left\{ \begin{array}{l}
d_{ij}^{ASB} \propto \sum_{\alpha} \dot{\gamma}^{ASB,\alpha} M_{ij}^{\alpha} \\
\omega_{ij}^{ASB} \propto \sum_{\alpha} \dot{\gamma}^{ASB,\alpha} T_{ij}^{\alpha} \\
d_{ij}^{MV} \propto \sum_{\alpha} \dot{\gamma}^{MV,\alpha} M_{ij}^{*\alpha} + \frac{\dot{\epsilon}_m^{MV}}{3} I_{ij} \\
\omega_{ij}^{MV} \propto \sum_{\alpha} \dot{\gamma}^{MV,\alpha} T_{ij}^{*\alpha}
\end{array} \right. \quad (6)$$

296 In case of MV configurations accounting for the void expansion, an additional  
297 hydrostatic term appears in the strain rate, where  $\mathbf{I}$  is the identity tensor and  $\dot{\epsilon}_m^{MV}$  is the  
298 mean strain rate.

299 The kinematic consequences of the presence of ASB and MV are thus shown by the total  
300 inelastic strain rate  $\mathbf{d}^{in}$  resulting from the superposition of regular plastic strain rate  
301  $\mathbf{d}^{in(PP)}$  by the singular strain rates  $\mathbf{d}^{in(ASB)}$  and  $\mathbf{d}^{in(MV)}$ .

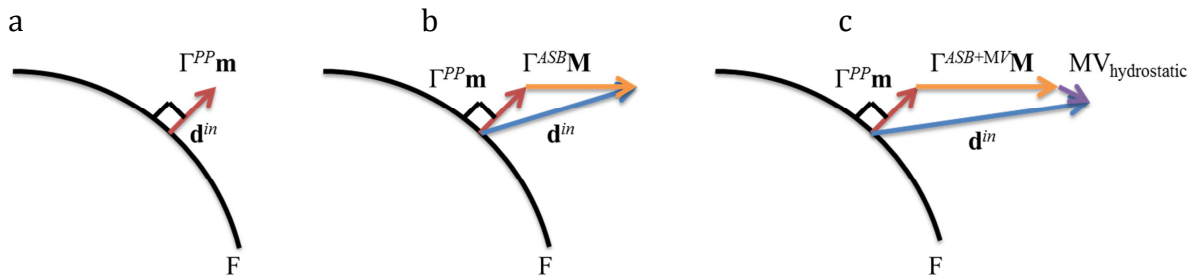
$$302 \quad d_{ij}^{in} = d_{ij}^{in(PP)} + d_{ij}^{in(ASB)} + d_{ij}^{in(MV)} \quad (7)$$

303 A schematic representation of the kinematics is shown in Fig. 7 where F in all the three  
304 cases represents the plastic potential in absence of the deterioration mechanisms. Fig.  
305 7a shows the case of regular plasticity before the appearance of ASB during which the  
306 total inelastic strain rate reduces to  $\mathbf{d}^{in} = \mathbf{d}^{in(PP)} = \Gamma^{PP} \mathbf{m}$  where  $\Gamma^{PP}$  is the regular plastic  
307 yield intensity and  $\mathbf{m} = (3/2) \times (\mathbf{s} / \sigma_{eq})$  the yield direction (normality rule) with  $\sigma_{eq}$   
308 designating Mises equivalent stress and  $\mathbf{s}$  the deviatoric part of the Kirchhoff stress  
309 tensor. The ASB formation in Fig. 7b superposes an additional strain rate  
310  $\mathbf{d}^{in(ASB)} = \Gamma^{ASB} \mathbf{M}$ , causing progressive deviation of the yield direction and loss of  
311 coaxiality between inelastic strain rate and stress deviator (general case). Finally, the  
312 micro-voiding in Fig. 7c further contributes to the yield intensity  $\Gamma^{MV}$  (assuming  
313 collinearity of ASB and MV orientations, see Fig. 4) and keeps on deviating the yield  
314 direction, provoking in particular a plastic expansion termed as  $MV_{hydrostatic}$ .

315 For the different possible configurations of MV in Fig. 3, the following lists the MV  
316 induced effects:

317 a) The first simple configuration assumes ideal spherical voids in Fig. 3a. In this  
318 case, the spherical MV contributes to isotropic degradation ( $Tr\dot{\mathbf{D}}$ ) and  
319 hydrostatic plastic strain rate ( $\mathbf{d}_v^p$ ) accounting for volume change only.

320 b) Then the second configuration considered shows elliptical voids which possess  
 321 an orientation  $(\mathbf{n}^*, \mathbf{g}^*)$  which may or may not be the same as that of the ASB (Fig.  
 322 3b). These types of elliptical voids contribute to anisotropic degradation rate (  
 323  $\dot{\mathbf{D}} = \dot{\mathcal{D}}_{MV} \mathbf{N}^*$ ;  $\mathbf{N}^* = \mathbf{n}^* \otimes \mathbf{n}^*$ ) and hydrostatic and deviatoric ( $\mathbf{d}_s^p$ ) strain rates (with  
 324 the corresponding tensor  $\mathbf{M}^* = (\mathbf{g}^* \otimes \mathbf{n}^*)^S$ ) accounting for volume and shape  
 325 changes respectively.  
 326 c) The third configuration in Fig. 3c presents micro-cracks with orientation  $\mathbf{n}^*, \mathbf{g}^*$   
 327 and which influence the anisotropic degradation rate ( $\dot{\mathcal{D}}_{MV}, \mathbf{N}^*$ ) only. When  
 328 crack opening is not taken into account, there is no MV related strain rate.



329 **Fig. 7** Illustration of yield direction evolution (a) before ASB onset (b) during adiabatic shear banding (c)  
 330 during micro-voiding. F is the regular plastic potential in the absence of deterioration mechanisms.

## 331 2.2 CONSTITUTIVE MODELLING

332 This section deals with the more specific form of the unified constitutive model  
 333 describing the ASB and the consequent MV induced deterioration. The  
 334 phenomenological model describing ASB induced failure originally developed by  
 335 Longère et al [21] has been conceptually improved by Longère and Dragon [22] to  
 336 incorporate the effects of micro-voiding (MV) in the post critical stage. The model used  
 337 in the present work is mostly based on the enlarged model in [22] while making some  
 338 simplifications/modifications. The role of dynamic recrystallization on the onset of ASB  
 339 in the materials susceptible to this mechanism has also been considered here. The aim of  
 340 the ASB+MV model is to clearly distinguish the contributions to the degradation coming  
 341 from the ASB on the one hand and the MV on the other hand. The constitutive model  
 342 presenting the dynamic recrystallization in the context of the model under consideration  
 343 can be found in [30].

344 The constitutive model developed here is a phenomenological one wherein the  
 345 consequences of the presence of the ASB at the global RVE scale on the regular  
 346 properties are described. The salient physical features described by the model are:

- 347 • softening influencing ASB onset:
- 348     • plastic dissipation induced thermal softening
- 349     • dynamic recrystallization (DRX) induced softening
- 350 • consequences:
- 351     • anisotropic material deterioration due to ASB+MV
- 352     • ASB+MV induced specific finite strain kinematics

353 In the following, the quantities/properties associated with the material inside the ASB  
 354 are termed as ‘singular’ and the quantities/properties associated with the material  
 355 outside the band are referred to as ‘regular’.

### 356 2.2.1 Large anisotropic strain framework

357 As very large strains and rotations are encountered during the ASB process, a finite  
 358 elastic-plastic deformation framework assuming moderate elastic deformation is  
 359 adopted. The objective corotational derivative  $\overset{\nabla}{\mathbf{A}}$  of a 2<sup>nd</sup> order tensor  $\mathbf{A}$  is given by

$$360 \quad \overset{\nabla}{\mathbf{A}} = \dot{\mathbf{A}} - \mathbf{W}\mathbf{A} + \mathbf{A}\mathbf{W} \quad (8)$$

361 where  $\mathbf{W}$  represents the rotation rate of anisotropy axes relative to ASB orientation.  
 362 Taking into account the negligible elastic rotation  $\boldsymbol{\omega}^e$  and relative small regular plastic  
 363 spin  $\boldsymbol{\omega}^{in(PP)}$ , see [21], the rotation rate  $\mathbf{W}$  is reduced to:

$$364 \quad \mathbf{W} = \boldsymbol{\omega} \cdot \left( \boldsymbol{\omega}^{in(ASB)} + \boldsymbol{\omega}^{in(MV)} \right) \quad (9)$$

365 where  $\boldsymbol{\omega}^{in(ASB)}$  and  $\boldsymbol{\omega}^{in(MV)}$  are the spin rates induced by ASB and MV respectively. In the  
 366 following, the tensors  $\mathbf{X}$  defined in the intermediate configuration are transported to  
 367 the current one denoted by a tilde  $\tilde{\mathbf{X}}$  via  $\tilde{\mathbf{X}} = \mathbf{Q}\mathbf{X}\mathbf{Q}^T$ ,  $\mathbf{W} = \dot{\mathbf{Q}}\mathbf{Q}^T$ .

### 368 2.2.2 Thermodynamic framework

369 In the framework of irreversible thermodynamics, the (regular) state variables to  
 370 account for are: the elastic strain tensor  $\mathbf{e}^e = \ln \mathbf{V}^e$ , the absolute temperature  $T$ , the  
 371 isotropic hardening variable  $\kappa$ . The singular temperature and other singular state  
 372 variables are not explicitly calculated in the model, but their effects on the material  
 373 degradation are embodied in the degradation tensor  $\tilde{\mathbf{D}}$ .

374 Helmholtz free energy  $\psi(\mathbf{e}^e, T, \kappa; \tilde{\mathbf{D}})$  is assumed as the state potential comprising of a  
 375 reversible part  $\psi^e(\mathbf{e}^e, T; \tilde{\mathbf{D}})$ , a stored energy part  $\psi^p(T, \kappa; \tilde{\mathbf{D}})$  and a purely thermal part  
 376  $\psi^T(T)$  as shown in (10). The presence of ASB affects both the elastic and plastic  
 377 properties.

$$378 \quad \psi(\mathbf{e}^e, T, \kappa; \tilde{\mathbf{D}}) = \psi^e(\mathbf{e}^e, T; \tilde{\mathbf{D}}) + \psi^p(T, \kappa; \tilde{\mathbf{D}}) + \psi^T(T) \quad (10)$$

379 Constitutive state laws derived from the state potential in (10) allows for expressing the  
 380 elastic Kirchhoff stress tensor  $\boldsymbol{\tau}$ , strain hardening force  $r$ , deterioration driving force  $\tilde{\mathbf{G}}$   
 381 and entropy  $s$ , as follows:

$$382 \quad \begin{aligned} \tau_{ij} &= \rho_0 \frac{\partial \psi}{\partial e_{ij}^e} = \rho_0 \frac{\partial \psi^e}{\partial e_{ij}^e} ; r = \rho_0 \frac{\partial \psi}{\partial \kappa} = \rho_0 \frac{\partial \psi^p}{\partial \kappa} \\ \tilde{G}_{ij} &= -\rho_0 \frac{\partial \psi}{\partial \tilde{D}_{ij}} = -\left( \rho_0 \frac{\partial \psi^e}{\partial \tilde{D}_{ij}} + \rho_0 \frac{\partial \psi^p}{\partial \tilde{D}_{ij}} \right) \\ \rho_0 s &= -\rho_0 \frac{\partial \psi}{\partial T} = -\left( \rho_0 \frac{\partial \psi^e}{\partial T} + \rho_0 \frac{\partial \psi^p}{\partial T} + \rho_0 \frac{d\psi^T}{dT} \right) \end{aligned} \quad (11)$$

383 where  $\rho_0$  designates the initial mass density. The model should satisfy the Clausius-  
 384 Duhem inequality in order to be consistent with the irreversible thermodynamics.  
 385 Therefore the intrinsic dissipation  $\mathcal{D}_{\text{int}}$  is expressed as:

$$386 \quad \mathcal{D}_{\text{int}} = \tau_{ij} d_{ji} - \rho_0 (\dot{\psi} + s\dot{T}) \geq 0 \quad (12)$$

387  $\mathcal{D}_{\text{int}}$  is decomposed into regular plasticity related term  $\mathcal{D}_{\text{reg}}$  and singular ASB+MV  
 388 induced term  $\mathcal{D}_{\text{sing}}$ . Using the Gibbs relation  $\rho_0 \dot{\psi} = \tau_{ij} d_{ij}^e + r \dot{\kappa} - \tilde{G}_{ij} \dot{\tilde{D}}_{ij} - \rho_0 s \dot{T}$  the  
 389 dissipation components are written as:

$$390 \quad \begin{aligned} \mathcal{D}_{\text{int}} &= \mathcal{D}_{\text{reg}} + \mathcal{D}_{\text{sing}} \geq 0 \\ \mathcal{D}_{\text{reg}} &= \tau_{ij} d_{ji}^p - r \dot{\kappa} \geq 0 ; \mathcal{D}_{\text{sing}} = \tau_{ij} d_{ji}^d + \tilde{G}_{ij} \overset{\nabla}{\tilde{D}}_{ji} \geq 0 \end{aligned} ; \begin{cases} d_{ij}^p = d_{ij}^{\text{in}(PP)} \\ d_{ij}^d = d_{ij}^{\text{in}(ASB)} + d_{ij}^{\text{in}(MV)} \end{cases} \quad (13)$$

391 In the context of generalised normality rule, the evolutions laws are correspondingly  
 392 derived from dissipation potentials namely the viscoplastic potential  $\phi_p$  and viscous-  
 393 degradation potential  $\phi_d = \phi_{\text{ASB}} + \phi_{\text{MV}}$  as:

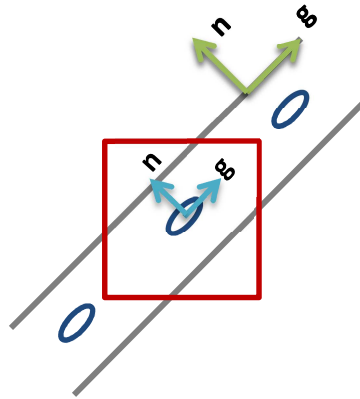


$$\begin{aligned}
d_{ij}^{in} &= \frac{\partial \phi_p}{\partial \tau_{ij}} = d_{ij}^{in(PP)} + d_{ij}^{in(ASB)} + d_{ij}^{in(MV)} \\
-\dot{\kappa} &= \frac{\partial \phi_p}{\partial r} \\
\overset{\nabla}{D}_{ij} &= \frac{\partial \phi_d}{\partial G_{ij}} = \overset{\nabla}{D}_{ij}^{ASB} + \overset{\nabla}{D}_{ij}^{MV}
\end{aligned} \tag{14}$$

395 It is now required to define a plastic potential H and/or a yield function F depending  
396 on whether non-associated (H ≠ F) or associated flow rule (H = F) is followed.

### 397 2.2.3 Constitutive equations

398 A more precise form of the constitutive equations adopted in the current study is  
399 presented in this section. In the following, for simplification, elliptical void shape is  
400 assumed with the orientation along the direction of the ASB as represented in Fig. 8.  
401 Hence the structural tensors in (1) corresponding to the ASB and MV are considered to  
402 be the same (N, M).



403  
404 **Fig. 8** Elongated micro-voids oriented in the direction of ASB

#### 405 2.2.3.1 Preliminary considerations

- 406 • *Elastic stress tensor*

407 Following the work of Halm and Dragon [31], see also Longère and Dragon [21], the  
408 elastic stress tensor in presence of anisotropic damage induced degradation is assumed  
409 of the form:

$$\begin{aligned}
410 \quad \tau_{ij} &= \rho_0 \frac{\partial \psi}{\partial e_{ij}^e} = \rho_0 \frac{\partial \psi^e}{\partial e_{ij}^e} = \lambda e_{kk}^e I_{ij} + 2\mu e_{ij}^e - \bar{\alpha} K \Delta T I_{ij} - a \left( e_{mn}^e \tilde{D}_{nm} I_{ij} + e_{kk}^e \tilde{D}_{ij} \right) - 2b \left( e_{ik}^e \tilde{D}_{kj} + \tilde{D}_{ik} e_{kj}^e \right) \\
411 & \hspace{20em} (15)
\end{aligned}$$

412 The form given to the stress tensor in (15) is such that it takes into account material  
413 degradation based on the orientation of the shear bands.  $\lambda$  and  $\mu$  represent the Lamé  
414 elasticity constants and  $K = (3\lambda + 2\mu)/3$  the bulk modulus.  $a$  and  $b$  are ASB+MV related  
415 constants inducing orthotropic elastic energy degradation – they may have different  
416 values for ASB and MV in a future work. In the thermal part,  $\bar{\alpha}$  represents the thermal  
417 expansion coefficient,  $T_0$  the initial temperature and  $\Delta T = T - T_0$  the rise in temperature.

418 Consider now the illustration in Fig. 4, the normal to the ASB is in direction 2, in which  
419 case, the deterioration tensor is calculated below:

$$\begin{aligned}
420 \quad \mathbf{n} &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \mathbf{D} = \mathcal{D} \begin{bmatrix} 0 & 0 & 0 \\ & 1 & 0 \\ sym & & 0 \end{bmatrix}; \quad \mathcal{D} = \mathcal{D}^{ASB} + \mathcal{D}^{MV} \hspace{5em} (16)
\end{aligned}$$

421 By neglecting the thermal dilatation, the components of  $\boldsymbol{\tau}$  for the above case (Fig. 4)  
422 deduced from (15)-(16) are:

$$\begin{aligned}
423 \quad \begin{cases} \tau_{11} = (\lambda + 2\mu) e_{11}^e + (\lambda - a\tilde{D}_{22}) e_{22}^e + \lambda e_{33}^e \\ \tau_{22} = (\lambda - a\tilde{D}_{22}) (e_{11}^e + e_{33}^e) + [(\lambda - 2a\tilde{D}_{22}) + 2(\mu - 2b\tilde{D}_{22})] e_{22}^e \\ \tau_{33} = (\lambda + 2\mu) e_{33}^e + (\lambda - a\tilde{D}_{22}) e_{22}^e + \lambda e_{11}^e \end{cases} \hspace{5em} (17)
\end{aligned}$$

$$\begin{aligned}
424 \quad \begin{cases} \tau_{12} = 2(\mu - b\tilde{D}_{22}) e_{12}^e \\ \tau_{13} = 2\mu e_{13}^e \\ \tau_{23} = 2(\mu - b\tilde{D}_{22}) e_{23}^e \end{cases}
\end{aligned}$$

425 The resulting (symmetric) stiffness matrix  $\mathbf{C}$ ,  $\boldsymbol{\tau} = \mathbf{C} : \mathbf{e}^e$ , reads

$$426 \quad [C] = \begin{bmatrix} \lambda + 2\mu & (\lambda - a\mathcal{D}) & \lambda & 0 & 0 & 0 \\ & (\lambda - 2a\mathcal{D}) + 2(\mu - 2b\mathcal{D}) & (\lambda - a\mathcal{D}) & 0 & 0 & 0 \\ & & \lambda + 2\mu & 0 & 0 & 0 \\ & & & 2(\mu - b\mathcal{D}) & 0 & 0 \\ & & & & 2\mu & 0 \\ sym & & & & & 2(\mu - b\mathcal{D}) \end{bmatrix} \quad (18)$$

427 Failure under shear loading yields an upper bound for deterioration intensity  $\mathcal{D}_c$  as shown  
428 below:

$$429 \quad \tau_{12} = 2(\mu - b\mathcal{D}_c)e_{12}^e = 0 \quad \rightarrow \quad \mathcal{D}_c = \frac{\mu}{b} \rightarrow \mathcal{D}_c (b = \mu) = 1 \quad (19)$$

430 A reduction in the material strength is seen in the direction normal to the ASB ( $\tau_{22}$ ) and  
431 in the corresponding shear direction ( $\tau_{12}$ ). It is seen that the constants  $a < 0$  and  $b > 0$   
432 contribute to the reduction in Young's modulus  $E_{22}$  whereas only  $b$  acts in the  
433 reduction of the shear modulus  $\mu_{12}$ .

434 • *Strain hardening force*

435 Experimental observations have shown that in absence of ASB and MV the coupling  
436 between strain hardening and thermal softening may be described in a product form  
437 allowing for using the method of separation of variables

$$438 \quad r(\kappa, T) = \rho_0 \frac{\partial \psi}{\partial \kappa} = \rho_0 \frac{\partial \psi^p}{\partial \kappa} = h'(\kappa) g(T) \quad (20)$$

439 The form of the thermal softening corresponding to steel is shown in (21) where  $\nu$  is the  
440 material specific thermal softening constant

$$441 \quad g(T) = \exp(-\nu T) \quad (21)$$

442 For materials such as titanium alloys, in accordance with experimental observations, the  
443 function in (22) satisfies the thermal softening characteristics with  $T_{ref}$  being a reference  
444 temperature such that  $g(T_{ref}) = 0$ .  $\langle \cdot \rangle$  are Macauley brackets defined as  $\langle x \rangle = \max(x, 0)$ .

445 
$$g(T) = \left\langle 1 - \left( \frac{T}{T_{ref}} \right)^t \right\rangle \quad (22)$$

446 where  $t$  designates a constant. The strain hardening function is expressed in (23) where  
 447  $R_\infty$  is related to the saturation of isotropic hardening (Voce law).

448 
$$h'(\kappa) = R_\infty [1 - \exp(-k\kappa)]; h(\kappa) = R_\infty \left[ \kappa + \frac{1}{k} \exp(-k\kappa) \right] \quad (23)$$

449 We are here assuming that this product form can be extended for ASB and MV

450 
$$r(\kappa, T; \tilde{\mathbf{D}}) = \rho_0 \frac{\partial \psi}{\partial \kappa} = \rho_0 \frac{\partial \psi^p}{\partial \kappa} = h'(\kappa) g(T) w(\tilde{\mathbf{D}}) \quad (24)$$

451 In the following equations the thermal softening function in (21) is used. The  
 452 deterioration function  $w(\tilde{\mathbf{D}})$  is expressed below where  $\chi_1$  and  $\chi_2$  are ASB+MV related  
 453 deterioration constants. The form of the deterioration function  $w(\tilde{\mathbf{D}})$  is adopted of the  
 454 same exponential form as the thermal softening function in (21) as the singular  
 455 temperature rise within the shear band is the main contributor to the material  
 456 deterioration. The expression of  $w(\tilde{\mathbf{D}})$  in (25) is tentatively retained for other materials  
 457 such as Ti alloys even though the form of their thermal softening function is different.

458 
$$w(D_{ij}) = \exp\left(-\chi_1 \tilde{D}_{ii} - \frac{\chi_2}{2} \tilde{D}_{ij} \tilde{D}_{ji}\right) \quad (25)$$

### 459 2.2.3.2 State potential

460 According to the preliminary considerations above, the state potential reads after  
 461 integration of the constitutive state laws in (15) and (24) with respect to their different  
 462 arguments:

463 
$$\psi(\mathbf{e}^e, T, \kappa; \tilde{\mathbf{D}}) = \psi^e(\mathbf{e}^e, T; \tilde{\mathbf{D}}) + \psi^p(T, \kappa; \tilde{\mathbf{D}}) + \psi^T(T)$$

464 
$$\begin{cases} \rho_0 \psi^e(\mathbf{e}^e, T; \tilde{\mathbf{D}}) = \frac{1}{2} \mathbf{e}^e : \mathbf{C}(\tilde{\mathbf{D}}) : \mathbf{e}^e - \bar{\alpha} K Tr(\mathbf{e}^e) \Delta T \\ \rho_0 \psi^p(T, \kappa; \tilde{\mathbf{D}}) = h(\kappa) g(T) w(\tilde{\mathbf{D}}) \\ \rho_0 \psi^T(T) = -\rho_0 \frac{C}{2T_0} \Delta T^2 \end{cases} \quad (26)$$

465 where  $C$  designates the specific heat.

### 466 2.2.3.3 Constitutive state laws

467 The complete expressions of the conjugate forces associated to the state variables,  
468 considering steel oriented thermal softening function, is shown below:

$$\begin{cases}
 \tau_{ij} = \lambda e_{kk}^e I_{ij} + 2\mu e_{ij}^e - \bar{\alpha} K \Delta T I_{ij} - a \left( e_{mn}^e \tilde{D}_{nm} I_{ij} + e_{kk}^e \tilde{D}_{ij} \right) - 2b \left( e_{ik}^e \tilde{D}_{kj} + \tilde{D}_{ik} e_{kj}^e \right) \\
 r = R_\infty \left[ 1 - \exp(-k\kappa) \right] \exp(-\nu T) \exp \left( -\chi_1 \tilde{D}_{kk} - \frac{\chi_2}{2} \tilde{D}_{kl} \tilde{D}_{lk} \right) \\
 \tilde{G}_{ij} = a e_{kk}^e e_{ij}^e + 2b e_{ik}^e e_{kj}^e + R_\infty \left[ \kappa + \frac{1}{k} \exp(-k\kappa) \right] \\
 \quad \cdot \exp(-\nu T) \exp \left( -\chi_1 \tilde{D}_{kk} - \frac{\chi_2}{2} \tilde{D}_{kl} \tilde{D}_{lk} \right) \left[ \chi_1 I_{ij} + \chi_2 \tilde{D}_{ij} \right]
 \end{cases} \quad (27)$$

470 The expression of the entropy is not detailed here. The diminution of the stress  $\boldsymbol{\tau}$  is  
471 caused by the isotropic heating and ASB+MV induced anisotropic degradation. The  
472 competition between hardening and softening is embodied in the isotropic hardening  
473 conjugate force  $r$ : the strain hardening capability  $\partial r / \partial \kappa$  is decreased by the thermal  
474 softening function  $g(T)$ . In addition, as the deterioration  $\mathbf{D}$  evolves, the deterioration  
475 related function  $w(\tilde{\mathbf{D}})$  magnifies the loss of the strain hardening capability as illustrated  
476 in Fig. 5. In isothermal conditions and in the absence of deterioration,  $r$  reaches a  
477 saturation stress  $R_\infty \exp(-\nu T_0)$  relative to isotropic hardening. The deterioration  
478 conjugate force  $\tilde{\mathbf{G}}$  has contributions from the reversible part  $\psi^e$  and stored energy part  
479  $\psi^p$  of the free energy which correspondingly represent the elastic and stored energy  
480 release rate.  $\tilde{\mathbf{G}}$  in the following is contained in  $\Delta G_{ASB}$  and  $\Delta G_{MV}$  through the scalar  
481 quantity  $G$  obtained by the projection of  $\tilde{\mathbf{G}}$  onto the band plane. In particular, the values  
482 of  $G$  at the ASB and MV onset are considered namely  $G_{ASB}$  and  $G_{MV}$  respectively and the  
483 quantities  $\Delta G_{ASB}$  and  $\Delta G_{MV}$  which are defined as:

$$484 \quad G = \tilde{G}_{ij} N_{ji} ; \Delta G_{ASB} = \langle G - G_{ASB} \rangle ; \Delta G_{MV} = \langle G - G_{MV} \rangle \quad (28)$$

485 It is to be noted that the deterioration conjugate force exists and evolves even before the  
486 deterioration onset. A finite supply of local energy release rate ( $G_{ASB}, G_{MV}$ ) is required to  
487 activate the deterioration (ASB, MV). The deterioration onset criteria and calculation of  
488  $G_{ASB}$  and  $G_{MV}$  are defined in section 2.3.

489 **2.2.3.4 Yield function**

490 Various void induced damage models exist in the literature. Lemaitre [32] modelled  
 491 ductile damage considering non-dilatant effect of void growth. The dilatant void growth  
 492 models of Rousselier [33] and GTN (Gurson-Tvergaard-Needleman) ([34],[35]) are  
 493 widely used damage plasticity models to describe ductile damage. The ASB induced  
 494 degradation, in the present study, was initially described using a thermo/elastic-  
 495 viscoplastic-deterioration (TEVPD) model. Now, in order to describe the post critical  
 496 micro-voiding, rather than using another plasticity-ductile damage coupled model, a  
 497 ‘unified approach’ is adopted wherein the MV induced consequences are merged within  
 498 the TEVPD framework without distorting much its formal structure. The unified  
 499 enlarged model has some analogies with Rousselier model [33] for its dependence on  
 500 hydrostatic stress via an exponential function that notably allows for reproducing void  
 501 growth under shear loading.

502 The yield function  $F_{total}$  when both the ASB and MV mechanisms are in operation reads:

503 
$$F_{total} = \sigma_{eq} + 3\xi \Delta G_{MV}^2 \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) - (R_0 + r) = \sigma_v \geq 0 \quad (29)$$

504 where  $\sigma_{eq}$  is the transformed equivalent stress accounting for the anisotropy induced in  
 505 the plastic flow by evolving ASB and MV,  $(R_0 + r)$  the rate independent yield stress with  
 506  $r$  the strain hardening force,  $\sigma_v$  the strain rate induced overstress,  $\sigma_m = \tau_{kk} / 3$  the mean  
 507 stress and  $\bar{\sigma}$  a reference stress.  $R_0$  is expressed in (30) with  $R_{int}$  being the internal  
 508 stress (thermal softening function related to steel).

509 
$$R_0 = R_{int} \exp(-vT) \exp\left(-\chi_1 \tilde{D}_{kk} - \frac{\chi_2}{2} \tilde{D}_{kl} \tilde{D}_{lk}\right) \quad (30)$$

510 The second term of the expression of  $F_{total}$  containing  $\sigma_m$  corresponds to the expansion of  
 511 micro-voids with  $\xi$  being the related material constant. The  $\Delta G_{MV}$  in this term has been  
 512 modified to  $\Delta G_{MV}^2$  from the previous work ([22],[36],[37]) in order to be consistent with  
 513 the MV incipience while deriving the deterioration rate expressed in (39)<sub>2</sub> below.

514 **2.2.3.5 Equivalent stress**

515 As mentioned above, a transformed equivalent stress  $\sigma_{eq}$  is used here which  
 516 incorporates the regular plasticity and the anisotropy induced in the plastic flow  
 517 provoked by the formation (onset and evolution) of the strongly oriented deterioration  
 518 mechanisms at stake (ASB+MV) through a 4<sup>th</sup> order tensor  $\mathbf{P}$  as shown below:

$$519 \quad \sigma_{eq} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{P} (\Delta G_{ASB}, \Delta G_{MV}) : \mathbf{s} ; \mathbf{P} = \mathbf{I} + 2(\eta_{ASB} \Delta G_{ASB}^2 + \eta_{MV} \Delta G_{MV}^2) \mathbf{M} \otimes \mathbf{M}} \quad (31)$$

520 where  $\mathbf{s}$  is the deviatoric part of the Kirchhoff stress tensor,  $\mathbf{I}$  the 4<sup>th</sup> order identity  
 521 tensor,  $I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ , and  $\eta_{ASB}$  and  $\eta_{MV}$  material constants relative to the  
 522 corresponding mechanisms. A power of 2 is used for  $\Delta G_{ASB}$  and  $\Delta G_{MV}$  terms, see their  
 523 definitions in (28), in the above expression in order to ensure the concomitance of the  
 524 deterioration induced rates while deriving their expressions later in (37) and (39)  
 525 below. The transformed equivalent stress  $\sigma_{eq}$  in (31) may also be expressed of the form:

$$526 \quad \sigma_{eq} \equiv \sqrt{\sigma_{VM}^2 + \sigma_{ASB}^2 + \sigma_{MV}^2} ; \begin{cases} \sigma_{VM} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}} \\ \sigma_{ASB} = \sqrt{3\eta_{ASB} \Delta G_{ASB} \tau_{res}} \\ \sigma_{MV} = \sqrt{3\eta_{MV} \Delta G_{MV} \tau_{res}} \end{cases} \quad (32)$$

527 where  $\tau_{res} = |\mathbf{s} : \mathbf{M}|$  represents the shear stress resolved along the band plane.  $\sigma_{eq}$  in (32)  
 528 therefore has contributions from the regular and singular mechanisms and is composed  
 529 of the regular plastic von Mises equivalent stress  $\sigma_{VM}$  and the singular stresses induced  
 530 by ASB -  $\sigma_{ASB}$  and then by MV-  $\sigma_{MV}$ .

531 One can note that, in the absence of MV ( $\Delta G_{MV} = 0$ ),  $F_{total}$  in (29) reduces to  $F_{PP+ASB}$  i.e.  
 532 yield function in the presence of ASB only. Further in the absence of ASB ( $\Delta G_{ASB} = 0$ ),  
 533 Von Mises yield criterion  $F_{PP}$  is followed. The different possible cases of the yield  
 534 function in the presence or absence of ASB and MV are shown in Table 1.

535 The expressions for the singular contributions to the equivalent stress are modified here  
 536 from the earlier versions in order to take into account the loss of strength of the material  
 537 due to ASB and the subsequent MV in the direction normal to the band during tension  
 538 loading by adding the stress term  $\sigma_N = \langle \boldsymbol{\tau} : \mathbf{N} \rangle = \max(0, \boldsymbol{\tau} : \mathbf{N})$  in the expression of the  
 539 transformed equivalent stress:

540

$$\hat{\sigma}_{eq} \equiv \sqrt{\sigma_{VM}^2 + \hat{\sigma}_{ASB}^2 + \hat{\sigma}_{MV}^2}; \begin{cases} \hat{\sigma}_{ASB} = \sqrt{\eta_{ASB} (3\tau_{res}^2 + \alpha\sigma_N^2)} \Delta G_{ASB} \\ \hat{\sigma}_{MV} = \sqrt{\eta_{MV} (3\tau_{res}^2 + \alpha\sigma_N^2)} \Delta G_{MV} \end{cases}; \quad (33)$$

$$F_{total} = \hat{\sigma}_{eq} (\mathbf{s}, \Delta G_{ASB}, \Delta G_{MV}) + 3\xi \Delta G_{MV}^2 \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) - (R_0 + r) = \sigma_v \geq 0$$

541 **Table 1** Form of yield function in the presence or absence of ASB and MV

Operating Mechanisms	$\Delta G$	$\sigma_{eq}^2$	$F = \sigma_v \geq 0$
<i>PP + ASB + MV</i>	$\Delta G_{ASB} \neq 0$ $\Delta G_{MV} \neq 0$	$\sigma_{VM}^2 + \sigma_{ASB}^2 + \sigma_{MV}^2$	$F_{total} = \sigma_{eq} (\mathbf{s}, \Delta G_{ASB}, \Delta G_{MV}) + 3\xi \Delta G_{MV}^2 \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) - (R_0 + r)$
<i>PP + ASB</i>	$\Delta G_{ASB} \neq 0$ $\Delta G_{MV} = 0$	$\sigma_{VM}^2 + \sigma_{ASB}^2$	$F_{PP+ASB} = \sigma_{eq} (\mathbf{s}, \Delta G_{ASB}, 0) - (R_0 + r)$
<i>PP</i>	$\Delta G_{ASB} = 0$ $\Delta G_{MV} = 0$	$\sigma_{VM}^2$	$F_{PP} = \sigma_{eq} (\mathbf{s}, 0, 0) - (R_0 + r)$

542 The value of the coefficient  $\alpha$  is assigned 3 in the present work. The scalar function  
543 preceding  $\mathbf{M} \otimes \mathbf{M}$  in the original expression of  $\mathbf{P}$  (see equation (36) in [22]) has been  
544 limited here to a single term. In a certain way, the a posteriori addition of  $\sigma_N$  in the  
545 transformed equivalent stress allows for palliating the aforementioned truncation.  
546 During tension loading in the direction normal to the shear band, presumably the entire  
547 load is carried by the shear bands. During normal compression loading, when  $\tau_{res} = 0$   
548 and  $\sigma_N = 0$ , the RVE is supposed to contain no shear bands and the entire load is carried  
549 by the regular matrix material.

550 It is to be remarked that ASB induced deterioration is in operation during the loading  
551 duration when the shear band is still soft. With longer duration or during interrupted  
552 loading, the material within the ASB would possibly undergo phase transformation due  
553 to quenching by the surrounding regular material and become harder. In this case, the  
554 material regains its near original strength except for the irreversible MV deterioration.  
555 This remark is to be taken into account in the prospective work.

### 556 2.2.3.6 Viscoplastic and viscous-damage potential

557 The existence of viscoplastic potential  $\phi_p$  and viscous-deterioration potential  $\phi_{ASB}, \phi_{MV}$   
558 of Norton-Perzyna's type is assumed in the form of power law:



559 
$$\phi_p = \frac{Y}{n+1} \left\langle \frac{H}{Y} \right\rangle^{n+1} ; \phi_{ASB} = \frac{Z}{m+1} \left\langle \frac{H}{Z} \right\rangle^{m+1} ; \phi_{MV} = \frac{W}{q+1} \left\langle \frac{H}{W} \right\rangle^{q+1} \quad (34)$$

560  $Y$  and  $n$  are viscous constants corresponding to the regular plasticity;  $Z, m$  and  $W, q$   
 561 are time-dependence related constants corresponding to the ASB and MV degradation  
 562 mechanisms respectively.

563 **2.2.3.7 Evolution laws**

564 A non-associated plasticity is followed such that the plastic potential  $H = F_{total} (\sigma_N \equiv 0)$   
 565 as expressed below.

566 
$$H = \sigma_{eq} + 3\xi \Delta G_{MV}^2 \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) - (R_0 + r) \quad (35)$$

567 The viscoplastic multiplier  $\Lambda_p$  and viscous-deterioration multipliers  $\Lambda_{ASB}, \Lambda_{MV}$  are  
 568 expressed below.

569 
$$\Lambda_p = \left\langle \frac{\partial \phi_p}{\partial F} \right\rangle = \left\langle \frac{H}{Y} \right\rangle^n ; \Lambda_{ASB} = \left\langle \frac{\partial \phi_d}{\partial F} \right\rangle = \left\langle \frac{H}{Z} \right\rangle^m ; \Lambda_{MV} = \left\langle \frac{\partial \phi_d}{\partial F} \right\rangle = \left\langle \frac{H}{W} \right\rangle^q \quad (36)$$

570 The detailed rate equations derived from the plastic potential according to the normality  
 571 rule are shown below. In addition to the deviatoric component, the MV induced strain  
 572 rate  $\mathbf{d}^{in(MV)}$  is also composed of a hydrostatic component. The MV expansion described  
 573 by the dependence on the mean stress according to  $\exp(\sigma_m / \bar{\sigma})$  was initially proposed by  
 574 Rice and Tracey [38].

575 
$$\left\{ \begin{array}{l} d_{ij}^{in(PP)} = \frac{3}{2} \Lambda_p \frac{s_{ij}}{\sigma_{eq}} \\ d_{ij}^{in(ASB)} = 3\Lambda_p \frac{\eta_{ASB} \Delta G_{ASB}^2 \tau_{res}}{\sigma_{eq}} M_{ij} \\ d_{ij}^{in(MV)} = 3\Lambda_p \left[ \frac{\eta_{MV} \Delta G_{MV}^2 \tau_{res}}{\sigma_{eq}} M_{ij} + \frac{\xi}{3\bar{\sigma}} \Delta G_{MV}^2 \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) I_{ij} \right] \end{array} \right. \quad (37)$$

576 
$$\dot{\kappa} = \Lambda_p \quad (38)$$

577

$$\begin{cases} D_{ij}^{\nabla ASB} = \frac{3\tau_{res}^2}{\sigma_{eq}} \Lambda_{ASB} \eta_{ASB} \Delta G_{ASB} N_{ij} \\ D_{ij}^{\nabla MV} = 3\Lambda_{MV} \left[ \frac{\tau_{res}^2}{\sigma_{eq}} \eta_{MV} \Delta G_{MV} + 2\xi \Delta G_{MV} \exp\left(\frac{\sigma_m}{\bar{\sigma}}\right) \right] N_{ij} \end{cases} \quad (39)$$

578 It is to be noted that in the present study void expansion is modelled irreversibly and the  
 579 void closure which may take place during change of the loading sign is not taken into  
 580 consideration. The inclusion of  $\Delta G_{MV}^2$  in the void expansion term of the yield function  
 581 translates into  $\Delta G_{MV}^2$  and  $\Delta G_{MV}$  in the corresponding terms of  $d_{ij}^{in(MV)}$  and  $D_{ij}^{\nabla MV}$   
 582 respectively. This ensures the systematic concomitance of the deviatoric and hydrostatic  
 583 terms of the rate equation relevant to MV.

584 The components of the deterioration induced spin rates can be deduced from (6) and  
 585 (37) as:

586

$$\begin{aligned} \omega_{ij}^{in(ASB)} &= 3\Lambda_p \frac{\eta_{ASB} \Delta G_{ASB}^2 \tau_{res}}{\sigma_{eq}} T_{ij} \\ \omega_{ij}^{in(MV)} &= 3\Lambda_p \frac{\eta_{MV} \Delta G_{MV}^2 \tau_{res}}{\sigma_{eq}} T_{ij} \end{aligned} \quad (40)$$

### 587 2.2.3.8 Viscous stress

588 The viscous stress in (35) is assumed of the form:

589

$$\sigma_v = Y \dot{\kappa}^{1/n} \quad (41)$$

590 It is reminded that  $Y$  and  $n$  are viscous constants corresponding to the regular plasticity  
 591 and as such do not depend on D.

### 592 2.2.3.9 Temperature rise

593 The regular temperature rate is calculated from the intrinsic plastic dissipation  
 594 expressed in (13) assuming adiabatic conditions as:

595

$$\rho c \dot{T} = \tau_{ij} : d_{ji}^p - r \dot{\kappa} \quad (42)$$

596 As the dissipation progressively concentrates within the band during the ASB process,  
 597 the regular temperature rate accordingly vanishes and the regular temperature  
 598 saturates; further cooling during unloading should be accounted for in prospective  
 599 works.

## 600 2.3 DETERIORATION INCIPIENCE CRITERIA

601 The mechanisms leading to onset of the ASB and MV and the corresponding criteria  
 602 adopted are discussed in the current section.

### 603 2.3.1 Softening mechanisms triggering the ASB formation

604 A criterion for the incipience of the ASB based on the physics of the mechanism is  
 605 essential. In the literature, several arbitrary ASB incipience criteria have been postulated  
 606 for example based on critical strain, see Zhou et al. [39], strain rate, see Bonnet-  
 607 Lebouvier et al. [14], temperature rise, see Teng et al. [15], and energy, see Dolinski et al.  
 608 [40]. Although the above mentioned engineering oriented criteria are easy to  
 609 implement, they strongly depend on the geometry and loading conditions. Hence there is  
 610 a need for a robust physics based criterion consistent with the constitutive equations.  
 611 Such a criterion may be obtained from the linear perturbation analysis, see e.g. Anand et  
 612 al. [41]. Longère et al. developed a criterion as shown in (43) based on simplified  
 613 analysis of material instability accounting for the competition between hardening and  
 614 softening mechanisms using the linear perturbation method. A detailed account of this  
 615 approach can be consulted in the references - Molinari [42], Longère et al. [21], Longère  
 616 and Dragon [43].

$$617 \quad J\left(\tau_{ij}, r, \dot{\kappa}; \frac{\partial r}{\partial \kappa}, \frac{\partial r}{\partial T}\right) = \sqrt{3}\tau - \left(r - \frac{1}{n}Y\dot{\kappa}^{\frac{1}{n}} + \rho_0 c_0 \left(\frac{\partial r}{\partial \kappa} / \left(-\frac{\partial r}{\partial T}\right)\right)\right) \geq 0 \quad (43)$$

618 where  $\partial r / \partial \kappa$  represents the strain hardening and  $\partial r / \partial T$  the thermal softening effects.  
 619 The deterioration process is actually assumed to run as soon as  $J = 0$ . This criterion  
 620 allows for determining  $G_{ASB}$  ( $= \mathbf{G} : \mathbf{N}$  at the instant of ASB incipience).

621 Two softening mechanisms triggering the ASB formation are considered in the present  
 622 study, viz. thermal softening and dynamic recrystallization.

### 623 2.3.1.1 Thermal softening

624 At high strain rates, adiabatic conditions can be assumed due to the limited time for heat  
625 transfer (see [44]). Thermal softening along with quasi-adiabatic condition plays an  
626 important role in provoking the ASB onset. The form of the thermal softening function  
627 can be adapted to different materials as shown in (21) and (22). It is therefore essential  
628 to accurately calculate the regular temperature rise causing the thermal softening. The  
629 plasticity induced regular temperature rise shown in (42) can also be expressed as:

$$630 \quad \rho c \dot{T} = \left( \frac{\sigma_{VM}^2}{\sigma_{eq}} - r \right) \dot{\kappa} \quad (44)$$

631 In most studies in the literature, the percentage of plastic work converted to heat i.e. the  
632 inelastic heat fraction  $\beta$  is assumed to be constant; for example  $\beta = 0.6$  in [45],  $\beta = 0.9$   
633 in [46]. However the experiments by Mason et al. [47] show that  $\beta$  is not a constant and  
634 varies with the strain, strain rate and temperature (see also [48],[49]) as is potentially  
635 the case in the present study. The form of the inelastic heat fraction can be deduced from  
636 (44) as:

$$637 \quad \rho c \dot{T} = \beta \sigma_{VM} \dot{\kappa} \quad ; \quad \beta = \frac{\sigma_{VM}}{\sigma_{eq}} - \frac{r}{\sigma_{VM}} \quad (45)$$

638 As  $r$  depends on strain and temperature and  $\sigma_{VM}$  is a function of strain, strain rate and  
639 temperature,  $\beta$  is expected to depend on strain, strain rate and temperature as well, see  
640 [50]. During the ASB process, the dissipation progressively concentrates within the  
641 bands. Correspondingly, the evolution of the regular temperature outside the bands  
642 diminishes and finally saturates.

### 643 2.3.1.2 Dynamic recrystallization

644 It has been shown that the microstructure also influences the ASB incipience, see eg. Liu  
645 et al. [28]. The micro-mechanisms contributing to the regular material softening and  
646 influencing the ASB onset, which are taken into consideration in the present study, are  
647 the dynamic recovery (DRC) and dynamic recrystallization (DRX). Many studies have  
648 shown that the DRX occurs as a consequence of the adiabatic shear banding  
649 ([51],[52],[53]). Contrarily, for certain alloys such as those of titanium, DRX is shown to  
650 contribute significantly to the softening leading to the very formation of ASB (see  
651 [25],[26]). The latter approach is considered in the present work and the competition  
652 between the DRX softening and thermal softening provoking the ASB initiation is

653 studied. The detailed development of the DRX related model can be found in the article  
 654 by Longère [30].

655 For the initial study on DRX, a critical value of the isotropic hardening variable  $\kappa_c$  is  
 656 imposed to initiate occurrence of DRX. According to Mohamed and Bacroix [54], the  
 657 stored energy at DRX onset should depend on temperature. But in the present study, for  
 658 simplification,  $\kappa_c$  is maintained constant and temperature independent. The DRC and  
 659 DRX induce a negative hardening effect (softening) and it is modelled in the hardening  
 660 term  $h'(\kappa)$  as shown in (46).

$$661 \quad h'(\kappa) = \tau_s(Y(\kappa)) \left[ 1 - \exp\left(-\frac{Y(\kappa)}{2} \kappa\right) \right] \quad (46)$$

662  $\tau_s$  is the DRC and DRX dependent saturation value of hardening stress.  $Y(\kappa)$  is a non-  
 663 dimensional quantity phenomenologically composed of DRC and DRX components  
 664 expressed below.

$$665 \quad Y(\kappa) = Y_{DRC} + Y_{DRX}(\kappa); \quad \tau_s(Y(\kappa)) = \frac{\eta}{Y(\kappa)} \quad (47)$$

666 where

$$667 \quad \begin{cases} Y_{DRC} = Y_0 \\ Y_{DRX} = Y_{\max} \left[ 1 - \exp\left(-\frac{\langle \kappa - \kappa_c \rangle}{\Delta \kappa_r}\right) \right] \end{cases} \quad (48)$$

668  $\eta$  and  $\Delta \kappa_r$  are kinetics related constants.  $Y_{DRC} = Y_0$  is considered a constant.  $Y_{\max}$   
 669 represents the saturation value of the quantity  $Y_{DRX}$ .

### 670 2.3.2 Mechanisms triggering the MV formation

671 The micro-voiding triggered during the loading and after unloading are distinguished  
 672 here.

- 673 • During loading

674 Micro-voids can nucleate during the ductile damage process in several ways. The voids  
675 primarily nucleate around inclusions [55] and also at the grain boundary triple points  
676 [56] in the material. The void nucleation may be triggered by the particle-matrix  
677 interface decohesion, the particle rupture or micro-cracking of the matrix around the  
678 particle [57].

679 • After unloading

680 As mentioned in section 2.1.1.2, the phase transformation due to quenching of the ASB  
681 after unloading provokes the formation of micro-crack at the ASB/matrix interface. The  
682 question arises whether the micro-cracks observed by Peirs et al. [1] within the ASB are  
683 formed during or after the loading.

684 The case treated in the current work is effect of MV during the characteristic loading  
685 time. In absence of further analytical tools or experimental data, the micro-voiding is  
686 tentatively assumed to initiate as soon as the  $\Delta G_{ASB}$  reaches a transition-to-  
687 voiding/critical level  $\Delta G_{ASB}^{crit}$  such that  $G_{MV} = \text{Tr}(\mathbf{GN})_{MV} = G_{ASB} + \Delta G_{ASB}^{crit}$ . The  $\Delta G_{ASB}^{crit}$  is  
688 calculated as  $\Delta G_{ASB}^{crit} = (\Omega - 1) * G_{ASB}$  where  $\Omega$  is tentatively an arbitrary constant greater  
689 than unity. After the MV initiation,  $\Delta G_{ASB}$  is maintained constant at the value of  $\Delta G_{ASB}^{crit}$   
690 and  $\Delta G_{MV}$  evolves taking the predominance as the degradation driving force.

### 691 3 NUMERICAL IMPLEMENTATION AND PERFORMANCE 692 EVALUATION

693 The model in itself does not represent the ASB and MV as entities in the finite elements  
694 but it phenomenologically reproduces their consequences in the elements. For a model  
695 to be considered predictive, it has to satisfy a complete Verification and Validation  
696 (V&V) procedure, (see [20]), which consists in numerically evaluating the performances  
697 of the model on initial boundary value problems (IBVP) with increasing complexity  
698 against corresponding experimental results. In the literature, the models describing ASB  
699 are widely calibrated and evaluated using a single IBVP. Indeed, the high strain rate  
700 loading of a hat shaped structure (HSS) is often used to this end, but this should not be  
701 considered as a sole evaluating configuration as the ASB orientation and path are  
702 entirely controlled by the specimen geometry and the predictability of the model cannot  
703 be sufficiently assessed.

704 In the (V&V) procedure, the model is first numerically implemented at the scale of a  
705 representative volume element (RVE) in order to determine the viscoplasticity and

706 viscous deterioration related constants for a given material from simple experimental  
 707 results such as those of standard tension/compression tests at various temperatures  
 708 and strain rates along with high strain rate torsion or/and shear tests. This is the step of  
 709 material constants **calibration**. The model performance is then evaluated and the  
 710 constants are further adjusted if needed by considering an initial-boundary value  
 711 problem (IBVP) for example using a structure, e.g. HSS [4] or Kalthoff and Winkler (KW)  
 712 impact specimen ([58],[59]), as the ASB path in these structures is known a priori. The  
 713 set of material constants is expected to satisfy the experimental results obtained from  
 714 both the simple tests and the tests on the structures. This is the step of **verification**. The  
 715 final challenging **validation** step is to test the model on a more complex structure where  
 716 the shear band path is a priori unknown, for example, ballistic impact of a thick plate [4],  
 717 high speed machining, etc.

718 In the present work, due to the lack of quantitative experimental results concerning the  
 719 MV, instead of the V&V, the feasibility and robustness of the enriched model is  
 720 demonstrated on the RVE and structural scale. The implementation of the model on the  
 721 complex IBVP namely simulation of high speed machining is a work in progress.

### 722 3.1.1 Numerical issues

723 Dealing numerically with this local softening phenomenon, resulting from a competition  
 724 between material hardening and softening mechanisms and which has severe  
 725 irreversible consequences at the structural level, still remains a challenging task. Indeed,  
 726 the aim is to control the evolution of this material instability all along its progress and  
 727 accordingly overcome the numerical instability the finite element computation code is  
 728 subject to in the softening regime and subsequent meshing dependence of the numerical  
 729 results.

730 Explicit time integration schemes are considered for both the initial-boundary value  
 731 problem and rate equations of the constitutive model. In other words, the finite element  
 732 computation code, namely LS-DYNA, is explicit and the rate equations are integrated  
 733 within a user material subroutine according to an explicit time integration scheme. The  
 734 numerical analyses are carried out using brick finite elements (continuum, 3-D, 8-node,  
 735 reduced integration).

736 The nonlinear constitutive model implemented as 'user material' subroutine is governed  
 737 by a system of differential equations which can be written as (see also [60])

$$738 \quad \dot{F} = A(F, t)F + f(F, t) \quad ; \quad F = F(v, \rho, \tau, r, \tilde{\mathbf{G}}, \tilde{\mathbf{D}}, T) \quad (49)$$

739 where  $A(F,t)$  represents a linear differential operator, and  $f(F,t)$  a nonlinear  
740 function. The forward finite difference method has been used to integrate the above  
741 equations. In the framework of the present study, numerical stability and convergence  
742 are furthermore controlled by an adaptive time sub-increment procedure based on the  
743 maximum strain increment  $\Delta\bar{\epsilon}$  principle (see [61]):

$$744 \quad \bar{\Delta t} = \frac{\Delta\bar{\epsilon}}{\dot{\bar{\epsilon}}} ; \dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} d_{ij} d_{ji}} \quad (50)$$

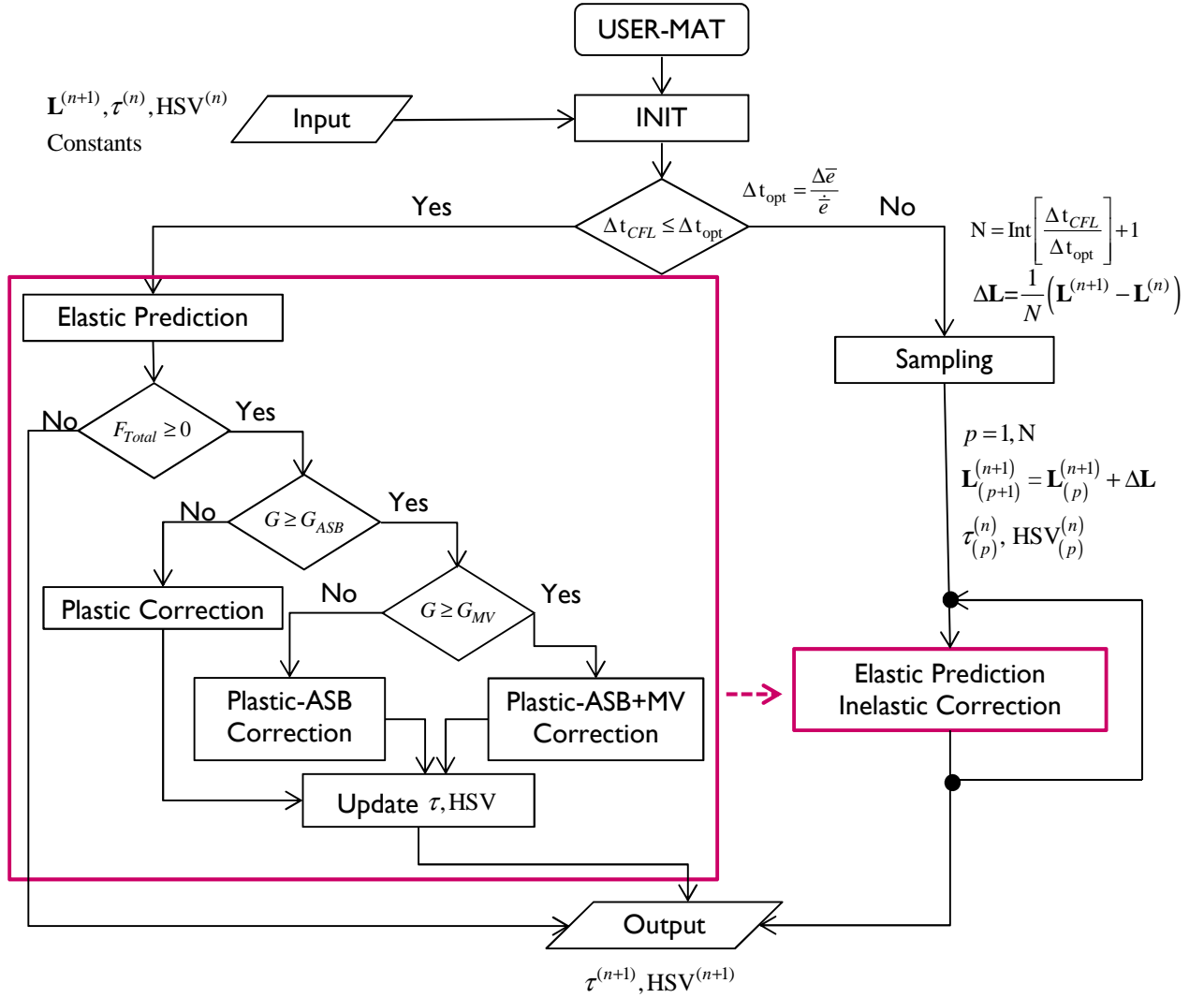
745 where  $\bar{\Delta t}$  represents the adaptive time sub-increment,  $\dot{\bar{\epsilon}}$  a scalar measure of the  
746 current strain rate, and  $\Delta\bar{\epsilon}$  the numerical admissible maximum strain increment.  
747 According to (XX), the value of  $\bar{\Delta t}$  is dependent on the current strain rate in the element.  
748 We are designating  $\Delta t_{CFL}$  the time increment imposed by the explicit computation code  
749 LS-DYNA calculated with respect to Courant-Friedrichs-Lewy [62] condition for wave  
750 propagation. As long as  $\Delta t_{CFL} < \bar{\Delta t}$ , the value of  $\bar{\Delta t}$  is forced to be equal to the value of  
751  $\Delta t_{CFL}$ , which ensures the numerical stability and convergence from the viewpoint of the  
752 numerical integration of the constitutive equations. At high strain rate, the value of  $\bar{\Delta t}$   
753 could become lower than the value of  $\Delta t_{CFL}$ . In this case, and for the element concerned,  
754 the sampling procedure is activated, using an optimised integration time increment such  
755 as  $\bar{\Delta t}_{opt} = \frac{\Delta t_{CFL}}{N}$ ,  $N = Int\left(\frac{\Delta t_{CFL}}{\Delta t}\right) + 1$ . The optimised integration time step is determined  
756 as follows :

$$757 \quad \bar{\Delta t}_{opt} = \frac{\Delta t_{CFL}}{N} \quad \text{with} \quad \begin{cases} N = 1 & \text{if} \quad \Delta t_{CFL} \leq \bar{\Delta t} \\ N = Int\left(\frac{\Delta t_{CFL}}{\Delta t}\right) + 1 & \text{if} \quad \Delta t_{CFL} > \bar{\Delta t} \end{cases} \quad (51)$$

758 This procedure involves a sub loop for the concerned elements only. Considering the  
759 great number of elements, this last point is crucial with regards to the calculation and  
760 (storage and dynamic) memory costs.”

761 The flow chart describing the different steps of the algorithm is depicted in Fig. 9.





762

763

Fig. 9 Flow-chart of the algorithm

764 **3.1.1.1 Regularization and mesh dependence**

765 The viscosity (rate dependency of both plasticity and degradation mechanisms)  
 766 contributes to regularize the boundary value problem in the softening regime, see [63],  
 767 [64]. Yet, as the ASB induces a very strong local softening, additional measures need to  
 768 be applied to overcome the numerical instability. For this purpose, an adaptive time sub-  
 769 increment procedure based on maximum strain increment principle (see [61],[29]) is  
 770 imposed which consists in partitioning the global time increment to obtain a local time  
 771 increment used to integrate the constitutive equations on the concerned elements.

772 Due to the usage of large scale postulate in the model, an a priori knowledge of the shear  
 773 band path and hence mesh refining in the selected regions is not required. A study on  
 774 the influence of the mesh size of the initial model describing the effect of ASB was  
 775 carried out by Longère et al. [29] and [4]. The large scale postulate requires the ASB to

776 be embedded within the finite element which in turn implies that it is not feasible to use  
777 mesh of any given size. It was shown that a mesh size of approximately five times the  
778 shear band ( $\sim 500\mu\text{m}$  for high strength steel) width satisfies the scale postulate. A very  
779 coarse meshing would give rise to a dispersed area of deteriorated finite elements.

#### 780 *3.1.1.2 Band orientation*

781 According to experimental observations, the ASB is oriented in the plane of the  
782 maximum shear stress. This requires the information of the principal stress values. With  
783 the knowledge of the plane of maximum shear stress, the resolved shear stress  $\tau_{res}$  and  
784 normal tension stress  $\sigma_N$  are calculated.

#### 785 *3.1.1.3 Stress triaxiality*

786 The ASBs are formed and propagate under conditions of negative or zero stress  
787 triaxiality. Accordingly, in the user material subroutine, an external condition is imposed  
788 such that the incipience of ASB and the evolution of  $\mathbf{D}^{\nabla ASB}$  take place only under positive  
789 or nil pressure.

790 On the other hand, different arguments have been proposed in the literature regarding  
791 the stress triaxiality conditions for the formation and growth of micro-voids. Bao and  
792 Wierzbicki [65] stated that the micro-voids occur only under positive stress triaxiality  
793 and they transform into micro-shear decohesion under an asymptote of -1/3 negative  
794 stress triaxiality. However, Longère et al. [27] demonstrated that the micro-voids could  
795 initiate and grow around inclusions in the material even under shear pressure loading.  
796 The latter view point is adopted herein and hence in the user material subroutine, the  
797 MV mechanism is allowed to operate both under negative and positive pressure  
798 conditions. The MV expansion is also minimally active in the negative stress triaxiality  
799 condition, thanks to its exponential dependence on the mean stress whose form is  
800 adopted from Rousselier [33] which itself is derived from Rice and Tracey [38].

#### 801 *3.1.1.4 Crack formation*

802 A material failure criterion is required in order to numerically process the element  
803 deletion. A theoretical upper bound for  $\mathcal{D}$  from (2) exists considering melting point of  
804 the material as the maximum temperature limit which is explained in [21]. The crude  
805 theoretical approximation for critical deterioration intensity  $\mathcal{D}_c = \mu / b$  is shown in (19).

806 Numerically implementing the  $\mathcal{D}_c$  as a threshold for element failure is not feasible as the  
807 elements would undergo excessive distortion (even though LS-DYNA is able to deal with  
808 large distortion) at such high values of  $\mathcal{D}$ . Therefore currently an upper limit for  $Tr\mathcal{D}$  is  
809 imposed in the user subroutine as a criterion for element deletion. It is to be noted that  
810 the numerical simulation of the crack propagation still remains a challenge and it could  
811 be dealt with X-FEM in the future, see for example [66], [67], [68], [69].

## 812 3.2 NUMERICAL IMPLEMENTATION ON RVE SCALE

813 The enlarged ASB+MV model is herein numerically implemented on a 3 dimensional  
814 RVE i.e. a single brick finite element of  $H$ -length. The FE is clamped on the bottom and  
815 tangentially loaded on the top with a velocity  $V$  producing simple shearing. The shear  
816 strain  $\Gamma$  used in the following is defined as  $\Gamma = \int \dot{\Gamma} dt$  where  $\dot{\Gamma} = V / H$ . Two cases are  
817 studying in the following corresponding to two typical materials susceptible to ASB: high  
818 strength steel for which the ASB onset is controlled by thermal softening and high  
819 strength titanium alloy for which there is a competition between/combination of  
820 thermal softening and DRX for the ASB initiation.

### 821 3.2.1.1 High strength steel: Thermal softening-controlled ASB onset

822 The material under consideration is high strength 30 Ni-Cr-Mo 6-6 type steel. The  
823 constants pertaining to plasticity and hardening are mostly obtained from [4]. The  
824 constants related to the ASB and MV induced softening are assigned arbitrary values in  
825 the current performance study.

826 **Table 2** Material properties for 30 Ni-Cr-Mo 6-6 type steel

827 Elastic - viscoplastic constants:

$\rho_0$ (kg/m <sup>3</sup> )	$c_0$ (J/kg.K)	$E$ (MPa)	$\nu$	$R_\infty$ (MPa)	$R_{int}$ (MPa)	$Y$ (MPa. s <sup>1/n</sup> )	$n$
7800	420	200e+3	0.33	400	920	60	6

828 Thermal properties:

$k$	$\nu$ (°C <sup>-1</sup> )	$T_0$ (°C)	$\bar{\alpha}$ (K)	$\dot{\epsilon}_{crit}$ (/s)
10	1.1e-3	20	1e-6	100

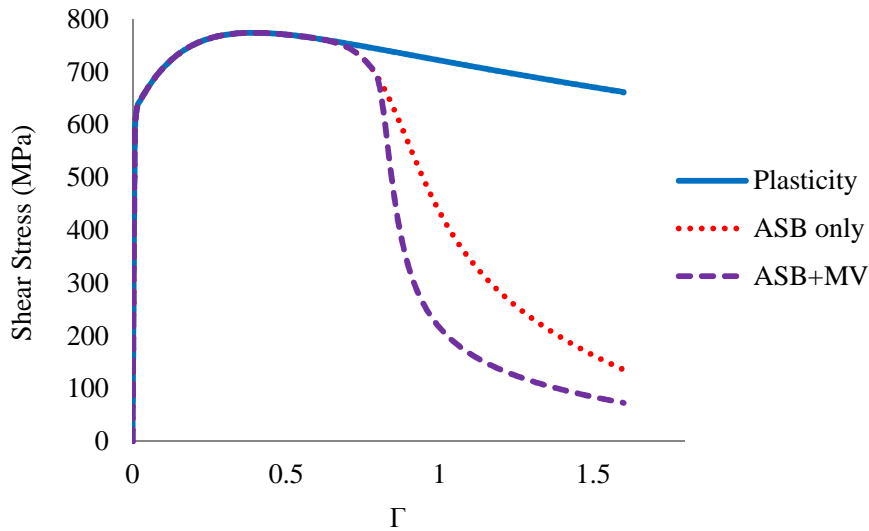
829 ASB related constants:

$Z$ (MPa. s <sup>1/n</sup> )	$m$	$\eta_{ASB}$ (MPa <sup>-2</sup> )	$\chi_1$	$\chi_2$	$a$ (MPa)	$b$ (MPa)
15	2	0.01	0.04	0.04	0	15e+3

830 MV related constants:

$W$ (MPa. s <sup>1/n</sup> )	$q$	$\eta_{MV}$ (MPa <sup>-2</sup> )	$\xi$	$\Omega$
20	2	0.08	1	1.5

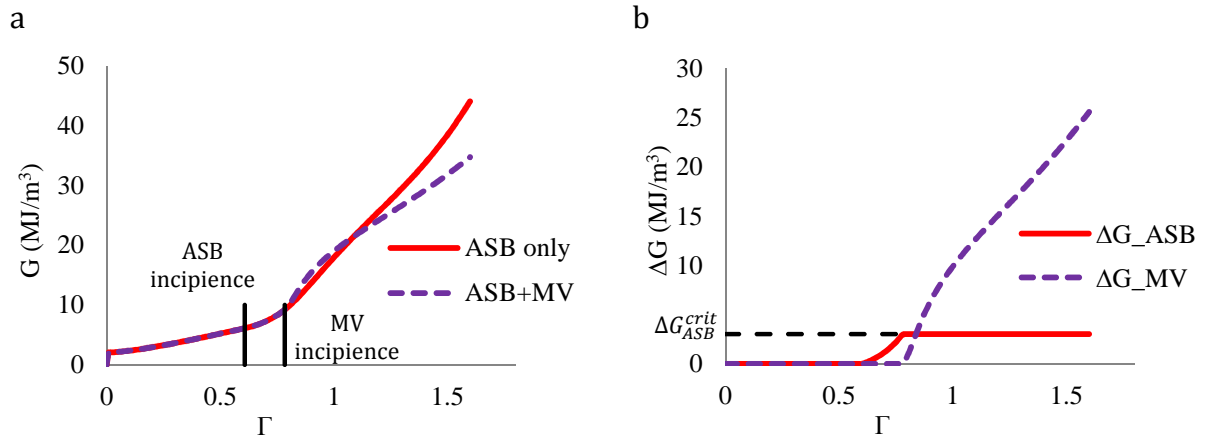
831 The shear stress – shear strain plot for three different cases of material model are shown  
832 in Fig. 10. The response of polycrystalline plasticity material without any localization is  
833 shown by the solid blue line. The thermal softening induced slight and smooth drop in  
834 strength is seen in this case. The model considering ASB induced deterioration is shown  
835 by the red dotted line in which we see a strong drop in strength after ASB onset. The  
836 purple dashed line represents the enriched ASB+MV model in which a relatively more  
837 abrupt drop in strength is observed due to the additional MV contribution. The drop in  
838 stress is attenuated by the viscosity of both ASB and MV. In other words, increasing the  
839 velocity, results in a less brutal drop in stress with a smoother slope. The inertial effects  
840 which are known to play an important role during void growth, see e.g. [70], are not  
841 explicitly accounted for in the present model, but their regularizing consequences are at  
842 least partly reproduced thanks to the viscous feature of the MV process. The constants  
843 pertaining to the MV in Table 2 are chosen in such a way so as to magnify its effect on  
844 the material and kinematics for the sole purpose of visualization.



845 **Fig. 10** Shear stress vs Shear strain plots for models with i) polycrystalline plasticity only, ii) plasticity  
846 +ASB and iii) plasticity+ ASB+MV  
847

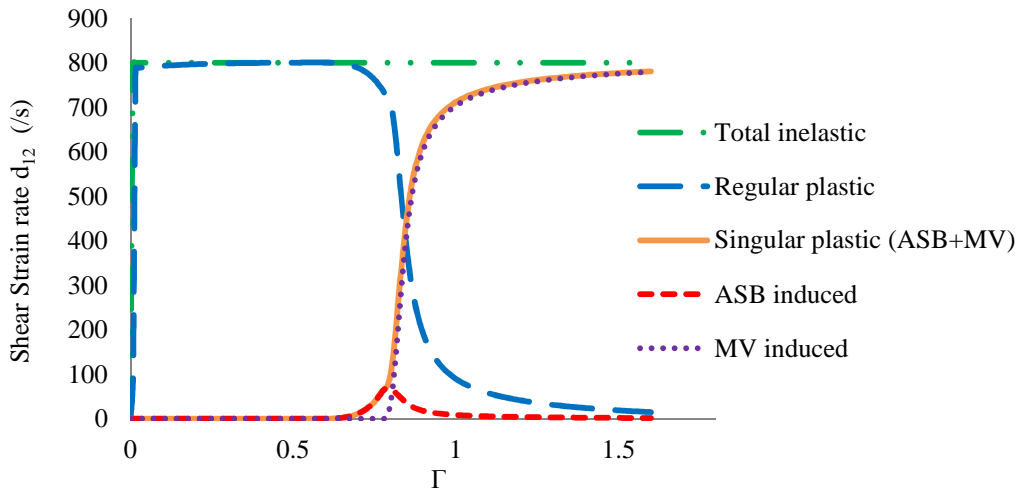
848 The evolution of the deterioration driving force containing quantities  $G = \tilde{\mathbf{G}}:\mathbf{N}$  and  $\Delta G$   
849 are shown in Fig. 11. Comparing  $G$  versus shear strain for ASB and ASB+MV cases, in Fig.  
850 11a, the additional MV induced contribution to  $G$  after its onset in the ASB+MV model  
851 can be clearly seen. For the ASB+MV model, the progression of  $\Delta G_{ASB}$  and  $\Delta G_{MV}$  is  
852 plotted in Fig. 11b. After attaining the critical value  $\Delta G_{ASB}^{crit}$ ,  $\Delta G_{ASB}$  remains constant  
853 while  $\Delta G_{MV}$  initiates and grows. The fact that  $\Delta G_{ASB}$  is non zero when MV operates  
854 allows the ASB-related process to operate until vanishing. In other words, MV becomes  
855 progressively and not suddenly predominant, as shown later.

856



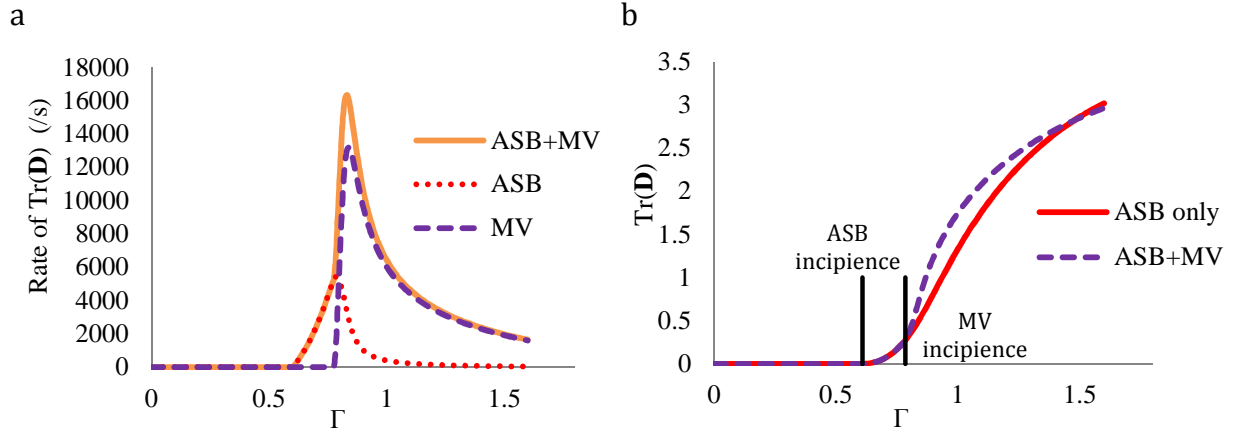
858 **Fig. 11** Deterioration driving force vs. Shear strain (a)  $G:N$  for models with ASB only and ASB+MV ; (b)  $\Delta G$   
 859 for the ASB+MV model showing the two components

860 For the ASB+MV model, the total inelastic strain rate  $d_{12}^{in}$  and its various components vs  
 861 the shear strain are shown in Fig. 12. Initially, the  $d_{12}^{in}$  is composed of the “regular”  
 862 polycrystalline plastic strain rate. At the onset of shear localization regular contribution  
 863 diminishes and the “singular” plastic strain rate becomes progressively predominant.  
 864 The singular part is composed initially of the ASB induced strain rate  $d_{ij}^{in(ASB)}$  and later by  
 865 the MV induced strain rate  $d_{ij}^{in(MV)}$ .



866 **Fig. 12** Inelastic shear strain rate with its components for the ASB+MV model  
 867

868 The evolution of the total deterioration rate  $Tr(\mathbf{D}^{\nabla ASB+MV})$  with its ASB and MV  
 869 components are presented in Fig. 13a. Finally, Fig. 13b shows the trace of the  
 870 deterioration tensor for ASB and ASB+MV models. The significant addition to the  $Tr\mathbf{D}$   
 871 due to MV is clearly seen.



872 **Fig. 13** (a) ASB+MV model: Deterioration rate with its components; (b) TrD for ASB only and ASB+MV  
873 models

### 874 3.2.1.2 High strength titanium alloy: Influence of dynamic recrystallization

875 The DRX mechanism is predominantly found to occur in Ti-6Al-4V as a potential cause of  
876 softening leading to the formation of ASBs [25]. In this section the enriched ASB+MV  
877 constitutive model containing DRX aspects is implemented on the RVE scale i.e. a single  
878 brick finite element for the Ti6Al4V alloy. The material properties concerning  
879 viscoplasticity and the thermal properties are obtained from experimental results as  
880 observed in [30]. The ASB, MV and DRX constants are assigned arbitrary values for the  
881 current parametric study.

882 **Table 3** Material properties for Ti-6Al-4V

883 Elastic - viscoplastic constants:

$\rho_0$ (kg/m <sup>3</sup> )	$c_0$ (J/kg.K)	$E$ (MPa)	$\nu$	$R_\infty$ (MPa)	$R_{int}$ (MPa)	$Y$ (MPa. s <sup>1/n</sup> )	$n$
4500	460	113e+3	0.34	972	1080	400	44

884 Thermal properties:

$T_0$ (°C)	$\bar{\alpha}$ (/K)	$T_{ref}$ (K)	$t$	$\dot{\epsilon}_{crit}$ (/s)
20	1e-6	923	0.6	100

885 ASB related constants:

$Z$ (MPa. s <sup>1/n</sup> )	$m$	$\eta_{ASB}$ (MPa <sup>-2</sup> )	$\chi_1$	$\chi_2$	$a$ (MPa)	$b$ (MPa)
100	2	0.01	0.04	0.04	0	15e+3

886 MV related constants:

$W$ (MPa. s <sup>1/n</sup> )	$q$	$\eta_{MV}$ (MPa <sup>-2</sup> )	$\xi$	$\Omega$
75	2	0.05	1	2

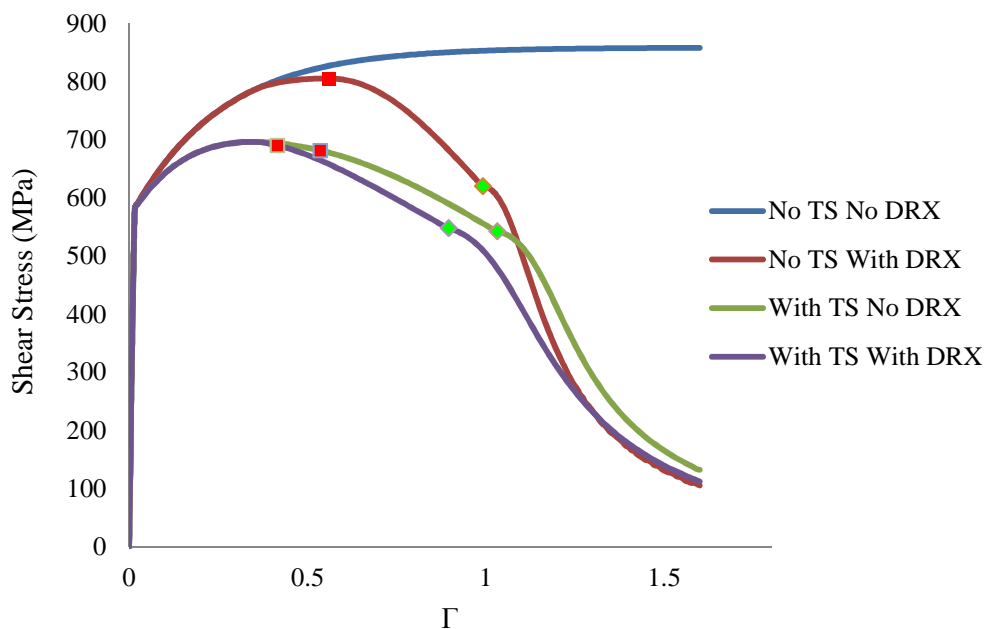
887 DRX related constants:

$Y_0$	$Y_{max}$	$\kappa_c$	$\Delta\kappa_r$	$\eta$ (MPa)
14	14	0.2	0.8	1.36e+4

888 In line of the work by [30] dealing with ASB only, the competitive influence of the two  
889 softening mechanisms provoking the formation of the ASB, namely thermal softening  
890 and DRX induced softening, and their effects on MV are studied here. Four different  
891 cases are numerically studied here :

- 892 (1) no thermal softening and no DRX (No TS & No DRX)
- 893 (2) no thermal softening and with DRX (No TS & With DRX)
- 894 (3) with thermal softening and no DRX (With TS & No DRX)
- 895 (4) with thermal softening and with DRX (With TS & With DRX)

896 The DRX related constants are chosen such that the ASB incipience and consequently the  
897 start of the drop in strength take place almost at the same time for cases (2) and (3). Fig.  
898 14 shows the plot of shear stress vs shear strain for the four different cases. For the first  
899 case, when no softening mechanism comes into play, no material degradation takes  
900 place as expected. The ASB incipience instant is marked by a red spot and MV incipience  
901 by a green one on the curves. It is seen that as calibrated, the ASB incipience takes place  
902 more or less simultaneously for cases (2) and (3) i.e. cases incorporating thermal  
903 softening only or DRX softening only. When both the softening mechanisms act in case  
904 (4), it is seen that the ASB onset takes place much earlier. The MV onset on the other  
905 hand is not directly influenced by the softening mechanisms but it depends on the value  
906 of  $G_{ASB}$  according to the current model.

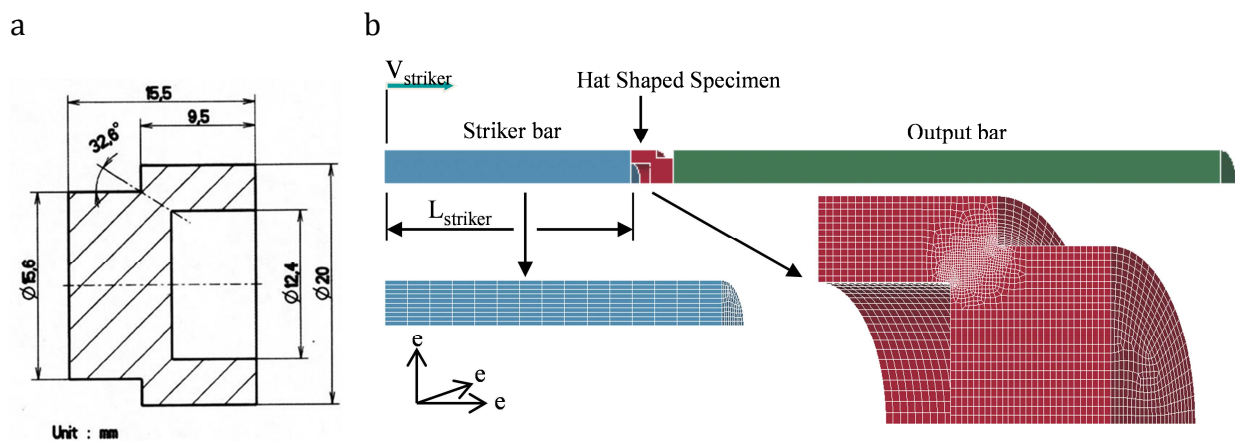


907

908 **Fig. 14** Shear stress vs shear strain for different cases of thermal softening and DRX induced softning.

### 909 3.3 APPLICATION TO INITIAL BOUNDARY VALUE PROBLEMS

910 Following the V&V principle, the feasibility of the enriched ASB+MV model on an IBVP is  
911 demonstrated by simulating the dynamic shearing of a hat shaped structure (HSS). A  
912 Couque type HSS [71] (see Fig. 15a) is used to this end. When the HSS is impacted, the  
913 inclined gauge section permits constant positive pressure and shear loading creating  
914 negative stress triaxiality conditions and thereby making it conducive for ASBs to form.  
915 Thus the ASB trajectory (i.e. gauge section) is known a priori and the HSS is suitable for  
916 verification of the model. Simulations are carried out for impact of HSS in a direct  
917 Hopkinson bar setup made up of tungsten, with a striker bar velocity of 30 m/s; see Fig.  
918 15.



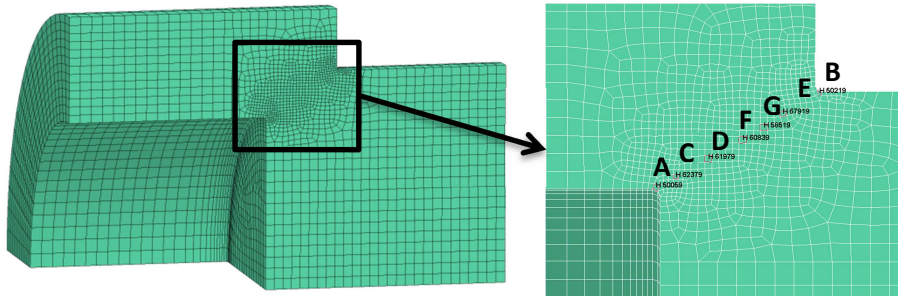
919 Fig. 15 (a) Hat Shaped Structure; (b) Direct Hopkinson bar setup. After Longère et al. [4].

#### 920 3.3.1 Case of High Strength Steel

921 As it is the case of steel, only thermal softening is taken into account as the softening  
922 mechanism leading to the formation of ASB. The ASB and MV related material constants  
923 used here for 30 Ni-Cr-Mo 6-6 steel are shown in Table 2. As in the elementary  
924 responses of the RVE examined in section 3.2, two configurations are studied in the  
925 following: (i) configuration describing ASB effect only, i.e. with MV onset being  
926 deactivated, and (ii) configuration reproducing both ASB and MV effects where ASB-  
927 related constants are the same as those used in (i).

928 Firstly, the resolved shear stress  $\tau_{res}$  is examined as function of time for the elements in  
929 the gauge section, as shown in Fig. 16, of the HSS. The ASB model shows a relatively  
930 mild drop in strength in Fig. 17a after the ASB onset whereas the loss of strength is  
931 accelerated by the presence of MV in Fig. 17 for the ASB+MV model.

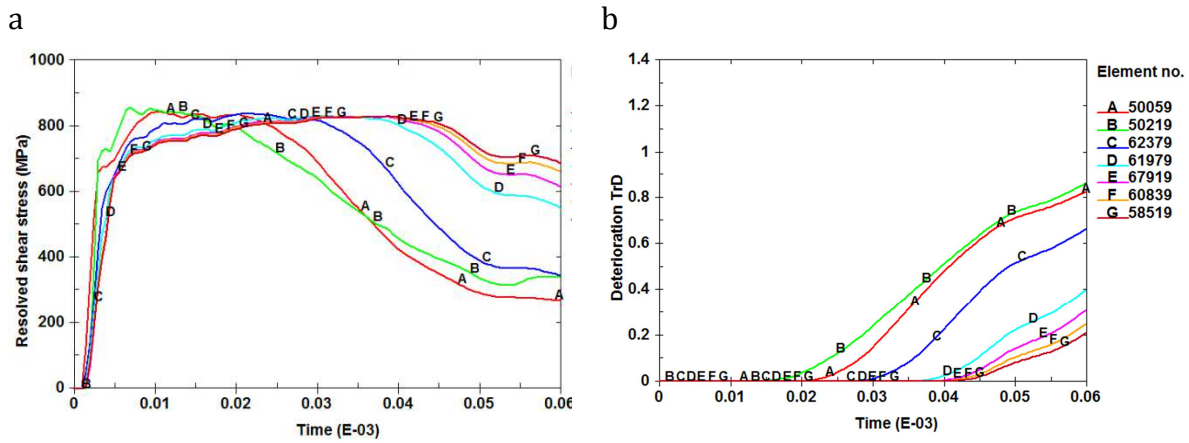




932

933

Fig. 16 Finite elements in the gauge section selected for plots.

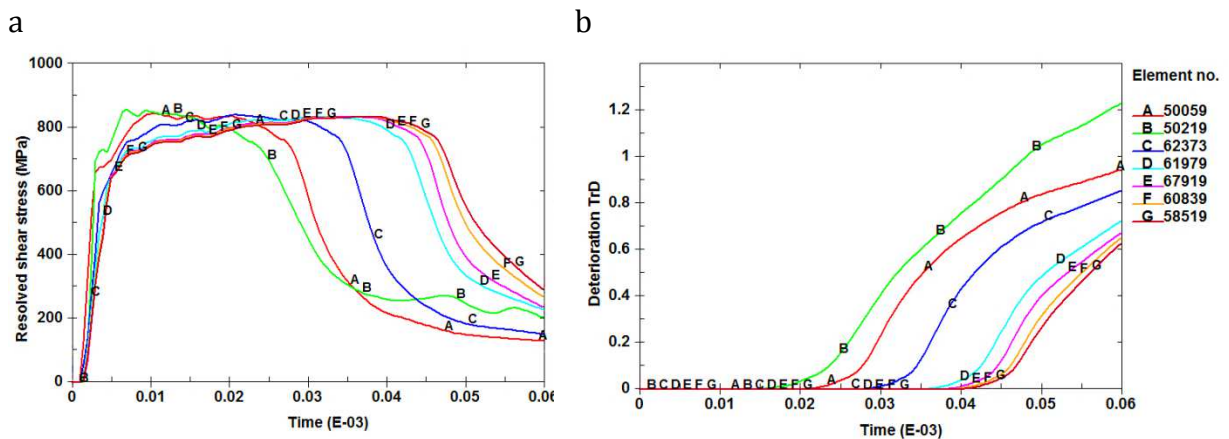


934  
935

Fig. 17 Numerical simulation of the dynamic loading of hat shaped structure. Model: ASB only. a) Resolved shear stress history. b) Deterioration variable history. See Fig. 16. for the location of the elements.

936  
937  
938

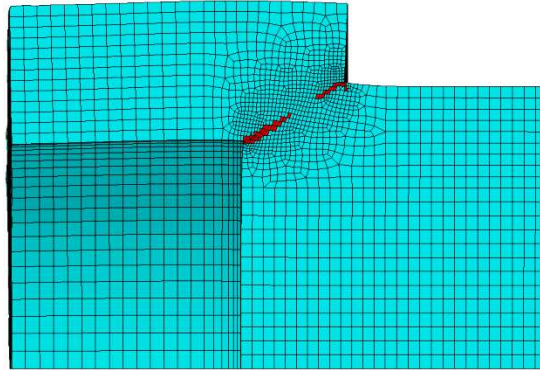
The evolution of the deterioration variable  $TrD$  for ASB and ASB+MV is plotted in Fig. 18,18b. As expected, the deterioration intensity is significantly higher for the ASB+MV case when compared with the ASB only model.



939  
940  
941

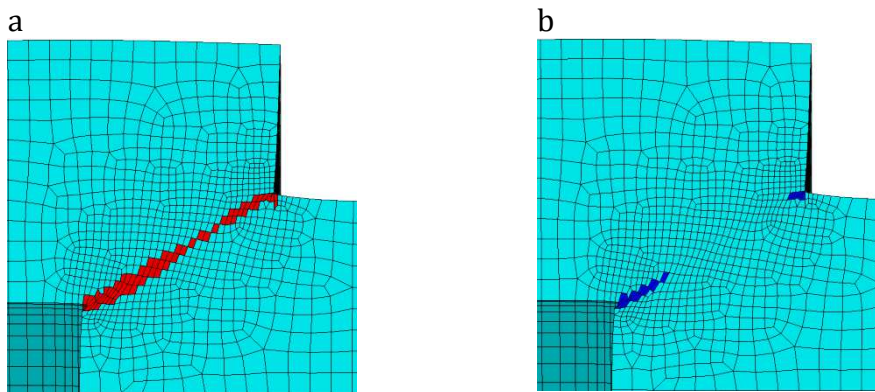
Fig. 18 Numerical simulation of the dynamic loading of hat shaped structure. Model: ASB + MV. a) Resolved shear stress history. b) Deterioration variable history. See Fig. 16. for the location of the elements.

942 The numerical deterioration maps shown in the following give an indication of the  
943 presence of ASB/MV in the finite elements. In Fig. 19, for the model considering ASB  
944 only, the elements in red contain ASB. Likewise for the ASB+MV model in Fig. 20, the  
945 elements with ASB are highlighted in red (Fig. 20a) and the ones containing MV in blue  
946 (Fig. 20b).



947

948 **Fig. 19** Model with ASB only: Deterioration indicator at  $39\mu\text{s}$ . Elements in red are subjected to ASB.



949 **Fig. 20** Model with ASB+MV: Deterioration indicator at  $39\mu\text{s}$ . a) Elements in red are subjected to ASB. b)  
950 Elements in blue are subjected to MV.

951 These numerical results bring two comments. Firstly, at the same time, considering ASB  
952 only leads to a partial propagation of ASB in the gauge section (Fig. 19), whereas  
953 considering both ASB+MV leads to a complete propagation of ASB (Fig. 20). MV in ASB  
954 wake thus contributes in propagating the ASB farther. Secondly, according to Fig. 20 and  
955 assuming that the loading ends at the time considered, although ASB has propagated  
956 throughout the whole structure, MV zones cover very limited areas, viz. near the  
957 specimen corners. . The material in the ligament between the MV zones, which has been  
958 subject to ASB, may recover its initial properties after unloading due to phase  
959 transformation. In other words, the remaining effect of ASB+MV may reduce to MV alone  
960 with only local parts of the structure affected. In another scenario, the quenching of the  
961 material of ASB after unloading and cooling may cause micro-cracking, leading to a  
962 complete fracture of the structure.

### 963 3.3.2 Case of Ti6Al4V

964 As mentioned in Section 2.3.1.2, the works of Rittel and co-workers [25] suggest that, for  
965 materials such as high strength Ti alloys, the phenomenon of dynamic recrystallization  
966 (DRX) potentially needs to be taken into account in ASB initiation. Unlike the common  
967 assumption that DRX is a consequence of ASB, they argue that the DRX not only precedes  
968 the ASB but is also a dominant microstructural factor in the very generation of the band.  
969 Osovski et al. [45] concluded that the respective influence of thermal and DRX induced  
970 softening depends on the chronology of the process and the material considered: in case  
971 of Ti6Al4V, DRX develops earlier and overrides the thermal softening effects; on the  
972 other extreme case of pure titanium which does not undergo DRX, the thermal softening  
973 plays a dominant role in the ASB initiation. In the line of their work, the competition  
974 occurring between thermal and DRX induced softening is analyzed here on a structural  
975 scale. The materials constants of Ti6Al4V are shown in Table 3.

976 In order to study the potential effect of DRX on ASB formation and evolution, two cases  
977 are examined in the simulation of HSS:

978 (1) no thermal softening and with DRX (No TS & With DRX)

979 (2) with thermal softening and with DRX (With TS & With DRX)

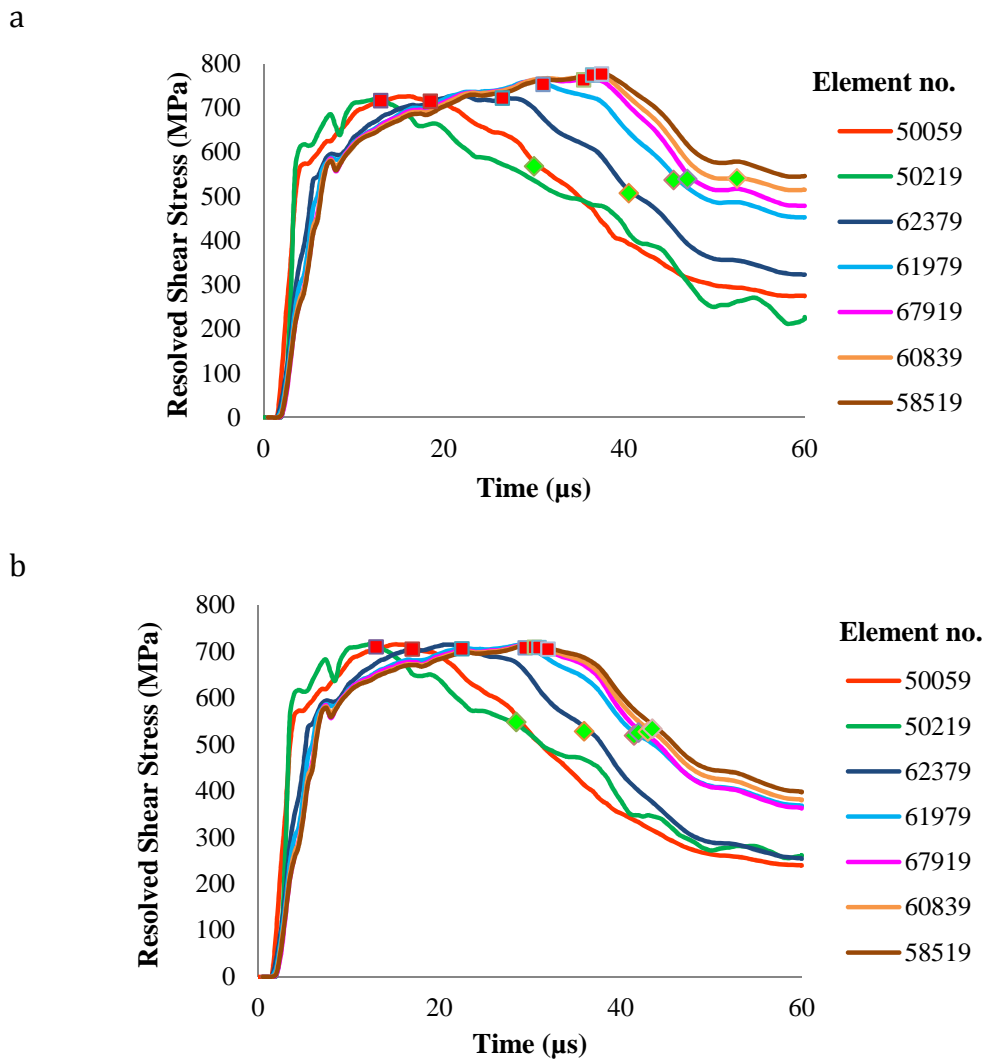
980 These configurations have to be considered together with those studied in Section  
981 3.2.1.2.

982 The resolved shear stresses for both cases for the selected elements in the gauge section  
983 (as shown in Fig. 16) of the HSS are plotted in Fig. 21a,b. In the graphs, the red marks  
984 indicate the ASB onset and the green ones MV onset. While examining the two cases for  
985 the instant of ASB onset, it is clearly seen that the case with combined DRX and thermal  
986 softening (Fig. 21b) shows relatively early onset of ASB when compared to the case of  
987 DRX induced softening only (Fig. 21a). Consequently the MV onset also takes place  
988 earlier for the former case than the latter. The results seen here is similar to the ones  
989 observed in the corresponding cases for the single FE in Chapter 2, Section 2.5.2.2.

990 The deterioration maps showing the elements with ASB and MV activated respectively at  
991 a given instant (viz. 45 $\mu$ s) for the above two cases are shown in Fig. 22. In the case  
992 considering DRX induced softening solely (Fig. 22a), fewer elements show ASB and MV  
993 at the given instant. The combined influence of thermal and DRX induced softening  
994 seems to have accelerated the ASB and MV onset which is seen in the deterioration map  
995 (Fig. 22b) in which relatively more elements are seen with the presence of ASB and MV.

996 As a consequence, neglecting the thermal softening and considering only DRX induced  
997 softening for materials such as Ti6Al4V as proposed by Osovski et al. [45], would lead to  
998 delay of ASB initiation and underestimation of the ASB propagation. However more

999 experimental evidence is needed to characterize each material in terms of the respective  
 1000 influence of thermal and DRX induced softening on the shear localization onset.



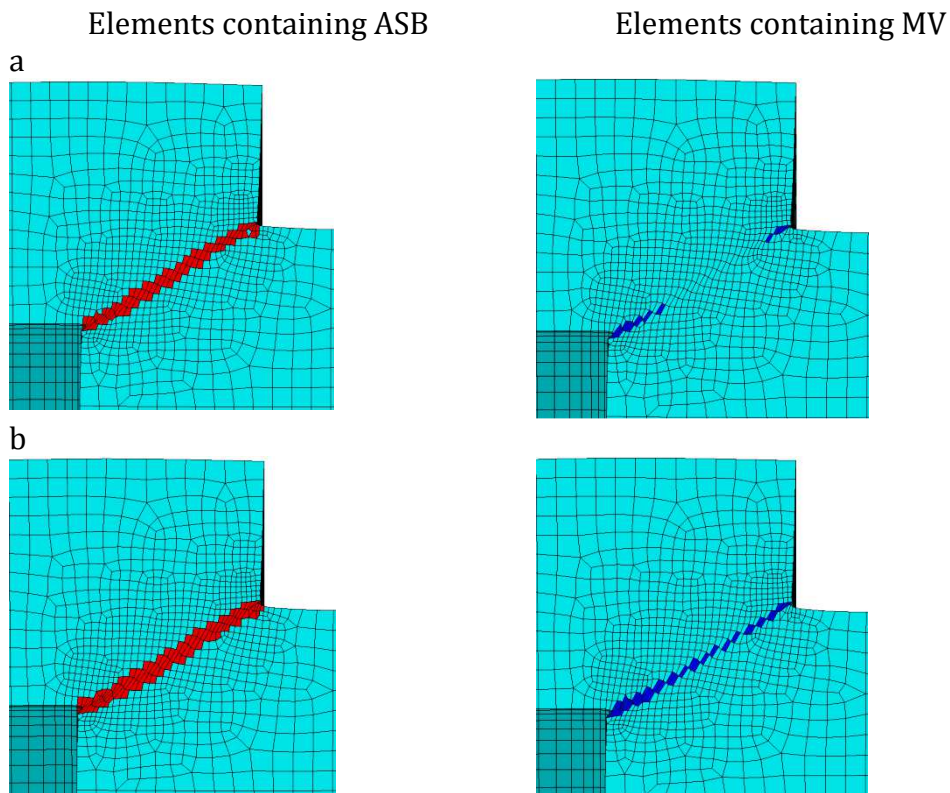
1001 **Fig. 21** Model ASB+MV: Deterioration variable history of elements in the gauge section, see **Fig. 16.** a)  
 1002 Case of no thermal softening and with DRX b) Case considering thermal softening and DRX.

### 1003 3.3.3 Experimental vs. numerical results confrontation

1004 In order to assess the prediction ability of the model to describe ASB and further micro-  
 1005 voiding, numerical results are compared with experimental ones. Material constants are  
 1006 reported in Table 4. Two configurations extracted from [4] are considered in the present  
 1007 work:

- 1008 • moderate impact configuration with  $L_{striker}=90\text{mm}$  and  $V_{striker}=23.7\text{m/s}$ , see Fig.  
 1009 23, leading to incomplete ASB and MV propagation, see macrograph,
- 1010 • severe impact configuration with  $L_{striker}=90\text{mm}$  and  $V_{striker}=23.7\text{m/s}$ , see Fig. 24,  
 1011 leading to complete ASB and MV propagation, see macrograph.

1012 In the moderate impact configuration, see Fig. 23 left, ASBs are shown to form from the  
 1013 specimen corners and propagate partially. A small crack has formed in the ASBs wake. In  
 1014 Fig. 23 right related to the numerical simulation, the same trend is observed where the  
 1015 damaged area length is slightly smaller than the ASB length. In the severe impact  
 1016 configuration, see Fig. 24 left, ASBs initiated from the corners join together to form a  
 1017 single band as do the cracks, leading to the complete fracture of the specimen. The same  
 1018 trend is observed once again in the numerical simulations. The comparison of the  
 1019 experimental macrograph, on the left, with the ASB and MV indicator maps, on the right,  
 1020 and of the history of the experimental and numerical axial stress on the left allows for  
 1021 concluding to a fair agreement.



1022 **Fig. 22** Model with ASB+MV: Deterioration indicator at 45 $\mu$ s. Elements in red are subjected to ASB.  
 1023 Elements in blue are subjected to MV. a) Case of no thermal softening and with DRX b) Case considering  
 1024 thermal softening and DRX.

### 1025 3.3.4 Discussion/assessment

1026 The realistic description of initiation and propagation of ASB and MV and their  
 1027 consequences on the structural material in a wide range of situations implies modelling  
 1028 as close as possible to the complex physics of the mechanisms while being tractable and  
 1029 applicable to initial boundary value problems. The enriched ASB+MV three-dimensional  
 1030 constitutive model is herein demonstrated to be feasible on a structural scale and shows  
 1031 the desired qualitative performance on the IBVP.

1032 **Table 4** Material properties for 30 Ni-Cr-Mo 6-6 steel

1033 Elastic - viscoplastic constants:

$\rho_0$ (kg/m <sup>3</sup> )	$c_0$ (J/kg.K)	$E$ (MPa)	$\nu$	$R_\infty$ (MPa)	$R_{int}$ (MPa)	$Y$ (MPa. s <sup>1/n</sup> )	$n$
7800	420	200e+3	0.33	400	920	60	6

1034 Thermal properties:

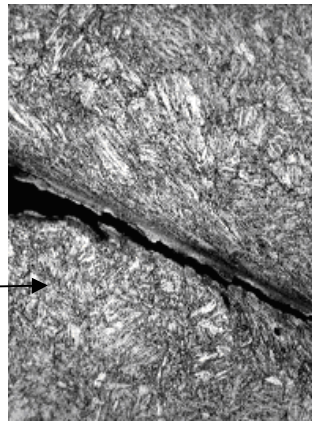
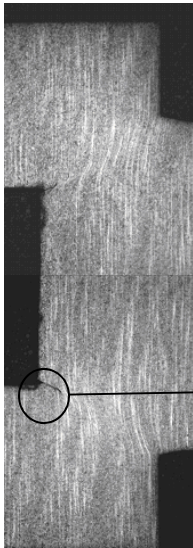
$k$	$\nu$ (°C <sup>-1</sup> )	$T_0$ (°C)	$\bar{\alpha}$ (/K)	$\dot{\epsilon}_{crit}$ (/s)
10	1.1e-3	20	1e-6	100

1035 ASB related constants:

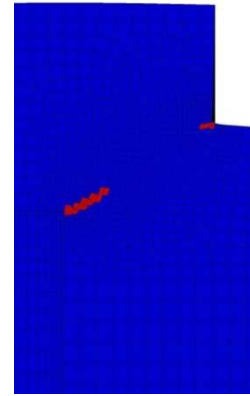
$Z$ (MPa. s <sup>1/n</sup> )	$m$	$\eta_{ASB}$ (MPa <sup>-2</sup> )	$\chi_1$	$\chi_2$	$a$ (MPa)	$b$ (MPa)
19	2	0.12	0.05	0.05	0	15e+3

1036 MV related constants:

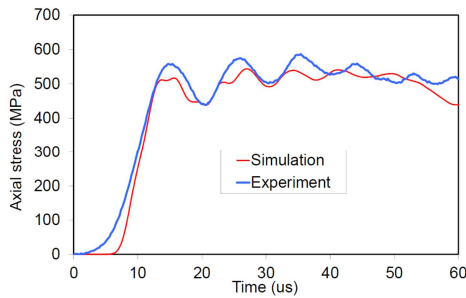
$W$ (MPa. s <sup>1/n</sup> )	$q$	$\eta_{MV}$ (MPa <sup>-2</sup> )	$\xi$	$\Omega$
20	2	0.08	1	1.5



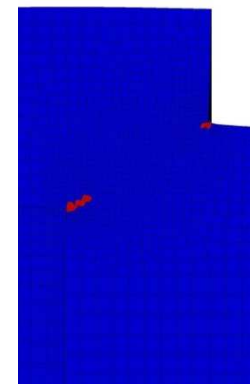
Macrograph



ASB indicator map



Axial stress transmitted into the output bar



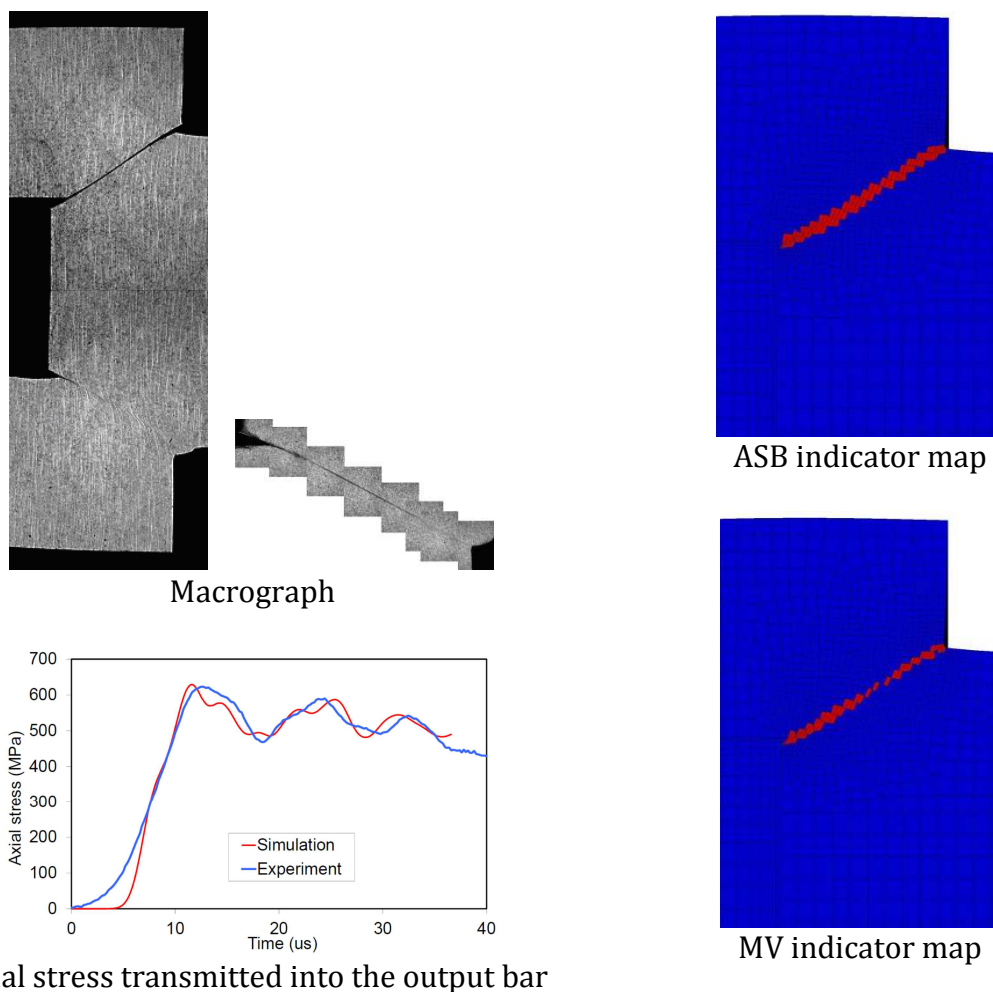
MV indicator map

1037 **Fig. 23** Configuration with  $L_{striker}=90\text{mm}$  ;  $V_{striker}=23.7\text{m/s}$ . Incomplete ASB and MV propagation. left: Post-  
 1038 mortem macrograph and Axial experimental and numerical stress transmitted into the output bar ; right:  
 1039 Numerical deterioration map (100  $\mu\text{s}$ ).

1040 • ASB only vs. ASB+MV

1041 While comparing the two models exhibiting ASB only and ASB+MV when implemented  
 1042 on a structural scale, the following conclusion can be drawn. The model with ASB and  
 1043 MV activated exhibits increased deterioration intensity and therefore accelerates the  
 1044 drop in strength of the material which is the expected behaviour. Also, while comparing  
 1045 with the ASB only model, the presence of MV advances the ASB propagation.

1046 Not all elements containing ASB show the presence of MV at a given instant. This would  
 1047 imply that, upon loading at that given instant, if the material within the ASB recovers its  
 1048 initial properties due to phase transformation, the only deteriorated region would be the  
 1049 elements containing MV. It is thus essential to distinguish the two mechanisms of ASB  
 1050 and MV.



Axial stress transmitted into the output bar

1051 **Fig. 24** : Configuration with  $L_{striker}=40\text{mm}$  ;  $V_{striker}=35.6\text{m/s}$ . Complete ASB and MV propagation. left:  
 1052 Post-mortem macrograph and Axial experimental and numerical stress transmitted into the output bar ;  
 1053 right: Numerical deterioration map ( $40\ \mu\text{s}$ )

1054

- 1055 • DRX vs. DRX+TS

1056 In case of materials susceptible to DRX such as high strength Ti alloys, in addition to  
1057 thermal softening, the DRX induced softening plays a role on the ASB onset. The  
1058 influence of the two softening mechanisms has been studied here concerning two cases  
1059 namely DRX only and DRX+TS. It is seen that the combined DRX+TS leads to a much  
1060 earlier onset of ASB and consequently MV when compared to the case with DRX induced  
1061 softening only. It is therefore important to consider both the softening mechanisms in  
1062 order to be conservative in the engineering approach.

## 1063 4 CONCLUSION

1064 A physics motivated phenomenological model describing ASB and MV effects developed  
1065 in a large scale postulate has been applied herein in accordance with the physics of the  
1066 shear localization and subsequent ductile damage process. The two main effects of  
1067 ASB+MV induced deterioration reproduced by the unified model are: material  
1068 consequence, viz. anisotropic deterioration of the material elastic and plastic properties;  
1069 and kinematic consequence, viz. progressive deviation of the plastic flow in the band  
1070 plane and then according to the micro-void orientation. The present enlarged ASB+MV  
1071 model is numerically implemented as user material in the engineering finite element  
1072 computation code LS-DYNA. The model has been successfully demonstrated to be  
1073 numerically feasible at the representative volume element scale and also on a structure  
1074 namely the hat shaped structure (HSS). The ASB+MV model distinguishes the ASB and  
1075 MV induced effects in the late pre-failure stage and the supplementary MV contribution  
1076 to the deterioration could be observed.

1077 The qualitative analysis of the ASB+MV model is done by comparing with ASB only  
1078 model in which the MV is deactivated while maintaining the ASB related constants. On  
1079 the scale of the RVE, the ASB+MV model demonstrates the desired behaviour by  
1080 showing supplementary MV induced material deterioration and kinematics. On the HSS,  
1081 it is observed that the presence of MV accelerates the propagation of ASB which has not  
1082 been shown in literature and thus constitutes a novel result. Also, at a given instant, not  
1083 all elements containing the ASB reach the criteria for MV onset; only the corner elements  
1084 show the presence of MV. One could infer the following possible scenarios from the  
1085 given state of elements on unloading: the material within the ASB might recover its  
1086 initial properties after being quenched by the surrounding material and thus phase  
1087 transformed leaving only the effect of MV in the respective elements in the corners; or  
1088 the cooling on unloading might cause micro-cracking within the ASB leading to complete  
1089 fracture. The respective influence of thermal and dynamic recrystallization (DRX)  
1090 induced softening is also studied on the RVE and structural scale. In both the cases it is  
1091 shown that the combined contribution of thermal and DRX softening brings about an  
1092 earlier ASB onset as expected. Considering only one of the softening mechanisms would



1093 delay the ASB onset and underestimate the ASB propagation, which is in contradiction  
1094 with some assumptions in literature considering that DRX is the unique mechanism  
1095 controlling the ASB onset particularly in Ti-6Al-4V titanium alloy.

1096 The challenge still remains in terms of identifying the instant of MV incipience from  
1097 experiments and hence to distinguish ASB and MV related material constants. Also, in  
1098 order to account for the different orientation of the MV with respect to ASB,  
1099 experimental observations are required.

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