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Link dimensioning of hybrid FSO/fiber networks resilient to adverse weather conditions

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Abstract

The presented paper deals with wireless networks composed of FSO (free space optics) links supported by terrestrial optical fiber connections in order to increase resilience to adverse weather conditions. An FSO link realizes, at low cost, a broadband optical transmission system established by means of two parallel light beams connecting a pair of (remote) transceivers placed in the line of sight. However, a major disadvantage of FSO links (with respect to fiber links) is their sensitivity to weather conditions: bad weather affects optical channel, resulting in substantial degradation of the transmission power at the receivers, which in general leads to capacity degradation on multiple FSO links. When the transmission power received at the end node of an FSO link is decreased, the signal modulation and coding scheme (MCS) at the transmitter should be adjusted accordingly; in effect, an FSO link can be kept operational, but with decreased capacity, even if the channel is affected. Yet, since in severe weather conditions some FSO links may not be able to realize any reliable transmission at all (whichever MSC is used), the network may become disconnected and a portion of traffic lost. Therefore, it is reasonable to consider using fibers instead of free space light beams since weather insensitive (and high capacity) fibers installed on the key links will ensure network connectivity in all weather states. Certainly, the so obtained connectivity does not come for free because of the high cost of the optical fiber connections as compared to the FSO systems and hence the number of fibers should be kept at minimum. Thus, for designing hybrid FSO/fiber networks we need an optimization model for finding cheapest configurations of links (and their capacities) that will be able to carry the demanded traffic on acceptable level in all weather states foreseen in network operation. This is not a simple task as it requires not only robust network optimization methods but also a tractable way of characterizing possible weather states (which are not known in advance) and their influence on FSO link capacity. In the paper we present an approach to the so described task and illustrate its effectiveness by means of a numerical study.

Keywords: robust optimization, multicommodity flows, resilient and survivable networks, free space optics, variable link capacity, linear and mixed integer programming

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1. Introduction

Motivation. This paper is focused on hybrid FSO (Free Space Optics) wireless networks experiencing link capacity degradations due to changing weather conditions. FSO is a well-established broadband wireless optical transmission technology, where the communication links are established by means of two parallel laser (light) beams sent between a pair of transceivers placed in the line of sight. FSO links are considered as an alternative to radio links for example in metropolitan Wireless Mesh Networks (WMN) [1] and [2]. FSO networks have some noteworthy properties such as transmission range of several kilometers, high transmission bandwidth, low cost (as compared with the fiber optical technology), immunity to electromagnetic interference, etc. Yet, a major disadvantage of FSO links is vulnerability to weather conditions such as fog, rain and snow, causing a substantial loss of the transmission power over the optical channel. When the network is not equipped with resilience mechanisms for ensuring and maintaining an acceptable level of service, weather degradation may lead to significant data losses and, during severe weather degradation, even to network disconnection [3].

One of the two resilience mechanisms considered in this paper is based on the notion of link margin, the quantity representing the additional amount of attenuation that can be accepted on an FSO link without losses in the transmitted data [2]. The use of this notion makes it possible to determine proper modulation and coding schemes (MCS) applied at the transmitters of the affected FSO links for a given weather condition. The fraction of the maximally attainable (i.e., nominal) link capacity that is lost due to a change of its MCS is referred to as link degradation ratio. As illustrated in [3], weather conditions may significantly reduce capacity of a subset of network links. Moreover, in some extreme cases link degradation ratio on some links may become equal to 1 leading to network disconnection (such disconnections can be observed for 10 – 15% of the weather states during a one year period). To deal with such unacceptable disconnections we consider another resilience mechanism. It consists in installing, instead of FSO systems, a limited number of (weather insensitive) optical fibers on the key links in order to secure connectivity of the so obtained (hybrid FSO/fiber) network in all the foreseen weather conditions.

Certainly, the combination of the two above described network mechanisms, i.e., MCS control and the use of optical fibers, will assure network resilience only when network links are properly dimensioned. This simply means that in all the foreseen weather conditions the current capacity available on the links (both FSO and fiber) is sufficient to carry the demanded traffic on an acceptable level. Introducing and studying a particular optimization model that takes the above requirement into account and at the same time minimizes the total cost of network links (note that this will limit the use of fibers which are more expensive to install/hire than the FSO links) is the main goal and motivation of this paper.

Problem studied. The network optimization problem studied in this paper is as follows: how to dimension the network links at the lowest cost, so that each link is equipped either with a set of several FSO modules or a fiber, and at the same time assure an acceptable level of the demand traffic satisfaction for all observed/predicted weather conditions. The problem is formulated as a mixed integer program (MIP) using the node-arc formulation (called

node-link formulation in [4]). The link dimensioning model assumes Global Rerouting traffic protection mechanism [5] (called also Unrestricted Reconfiguration in [4]).

45 An important part of our optimization approach is translating weather states into network states characterized by the so called link degradation ratios. Typically, a given weather condition affects a subset of network links, and each affected link loses a portion (fraction) of its nominal capacity – the quantity called link degradation ratio above. Typical values of link capacity degradation ratios are 0.25, 0.50 and 1, and each particular weather condition
50 that may occur defines a network state referred to as link degradation state. Certainly, the set of such states used in optimization, called the reference set of (link degradation) states, needs to be somehow predicted. Such a prediction can be made using historical weather records for the network area observed in the past, and possibly long term weather forecasts. Yet, such obtained reference set will most likely contain an excessive number of states and
55 at the same time will not contain all the states that will appear in the reality. Thus, in our optimization approach we use a different characterization of the reference set of states, called called uncertainty sets (or uncertainty polytopes). A certain type of such sets were studied in [3]. In this paper we propose another version of uncertainty sets that aim on estimating the real reference set of future states more accurately than those described in
60 [3]. Our alternative characterization of link degradation states captures the incidence of links and nodes for a given weather state. That is, we consider simultaneous degradations of subsets of nodes (degradation of a node implies degradation of all links incident to it), instead of simultaneous degradation of subset of links. As shown in the numerical section, this allows achieving more cost-efficient link dimensioning with the same or even higher
65 traffic satisfaction, as compared to the previous characterization.

Related work. Failures in communication networks occur due to a variety of both external and internal reasons of different range and scale. For example, these can be natural disasters such as floods, earthquakes and fires, weather phenomena like tornadoes and snow storms, and technology-related disasters such as power blackouts or equipment (node or link) breakdowns.
70 All of them decrease the traffic handling capability of the network, sometimes causing a whole network subarea to be turned off.

Large-scale failures caused by disasters are by far more dynamic and broader in terms of range than the failures traditionally considered in network optimization. The large-scale failures frequently result in the so-called region failures, i.e., simultaneous failures of network
75 elements located in a specific geographical area. Examples of research in this area can be found in [6], where the authors propose to use spectral graph theory to characterize network survivability regarding geographically correlated failures. They present a network survivability evaluation model employing an optimization problem to find the most vulnerable geographic locations. In line with this, the authors of [7] present assessments on the vul-
80 nerability of optical networks and a comparative analysis of network robustness for different synthetic and real optical network topologies under various types of massive failures using the Girona Network Robustness Simulator (GNRS). Another example is the work of [8] and [9], where major disaster are considered for wired network optimization. As for wireless mesh networks, new measures of survivability for a regional failure scenario, including the
85 region failure survivability function and the expected percentage of total flow delivered after a failure as a function of the region radius are discussed in [10].

On the other hand, the “traditional” failure scenarios, aiming at covering the equipment failures, assume usually single total failures of links or nodes, and sometimes multiple simultaneous total link failures (the so called shared risk link groups). The main resilience mechanism considered in this case is, roughly speaking, flow rerouting taking advantage of spare link capacity foreseen in network dimensioning. A lot of research has been devoted to this area, but almost exclusively for wired networks; for a summary see [4]. As far as wireless networks are concerned, there is some work on resilient network optimization but again typically taking into account isolated failure events, involving a (total) failure of one network link/node at a time (see [11]). Furthermore, in [12], the authors investigate wireless mesh networks and their sensitivity to weather conditions, in particular the heavy rainstorms that may lead to important signal attenuation. The contributions of that work include presentation of a MIP model of weather-resistant links formulation problem followed by an analysis of its computational complexity.

It happens, however, that the above characterized research is hardly relevant to the subject of this paper, which is wireless network resilience to adverse weather conditions. In fact, we are not treating weather changes as a factor causing network (in particular link capacity) failures. Although sometimes very bad weather (like heavy snow) can cause reliable data transmission on a link impossible, we treat the changing weather condition as natural, always present, element of network operation (this is why we do not use the term “link failure” but rather “link degradation” in the paper). In any case, even if we treat the weather degradation as multiple but partial link capacity failures we observe that this case has been treated to a very small extent in the literature. From this perspective, the work presented in the current paper is inspired mostly by the optimization approach described in [13]. There, however another routing strategy (called flow thinning) was used and optimization based on uncertainty sets was not exploited (this was to some extent done in a recent paper [14]).

Finally, we note that as far the hybrid feature is concerned, an optimization approach related to the FSO network link dimensioning problem was presented in [15], where disjoint path routing to ensure protection against link degradation in hybrid FSO/fiber networks was considered.

Contribution. The presented paper, a major enlargement of its conference predecessor [16], extends the optimization approach developed in [3] where we studied the FSO network link dimensioning problem taking into account resilience against link degradations caused by adverse weather conditions, at the same considering uncertainty in weather states modeling. In the current paper we first extend the work of [3] to hybrid FSO/fiber network configurations, and then we propose an enhanced characterization of (link degradation) uncertainty states that captures the phenomenon of the proximity of simultaneously affected wireless links. Finally, we present results of an extensive original numerical study that, among other issues, shows to what extent the network dimensioned with our algorithms can handle real weather conditions. These extensions are original and constitute a novelty in resilient network modeling and optimization.

Paper organization. In Section 2 we present a mathematical formulation of the main dimensioning problem for resilient hybrid FSO/fiber networks, together with a solution approach

130 (based on a cut generation algorithm), that assumes an explicit list of link degradation states. In Section 3 we study an optimization approach based on the so-called uncertainty polytopes that are useful for the cases when the explicit list of degradations states is hardly achievable; for doing that we introduce the concept of geographic-aware uncertainty set. Then, in Section 4, we describe the optimization methodology (based on the previously
 135 introduced algorithms) that we propose for dealing with real network instances. Next, in Section 5, we present an extensive numerical study for two network instances that illustrate the effectiveness of the proposed methodology. Finally, in Section 6, we give concluding remarks.

2. Optimization approach for an explicit reference set of states

140 As already mentioned, the problem studied in this paper is motivated by the necessity of dealing with the degradation of the FSO data transmission caused by weather conditions. A solution is to control the current signal MCS applied at the transmitter of the affected FSO link to secure correct data transmission. This, however, results in decreasing effective transmission data rates. In the sequel, the fraction of the maximal attainable (i.e., nominal)
 145 link capacity (bit-rate) lost due to a change of its MCS is called *link degradation ratio*. As discussed in [3], weather conditions may have an significant impact on link capacity, in particular they may even cause network disconnection. This may occur in severe cases approximately in 10 – 15% of hourly states during a one year period. One way to reduce this impact is to install a (limited) number of terrestrial fibers to assure connectivity. In
 150 fact, optimizing hybrid networks combining the use of FSO with terrestrial fibers is the main issue underlying the studies of this paper.

In such a hybrid network, a link is composed of either a set of parallel FSO transmission systems, or a high capacity optical transmission system (for example a DWDM system of capacity of the order of 10 Tbps in each direction) installed on a fiber. In both cases the
 155 transmission systems are of the full duplex type. An FSO transmission system is established by means of two parallel oppositely directed light beams connecting a pair of nodes placed in the line of sight, each equipped with an appropriate transceiver. Each such light beam carries the data with a given nominal bit-rate of 1–20 Gbps [17].

In order to consider capacity degradation of the FSO links in optimization modeling, we
 160 need to estimate link degradation ratios under various weather conditions that can occur during network lifetime. In the following, we consider four FSO link operation modes with the degradation ratios 0, 0.25, 0.50, 1 (the first ratio corresponds to fully operational mode while the last ratio to total link degradation). Depending on the current channel condition induced by the actual weather between the end nodes of a link, a proper modulation and
 165 coding scheme is selected and translated (using the link margin value) to the link degradation ratio taking one of four values fixed to 0, 0.25, 0.50 or 1. Thus, for optimization purposes, each weather state (in the considered set of weather states \mathcal{W}) is converted into a *link degradation state* described by a vector of the corresponding link degradation ratios. The set of all such (different) vectors constitutes what we call *reference degradation set* (constituted
 170 of link degradation states) denoted by \mathcal{S} . Observe that the fiber links, contrary to FSO links, are not subject to capacity degradation and hence are assumed to be 100% reliable.

In our state modeling methodology we analyze the weather records observed over a time period, typically over one year. For each hourly period (out of $365 \times 24 = 8760$ periods) we translate the observed weather conditions into the link degradation ratios – in effect, each of the hourly periods is assigned its link degradation ratio vector. Next, we identify the subset of periods with different link degradation ratio vectors to form the proper reference degradation set \mathcal{S} .

2.1. Notation

The considered network is modeled by means of a bi-directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ composed of the set of nodes \mathcal{V} , the set of (bi-directed) links $\mathcal{E} \subseteq \mathcal{V}^{[2]}$ (where $\mathcal{V}^{[2]}$ denotes the family of all 2-element subsets of the set of nodes \mathcal{V}), and the set of (directed) arcs $\mathcal{A} \subseteq \mathcal{V}^2 \setminus \{(v, v) : v \in \mathcal{V}\}$. The end nodes of a given link $e \in \mathcal{E}$ are denoted (slightly abusing the notation) by $v(e)$ and $w(e)$, i.e., when $e = \{v, w\}$ then $v(e) = v$ and $w(e) = w$. Analogously, the originating node of a given arc $a \in \mathcal{A}$ will be denoted by $o(a)$ and its terminating node by $t(a)$, i.e., when $a = (v, w)$ then $o(a) = v$ and $t(a) = w$. For each node $v \in \mathcal{V}$ we define $\delta(v) := \{e \in \mathcal{E} : v = v(e) \text{ or } v = w(e)\}$ (the set of links incident with node v), $\delta^+(v) := \{a \in \mathcal{A} : o(a) = v\}$ (the set of arcs outgoing from node v), and $\delta^-(v) := \{a \in \mathcal{A} : t(a) = v\}$ (the set of arcs incoming to node v). In the sequel, the number of nodes $|\mathcal{V}|$, the number of links $|\mathcal{E}|$ and the number of arcs $|\mathcal{A}|$ will be denoted by V , E , and A , respectively. Note that we use the sign “:=” for “equal by definition”.

Observe that since the graph is bi-directed, each link $e \in \mathcal{E}$ is built of two oppositely directed arcs $a(e) = (v(e), w(e))$, $a'(e) = (w(e), v(e)) \in \mathcal{A}$. Conversely, each arc $a = (v, w) \in \mathcal{A}$ has its oppositely directed counterpart $a' = (w, v) \in \mathcal{A}$ and these two arcs compose the link $e = \{v, w\} \in \mathcal{E}$ that is denoted by $e(a)$ or $e(a')$.

As already mentioned, we are considering hybrid optical networks with two types of links, FSO (free space) links and fiber (terrestrial) links. We assume that the cost of installing a fiber link between two given nodes is substantially higher as compared to an FSO link, but it guarantees high (full duplex) transmission rate of L Gbps on each of the two oppositely directed arcs composing the link. The capacity of an FSO link is modular and each module provides (full duplex) transmission rate equal to M Gbps on each of the two oppositely directed arcs of the link. Note that the fiber link transmission capacity L is typically provided by means of an optical transmission system (such as a DWDM full duplex system with bit-rate of the order of 10 Tbps) while the FSO link transmission capacity is achieved by means of parallel full duplex FSO systems between several transceiver pairs installed at the end nodes of the link. Typically, each such system can provide bit-rate M of the order of 10 Gbps.

The capacity of a link is thus provided either by several FSO modules or of just one but very large fiber module (L is 1-3 orders of magnitude greater than M). When the link capacity is optimized, the number of FSO modules installed on link $e \in \mathcal{E}$ is expressed by a variable y_e while the presence of a fiber on link e is determined by a variable q_e . That is, y_e is a non-negative integer variable determining the FSO link capacity equal to $M \cdot y_e$ while q_e is a binary variable indicating the presence ($q_e = 1$) or lack ($q_e = 0$) of a fiber on link e . In consequence, each of the two oppositely directed arcs of link e has capacity $M y_e + L q_e$. To each link $e \in \mathcal{E}$ there corresponds a non-negative cost $\xi(e)$ for the FSO module, and $\kappa(e)$ for the fiber module; it is assumed that $\kappa(e) \gg \xi(e)$. Hence, the total cost of the network is given by $\sum_{e \in \mathcal{E}} (\xi(e) y_e + \kappa(e) q_e)$. When link capacities in the network are given and fixed,

then, for each link $e \in \mathcal{E}$, $c(e)$ will denote its FSO capacity (equal to 0 or to a multiple of M), and $c'(e)$ its fiber capacity (0 or L). Hence, the total capacity of e is equal to $c(e) + c'(e)$; note that actually one of these quantities will be equal to 0. In the following we will use the vectors $y := (y_e, e \in \mathcal{E})$, $q := (q_e, e \in \mathcal{E})$, $c := (c(e), e \in \mathcal{E})$, and $c' := (c'(e), e \in \mathcal{E})$ to denote the capacity variables or parameters.

Traffic demands are directed and are represented by the set \mathcal{D} (the number of demands $|\mathcal{D}|$ will be denoted by D). Each demand $d \in \mathcal{D}$ is characterized by an ordered pair of nodes $(o(d), t(d))$ (its origin and termination) and the volume $h(d)$ (a parameter) that has to be realized from $o(d)$ to $t(d)$. Demand volumes and link capacity modules are expressed in the same units.

FSO links are subject to capacity degradation. A particular configuration of FSO link degradations is referred to as a link degradation state, and the set of the considered link degradation states (referred to as state set) (or state list) is denoted by \mathcal{S} (the number of states in \mathcal{S} , i.e., $|\mathcal{S}|$, is denoted by S). When a link, link e , say, is degraded in a particular state $s \in \mathcal{S}$, all FSO systems realized on e lose the same fraction of their capacity that is specified by a (link) degradation ratio denoted by $\beta(e, s)$ from a given set of degradation ratio values \mathcal{R} , for example $\mathcal{R} = \{0, 0.25, 0.5, 1\}$. In consequence, each degradation state $s \in \mathcal{S}$ can be identified with a vector of link degradation ratios $\beta(s) = (\beta(e, s), e \in \mathcal{E})$, where $\beta(e, s) \in \mathcal{R}$ is the fraction of capacity of link e *not available* in state s . Since the fiber link, if provided, is always fully available ($\beta(e, s) = 0$ for a fiber link e), the capacity of link $e \in \mathcal{E}$ available in state $s \in \mathcal{S}$ is equal to $(1 - \beta(e, s))My_e + Lq_e$. Note that if link e is selected to be the fiber link ($q_e = 1$) then there is no use to provide any FSO transmission systems on it ($y_e = 0$) because e is always available and its capacity is assumed sufficient to carry any traffic load induced by the traffic matrix and the demand routing used.

Finally, we assume that in any link degradation state $s \in \mathcal{S}$, each demand $d \in \mathcal{D}$ can be routed in a bifurcated way along all possible paths from $o(d)$ to $t(d)$, and the traffic flows in the particular states are independent of each other. This means that the network applies the so-called *Global Rerouting* (called also *Unrestricted Reconfiguration*) mechanism [5].

2.2. Problem formulation

The basic problem considered in this paper can be formulated as a mixed-integer program (MIP) in the node-arc notation (using the arc-flows variables $x := (x_{ad}^s, a \in \mathcal{A}, d \in \mathcal{D}, s \in \mathcal{S})$ and the link capacity variables $y = (y_e, e \in \mathcal{E})$, $q = (q_e, e \in \mathcal{E})$.) The notation is summarized in Table 1.

parameters	description
\mathcal{V}	set of nodes v ($v \in \mathcal{V}$)
\mathcal{A}	set of (directed) arcs a ($a \in \mathcal{A}$)
\mathcal{E}	set of (bi-directed) links e ($e \in \mathcal{E}$, composed of two oppositely directed arcs)
$\delta(v)$	the set of links incident to node v
$\delta^+(v), \delta^-(v)$	set of arcs outgoing from (incoming to) node $v \in \mathcal{V}$
$a(e), a'(e)$	two oppositely directed arcs corresponding to link $e \in \mathcal{E}$
$e(a)$	bi-directed link e corresponding to arc $a \in \mathcal{A}$
$c(e)$	capacity of FSO link $e \in \mathcal{E}$ (when fixed and given)
$c'(e)$	capacity of fiber link $e \in \mathcal{E}$ (when fixed and given)
M	capacity module of FSO link
L	capacity module of fiber link
$\xi(e)$	cost of one FSO capacity module on link $e \in \mathcal{E}$
$\kappa(e)$	cost of one fiber capacity module on link $e \in \mathcal{E}$ ($\kappa(e) \gg \xi(e)$)
\mathcal{D}	set of traffic demands d ($d \in \mathcal{D}$)
\mathcal{S}	set of degradation states s ($s \in \mathcal{S}$)
\mathcal{R}	given set of possible failure ratios ($\mathcal{R} = \{0, 0.25, 0.5, 1\}$)
$\beta(e, s)$	fraction of capacity lost on link $e \in \mathcal{E}$ in state $s \in \mathcal{S}$ ($\beta(e, s) \in \mathcal{R}$)
variables	description
x_{ad}^s	flow carrying traffic of demand d on arc a in state s (no-negative continuous)
y_e	number of modules installed on link e (non-negative integer)
q_e	binary variable indicating the presence ($q_e = 1$) or lack ($q_e = 0$) of a fiber on link e
$\lambda_d^v, \pi_a, \Pi_e$	dual variables
evaluation metrics	description
C^*	total optimal cost of the network
AT	average carried traffic

Table 1: Basic parameters and variables of the optimization model.

Formulation of the optimization problem in question is as follows:

250

Problem P(S) (main problem in node-arc formulation):

$$C(\mathcal{S}) = \min \sum_{e \in \mathcal{E}} (\xi(e)y_e + \kappa(e)q_e) \quad (1a)$$

$$\begin{aligned} \sum_{a \in \delta^+(v)} x_{ad}^s - \sum_{a \in \delta^-(v)} x_{ad}^s = \\ = \begin{cases} h(d) & \text{if } v = o(d) \\ -h(d) & \text{if } v = t(d), \\ 0, & \text{otherwise} \end{cases} \quad d \in \mathcal{D}, v \in \mathcal{V}, s \in \mathcal{S} \end{aligned} \quad (1b)$$

$$\sum_{d \in \mathcal{D}} x_{ad}^s \leq (1 - \beta(e(a), s))My_{e(a)} + Lq_{e(a)}, \quad a \in \mathcal{A}, s \in \mathcal{S} \quad (1c)$$

$$x \geq 0 \text{ continuous; } y \geq 0 \text{ integer; } q \text{ binary.} \quad (1d)$$

Above, x_{ad}^s is the flow on arc a dedicated to carry the traffic of demand d in state s . The objective of P(S), i.e., minimizing the cost of links, is specified in (1a). Constraints (2b) are

the flow conservation equations for each demand d at each node v in each state s , assuring the realization of $h(d)$ for each d in each state s . Finally, the capacity constraint (1c) ensures that the capacity of link e is not exceeded in any state s .

Note that because of high costs $\kappa(e)$ of the fiber links (that by assumption are significantly larger than costs $\xi(e)$ of the FSO links), the optimal solution of problem $P(\mathcal{S})$ will, first of all, minimize the total cost (equal to $\sum_{e \in \mathcal{E}} \kappa(e)q_e$) of those fiber links that are necessary to make the network (demand-wise) connected in all states in \mathcal{S} .

2.3. Solution methodology

Because of modular links, problem $P(\mathcal{S})$ is \mathcal{NP} -hard even for a polynomial number of states in set \mathcal{S} (see [4]). The direct approach to $P(\mathcal{S})$ requires a MIP solver that makes use of the branch-and-bound (B&B) approach. B&B requires, at each B&B node, solving a linear subproblem (a relaxation of $P(\mathcal{S})$ with y and q relaxed to be continuous, some entries in y additionally bounded and some entries in q fixed, see [18]) involving a large number of arc-flow variables $x := (x_{ad}^s, a \in \mathcal{A}, d \in \mathcal{D}, s \in \mathcal{S})$ which may lead to excessive computation times. For this reason, we have developed a more efficient approach based on the Benders decomposition method [19]. This approach falls into the category of cut generation algorithms [18] and is based on successive iterations, each consisting of solving a *master problem* and a set of *feasibility tests*. The master problem contains a set of capacity constraints that are successively extended with new cuts found by means of feasibility tests performed on the current optimal capacity solution of the master. This is explained in detail below.

2.3.1. Feasibility testing

Let \mathcal{C} denote the set of all pairs (c, c') of capacity vectors feasible for (1) and suppose we wish to test whether a given pair (c, c') is in \mathcal{C} . Because the demand routing in a particular link degradation state is independent of the demand routing in the remaining states, we can perform the feasibility test in question by checking the feasibility separately for each state. This is done by means of the following linear programming problem.

Problem F(c, c', s) (feasibility of (c, c') in state s):

$$O(c, c', s) = \min \sum_{e \in \mathcal{E}} z_e \tag{2a}$$

$$\begin{aligned} \sum_{a \in \delta^+(v)} x_{ad} - \sum_{a \in \delta^-(v)} x_{ad} = \\ = \begin{cases} h(d) & \text{if } v = o(d) \\ -h(d) & \text{if } v = t(d), \\ 0 & \text{otherwise} \end{cases} \quad d \in \mathcal{D}, v \in \mathcal{V}, s \in \mathcal{S} \end{aligned} \tag{2b}$$

$$\sum_{d \in \mathcal{D}} x_{ad} \leq (1 - \beta(e(a), s))c(e(a)) + c'(e(a)) + z_{e(a)}, \quad a \in \mathcal{A} \tag{2c}$$

$$x, z \geq 0 \text{ and continuous.} \tag{2d}$$

Above, $O(c, c', s)$ expresses the minimum of the sum of links' overloads measured by the components of the vector $z := (z_e, e \in \mathcal{E})$. The test is proper since (c, c') is feasible for s if, and only if, $O(c, c', s) = 0$. If the result of the test is negative (i.e., $O(c, c', s) > 0$) we need to find an inequality that separates (c, c') from \mathcal{C} . This can be done by considering the dual to

285 formulation (2) (whose variables $\lambda := (\lambda_d^v, v \in \mathcal{V}, d \in \mathcal{D})$ and $\pi := (\pi_a, a \in \mathcal{A})$ correspond to the primal constraints (2b) and (2c), respectively):

Problem D(c, c', s) (dual to F(c, c', s)):

$$W(c, c', s) = \max \left\{ \sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) + \right. \\ \left. - \sum_{e \in \mathcal{E}} \Pi_e ((1 - \beta(e, s))c(e) + c'(e)) \right\} \quad (3a)$$

$$\Pi_e = \pi_{a(e)} + \pi_{a'(e)}, \quad e \in \mathcal{E} \quad (3b)$$

$$\Pi_e \leq 1, \quad e \in \mathcal{E} \quad (3c)$$

$$\lambda_d^{o(d)} = 0, \quad d \in \mathcal{D} \quad (3d)$$

$$\lambda_d^{t(a)} - \lambda_d^{o(a)} \leq \pi_a, \quad a \in \mathcal{A}, d \in \mathcal{D} \quad (3e)$$

$$\Pi, \pi \geq 0 \text{ continuous, } \lambda \text{ continuous.} \quad (3f)$$

Let $\lambda^*, \pi^*, \Pi^* = (\Pi_e^*, e \in \mathcal{E})$ be an optimal solution of problem D(c, c', s). Since, by the strong duality property [20], $W(c, c', s) = O(c, c', s)$ (where $O(c, c', s)$ is defined by (2a)), the inequality that separates (c, c') from \mathcal{C} (provided $W(c, c', s) > 0$) is as follows:

$$\sum_{e \in \mathcal{E}} \Pi_e^* (1 - \beta(e, s)) M y_e + \sum_{e \in \mathcal{E}} \Pi_e^* L q_e \geq \sum_{d \in \mathcal{D}} \lambda_d^{t(d)*} h(d). \quad (4)$$

2.3.2. A cut generation algorithm for an explicit state set \mathcal{S}

An iterative algorithm for solving (1) is given below. It results from applying Benders' decomposition [19] to P(\mathcal{S}) (this particular application can be found in [4]). In each iteration, first the master problem involving only the capacity variables y, q is solved, and then its optimal solution (y^*, q^*) is tested for feasibility with respect to P(\mathcal{S}). If the test is positive, the algorithm is stopped and solution (y^*, q^*) is optimal for P(\mathcal{S}). If not, new inequalities deduced from the feasibility tests are added to the master problem and the algorithm is iterated. (Below, the notation $(y, q) \in \Omega$ means that variables y and q fulfill all inequalities in the set of inequalities Ω .)

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A1: cut generation algorithm for state set \mathcal{S}

Input: *network topology (nodes/links), traffic demands matrix, set of degradation states \mathcal{S} .*

Output: *number of FSO modules for each link, location of terrestrial fibers.*

Step 0: $\Omega := \{y \geq 0\}$.

Step 1: Solve the master problem:

$$\text{minimize } \sum_{e \in \mathcal{E}} (\xi(e)y_e + \kappa(e)q_e) \quad (5a)$$

$$\text{subject to } (y, q) \in \Omega \quad (5b)$$

y integer, q binary

305 and put $c(e) = M y_e^*, c'(e) = L q_e^*, e \in \mathcal{E}$, where (y^*, q^*) is an optimal solution of (5).

Step 2: For each $s \in \mathcal{S}$ solve the feasibility test (3) and if $W(c, c', s) > 0$, then add inequality (4) to Ω .

Step 3: If no inequalities have been added to Ω in Step 2, then stop: (y^*, q^*) is an optimal solution for problem (1). Otherwise go to Step 1.

310 3. K -set optimization approach

It is important to note that algorithm A1, and, for that matter, also the direct solution of problem (1), can become ineffective when the state set \mathcal{S} contains a large number of states in the (explicit) state list \mathcal{S} , as this can lead to an excessive number of feasibility tests (3) to be performed in each iteration of A1, or too many flow variables in (1). Another point is that a representative state set \mathcal{S} may be difficult to construct or it may not represent the precise set of degraded states that may occur in the future. Hence, it would be advantageous to have a way to deal with achieving network resilience to adverse weather conditions at a minimum cost more effectively.

Such a way was proposed in [3], where we used a special kind of “idealized” state sets, called K -sets, parameterized by an integer value K less than or equal to the number E of links in the network. For a given $0 \leq K \leq E$, the K -set contains the states corresponding to all combinations of K , or less, links simultaneously degraded. The proposed approach consists of using K -sets (for a set of selected values of parameter K) as an input (instead of the true state set \mathcal{S}) in the resilient network dimensioning problem, and then testing the so obtained optimal link capacities (called the robust solution) on the reference degradation set S . In Section 5, such a K -set will be called *link K -set* and denoted by “link(K)”.

In the present study we consider an enhanced type of a K -set, where instead of link degradations we consider node degradations. The main motivation for this enhancement comes from the nature of degradation caused by the nature of weather conditions. That is, when bad weather hits a given subarea of the network area then it affects the transceivers in all nodes in the subarea and, in consequence, all links incident to the affected nodes are degraded simultaneously. This results in a sort of geographic-aware uncertainty sets, which are expected to cover the set \mathcal{S} more effectively than link K -sets.

Such a K -set, denoted with $\mathcal{Q}(K)$, is defined, for any integer $K = 1, 2, \dots, V$, as the set of binary vectors $\mathcal{Q}(K) := \{u = (u_v, v \in \mathcal{V}) : \sum_{v \in \mathcal{V}} u_v \leq K, u_v \in \{0, 1\}\}$. Each $u \in \mathcal{Q}(K)$ determines a state $s(u)$ by assuming that, for each $e \in \mathcal{E}$, the degradation ratio $\beta(e, s(u))$ is defined through equality $1 - \beta(e, s(u)) = (1 - \beta(e)u_{v(e)})(1 - \beta(e)u_{w(e)})$ for a given fixed fraction $\beta(e)$ ($0 < \beta(e) \leq 1$). It follows that $\beta(e, s(u)) := \beta(e)(u_{v(e)} + u_{w(e)}) - \beta(e)^2 u_{v(e)} u_{w(e)}$, $e \in \mathcal{E}$. Thus, the support of vector u , i.e., $\{e \in \mathcal{E} : \beta(e, s(u)) > 0\}$ determines the set of links degraded in state $s(u)$, where each such link is degraded with ratio $\beta(e)$ (when only one of its end nodes is degraded) or $2\beta(e) - \beta(e)^2$ (when both end nodes of e are degraded). The set $\{s(u) : u \in \mathcal{Q}(K)\}$ of the states determined by K -polytope $\mathcal{Q}(K)$ will be denoted by $\mathcal{S}(K)$. In Section 5, the so described K -set will be called *node K -set* and denoted by “node(K)”.

The solution approach for the problem involving the state set $\mathcal{S}(K)$ is based on algorithm A1 formulated in Section 2.3.2. The difference is that now a modified feasibility test to generate the cuts is used: instead of feasibility tests (3) executed for each state s on the

explicit list of states \mathcal{S} , only one test is considered:

$$W(c, c') = \max \left\{ \sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \Pi_e (c(e) + c'(e)) + \sum_{e \in \mathcal{E}} \Pi_e \beta(e) c(e) (u_{v(e)} + u_{w(e)}) - \sum_{e \in \mathcal{E}} \Pi_e \beta(e)^2 c(e) u_{v(e)} u_{w(e)} \right\} \quad (6a)$$

$$\Pi_e = \pi_{a(e)} + \pi_{a'(e)}, \quad e \in \mathcal{E} \quad (6b)$$

$$\Pi_e \leq 1, \quad e \in \mathcal{E} \quad (6c)$$

$$\lambda_d^{o(d)} = 0, \quad d \in \mathcal{D} \quad (6d)$$

$$\lambda_d^{t(a)} - \lambda_d^{o(a)} \leq \pi_a, \quad a \in \mathcal{A}, d \in \mathcal{D} \quad (6e)$$

$$\sum_{v \in \mathcal{V}} u_v \leq K \quad (6f)$$

$$\Pi, \pi \geq 0 \text{ continuous}; \lambda \text{ continuous}; u \text{ binary.} \quad (6g)$$

Certainly, to obtain a proper mixed-integer programming formulation we need to eliminate non-linearities in (6a) induced by products $\Pi_e \cdot u_{v(e)}$, $\Pi_e \cdot u_{w(e)}$ and $\Pi_e \cdot u_{v(e)} \cdot u_{w(e)}$, where $u_{v(e)}, u_{w(e)}$ are binary and Π_e is between 0 and 1. This can be easily done by introducing, for each such product, a continuous non-negative variable X'_e, X''_e and Y_e , respectively, together with additional constraints $X'_e \leq \Pi_e, X'_e \leq u_{v(e)}, X'_e \geq \Pi_e + u_{v(e)} - 1, X''_e \leq \Pi_e, X''_e \leq u_{w(e)}, X''_e \geq \Pi_e + u_{w(e)} - 1, Y_e \leq \Pi_e, Y_e \leq u_{v(e)}, Y_e \leq u_{w(e)}, Y_e \geq \Pi_e + u_{v(e)} + u_{w(e)} - 2$ that imply $X'_e = \Pi_e u_{v(e)}, X''_e = \Pi_e u_{w(e)}$ and $Y_e = \Pi_e u_{v(e)} u_{w(e)}$.

Note that for any fixed feasible u the value of the objective function defined by the right-hand side of (6a) maximized over λ, π, Π is equal to $W(c, c', s)$ (see (3a)) for $s = s(u)$. Since in (6) we are maximizing also over u , we finally observe that $W(c, c') = \max_{s \in \mathcal{S}(K)} W(c, c', s)$, as required.

Now assume that $W(c, c') > 0$. Then, the Benders inequality induced by the optimal solution $\lambda^*, \pi^*, \Pi^*, u^*$ of (6) is as follows:

$$\sum_{e \in \mathcal{E}} \Pi_e^* (1 - \beta(e) (u_{v(e)}^* + u_{w(e)}^*) + \beta(e)^2 u_{v(e)}^* u_{w(e)}^*)) M y_e + \sum_{e \in \mathcal{E}} \Pi_e^* L q_e \geq \sum_{d \in \mathcal{D}} \lambda_d^{t(d)*} h(d). \quad (7)$$

Given the above, for the purpose of the K -set approach, algorithm A1 is modified by replacing the feasibility tests (3) performed for all $s \in \mathcal{S}$ with just one feasibility test (6). Clearly, in such a modified algorithm (called A2 in the following) the feasibility test in Step 2 delivers only one Benders inequality (7) (in A1 up to S inequalities can be generated) – the one that corresponds to the state $s(u^*)$ for which, as already mentioned, the value of $W(c, c', s(u))$ resulting from the feasibility test (3) is maximal over all $s(u) \in \mathcal{S}(K)$.

Observe that in general the state set $\mathcal{S}(K)$ does not have to reflect any realistic set of states, in particular the values $\beta(e, s(u))$ need not correspond to real degradation ratios from the set \mathcal{R} defined in Section 2.1. In fact, set $\mathcal{S}(K)$ is meant to be used for network dimensioning so that for properly chosen parameters K and $\beta(e), e \in \mathcal{E}$, the resulting link capacities y, q will be sufficient to carry the demand traffic in a given realistic reference degradation state \mathcal{S} and the cost of y, q will be near to the optimal cost $C(\mathcal{S})$ for problem (1). Our aim is to help network control to achieve a carefully designed and operated network. Our concern is not the short-term operational time-horizon, but the long-term planning horizon, and our aim is to elaborate a link dimensioning procedure best suited to most of the expected weather conditions. In this aspect, the proper choice of parameters K and $\beta(e)$ means the best parameter combination giving a low cost and high traffic satisfaction

with respect to the reference degradation set composed of link degradation sets induced by weather states recorded during a given time period (what we may call a realistic degradation set). We show in our numerical results that for different values of K we have different values of traffic satisfaction for different total link cost. This information may be useful to network managers to decide on appropriate link dimensioning according to their economical strategy.

Finally, we note that formulation (6) can be extended to handle combinations of both links and nodes degradations in a unique K -set. We recall that the problem in hand is \mathcal{NP} -hard and remains hard to solve for large size instances. Clearly, algorithm A2 is computationally more efficient than A1 as it replaces the feasibility tests (3) performed for all $s \in \mathcal{S}$ in A1 with a single test (6).

4. Network optimization procedure

When it comes to applying the above described algorithms A1 and A2, computational effectiveness should be considered. It turns out that keeping both vectors of decision variables (y and q) in the master problem (5) would lead to excessive computation times when applying A1 and A2 directly to the network instances of the sizes considered in the numerical study presented in Section 5. Therefore in the actual implementation used in the study, we took advantage of two assumptions: (i) the cost of the fiber link (equipped with an optical transmission system) is much higher than the cost of an FSO system, i.e., $\kappa(e) \gg \xi(e), e \in \mathcal{E}$, and (ii) the capacity L of the fiber is virtually infinite since an optical fiber transmission system would be sufficient to carry even the sum of all demand volumes, i.e., $\sum_{d \in \mathcal{D}} h(d)$. Thus, in practice we will install only a few fibers (at most $V - 1$), just to ensure full demand connectivity (or, at least, high connectivity) in all states in \mathcal{S} , where demand connectivity in a given state s means that for each demand there exists a path between the demand's end nodes composed of links with degradation ratios $\beta(e, s)$ strictly less than 1 (in such a case, the state is called *connected*, and *disconnected* otherwise). In fact, in the case when all states are connected, the fibers will not be installed at all because of their high cost. Clearly, when a fiber link is installed then no additional FSO systems are needed to be installed on that link.

Taking the above into account, in our numerical experiments (and thus in executing algorithms A1 and A2) we assume that vector q specifying the assignment of fibers is predefined, i.e., fixed and given, so that only the vector y of FSO link capacities represents the problem variables. More specifically, we run numerical experiments for instances where the subset of fiber links has been fixed, and we do this separately for consecutive subsets with increasing number of (fiber) links, starting from one-link subsets, and choose the cheapest subset that assures network connectivity. Thus, there is no need for variables to determine the subset of links where the fiber is installed. A detailed explanation of the optimization procedure used for the numerical study described in Section 5 is given below.

Network optimization procedure - Phase 1

1. For a given set of states \mathcal{S} (with pairwise different link degradation ratio vectors $\beta(s) := (\beta(e, s), e \in \mathcal{E}), s \in \mathcal{S}$) we first identify all states that contain disconnected demands assuming that no fiber links are used ($q = 0$). We denote the set of such states with $\mathcal{S}''(0)$ and call it *the set of disconnected states*. The complementary set, denoted with $\mathcal{S}'(0) := \mathcal{S} \setminus \mathcal{S}''$, is called *the set of connected states*.
2. We apply algorithm A1 assuming $q = 0$ for set $\mathcal{S}'(0)$. Let the resulting link capacity vector be denoted by $y(0)$.
3. Assuming $c = y(0)$ and $c' = 0$, for each state $s \in \mathcal{S}''(0)$ we solve the following version of problem F(c, c', s):

$$Q(c, c', s) = \min \sum_{d \in \mathcal{D}} z_d \quad (8a)$$

$$\begin{aligned} & \sum_{a \in \delta^+(v)} x_{ad} - \sum_{a \in \delta^-(v)} x_{ad} = \\ & = \begin{cases} h(d) - z_d & \text{if } v = o(d) \\ -h(d) + z_d & \text{if } v = t(d), \quad d \in \mathcal{D}, v \in \mathcal{V} \\ 0 & \text{otherwise} \end{cases} \quad (8b) \end{aligned}$$

$$\sum_{d \in \mathcal{D}} x_{ad} \leq (1 - \beta(e(a), s))c(e(a)) + c'(e(a)), \quad a \in \mathcal{A} \quad (8c)$$

$$x, z \geq 0 \text{ and continuous.} \quad (8d)$$

In this way we obtain the values $T(s, 0) := Q(c, c', s)$, which express the total *traffic not carried* in the states $s \in \mathcal{S}''(0)$ when no fibers are provided. Next, we compute the quantity $T(0) := \sum_{s \in \mathcal{S}''(0)} n(s) \cdot T(s, 0)$ where $n(s)$ denotes the number of real weather states that correspond to the same state s in the reference set \mathcal{S} . Using $T(0)$ we compute an important quality measure, namely the *fraction of carried traffic* by the network averaged over all weather states in \mathcal{W} considered in the optimization process. This value is given by the formula $AT(0) := \frac{H - T(0)}{H}$ where $H := \sum_{s \in \mathcal{S}} \sum_{d \in \mathcal{D}} n(s)h(d)$.

4. We look for the best network solution in terms of extending demand connectivity achieved by replacing a selected subset of $m \geq 1$ FSO links by fiber links. We first consider adding only one fiber link ($m = 1$). For that we simply examine each FSO link in \mathcal{E} one by one. For a given fixed fiber link e , we update the sets \mathcal{S}' and \mathcal{S}'' , and apply algorithm A1 as above but assuming $q = 0$ except for $q_e = 1$. Then for the so obtained $y(1)$ we compute $T(1)$ in the same way as $T(0)$. After doing that for all links we finally select the link e^* with the smallest $T(1)$, i.e., the smallest amount of traffic lost, and select the so defined network configuration for the optimal solution. The fraction of carried traffic is then defined as $AT(1) := \frac{H - T(1)}{H}$.
5. We repeat the above calculations for $m = 2$, i.e., for every pair of $(e, e') \in \mathcal{E}^{[2]}$, and find the best placement of a pair of fiber links (e^*, e'^*) that results in the smallest amount $T(2)$ of traffic lost. The fraction of carried traffic $AT(2)$ is defined analogously to $AT(1)$, using $T(2)$ instead of $T(1)$.
6. For $m \geq 3$ we apply a simplified approach: the choice of the m -th fiber is done by adding the best choice fiber link to the set of $m - 1$ fiber links already found for the $m - 1$ case. The values of $AT(m)$ are calculated as before, using appropriate $T(m)$ values. Clearly, once $T(m)$ becomes equal to 0, Phase 1 is terminated and the so obtained selection of fiber links is used in Phase 2.

7. We optimize the network using algorithm A2 for selected cases of link and node K -sets, fixing the sets of $m = 0, 1, \dots$ fiber links found in Phase 1. Since in all cases we assume that for all FSO links the link degradation ratio $\beta(e)$ is below 0.5, we use a different formula for $\beta(e, s(u))$ when $u_{v(e)} = u_{w(e)} = 1$ (i.e., when both end nodes of link e are degraded in the considered node K -set), namely $\beta(e, s(u)) = 2\beta(e)$ (instead of $\beta(e, s(u)) = 2\beta(e) - \beta(e)^2$). Thanks to that we can reduce the computation time of the feasibility test (6) whose objective function is now modified as follows:

$$W(c, c') = \max \left\{ \sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \Pi_e ((1 - \beta(e)(u_{v(e)} + u_{w(e)}))c(e) + c'(e)) \right\}. \quad (9)$$

8. The vectors y obtained for the considered list of K -sets and the three cases of $m = 0, 1, 2$ fibers (in each case vector q is predefined), are tested for the traffic loss $T(m)$ they induce for the original state set \mathcal{S} in the same way as in Phase 1. Then the fraction of carried traffic $AT(m)$ is calculated as in Phase 1.

5. Numerical study

Below we present results of a numerical study illustrating the performance of our optimization approach for various weather scenarios. Two network instances are studied – a medium-size network instance taken from the literature, and our own network instance prepared for the purpose of paper [3]. All numerical calculations were performed on an Intel (R) Core (TM) i7-4610M, 3.0 GHz computer with 16 GB of RAM. The algorithms were coded in Java and the optimization problems were solved with IBM ILOG CPLEX 12.6 (using Concert Technology) running with the default setting.

For obtaining the results reported below we applied the network optimization procedure described in Section 4 with one exception: we were solving the main problem $P(\mathcal{S})$ (problem (1)) directly with CPLEX instead of using algorithm A1. For all the considered K -sets the link degradation ratio $\beta(e) = 0.25$ was assumed uniformly for all FSO links $e \in \mathcal{E}$.

5.1. Paris Metropolitan Area network

In this section we will study of a realistic FSO network instance with 12 nodes and 21 links, depicted in Figure 1. We call this instance PMAN (Paris Metropolitan Area network) since it was created using the real data for the Paris Metropolitan Area. It should be noted that although all the data used to construct the PMAN instance are real, the instance itself does not exist in reality and was elaborated only for our own purpose. A full characterization of PMAN is available in [3].

We examined the weather conditions in Paris Metropolitan Area observed during one year period from January 1, 2016 until December 31, 2016 available in [21] and extracted a set \mathcal{W} of all $366 \times 24 = 8784$ hourly weather states. Then we translated each such obtained weather state to the corresponding link degradation states. We observed that among all 8784 states there are 7531 connected states (85.74%) and 1253 (14.26%) disconnected states. Among the 7531 connected states, 68.46% are *nominal states* with no degraded links. **There are 20.95% states where the nominal capacity of all affected links is reduced by 25% (degradation ratio**

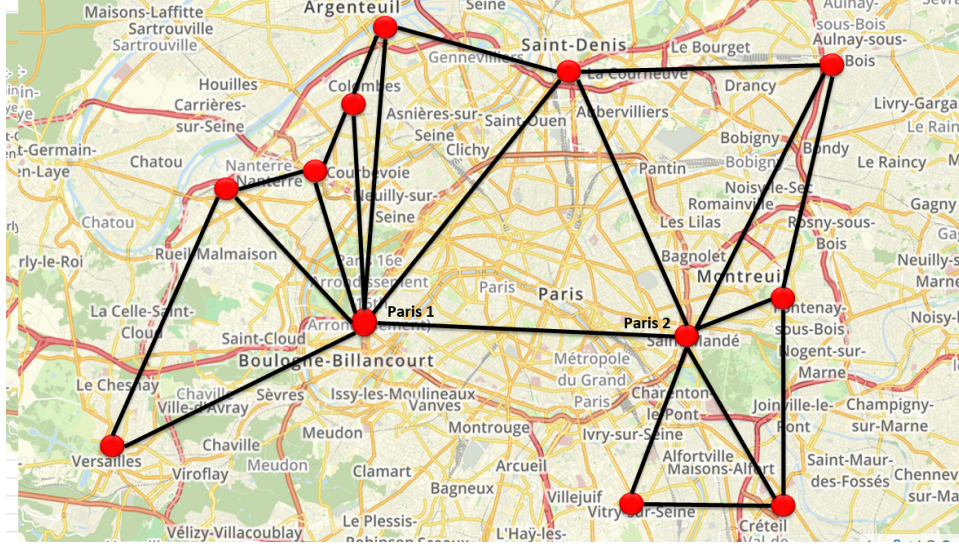


Figure 1: Paris Metropolitan Area network (PMAN).

0.25). In the remaining 10.59% of the states there are links degraded with ratio 0.5 and/or 1. Finally, we have formed the reference set of states \mathcal{S} (i.e., the set of all pairwise different link degradation states $\beta(s)$) that turns out to contain $S = 539$ states. Within \mathcal{S} , there are $S'(0) = 206$ connected states (forming set $\mathcal{S}'(0)$, see step 1 in Phase 1 of the network optimization procedure described in Section 4) and $S''(0) = 333$ states (forming the set of disconnected states $\mathcal{S}''(0)$).

The individual traffic demands in PMAN (the demands are directed and there are 132 of them) are of the order of several Gbps, except for the traffic between Paris 1 and Paris 2 with much higher demand of 74.38 Gbps (in each direction). The full traffic matrix for PMAN can be found in [3]. An $M = 10$ Gbps FSO module is assumed, as for example the module of the LightPointe's AireLink 80 10Gbps system [17]; the cost of one such module on each link is equal to 1, i.e., $\xi(e) = 1, e \in \mathcal{E}$.

Table 2 shows the influence of using fiber links on the FSO link cost $C^*(m)$ (where $C^*(m) := \sum_{e \in \mathcal{E}} y_e^*$ and y^* is the vector of optimal sizes of FSO links when m fibers selected in Phase 1 are installed), and on the fraction of the average carried traffic $AT(m)$, $m = 1, 2, \dots, 11$. The consecutive rows depict, for each m , the values of $C^*(m)$ and $AT(m)$, and, additionally, the percentage of disconnected states in \mathcal{W} and the total computational time. To obtain these results we dimensioned the network applying Phase 1 of the optimization procedure described in Section 4 for appropriate sets of connected states \mathcal{S}' and sets of disconnected states \mathcal{S}'' . Since it happens that the first choice fiber link e^* is placed between Paris 1 and Paris 2, the FSO link cost for $m = 1$ is drastically decreased with respect to the case with no fibers ($m = 0$). This is because the cost of the fiber links is not counted in $C^*(m)$ and the fiber links do not suffer from capacity degradation. In effect, when the fiber between Paris 1 and Paris 2 is provided, the large traffic demand between these nodes (plus some other traffic) is carried with no cost. Adding consecutive fibers keeps on decreasing the FSO cost until all demands become connected in all states in \mathcal{S} (using only the fiber links), which happens for $m = 11$. Next we observe that the carried traffic grows by 2 percentage points when the first fiber is added (from 95.19% to 97.16%), and the percentage of disconnected

states in \mathcal{W} drops by 4.4 percentage points. When m is increased beyond 1, the carried traffic grows monotonically (but slowly) to 100% while the percentage of disconnected states decreases gradually but not monotonically to 0%. This happens because in Phase 1 of the optimization procedure we do not control the number of disconnected states, as we do it only for the carried traffic. The computation time ranges from 227 to 12650 seconds, and in general tends to grow with m . As a conclusion we may state that adding only one or two fibers to PMAN would be beneficial as this will non-negligibly increase the carried traffic (and connectivity) and at the same significantly decrease the number of installed FSO systems. Installing more fibers could of course be considered but benefit from it would be low.

Table 2: Impact of fiber links, PMAN dimensioned for $\mathcal{S}'(0)$.

number of fibers m	FSO cost $C^*(m)$	carried traffic $AT(m)$ [%]	disconnected states [%]	time [s]
$m=0$	145	95.19	19.75	227.1
$m=1$	42	97.16	14.36	1251.8
$m=2$	36	97.61	10.83	1859.3
$m=3$	26	97.75	12.91	1726.1
$m=4$	21	98.26	11.67	1000.9
$m=5$	20	98.47	12.01	2269.9
$m=6$	18	98.60	10.85	6109.8
$m=7$	14	98.89	10.59	3380.4
$m=8$	13	98.99	7.10	6518.7
$m=9$	12	99.15	5.92	12650.3
$m=10$	4	99.71	4.06	4999.7
$m=11$	0	100.00	0.00	5593.1

Table 3 illustrates efficiency of using node (and link) K -sets assuming link degradation ratio $\beta(e)$ equal to 0.25 uniformly for all links. Three cases are considered: 0-fiber ($m = 0$: no fibers added), 1-fiber ($m = 1$: one fiber added) and 2-fibers ($m = 2$: two fibers added). For each case, the cost, fraction of carried traffic, and computation time are given in the consecutive rows of the table. The results for the explicit state list \mathcal{S}' are taken from the rows $m = 0, 1, 2$ in Table 2. The results for the node K -sets (namely node(1), node(2)) were calculated by means of Phase 2 of the optimization procedure, while the results for the link K -sets (link(3), link(4), link(6), link(7)) were computed by an algorithm analogous to A2 used in [3].

Table 3: Impact of K -sets, PMAN.

	0-fiber case ($m=0$)			1-fiber case ($m=1$)			2-fibers case ($m=2$)		
	$C^*(0)$	$AT(0)$ [%]	time [s]	$C^*(1)$	$AT(1)$ [%]	time [s]	$C^*(2)$	$AT(2)$ [%]	time [s]
set S'	145	95.19	227.1	42	97.16	1251.8	36	97.61	1859.3
link(3)	37	93.15	41.1	21	96.19	39.8	19	96.72	40.1
link(4)	37	93.21	35.5	21	96.29	36.3	19	97.05	38.2
node(1)	36	93.19	36.9	20	96.19	37.4	18	96.85	37.2
node(2)	37	93.25	42.4	21	96.30	44.6	20	97.10	44.8
link(6)	37	93.16	37.5	21	96.26	37.7	20	97.03	37.5
link(7)	37	93.26	39.3	22	96.94	40.3	20	97.51	39.8

The results of Table 3 for the 0-fiber case ($m = 0$) are shown in a graphical form in Figure 2. For completeness, Figure 2 contains additional results (for link(0), link(1), link(2), link (5), link(8), link(9)). Note that link(0) corresponds to the case when only one state is considered, namely the nominal state (all links fully available), so that its cost $C^* = 32$ is the lower bound for the cost for all other cases.

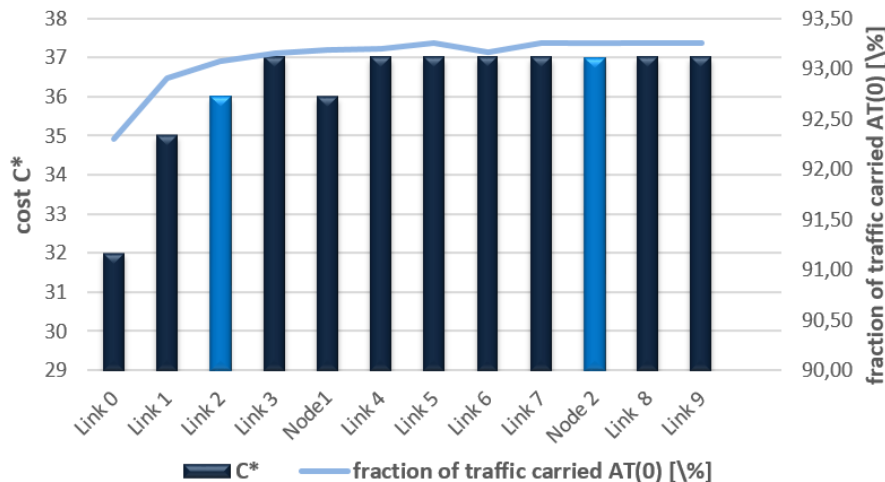


Figure 2: FSO link cost C^* and carried traffic, PMAN.

Table 3 reveals that in fact all the considered K -sets (except for link(0)) give virtually the same fraction of carried traffic for all three values of m ($m = 0, 1, 2$), which are (around) 93.1%, 96.2%, and 97%, respectively. These values are less than the corresponding values for the explicit state list case (in the row named set S') by, respectively, 2, 1, and 0.5 percentage points. These are acceptable values from the quality of traffic handling viewpoint. Moreover, the costs C^* for the K -sets (except for link(0)) are almost the same, and substantially smaller than for the case of the explicit list S' . Taking this into account, and the fact that the K -set approach requires substantially shorter (of at least one order of magnitude) computation times as compared to the explicit state list approach, the former approach is a reasonable alternative to the latter, especially when the weather state set \mathcal{W} , and thus the reference state set \mathcal{S} are not available and/or hard to forecast. Examining Figure 2 we also note that

540 the node K -set node(1) should be recommended for PMAN dimensioning because of its low cost $C^* = 36$ which gives the gain equal to $\frac{37-36}{37-32} = 20\%$ with respect to other K -sets whose cost is equal to 37 (except for link(1)). The case link(1) (with cost $C^* = 35$) could also be considered for the dimensioning with cost gain $\frac{37-35}{37-32} = 40\%$ but at the expense of a slightly decreased fraction of carried traffic. Moreover, for the 1-fiber and 2-fiber cases the use of
 545 node(1) becomes even more beneficial cost-wise.

5.2. Network *nobel-germany*

In this section we will discuss results for a moderate-size network example derived from the instance called *nobel-germany* that can be found in SNDlib [22]. The instance is composed of 17 nodes and 26 undirected links, and there are 121 undirected traffic demands (so that only 89% of all $\frac{17 \times 16}{2} = 136$ undirected pairs of nodes generate demand) expressed in Mbps (detailed data is available at sndlib.zib.de). As for PMAN, the cost of one FSO capacity module on each link is 1 but for this particular instance, the FSO capacity module $M = 1$ Mbps (instead of 10 Gbps) is assumed. This is done to match the values of traffic demands (given in SNDLib) which are of the order of several Mbps. Note that if the traffic demands were scaled up by the factor of 1000 (and expressed in Gbps) then the results presented below would be valid for the (realistic) module size $M = 10$ Gbps as well.
 555

Observe that since the links and demands in *nobel-germany* specified in SNDlib are undirected, the formulations of the master problem and the feasibility tests formulated in the previous sections must be (slightly) adjusted. A way to do it is described in [3] (Section 6.1.2).

560 Using the weather records from [21], three different weather scenarios were constructed following the guidelines of the Köppen-Geiger climate classification system [23]: the first scenario, typical for southern Europe (called “Mediterranean climate” scenario), the second, typical for middle Europe (called “moderate continental climate” scenario), and the third, typical for northern Europe (called “cold continental climate” scenario). These scenarios were then used to estimate link degradation state for each hour of a typical year for *nobel-germany*.
 565 A detailed description of such an estimation process is given in [24]. Selected characteristics of link degradation states for *nobel-germany* corresponding to all $365 \times 24 = 8760$ hours of the year are given in Table 4.

Table 4: Distribution of link degradation states

	Mediterranean climate	moderate continental climate	cold continental climate
nominal states	68.46%	47.96%	27.47%
degradation ratio 25%	20.95%	25.48%	30.01%
degradation ratio 50%	3.76%	9.45%	15.18%
degradation ratio 100%	0.6%	4.01%	7.43%
various degradation ratios	6.23%	13.01%	19.91%

For each of the above weather scenarios (which in fact correspond to the sets of weather states \mathcal{W}) we performed the same calculations as for PMAN (Section 5.1), except for the
 570 calculations reported in Table 2 which are skipped. That is, we repeated the calculations for PMAN and organized them in a way analogous to the one used in Table 3 and Figure 2. The results for *nobel-germany* are reported below and are presented in:

- Table 5 and Figure 3 for scenario “Mediterranean climate”
- Table 6 and Figure 4 for scenario “moderate continental climate”
- Table 7 and Figure 5 for scenario “cold continental climate”.

Table 5: Impact of using K -set node (*nobel-germany*/Mediterranean climate).

	0-fiber case (m=0)		1-fiber case (m=1)		2-fibers case (m=2)	
	$C^*(0)$	$AT(0)$ [%]	$C^*(1)$	$AT(1)$ [%]	$C^*(2)$	$AT(2)$ [%]
set S'	388	99.84	312	99.85	250	99.89
link(2)	144	99.53	133	99.58	123	99.60
link(3)	149	99.58	135	99.64	124	99.65
node(1)	139	99.52	129	99.59	119	99.63
node(2)	152	99.62	140	99.67	130	99.68
link(7)	155	99.59	145	99.62	137	99.65
link(8)	155	99.63	145	99.67	137	99.68
link(9)	156	99.63	146	99.68	137	99.68

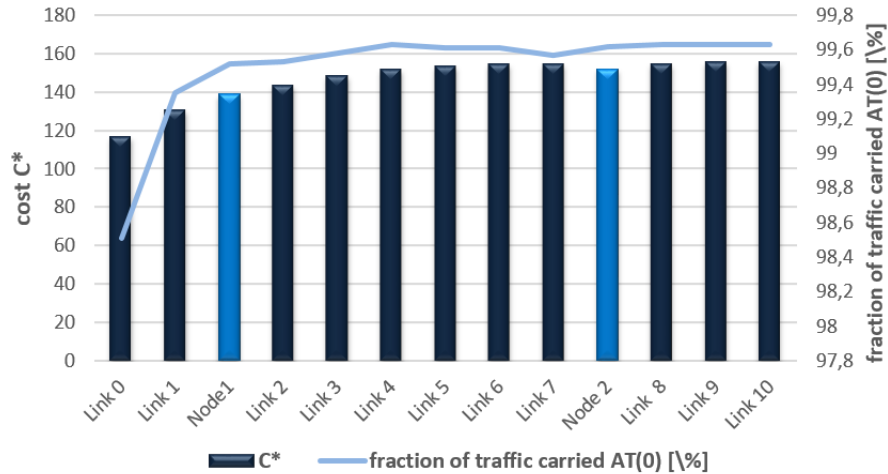


Figure 3: Cost and carried traffic for 0-fiber case (*nobel-germany*/Mediterranean climate).

Table 6: Impact of using K -set node (*nobel-germany/moderate* continental climate).

	0-fiber case (m=0)		1-fiber case (m=1)		2-fibers case (m=2)	
	$C^*(0)$	$AT(0)$ [%]	$C^*(1)$	$AT(1)$ [%]	$C^*(2)$	$AT(2)$ [%]
set S'	419	99.04	401	99.05	319	99.22
link(2)	144	95.01	133	95.26	123	95.68
link(3)	149	95.60	135	95.95	124	96.24
node(1)	139	95.08	129	95.44	119	95.99
node(2)	152	96.06	140	96.35	130	96.73
link(7)	155	95.74	145	96.01	137	96.30
link(8)	155	96.12	145	96.39	137	96.74
link(9)	156	96.13	146	96.42	137	96.78

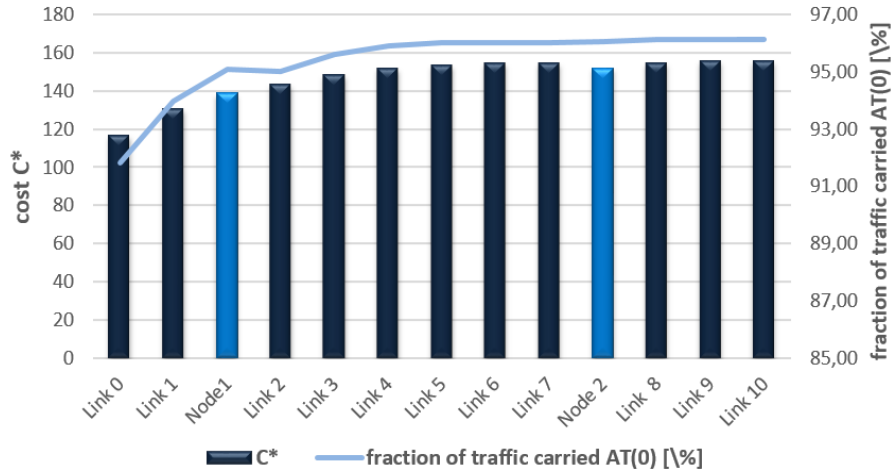


Figure 4: Cost and carried traffic for 0-fiber case (*nobel-germany/moderate* continental climate).

Table 7: Impact of using K -sets (*nobel-germany/cold* continental climate).

	0-fiber case (m=0)		1-fiber case (m=1)		2-fibers case (m=2)	
	$C^*(0)$	$AT(0)$ [%]	$C^*(1)$	$AT(1)$ [%]	$C^*(2)$	$AT(2)$ [%]
set S'	498	98.11	476	98.55	391	98.79
link(2)	144	88.17	133	88.60	123	89.34
link(3)	149	89.24	135	89.88	124	90.61
node(1)	139	88.32	129	88.86	119	89.75
node(2)	152	90.18	140	90.75	130	91.52
link(7)	155	89.89	145	90.41	137	91.07
link(8)	155	90.48	145	91.00	137	91.69
link(9)	156	90.55	146	91.10	137	91.77

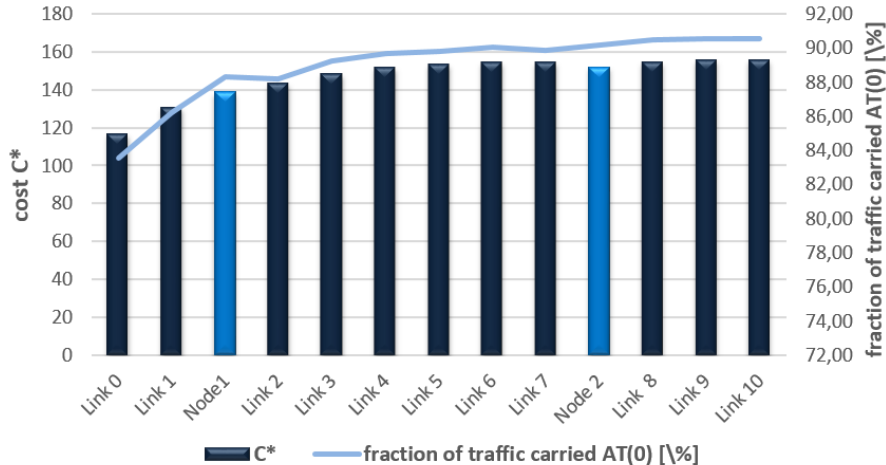


Figure 5: Cost and carried traffic for 0-fiber case (*nobel-germany*/cold continental climate).

Additionally, in Table 8 we show how the percentage of disconnected states decreases when one or two fibers are allowed.

Table 8: Percentage of disconnected states for different network configuration

	mediterranean climate	moderate continental climate	cold continental climate
0-fiber case	0.59%	4.13%	10.97%
1-fiber case	0.52%	4.01%	9.05%
2-fiber case	0.52%	3.90%	8.02%

When examining the above results, we first note that the results for PMAN discussed in Section 5.1 are of a bit different character than those presented above. The reason is that in PMAN the traffic demand between Paris 1 and Paris 2 dominates the rest of demands and therefore putting a fiber link between these two nodes is extremely advantageous. In the case of *nobel-germany* the traffic matrix is considerably more uniform.

As far as the impact of adding fibers is concerned, we observe that its effect on the FSO link cost C^* decrease is not that prominent as for PMAN, yet quite large. Also, although the impact on the fraction of carried traffic is marginal, for the “cold continental climate” scenario the percentage of disconnected states is non-negligible (Table 8).

Regarding the advantages of using K -sets in network dimensioning, we first of all notice that the link capacity cost obtained with K -sets decreases, in comparison with the network dimensioned for the explicit state list \mathcal{S}' , the FSO link cost (for all $m = 0, 1, 2$) significantly, as for PMAN. At the same time the K -sets maintain, but only for “Mediterranean climate” and “moderate continental climate” scenarios, acceptable fractions of carried traffic. For the “cold continental climate” scenario these fractions are not as good when compared to the explicit state list dimensioning. For *nobel-germany*, the FSO link cost achieved with node(1) is visibly the lowest for all cases. In the columns of the three corresponding tables we can notice that for node(1) we have $C^*(0) = 139$ (cost for $m = 0$), while all other values of $C^*(0)$ are greater or equal to 144. The same phenomenon can be noticed in the columns

for $m = 1$ and $m = 2$. Since the fraction of carried traffic for node(1) is acceptable for the “Mediterranean climate” and “moderate continental climate” cases, this particular node K -set can be recommended. For the “cold continental climate” case, however, when these fractions are considered under expectations of the network operator, node(2) or link(8) are a better option.

6. Concluding remarks

In the paper we have presented a robust optimization approach to modular dimensioning of hybrid FSO/fiber networks resilient to degradation of link capacities caused by weather conditions. In such networks, FSO wireless links are supported by terrestrial optical fiber connections in order to improve the resilience in question. In our approach, we first optimize link capacity (i.e., we solve the network dimensioning problem) taking into account all weather states for a given year, for which we produce a representative set of the link degradation states (called the reference degradation set). Then we dimension the network assuming a special (uncertainty) set of link degradation states represented by the so-called node K -set, where K specifies the number of areas (represented by network nodes) that can be simultaneously affected by bad weather. We do that for a selected set of values of parameter K , checking to what extent the demand traffic matrix is satisfied by the K -set solution in all states in the reference degradation set. Finally, we examine how good is the traffic satisfaction in question as compared to the satisfaction achieved with the solution obtained when all states in the reference set are considered in optimization explicitly.

We have tested our approach on a moderate-size network instance (*nobel-germany*) for three different weather scenarios. Also, we have studied effectiveness of our approach on a realistic network instance (based on the Paris Metropolitan Area data). The presented numerical study illustrates the advantage of using node K -sets over the previously used link K -sets. In parallel, we have studied the impact of adding fiber links to the network and analyzing both the traffic performance and cost efficiency of such hybrid FSO/fiber networks.

A natural extension of the above-described work is investigating enhancements of uncertainty sets in the spirit of geographic-aware node K -sets introduced in this paper. One direction of such enhancements is including the multiple degradation ratio case. Enhancing coverage of the real states with a more compact K -set is another promising research direction. Another potential future direction in line with this work is investigating node dimensioning. This kind of investigations will be subject to our research in the near future.

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