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A DSS approach for heterogeneous parallel machines scheduling with due windows, processor-&-sequence-dependent setup and proximate supply chain constraints

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This paper describes the basis of a Decision Support System (DSS) designed to support scheduling for a fertilizer production plant made up of several parallel lines of heterogeneous characteristics, with setup time depending on sequence and lines, and taking into account non-availability constraints; it also considers the primary impacts of schedule upstream and downstream of the supply chain which integrates the plant. Our DSS uses an original MILP scheduling formulation whose implementation, through an Algebraic Modeling Languages (AML), facilitates interaction with the scheduler to formulate an instantiated problem whose optimal solution is acceptable.

Keywords: Decision Support System, Optimization, Scheduling, Heterogeneous Parallel Processors, Sequence Dependent Setup, Non-Availability Constraints. Supply chain

I. Introduction

OCP SA (<http://www.ocpgroup.ma/fr>) accounts for one third of world phosphate exports across product segments. The Jorf fertilizer production plant, located near the sea at the end of the Northern axis of OCP's supply chain (SC), is the Moroccan group's largest plant. It produces some twenty one fertilizer references at 7 parallel lines (3 "107" identical lines and 4 "07" identical lines); 7 out of 21 references may only be produced at a single type of line, which yields 35 routings. Fertilizer demand is seasonal and produced to order, with shipping performed by boat at the harbour located near the fertilizer plant. The subject of our paper is the issue of scheduling a set of orders for the production lines starting from a date, without

interfering with orders in process at that date. Each order is characterized by a fertilizer reference to be manufactured, a tonnage and a time window during which production must be completed. The schedule must factor in production stoppage required by preventive maintenance operations, and such stoppage has no impact on ongoing production time. Order fulfillment time varies from line to line, in particular due to their different technical features and due to production rate modulation options. The schedule must also factor in substantial cleaning operation time between two batches of production of different quality on the same line. Such line preparation lead time depends on the line itself and on the previously produced reference.

The proposed schedule has immediate upstream and downstream consequences that may make it unfeasible due to pump priming issues (upstream) or finished product inventory saturation (downstream):

- Fertilizer manufacturing requires, among other inputs, sulfuric acid and phosphoric acid, the latter using sulfuric acid as a component. Jorf's site SC comprises production units for both acids. Moreover, the phosphoric acid that it produces is also exported and shipped and both acids may be used by the Joint Ventures (JV) also operating at Jorf. An analysis of the fertilizer BOM's reveals a strong dispersion on the consumption rate for these inputs (for example, production of one tonne of NPK 14-23-14 1B fertilizer requires nearly 7 times more pure phosphoric acid than one tonne of MAP Std (11-52) fertilizer). This means that the adopted scheduling has a strong impact on consumption rate for these acids and that the schedule is only workable if it does not deplete inventory for these raw materials.
- The manufactured fertilizers are carried by conveyer belts out of the six storage areas. The storage areas are dedicated to the fertilizer references belonging to a particular family. Area capacity varies depending on the number of stored references, as any mixing of fertilizers is highly hazardous. The allocation of such areas to product families changes over time and

several areas may be dedicated to a single family. Stored fertilizers are moved on conveyer belts to be loaded on the boats to fulfill orders, most of which concern a single reference. Loading takes a few days as per a schedule (that may be altered due to weather conditions). Residual storage capacity for a fertilizer reference therefore, varies over time. The chosen schedule determines fertilizer inventory build up and will not be workable if inventory build up is not matched by storage capacity.

Production stoppage risk due to pump priming issues (upstream) and/or saturated storage (downstream) serves to place the scheduling problem in the wider framework of SC management for the relevant fertilizer plant.

The coherence of operational and tactical decisions taken by the different entities of a SC is extremely difficult to guarantee, as a result of which substantial loss of efficiency and effectiveness may be sustained. The reasons are easy enough to explain. One is substantial decision-making autonomy, supported by local performance assessment systems that hampers multiple entity collaboration and promotes a local management focus. Another is the fact that centralized decision-making to eliminate causes of poor performances due to organizational factors is practically impossible to implement in a SC due to modelling complexity and calculation complexity. SC management generally involves a mix of local decisions and regulatory decisions taken at a higher level, upon detection (often too late) of the fact that the supply chain is no longer under control. Where the variety of products moving along the SC is not too great, one may restrict regulatory decisions through judicious recourse to buffer inventory. This solution, while enhancing decisional decoupling though it does not guarantee it, involves a loss of efficiency. An alternative consists in setting up decisional patterns that sharply reduce local decision-making, through upper-level regulation based on constraint trade-offs between the different links of the SC and a posteriori regulation for remedial measures to substantial disruptions. In this context, the Decision Support Systems (DSS) appear a useful

approach to improve the relevance of decisions taken in a SC. This is precisely the option explored in this paper with a restricted scope for a fertilizer plant scheduling exercise that is limited to the direct upstream (to avoid pump priming issues) and downstream constraints (to avoid saturation).

The bases for DSS were laid by Gorry and Scott Morton (1971) and transformed into a proper system by Keen and Scott Morton (1978). IS/IT development of course immensely increased the potential of DSS (Shim *et al.*, 2002; Eomi & Kim, 2006; Bhargava *et al.* 2007...) without altering its rationale. Basically DSSs comprise: 1) an interface to formally express a complex issue that is partially structured to define a structured problem; 2) one or multiple modules to solve the structured problem, usually based on optimization or simulation models (Power and Sharda, 2007); 3) an interface capable of exploring all the consequences of a solution obtained and of either adjusting it marginally or of validating it; 4) if none of these solutions is acceptable, the DSS then is used to define a new structured problem using the feedback from prior formulations that failed to deliver a satisfactory solution.

The DSS concept used to solve the scheduling issue of the Jorf fertilizer plant (Azzamouri *et al.*, 2016) is set forth under section IV. The structured problem that lies at the heart of module 2 of the DSS corresponds to the problem of a locally defined schedule that i) does not factor in the upstream or downstream constraints (model 1) or ii) factors these constraints in (model 2), although the operational value of the latter formulation is limited for numerical reasons. An analysis of literature (section II) reveals that none of the proposed formulations reflects the full complexity of the problem at hand. Both models are described under section III and their use is illustrated by an example. After this presentation of the DSSS (section IV), we conclude our paper with a few comments on its implementation

II. Literature review

The fact that the fertilizer production process is a continuous one does not mean that the scheduling problem is not of the discrete type since order production time at a given line is predetermined by the line itself and by the previously produced reference (which may require a preparation lead time prior to launching production). Note that additionally the different stages of a continuous manufacturing process are not relevant for scheduling purposes as these are implicitly defined at “line-end”. Each line may be viewed as a standalone processor and the processors are heterogenous since the fulfillment time of orders is not the same for all processors. The scheduling problem consists in allocating orders to processors and in sequencing the different orders allocated to each line, while taken into account all the above listed considerations.

We therefore looked at the literature dealing with the scheduling of orders for heterogenous processors. We have analyzed 40 recent articles published in the best journals, by using an analysis grid organized along five axes. This analysis is summarized in table 1 whose last line shows the characteristics retained simultaneously in our approach.

- ***Characteristics of processors.*** The production system is made of parallel processors. They can be *identical*, if any job can be processed by any machine with the same production time, or *heterogeneous*, in the opposite case. The *availability* of those processors may be permanent or not (preventive maintenance...); at the beginning of the schedule, all processors may be available or not (orders in progress).
- ***Characteristics of jobs.*** Jobs can be all *available* to be launched when schedule starts, or not (progressive arrivals). In both cases, the characteristics of all jobs are known. *Due dates constraints* may have to be met (through upper bounds or time windows) or not considered. *Preemption* may be authorized or not. In some papers, automatic splitting of the ordered quantity is integrated. The consequences of the solution, upstream and

downstream of the supply chain, may imply the introduction of additional constraints in the problem definition

- ***Characteristics linked simultaneously with jobs and processors.*** Heterogeneity implies differences of production times and the impossibility, for some processors, to treat some productions. Setup times, where applicable, may depend on the sequence and/or the processor.
- ***Optimization criteria.*** Few optimization criteria use an economic point of view; most relate to efficiency (makespan...). Due to lack of space, this aspect is not considered here.
- ***Paper's aims.*** A scientific paper is written with a specific aim. Three categories of targets can be identified: the *numerically-oriented* ones (new algorithm to solve a specific class of problems, analysis of the limits of existing algorithm...); the *model-oriented* ones (new formulation of a complex problem, sometimes preceded by a representative case study or followed by a small academic case study used to illustrate problem formalization); another category, not encountered in the surveyed papers, is description-oriented, devoted to explanation of complex, real situations.

Table 1. Main characteristics of the analysed papers

Paper# (see reference list)	Goals / Inputs section			Characteristics of machines			Characteristics of jobs							Joint characteristics of jobs and machines					
	Modeling / Problem formalization	Specific resolution algorithm	Use of general resolution methods	Topology of parallel processors Identical (I), Heterogeneous (H)	Availability level of parallel processors		Possibility of delayed arrival of some job	Jobs due dates		Taking account of the consumption of input in the problem formulation	Consideration of the storage constraint of production finished	Possibility of job preemption	Taking into account the priority of job	Preemption	Machine dependent production time	Setup time			Possible implementation of a job on any parallel machine
					Total at the beginning of the scheduling	Taking into account of shutdowns processors		Ignored	Taken into account							Nil	Machine dependent	Sequence dependent	
	Later date	time windows																	
1		X		H	X	X			X						X	X			X
2		X		I	X	X			X						X		X	X	X
3		X		I		X				X							X	X	X
4		X		I				X					X			X			X
5	X	X		H	X	X			X				X			X	X		X
6	X	X		H	X	X		X							X		X	X	
7	X	X		H	X	X			X						X		X	X	
8	X	X		H	X				X				X		X			X	X
9		X		H	X	X			X						X			X	X
10	X	X		H	X	X			X	X					X	X			X
11	X	X		I	X	X	X			X					X				X
12		X		I	X	X	X			X								X	X
13		X		I	X	X		X										X	X
14	X	X		I				X				X		X		X			X
15	X	X		I	X	X	X		X						X		X		X
16		X		H	X	X			X						X		X	X	X
17			X	H	X	X			X				X					X	X
18	X	X		I		X		X							X				
19	X	X		H	X				X						X				X
20	X	X		H	X	X			X						X		X	X	X
21		X		I				X								X			X
22		X		I				X					X			X			X
23		X		H	X	X			X						X		X	X	X
24	X	X		H	X	X		X							X		X	X	X
25				H															
26	X	X		H	X	X			X	X					X	X			X
27	X	X		I	X	X		X										X	X
28	X	X	X	H	X	X		X											X
29	X	X		H		X	X		X				X		X			X	X
30	X	X		H	X	X			X				X		X		X	X	X
31		X		I	X	X	X			X			X			X			X
32		X		I	X	X	X		X									X	X
33		X		I	X	X	X		X							X			
34	X	X		H	X	X		X							X		X	X	X
35	X	X		H	X	X		X										X	X
36	X	X		H	X	X		X							X			X	X
37		X		I	X	X	X		X				X		X			X	X
38		X		I	X	X			X				X					X	X
39		X		I	X			X					X			X			X
40	X		X	H	X	X			X						X		X	X	X
41 (Us)	X		X	H	X		X			X	X	X		X	X		X	X	

III. Definition of scheduling problem

Two models for MILP scheduling issues are proposed (§III.1) and their use is illustrated by a limited numerical example (§III.2). But the combinatory explosion of model 2 versus model

1 leads one to prefer model 1 in the DSS in order to have acceptable calculation time. We shall briefly review the definition of the economic valuation system for the decisions to be made, an aspect all too often left aside by operational research experts (§III.3)

III.1 Models of the scheduling problem

Two models for the scheduling problem have been proposed (Giard *et al.*, 2015; Azzamouri *et al.*, 2016). Model 1 is based on local reasoning and ignores possible unavailability of raw materials (upstream of the SC) and any saturation of finished product inventories (downstream of the SC). Model 2 takes these constraints explicitly into account.

The two models share multiple features, thus justifying joint presentation. The model parameters combine ranges and BOM information with information stemming from the orders to be fulfilled. This prior transformation device is explained in the notation table exhibit

The use of an Algebraic Modeling Languages (AML) that separates a generic model (that can include parametrization enabling optional relations) from the data, whose combination yields an instance of a problem to solve, is particularly appropriate in a DSS. Another advantage, leveraged here, is the possibility of introducing predicates to reduce the set of variables and relations generated when creating a problem instance, to the relevant set for the actual context.

Definition of orders

A set \mathcal{E}_O of O orders ($o = 1..O$) must be scheduled on several L non-identical parallel lines ($l = 1..L$). The L first orders ($o = 1..L$) are currently in progress at the beginning of the scheduling problem, as they were launched before; they define the set \mathcal{E}_O . The $(O - L)$ following orders ($o = L + 1..O$) are the new orders to be scheduled; they define the set \mathcal{E}_O . L fictitious orders are added, one per line ($o = O + 1..O + L$), to be the last scheduled order on each

line (they act like fictitious tasks in the classic MILP formulation of the project scheduling problem); the set \mathcal{E}_O^m ($o = L+1..O+L$) is made of the fictitious and new orders.

Table 2. Example of definition of a set of orders (with L=2 and O=7)

		Orders to launch or fictitious order \mathcal{E}_O^m								
		orders in progress or to launch \mathcal{E}_O								
orders in progress \mathcal{E}_O		new orders to launch \mathcal{E}_O^m					fictitious orders			
1	2	3	4	5	6	7	8	9		

Representation of time

In MILP, time is defined with periods (e.g. hours) and not with dates. As non-availability periods are planned in the lines (mainly for preventive maintenance purposes...), the absolute period index p ($p = 1..P$) must be replaced by a relative period index π_{lp} which depends on the line. Boolean $\psi_{lp} = 1$ if line l is not available during period p , and 0, if it is; then we can write $\pi_{lp|\psi_{lp}=0} = p - \sum_{t=1}^{t=p} \psi_{lt}$. The use of relative periods is mandatory in case of order due dates, taken into account in our models, and if there are time constraints on some inputs and/or some outputs. In the problem under study, work interrupted by maintenance can be resumed without changing total processing time.

Table 3 Example of definition of relative time

Non-availability periods (preventive maintenance...) if $\psi_{lp}=1$

		Period p (absolute time)									
ψ_{lp}		$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
Line/ l	$l=1$	0	0	0	1	1	0	0	0	0	0
	$l=2$	0	0	0	0	1	1	0	0	0	0

Relative time π_{lp} (depending on absolute time and non-availability)

		Period p (absolute time)									
π_{lp}		$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
Line/ l	$l=1$	1	2	3			4	5	6	7	8
	$l=2$	1	2	3	4			5	6	7	8

Decision variables

Two models are introduced. In both models, the orders inherit information from the references to produce. The table of notations, given in annex, makes explicit this mechanism. Boolean parameter $\beta_{lj} = 1$ if the output reference r required by order j (information given by table R_j) can be produced by line l . Production rate ω_{lj} of order j on line l copies that of reference r on line l . Setup times λ_{lij} involved by a change of output reference from one order to the following one on the same line; it copies that of the setup time involved by producing reference R_j after reference R_i on line l . The total production time θ_{lij} of order j following order i on line l is the sum of the setup time λ_{lij} and of the processing time φ_{lj} calculated as the quotient of the ordered quantity by the production rate of the reference r on line l . This preliminary transformation simplifies the two versions of our scheduling model.

In model 1, the binary decision variable $x_{lij} = 1$ if order j ($j \in \mathcal{E}_{O^m}$) is processed on line l just after order i ($i \in \mathcal{E}_O$); in this formulation, time is indirectly addressed through precedence constraints in production (see relation (9)). The variable x_{lij} exists only if $\beta_{li} = 1$ and $\beta_{lj} = 1$. Relation (1) defines the predicate \mathcal{H}_1 associated with the set of the decision variables x_{lij} that makes sense. This is very helpful when using Algebraic Modeling Languages (AML), like Xpress-IVE or GAMS.

$$l, i, j \mid i \in \mathcal{E}_O \wedge j \in \mathcal{E}_{O^m} \wedge i \neq j \wedge \beta_{li} = 1 \wedge \beta_{lj} = 1 \Rightarrow \mathcal{H}_1 = True \quad (1)$$

In model 2, the binary decision variable $x_{lijp} = 1$ if order j ($j \in \mathcal{E}_{O^m}$) is processed on line l just after order i ($i \in \mathcal{E}_O$), its last production period being p . Relation (2) defines the predicate \mathcal{H}_2 associated with the set of decision variables x_{lijp} that makes sense. Note that this predicate includes the due date boundaries.

$$l, i, j, p \mid i \neq j \wedge i \in \mathcal{E}_O \wedge j \in \mathcal{E}_{O^m} \wedge \beta_{li} = 1 \wedge \beta_{lj} = 1 \wedge I_{lj} \leq \pi_{lp} \leq S_{lj} \Rightarrow \mathcal{H}_2 = True \quad (2)$$

In both models, the delivery date y_j is a decision variable corresponding to the last period of production of order j . In model 1, this variable is real; it is integer in model 2. The links between y_j and x_{lij} or x_{lijp} are explained below.

Two other decision variables may be added in model 2. They reflect additional constraints and may be used in the optimization criterion. The first one is makespan M , defined as the latest delivery date among the scheduled orders (see relation 16). The second one relates to consumption of a critical input, pulled by the schedule, and is the peak Π of its consumption (see relation 13).

Assignment constraints

A new or fictitious order j ($j \in \mathcal{E}_{O^m}$) is allocated to a single line ($l = 1..L$). An order in progress at the beginning of the scheduling ($j \in \mathcal{E}_O$) cannot be followed by more than one new order and may be followed by no order if that line is not used in the schedule. This is enforced by relations (3) for model 1 and (4) for model 2.

$$\forall j \in \mathcal{E}_{O^m}, \sum_{l, i \mid \mathcal{H}_1 = True} x_{lij} = 1 \quad ; \quad \forall i \in \mathcal{E}_O, \sum_{l, i \mid \mathcal{H}_1 = True} x_{lij} \leq 1 \quad (3)$$

$$\forall i \in \mathcal{E}_{O^m}, \sum_{l, j, p \mid \mathcal{H}_2 = True} x_{lij\pi_p} = 1 \quad ; \quad \forall i \in \mathcal{E}_O, \sum_{l, j, p \mid \mathcal{H}_2 = True} x_{lij\pi_p} \leq 1 \quad (4)$$

Respect of delivery windows

The last period of production of order j is its delivery date y_j . This date is bounded by lower and upper dates (L_{lj} and U_{lj}) that are defined with the relative calendar of line l . These constraints are enforced for the new orders (\mathcal{E}_{O^m}) in model 1 by relation (5) and, in model 2, by

relation (2) which restrains the scope of variable x_{lijp} . Note that the delivery dates of the orders in progress are already known ($\forall j \in \mathcal{E}_O, y_j = L_{jj} = U_{jj}$).

$$\forall j \in \mathcal{E}_O, \sum_{l,i,j|\mathcal{Z}_1=True} L_{lj} \cdot x_{lij} \leq y_j \leq \sum_{l,i,j|\mathcal{Z}_1=True} U_{lj} \cdot x_{lij} \quad (5)$$

A new order j produced on line l must have a predecessor i (in progress or new) produced on the same line. Relation (6) for model 1 and (7) for model 2 enforce order i to be produced on line l if $x_{lij} = 1$ (model 1) or $\sum_p x_{lij\pi_p} = 1$ (model 2).

$$\forall j \in \mathcal{E}_O \wedge \forall l | \beta_{lj} = 1, \sum_{k \in \mathcal{E}_O | k \neq j \wedge \beta_{lk} = 1} x_{lkj} = \sum_{h \in \mathcal{E}_O | h \neq j \wedge \beta_{lh} = 1} x_{ljh} \quad (6)$$

$$\forall j \in \mathcal{E}_O \wedge \forall l | \beta_{lj} = 1, \sum_{k \in \mathcal{E}_O | k \neq j \wedge \beta_{lk} = 1} \sum_{p | L_{lj} \leq \pi_p \leq U_{lj}} x_{lkj\pi_p} = \sum_{h \in \mathcal{E}_O | h \neq j \wedge \beta_{lh} = 1} \sum_{p | L_{lj} \leq \pi_p \leq U_{lj}} x_{ljh\pi_p} \quad (7)$$

Non-overlapping constraints

Relations (8) for model 1 and (9) for model 2 prevent order j from being produced as long as the production of order i is in progress, when both orders are produced on the same line. In these relations, the number P of periods plays the role of the “big M ” constant.

$$\forall i \in \mathcal{E}_O, j \in \mathcal{E}_O | i \neq j, y_j - y_i \geq \sum_{l|\mathcal{Z}_1=True} \theta_{lij} \cdot x_{lij} - P \cdot (1 - \sum_{l|\mathcal{Z}_1=True} x_{lij}) \quad (8)$$

$$\forall i \in \mathcal{E}_O, j \in \mathcal{E}_O | i \neq j, y_j - y_i \geq \sum_{l,p|\mathcal{Z}_2=True} \theta_{lij} \cdot x_{lij\pi_p} - P \cdot (1 - \sum_{l,p|\mathcal{Z}_2=True} x_{lij\pi_p}) \quad (9)$$

Consideration of the upstream and downstream constraints (model 2)

Relations (10) to (16) are specific to model 2. Relation (10) links variables y_j and $x_{lij\pi_p}$ in model 2. Note that y_j is determined using the relative time definition in both models.

$$\forall j \in \mathcal{E}_0, y_j = \sum_{l,i,p | \mathcal{Z}_2 = \text{True}} \pi_{lp} \cdot x_{lij\pi_p} \quad (10)$$

Let's take into account the incidence of the schedule on the consumption of an input regarded as critical (upstream consequences) and on the use of the inventories of the different produced qualities of fertilizers (downstream consequences).

Relation (11) defines total consumption C_p during period p of the input regarded as critical. It is a linear expression of the decision variables and uses the consumption rate q_j of order j on line l (null if $\Psi_{lp} = 1$). C_p is calculated as the sum of the consumption of orders in progress (first part of the relation) and of new orders.

$$\forall p, C_p = \sum_{l,j \in \mathcal{E}_0 | l=j \wedge \Psi_{lp}=0 \wedge \pi_{lp} \leq U_j} q_{lj} + \sum_{l,j \in \mathcal{E}_0 | \beta_{lj}=1 \wedge \Psi_{lp}=0 \wedge \pi_{lj} \leq U_j} q_{lj} \cdot \sum_{t \geq p, i \in \mathcal{E}_0 | \beta_{li}=1 \wedge i \neq j \wedge L_j \leq \pi_{li} < \pi_{lp} + \varphi_j} x_{lij\pi_{li}} \quad (11)$$

Assuming that one knows the evolution K_p of the inventory of the input regarded as critical, leaving aside the incidence of the schedule to find, relation (12) prevents from the risk of production stoppage due to input stockout. Note that the generalization of the model to consider all inputs is obvious but it increases the size of the problem needlessly in most cases.

$$\sum_{t=1}^{t=p} C_p < K_p \quad (12)$$

One can determine the maximum of peak Π of consumption of the input regarded as critical with relation (13) if the optimization criterion is that of relations 25 or 26

$$\forall p, C_p \leq \Pi \quad (13)$$

Order j relates to quality output r (given by R_j) and several orders may relate to a single quality reference r . With production ratio p_{lj} on line l , total production P_{rp} , during period p is given by relation (14) which is a linear expression of the decision variables. Knowing the

evolution Λ_{rp} of the inventory of reference r , leaving aside the incidence of the schedule to find, and the maximum level Λ_r^{\max} that the inventory cannot exceed, relation (15) prevents a schedule to be unfeasible due to a saturation of the stock.

$$\forall p, r, P_{rp} = \sum_{l, j \in \mathcal{E}_0, |l=j \wedge R_j=r \wedge \Psi_{lp}=0 \wedge \pi_{lp} \leq U_j} P_{lj} + \sum_{l, j \in \mathcal{E}_0, |\beta_{lj}=1 \wedge R_j=r \wedge \Psi_{lp}=0 \wedge \pi_{lj} \leq U_j} P_{lj} \cdot \sum_{t \geq p, i \in \mathcal{E}_0, |\beta_{li}=1 \wedge i \neq j \wedge L_j \leq \pi_{li} < \pi_{lp} + \varphi_{lj}} x_{lij\pi_{li}} \quad (14)$$

$$\forall r, p, \sum_{t \leq p} P_{rp} + \Lambda_{rp} < \Lambda_r^{\max} \quad (15)$$

The introduction of makespan M leads to its determination by relation (16)

$$\forall j \in \mathcal{E}_C, \sum_{l, i, p, |\mathcal{Z}_2=True} P \cdot x_{lij\pi_{lp}} \leq M \quad (16)$$

Optimization criteria

The schedule cost is given by relation (17) for model 1 and (18) for model 2, where γ_{lij} is the cost of producing order j on line l , after order i . These costs does not take into account expenses which must be supported whatever the decision taken; this point will be discussed under §III.3.

$$C_1 = \sum_{l, i, j, |\mathcal{Z}_1=True} \gamma_{lij} \cdot x_{lij} \quad (17)$$

$$C'_1 = \sum_{l, i, j, p, |\mathcal{Z}_2=True} \gamma_{aij} \cdot x_{lij\pi_{li}} \quad (18)$$

Several schedules may have the same minimum value of C_1 , because slacks can exist between orders sequenced on the same line (in model 1, the decision variable y_j being real, the number of equivalent schedules may be infinite). Among them, schedules with the earliest delivery dates are usually preferred, to keep the productive system busy in the short term. This is obtained by adding a penalty K (relation 19), weighted by the coefficient α , to C_1 , which

gives criterion C_2 (relation 21) to be used in model 1; in model 2, the penalty K' (relation 20) is added to C'_1 , to give criterion C'_2 (relation 22). The value of α must be low enough not to modify the values of order variables x_{lij} or $x_{lij\pi_t}$. The aim is to drive the choice of schedule from the set of optimal schedules. The value of coefficient β_j ($0 \leq \beta_j \leq 1$) aims to pull order j toward its earliest schedule when $\beta_j = 1$ and to push it toward its earliest schedule; if $\beta_j = 0$. The desired result is not achieved when conflicts occur between orders scheduled on the same line; in this case, one needs to work out the best schedule among the optimal ones.

$$K = \left[\sum_{j \in C''} \beta_j \cdot (y_j - \sum_{l,i|\mathcal{Z}_1=True} L_{lj} \cdot x_{lij}) + \sum_{j \in C''} (1 - \beta_j) \cdot (\sum_{l,i|\mathcal{Z}_1=True} U_{lj} \cdot x_{lij} - y_j) \right] \quad (19)$$

$$K' = \left[\sum_{j \in C''} \beta_j \cdot (y_j - \sum_{l,i,j,p|\mathcal{Z}_2=True} L_{lj} \cdot x_{lij\pi_t}) + \sum_{j \in C''} (1 - \beta_j) \cdot (\sum_{l,i,j,p|\mathcal{Z}_2=True} U_{lj} \cdot x_{lij\pi_t} - y_j) \right] \quad (20)$$

$$C_2 = C_1 + \alpha \cdot K \quad (21)$$

$$C'_2 = C'_1 + \alpha \cdot K' \quad (22)$$

Using model 2, one may prefer to minimize the consumption peak of the input regarded as critical, regardless of cost considerations. This leads to minimize relation 23 or 24.

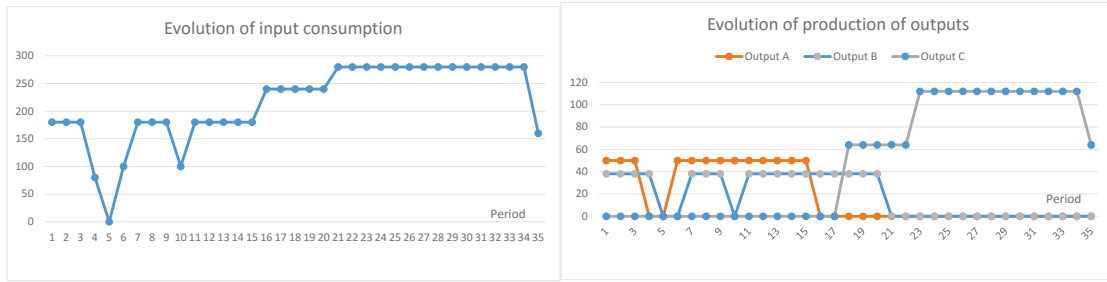
$$M \quad (23)$$

$$M + \alpha \cdot K' \quad (24)$$

One may also prefer to minimize the makespan, regardless of cost considerations, which leads to minimize relation 25 or 26

$$\Pi \quad (25)$$

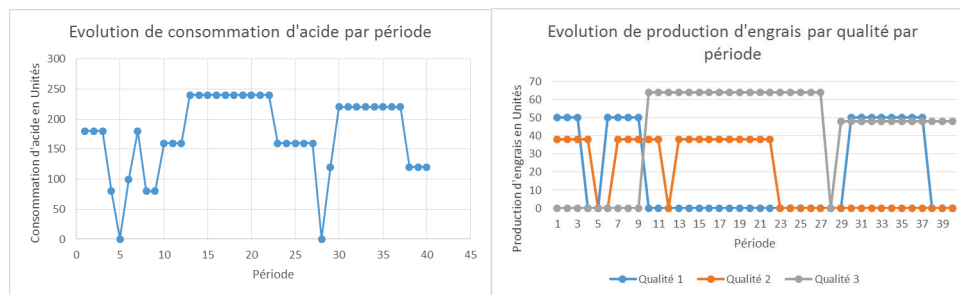
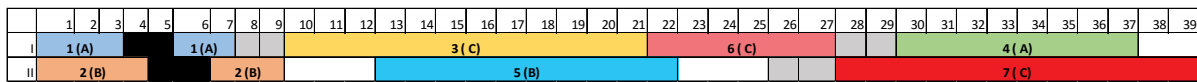
$$\Pi + \alpha \cdot K' \quad (26)$$



Figures 1: Optimal solution of model 1

Model 2

Using criterion (24) related to the consumption peak of the input regarded as critical, with $\alpha = 0.01$ and $\beta_j = 1, \forall j$, we obtain the optimal solution, illustrated by the Gantt in figure 2, where $x_{1,1,3,21} = x_{1,3,6,27} = x_{1,6,4,37} = x_{1,4,8,38} = x_{2,2,5,22} = x_{2,5,7,39} = x_{2,7,9,40} = 1$ (the other decision variables being nil). We note a cost increase of 10 due to additional production launch versus the schedule yielded by model 1 and a consumption peak dropping to 240. Use of this criterion leads to an efficiency loss, (poor use of plant 2) and postpones by 5 hours the completion date for all the orders.



Figures 2: Optimal solution of model 2

III.2 Valuation System Used

Search for a solution to reduce operating cost of a production system is pointless if the savings theoretically afforded by the new solution do not show up in the corporate results. In

other words, an optimization model that matches reality adequately but relying on an irrelevant cost system is devoid of operational value.

The downside of using inadequate management accounting was pointed out at the end of the 1980's and summarized by Johnson and Kaplan (1991), which led to the emergence of activity accounting (*Activity Based Costing*, ABC) popularized by many textbooks and in particular that of Kaplan & Cooper (1997). Rather than using standard costs, quotient of expenses by a produced quantity, this approach relies on the cost driver concept. Its use should be tailored to the context of transformation of a production system, since a number of expenses are not impacted by the decisions taken. It follows that one must develop an activity valuation model based on processes and stemming from the recommendations issue out of *Supply Chain Costing* (Seuring, 2002). Building a valorization system for operational management purposes in relation to hybrid production supply chains is a major issue (Degoun *et al.*, 2015).

Fertilizer plant scheduling decisions are taken with reference to a temporal timeframe that is limited to four weeks. In this timeline, the number of lines open and the number of shifts operating them are known; the scheduling decisions are designed to use this production potential, without any impact whatsoever on these expenses as these must be considered to be fixed on this time horizon. Setup time corresponds to line cleaning time. Payroll cost does not feature since it is part of fixed expenses for the period; on the other hand, the cost of raw material wasted in the cleaning process and that of the cleaning products (mainly water) must be taken into account; the cost driver therefore clearly is the switch to a new reference for that line. Moreover, lines 07 and 107 have different performances levels. The difference of manufacturing rate for a fertilizer reference only impacts order delivery time and thus on the production means used. Between these types of lines, one observes slight differences on production BOM's: to manufacture one tonne of a given fertilizer reference at workshop 07, versus workshop 107, one requires a little more raw materials (the difference can amount to up

to 0.5 %) and a little more power. Here the cost driver is clearly amount produced in a particular workshop. In principle, no other expense varies based on the selected scheduling solution.

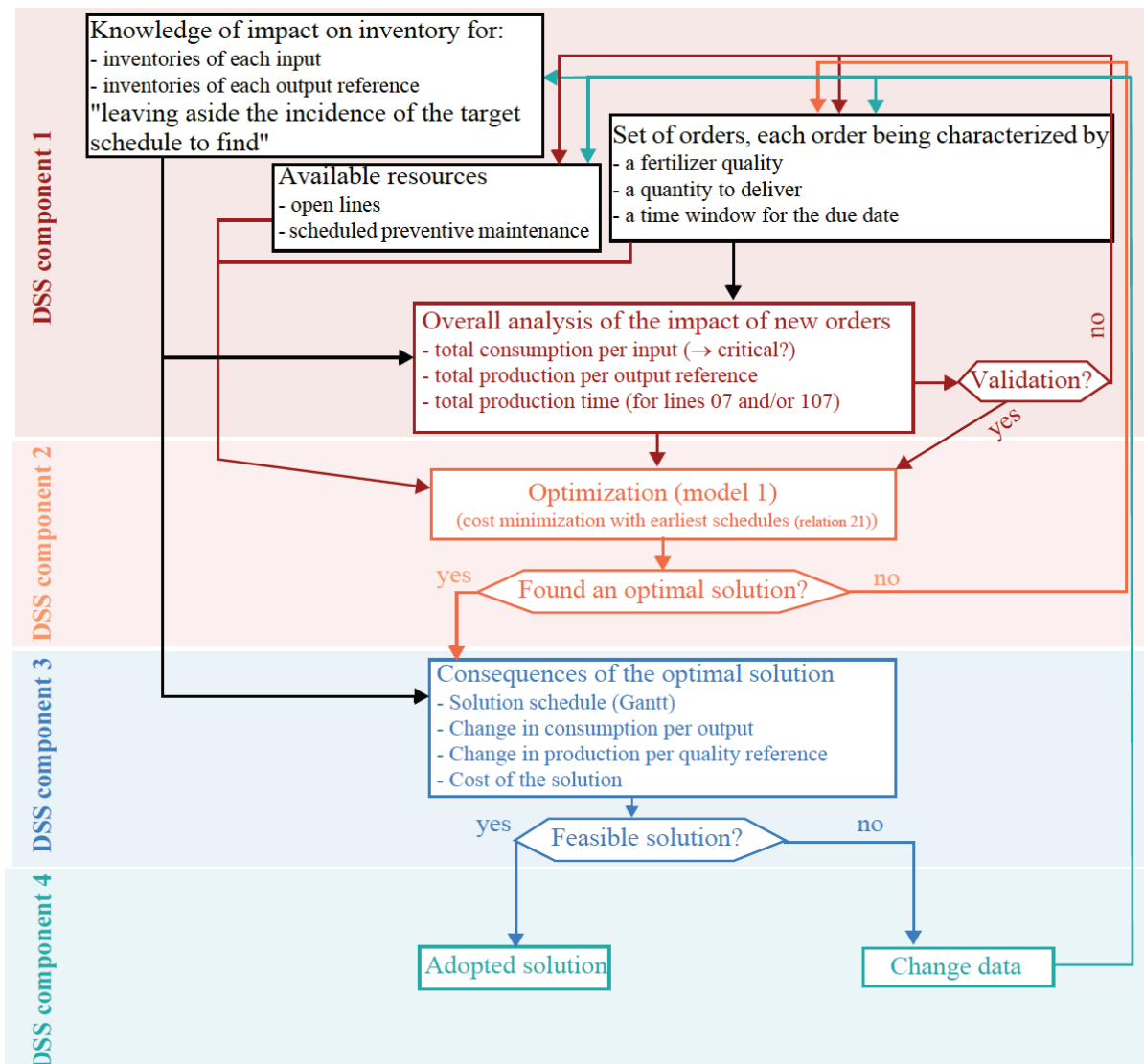
The inclusion of these drivers in schedule choice, actually yields possible differences in actual expenditure.

IV. DSS Description

The DSS prototype (Azzamouri *et al.*, 2016) rests on a relational database comprising a number of structural data (production system description, routings, BOMs, costs) and data used to define a semi-structured problem: orders, preventive maintenance, forecasted trends – excluding impact of the schedule to be obtained – of raw material inventories used and of fertilizer reference inventories. An initial Java DSS interface module is used to update the database implemented under MySQL.

The next module of this interface is used to define the semi-structured problem to be dealt with (component 1 of SAD), serving to perform an initial feasibility check, and then to define a structured problem to be solved with model 1 (the possible use of model 2 is not useful due to large calculation time) and to generate a data file to be used to create a problem instance of the generic problem defined under Xpress-IVE, before launching the problem resolution (component 2 of DSS). A third module uses the results of the optimization to assess feasibility in relation to impact upstream and downstream of SC (component 3 of DSS). If the solution is deemed not feasible, this leads to revising the issue to be optimized (component 4 of DSS). The retroaction of analysis of a solution that was found with model 1 (component 4 of SAD) on the reformulation of a semi-structured issue (component 1 of SAD) mainly concerns the definition of the date windows for completion of production of orders and on the weighting coefficients leading orders to be scheduled preferably either soonest or latest. It also may lead to a break up of orders in order to meet a delivery date where it appears that the production

potential is under-utilized or to adjust the preventive maintenance schedule. The following figure summarizes DSS operation.



OCP's CL comprises a number of parallel axes. Two of them, the Northern axis (Jorf) and the central axis possess a fertilizer production plant. The proposed approach can easily be tailored to enable global, therefore more efficient, management of fertilizer production in these plants.

V. Conclusion

A prototype of Decision Support System is about to be experimented with some actual problems, which is a mandatory step before its implementation in the fields. Two

complementary extensions are studied: the first one aims to merge the scheduling of two distant fertilizer plants to improve their capacity use (see above); the second aims to lengthen the schedule horizon to be able to include possible orders and help the sales department negotiate the best compromise between price and delivery time.

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VII. Table of notations

In this table, we present primary information of orders, BOM and routing used to calculate the model parameters (they are not used in the model). The retained time bucket is one hour in the table (in accordance with the data of the actual problem) but this unit can easily be changed.

INDEXES

o Subscript of an order ($o = 1..O$)
 Set \mathcal{E}_0 defined for $o = 1..L$; set \mathcal{E}_O defined for $o = L+1..O$; set \mathcal{E}_o defined for $o = O+1..O+L$; set \mathcal{E}_o defined for $o = L+1..O+L$

i, j Subscript of a scheduled order on a given line, order i being directly followed by order j

l Subscript of a parallel line ($l = 1..L$)

p Subscript of absolute period ($p = 1..P$)

π_{lp} Subscript of relative period for line l corresponding to the absolute period p ; it takes into account the periods during which line l is not available (preventive maintenance...)

$$\pi_{lp|\psi_{lp}=0} = p - \sum_{t=1}^{t=p} \psi_{lp}$$

r Subscript of an output reference
 r is used in the creation of the problem data but not in the model

TECHNICAL PARAMETERS

ψ_{lp} Binary parameter = 1 if line l is unavailable during period p

R_j Reference to produce for order j
 R_j is used in the creation of the problem data but not by the model

β'_{lr} Binary parameter = 1 if the output reference r required by order j can be produced by line l
 β'_{lr} is used in the creation of the problem data but not by the model

β_{lj} Binary parameter = 1 if the order j can be produced by line l ($\beta_{lj} = \beta'_{lR_j}$)

ω'_{lr} Production rate of output r on line l (in tonnes/hour)
 ω'_{lr} is used in the creation of the problem data but not by the model

ω_{lj} Production rate of order j on line l ($\omega_{lj} = \omega'_{lR_j}$)
 ω_{lj} is used in the creation of the problem data but not by the model

Q_j Quantity to produce for order j
 Q_j is used in the creation of the problem data but not by the model

ϕ_{lj} Processing time of order j on line l ($\phi_{lj} = \lceil Q_j / \omega_{lj} \rceil$), excluding setup time

Expressed in hours (with the definition of the production rate)

λ'_{lr_2} Setup time for producing the reference r_2 after the reference r_1 on line l
 λ'_{lr_2} is used in the creation of the problem data but not by the model

λ_{lij} Setup time for producing the reference j after the reference i on line l
 $\lambda_{lij} = \lambda'_{lR_iR_j}$ is used in the creation of the problem data but not by the model

θ_{lij} Total processing time of order j produced on line l after order j , including setup time
 $\theta_{lij} = \phi_j + \lambda_{lij}$

L_{lj} Lower boundary of the last period of production for order j (y_j); it uses the relative period index of line l where this order is produced

U_{lj} Upper boundary of the last period of production of order j (y_j); it uses the relative period index of line l where this order is produced

Λ_{rp} Evolution of the inventory of output r , leaving aside the incidence of the schedule to find
Used only in model 2

Λ_r^{\max} Maximum level that the inventory of output r cannot exceed
Used only in model 2

K_p Evolution of the inventory of the input regarded as critical, leaving aside the incidence of the schedule to be found
Used only in model 2

COSTS

Λ'_{lr_2} Setup cost induced by the production of the reference r_2 after the reference r_1 on line l
 Λ'_{lr_2} is used in the creation of the problem data but not by the model

Λ_{lij} Setup cost induced by the production of order j after order i on line l
 $\Lambda_{lij} = \Lambda'_{lR_iR_j}$ is used in the creation of the problem data but not by the model

Γ_{lr} Direct variable cost induced by the production of one hour of reference r on line l
 Γ_{lr} is used in the creation of the problem data but not by the model

γ_{lij}	Production cost of Q_j in line l $\gamma_{lij} = \Gamma_{lR_j} \cdot \phi_{lj} + \Lambda_{lij}$
α	Low coefficient applied to the penalty cost added to the objective function C_1 to drive the choice of the schedule among the set of optimal schedules, in case of slack in production schedule
β_j	Coefficient used to drag the schedule of order j towards its earliest value or its latest possible value in case of slack; $0 \leq \beta_j \leq 1$

DECISION VARIABLES

x_{lij}	Binary decision variable = 1 if order j ($j \in \mathcal{E}_{O^m}$) is processed on line l just after order i ($i \in \mathcal{E}_O$) <i>Used only in model 1</i>
x_{lijp}	Binary decision variable = 1 if order j ($j \in \mathcal{E}_{O^m}$) is processed on line l just after order i ($i \in \mathcal{E}_O$) <i>Used only in model 2</i>
y_j	Date of the end of production of order j ; it uses the relative period index π_{lp} of line l where this order is produced y_j is a real variable in model 1 and integer variable in model 2
M	Makespan ($M = \text{Max}(y_j)$) <i>Used only in model 2</i>
K	Maximum consumption of the input regarded as critical involved by the schedule solution <i>Used only in model 2</i>