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Design and Analysis of Several State-Feedback Fault-Tolerant Control Strategies for Semi-Active Suspensions^{*}

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Abstract: The design of State-Feedback Fault Tolerant control for Semi-Active Suspension Systems is considered in this work, that exploits diverse simple-to-implement approaches. The suspension damper is assumed to undergo multiplicative (time-varying) faults, that can be estimated by (four) modular fault estimation observers. With these fault estimations, active fault tolerant control (*FTC*) schemes can be synthesized, based on the reconfiguration of nominal State-Feedback policies. Seven approaches are discussed: *i)* Direct fault compensation; *ii)* Pole placement compensation; *iii)* Fault-dependent pole placement; *iv)* Linear-fault-dependent Linear Quadratic Regulator (*LQR*) design; *v)* Polynomially-fault-dependent *LQR* parameters; *vi)* *LQR* with Fault-dependent controlled outputs; *vii)* Heuristic (vehicle-oriented) fault-dependent *LQR* synthesis. The performances of these methods are analysed and compared through realistic and high-fidelity simulations. Results show the overall good operation of the latter approaches to compensate fault events and maintain performances.

Keywords: Semi-Active Suspensions, State-Feedback, Fault Estimation, *FTC*, *LQR*.

1. INTRODUCTION

Automotive engineering sector has recently come to know active safety and comfort features, such as controlled suspensions; Semi-Active (*SA*) suspension systems have proved to be efficient, while being less expensive and energy-consuming than purely active suspensions. This type of suspension can be sighted on new *top-cars* and in a good deal of academic and industrial research, as (Lu and DePoyster, 2002; Savaresi et al., 2010). They are able to influence the driving performance, enhancing ride comfort if smoothly controlled. Nonetheless, the main challenge when using *SA* suspensions is to handle the dissipativity constraints of the dampers whilst ride performances (such as passenger comfort) are enhanced: these dampers only dissipate energy, generating force in the same direction as the damping motion.

Most physical systems are subject to possible faults, failures and component malfunctions, as in vehicle suspensions, where the controlled damper might face such issue. For instance, the damping fluid might leak, providing smaller damping. These events imply performance degradation or even loss of control (instability). It has also to be remarked that if a fault occurs upon a given damper, the maximal force that it can provide also decreases (Hernandez-Alcantara et al., 2016). This issue, in many works, is neglected, considering that faults can be

compensated simply by increasing the amount of control action, which is rather contradictory.

Accordingly, attention has been considerably given to Fault Tolerant Control (*FTC*) schemes. *FTC* has the goal of allowing a system to recover performances if malfunctions occur (or, at least, guarantee some continuous stability). *FTC* schemes can be either passive or active, as seen in (Blanke et al., 1997). Passive approaches usually stand for more conservative (robustness-based) schemes, whereas Active approaches, on the other hand, reside in the continuous reconfiguration of the control law, whenever faults are detected. The adequate behaviour of active *FTC* systems depends on a solid Fault Detection and Diagnosis (*FDD*) system (or, at least, an efficient Fault Estimation (*FE*) scheme). Literature shows that the modular design (*FDD/FE* and *FTC* designed separately) presents its benefits, being more flexible for practical applications and easier to test and implement (Blanke et al., 1997; Seron and De Doná, 2015).

Fault Tolerant control applied for the case of *SA* suspension systems has been analyzed by rather few papers: (Tudon-Martinez et al., 2013) presents an *FTC* strategy with Linear Parameter Varying (*LPV*) accommodation; the work by (Nguyen et al., 2015) approaches this issue via *LPV* gain-scheduling. Yet, it must be remarked that these works are not so simple to implement, as they consider either high-order (*LPV*) models, resort to optimization or employ nonlinear control.

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The state-of-practice of controlled suspensions stand for simpler policies, on-off strategies (Skyhook, Ground-hook), min./max. approaches (*ADD* methods) and state-feedback methods (pole placement, Linear Quadratic Regulators (*LQR*) *etc*) Tseng and Hrovat (2015).

1.1 Problem Statement

Considering the given contextualization, this paper tackles the following issue: how to design a (simple-to-implement) Fault-Tolerant state-feedback (*SF*) control system for a vehicle with four *SA* suspensions? Faults are assumed to occur on the dampers (actuator element), such as oil leakages or physical deformation (Morato et al., 2019a). The controller must cope with these faults while maintaining desired control performances and abiding to the *SA* dissipativity constraints.

To do so, one analyses (seven) different adaptative *SF* laws, that adjust themselves autonomously according to the level of faults detected on the damper. The efforts herein were done to demonstrate that different approaches can be used to compensate for the effect of fault, while not disrespecting performance objectives and being implementable. Overall good results are obtained and illustrated with the aid of realistic simulations.

Remark that *SF* control is quite classical, with a very wide range of industrial practices and the topic of many academic works. Its application to the *SA* suspensions problem can achieve some good performances, as demonstrates (Unger et al., 2013).

The paper's organization is: a model that describes the vertical *SA* suspension dynamics is presented in Section 2, as well as assumptions on how to collect information about faults and states; the dilemma between faulty events and a car's driving performance is detailed in Section 3; the seven Fault Tolerant *SF* approaches are minutely explained in Section 4; results are discussed in Section 5.

2. NOTATION AND PRELIMINARIES

Firstly, the notation is reviewed and some preliminaries are recalled. The used vehicle, tire, spring and damper models are well known in literature and readers are invited to refer to (Savaresi et al., 2010) for more details. An automotive suspension comprises, basically, two components: a spring and a damping structure. These components have to work together to maintain the tire's contact to the ground. The goal of the damping structure is to reduce the effect of travelling upon a rough road by absorbing shock and helping with driving performance, ensuring a smoother and safer ride (comfort).

Throughout literature, there are some well-established models of vehicles and automotive suspension systems. In this work, a reduced-order Quarter-of-Vehicle model (*QoV*) is used for both analysis and control goals, considering the behaviour of each vehicle corner. The *QoV* model comprises the vertical displacement of each chassis corner ($z_{s_{ij}}$) and of each wheel ($z_{u_{s_{ij}}}$). The tire forces ($F_{t_{ij}}$) are considered as proportional to the wheel deflection, as gives Eq. (1), where $k_{t_{ij}}$ represents the stiffness coefficients of the tires and $z_{r_{ij}}$ stand for the road profile disturbances.

Each vertical suspension force (at each corner), represented by $F_{s_{ij}}$, is modelled by a spring and a damper with passive and *SA* parts, as describes Eq. (2), where u_{ij} , the control input, should satisfy some dissipativity constraints. Remark that $z_{def_{ij}} = z_{s_{ij}} - z_{u_{s_{ij}}}$ stands for the suspension deflection. The subscripts $i - j$ stand, respectively, for (front/rear)-(left/right) corners.

$$F_{t_{ij}}(t) = k_{t_{ij}} \overbrace{(z_{u_{s_{ij}}}(t) - z_{r_{ij}}(t))}^{\text{Wheel deflection}} \quad (1)$$

$$F_{s_{ij}}(t) = \underbrace{k_{ij} z_{def_{ij}}(t)}_{\text{Spring}} + \underbrace{c_{0_{ij}} \dot{z}_{def_{ij}}(t)}_{\text{Nominal damping}} + \underbrace{u_{ij}(t)}_{\text{Control input}} \quad (2)$$

Possible faults of *SA* dampers may occur due to internal oil leakages, physical deformation, due to electrical problems on the amplifiers used to control the dampers, or even to the presence of air in the damping fluid. In practice, this converts in an energetic *loss of effectiveness* (Hernandez-Alcantara et al., 2016). Henceforth, faults are here represented by a multiplicative factor α_{ij} upon each controlled damper force u_{ij} . The amount of damping force available in a faulty case ($\alpha_{ij} \neq 1$) should be smaller than when $\alpha_{ij} = 1$, further discussed in Section 3.

Remark 1. In a faultless situation, $\alpha_{ij} = 1$ and when the damper achieves an uncontrollable condition, i.e. with only passive ($c_{0_{ij}}$) behaviour, $\alpha_{ij} = 0$. Thus: $\alpha_{ij}(t) \in [0, 1]$.

The *state-space* representation of the (faulty) *QoV* model is found by taking the system states as Eq. (4) and the disturbances as the road profile given in Eq. (5). The outputs are the wheel and chassis accelerations, as depicts Eq. (6), easily measurable with on-board vehicle sensors (acceleration/inertial units), widely present in modern cars. Thus, for each corner:

$$\begin{aligned} \dot{x}^{ij}(t) &= A^{ij} x^{ij}(t) + B_1^{ij} w^{ij}(t) + B_2^{ij} \alpha_{ij}(t) u_{ij}(t) \\ \dot{y}^{ij}(t) &= C^{ij} x^{ij}(t) + D_1^{ij} w^{ij}(t) + D_2^{ij} \alpha_{ij}(t) u_{ij}(t) \end{aligned} \quad (3)$$

with constant matrices A^{ij} to D_2^{ij} .

$$x^{ij}(t) = [z_{s_{ij}}(t) \ \dot{z}_{s_{ij}}(t) \ z_{u_{s_{ij}}}(t) \ \dot{z}_{u_{s_{ij}}}(t)]^T \quad (4)$$

$$w^{ij}(t) = z_{r_{ij}}(t) \quad (5)$$

$$y^{ij}(t) = [\ddot{z}_{s_{ij}}(t) \ \ddot{z}_{u_{s_{ij}}}(t)]^T \quad (6)$$

Assumption 1. As a (decoupled) *QoV* model is considered for both analysis and control, *SF* laws can be designed, individually, for each corner of the vehicle. This provides simplicity (in particular to cope with *FTC* objectives) and a straightforward implementation that could be done on simple microcontrollers embedded to each *SA* suspension system. If a full vehicle model was considered, this would have greatly enlarged the computational burden without actually leading to better results, given that the mean variations of $\ddot{z}_{s_{ij}}$ and $\ddot{z}_{u_{s_{ij}}}$ can be entirely felt by the *QoV* model.

Assumption 2. Using the (faulty) *QoV* model and solely the available measurements $y^{ij}(t)$, given by Eq. (3), asymptotical extended-state observers can be synthesized in order to estimate **both faults α^{ij} and states $x^{ij}(t)$** , in such a way that *SF* control laws can be implemented based on \hat{x}^{ij} , see (Aubouet et al., 2010).

Remark 2. The chosen observer, from (Morato et al., 2019a), is a polytopic *LPV* scheme with **one** single parameter ($\rho = u_{ij}$), so the cost of its implementation is actually very low and it can run very fast, in real-time. Its computation consists in the sum of two simple linear models and there is no need for on-line optimization procedure. This observer is extensively discussed in that referred paper, where it is demonstrated that it can estimate states and faults with small error; therefore, results concerning its performances are omitted herein.

3. STATE-FEEDBACK *FTC*

To design adequate *FTC* laws, the dissipativity constraints of the *SA* dampers must be taken into account. These are, for each controllable damping force:

$$\underline{c}_{ij}(\alpha_{ij}(t))\dot{z}_{def_{ij}}(t) \leq u_{ij}(t) \leq \overline{c}_{ij}(\alpha_{ij}(t))\dot{z}_{def_{ij}}(t), \quad (7)$$

which can be equivalently expressed as¹:

$$u_{ij}(t) \in \mathcal{D}_{ij}(\dot{z}_{def_{ij}}, \hat{\alpha}_{ij}, t). \quad (8)$$

This means, in practice, that the controlled damping $u_{ij}(t)$ must always belong to the feasibility set \mathcal{D}_{ij} . The available damping force is related to the level of faults upon the damper: more faults lead to smaller forces and increased damping motion $\dot{z}_{def_{ij}}$, which means that \mathcal{D}_{ij} shrinks and shifts² according to α_{ij} , as suggests Morato et al. (2019b). Figure 1 illustrates which are the dissipativity constraints and how do they become stricter when faults occur³.

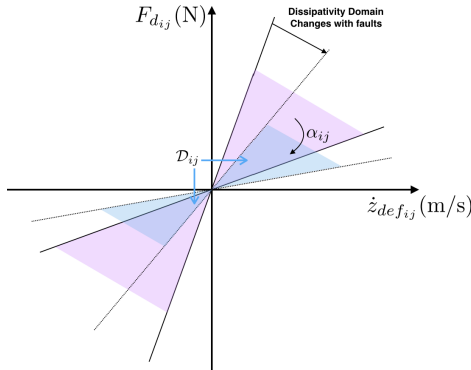


Fig. 1. *SA* Dissipative Forces Domain \mathcal{D}_{ij}

As a consequence, if the saturation constraint is not adapted, then the dissipativity condition of the *SA* damper is not guaranteed. Indeed, as says Nguyen et al. (2015), the required force could be outside the range of feasible “faulty” forces if no fault information ($\hat{\alpha}_{ij}$) is included in the control design. Thus, due to the described limitations, a fault-scheduled clipping function⁴ must be defined in such way that $u_{ij}(t)$ always lies inside \mathcal{D}_{ij} :

¹ \underline{c}_{ij} and \overline{c}_{ij} stand, respectively, for the minimal and maximal damping coefficients.

² By design, the min. / max. damping coefficients decrease with faults, shrinking the available set in size, but also shifts it sideways, as illustrates the Figure.

³ Take $F_{d_{ij}}(t) = F_{s_{ij}}(t) - k_{ij}z_{def_{ij}}(t)$.

⁴ The dissipativity constraints of the *SA* dampers, then, are usually handled using simple projection (i.e. saturation/clipping). Note that these kinds of approaches are easy to implement by using suspension-embedded microcontrollers (Morato et al., 2019b).

$$\text{clip}(u_{ij}(t), \mathcal{D}_{ij}) = \begin{cases} u_{ij}(t) & \text{if } u_{ij}(t) \in \mathcal{D}_{ij} \\ u_{ij}^\perp & \text{if } u_{ij}(t) \notin \mathcal{D}_{ij} \end{cases}, \quad (9)$$

where u_{ij}^\perp is the orthogonal projection of u_{ij} on \mathcal{D}_{ij} .

Hence, the **control objective** is to find a *SF* control law, for each *SA* suspension *QoV* model, so that:

$$\begin{aligned} \min_{u_{ij}(t)} J_{SF}^{ij}(\tau) \\ \text{s.t. } u_{ij}(t) \in \mathcal{D}_{ij}(\dot{z}_{def_{ij}}, \hat{\alpha}_{ij}, t) \end{aligned} \quad (10)$$

with $J_{SF}^{ij}(\tau) = \int_0^\tau (a_1 \ddot{z}_{s_{ij}}^2(t) + a_2 \ddot{z}_{us_{ij}}^2(t)) dt$. If these accelerations are minimized, a smoother and more comfortable drive is achieved (Kiencke and Nielsen, 2005). To do so, diverse policies can be used, such as pole placement and linear quadratic regulators. Notice that Fault-Tolerance (*FT*) is achieved iff the dissipativity constraint is respected.

This work considers *SF* control laws on the form:

$$\begin{aligned} u_{ij}(t) &= \text{clip}(u_{ij}^*(t), \mathcal{D}_{ij}), \\ u_{ij}^*(t) &= -K^{ij}(\cdot)\hat{x}^{ij}(t), \end{aligned} \quad (11)$$

with $u_{ij}^*(t)$ computed according to the control objectives, for each corner of the car. The closed-loop system is characterized by the pair $[(A^{ij} - B_2^{ij}K^{ij}), B_1^{ij}]$ if faultless and $[(A^{ij} - \alpha_{ij}B_2^{ij}K^{ij}), B_1^{ij}]$ if faulty⁵. Then, there should be continuity of eigenvalues between both conditions (so that performance goals J_{SF}^{ij} are maintained). But, given that $\|(1 - \alpha_{ij})B_2^{ij}K^{ij}\|_2$ grows bigger as $\alpha_{ij} \rightarrow 0$ (greater faults), Proposition 1 might not be guaranteed.

Proposition 1. Let $A \in \mathbb{R}^{n \times n}$ be a square matrix. Given $\epsilon > 0$, there exists $\delta > 0$ s.t. if a constant $\Delta A \in \mathbb{R}^{n \times n}$ satisfies $\|\Delta A\|_2 < \delta$, then the distance between the eigenvalues of matrices $(A + \Delta A)$ and A is smaller than ϵ : $\|\lambda(A + \Delta A) - \lambda(A)\|_2 < \epsilon$.

Then, if a fault occurs, some other stabilizing gain matrix $K^{ij}(\alpha_{ij})$ should be computed or a new feedback law $u_{ij}(t, \alpha_{ij})$ used in such a way that the (faulty) closed-loop (*CL*) system remains stable and that dissipativity constraints are obeyed (Eq. (8)), while each J_{SF}^{ij} is minimized, maintaining the comfort performance objective. In practice, this is equivalent to a continuity of eigenvalues between faulty and faultless situations. This *FT SF* control problem is represented by Figure 2 and tackled by the seven approaches presented next.

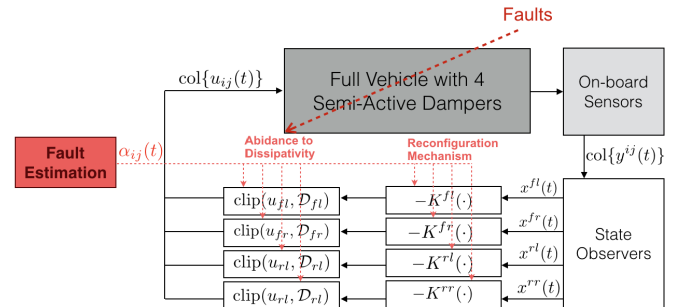


Fig. 2. *SF FTC*: *SA* Suspensions

⁵ For simplicity, clipping action was disregarded and $\hat{x}^{ij} = x^{ij}$.

4. PROPOSED *FTC* APPROACHES AND COMPARISONS

4.1 Approach i: State-Feedback Compensation

Structure: $u_{ij}^*(t) = -\frac{1}{\hat{\alpha}_{ij}} K^{ij} \hat{x}^{ij}(t)$.

Goal: Find a stabilizing matrix K^{ij} for the faultless condition, according to performance specifications (10) and reset the control law whenever a fault occurs.

Pros: Directly compensates the effect of α_{ij} upon the gain matrix K^{ij} . Computation is fast and direct.

Cons: The problem with this reconfiguration approach is that if $\alpha_{ij} \rightarrow 0$, then, $u_{ij}(t) \rightarrow \infty$. This is contradictory with the damper dissipativity constraints.

4.2 Approach ii: Pole Placement Re-design

Structure: $u_{ij}^*(t) = -K^{ij}(\hat{\alpha}_{ij})x^{ij}(t)$.

Goal: Find a more conservative performance when α_{ij} decreases. Considering that the nominal *CL* eigenvalues are $\lambda_{CL} = \text{col}\{p_1, \dots, p_n\}$, one can find $K^{ij}(\hat{\alpha}_{ij})$ so that the faulty *CL* eigenvalues are placed at $(\hat{\alpha}_{ij} \cdot \lambda_{CL})$, to account for graceful performance degradation⁶.

Pros: Consonant with faults. Computation is a simple polynomial of $K^{ij}(\hat{\alpha}_{ij})$ for the fixed interval $(0, 1)$ and a given number of interpolation points α_{ij}^k .

Cons: Too conservative, doesn't guarantee (8).

4.3 Approach iii: Affine Gain Matrix

Structure: Here, the *SF* gain is chosen as first-order affine on $\hat{\alpha}_{ij}$: $K^{ij}(\hat{\alpha}_{ij}) = K_0^{ij} + (1 - \hat{\alpha}_{ij})K_1^{ij}$, i.e. $K^{ij} = K_0^{ij}$ in faultless condition, i.e. $\alpha_{ij} = 1$.

Goal: Same as Ap. ii. With this approach, the *CL* matrix is $A_{CL}^{ij} = \underbrace{(A^{ij} - \hat{\alpha}_{ij}B_2^{ij}K_0^{ij})}_{\hat{A}^{ij}} - \underbrace{[\hat{\alpha}_{ij}B_2^{ij}(1 - \hat{\alpha}_{ij})]}_{\hat{B}^{ij}} K_1^{ij}$.

Then, if the pair $(\hat{A}^{ij}, \hat{B}^{ij})$ is controllable, the problem is solved by finding a gain matrix K_1^{ij} so that the *CL* system is stable for every $\hat{\alpha}_{ij}$, placing the *CL* poles on $(\hat{\alpha}_{ij}\lambda_{CL})$.

Pros: Same as Ap. ii: Should provide a more conservative result, knowing that the settling time performances of x^{ij} depend on the inverse of its eigenvalues. So, if the new poles are closer to 0, the stabilization of the faulty system becomes slower and, thus, $u_{ij}^*(t)$ may always be inside the dissipativity set $\mathcal{D}_{ij}(\cdot)$.

Cons: Same as Ap. ii.

4.4 Approaches iv-v: Fault-scheduled LQR

Goal: The Linear Quadratic Regulator (*LQR*) is another classical *SF* control law that is largely applied in industrial and academic works of vehicle suspension systems (Tseng and Hrovat, 2015). Its main idea is to find a *SF* matrix K^{ij} such that the following cost function is minimized:

$$J_{LQR} = \int_0^\infty ((x^{ij})^T Q^{ij} x^{ij} + u_{ij}^T R^{ij} u_{ij}) dt \quad (12)$$

Then, say weights Q_0^{ij} and R_0^{ij} are adjusted for the aimed control performances J_{SF}^{ij} at faultless conditions.

⁶ There is a direct relationship between the placement of the *CL* eigenvalues and the achieved performances, J_{SF}^{ij} (Kiencke and Nielsen, 2005; Morato et al., 2019b).

Structure: Re-configuring the control law in the presence of faults by taking parameter weights dependent of $\hat{\alpha}_{ij}$ (for performance adaptation in faulty case). This is:

- Ap. iv: $R^{ij}(\hat{\alpha}_{ij}) = \frac{1}{\hat{\alpha}_{ij}} R_0^{ij}$ and $Q(\hat{\alpha}_{ij}) = \hat{\alpha}_{ij} Q_0$;
- Ap. v: $R^{ij}(\hat{\alpha}_{ij}) = \frac{1}{\hat{\alpha}_{ij}^2} R_0^{ij}$ and $Q^{ij}(\hat{\alpha}_{ij}) = \hat{\alpha}_{ij}^2 Q_0^{ij}$.

Pros: Both approaches restrain the (max,min) bounds of $u^{ij}(t)$ with greater faults: iff $\hat{\alpha}_{ij} \rightarrow 0$, then $R^{ij}(\hat{\alpha}_{ij}) \rightarrow \infty$ and $\|u_{ij}^*(t)\|_\infty \rightarrow 0$, thus obeying to the dissipativity constraints, Eq. (8).

Cons: Not able to point out which will be the system's *CL* poles, only that they will be (Lyapunov sense) stable. Moreover, computation is more complex, yet offline.

4.6 Approach vi: Fault-scheduled LQR via LMIs

Goal: Linear Matrix Inequalities (*LMIs*) can also be used to solve the *LQR SF* stabilization problem. Instead of using the usual *Riccati* equations to find the gain matrix K^{ij} , one can also opt to solve an *LMI* problem. Considering $z^{ij}(t) = C_z x^{ij}(t) + D_z u_{ij}^*(t) \in \mathbb{R}^{n_z}$ as an auxiliary vector whose energy is to be minimized, the equivalent *LQR* is found with $Q^{ij} = C_z^T C_z > 0$ and $R^{ij} = D_z^T D_z > 0$, having full-rank C_z and D_z with $C_z^T D_z = 0$. This *LMI* problem is:

$$\min \lambda^{ij} \text{ s.t. : } \begin{bmatrix} \lambda & x_0^T \\ x_0 & Q \end{bmatrix} > 0 \quad (13)$$

$$\begin{bmatrix} (QA^T + AQ + Y^T B_2^T + B_2 Y) & (QC_z^T + Y^T D_z^T) \\ (C_z Q + D_z Y) & -\text{eye}(n_z) \end{bmatrix} < 0$$

Then, $K^{ij} = -Y^{ij}(Q^{ij})^{-1}$, and the cost function $J_{LQR} = \int_0^\infty \|z(t)\|^2 dt$ satisfies $J_{LQR} < x^{ij}(0)^T (Q^{ij})^{-1} x^{ij}(0)$.

Structure: The issues of a faulty situation are overlapped by choosing matrices C_z and D_z that vary according to fault events. Being C_{z_0} and D_{z_0} the nominal weight for the faultless case, then, the re-design can be found by taking: $D_z(\hat{\alpha}_{ij}) = \frac{1}{\sqrt{\hat{\alpha}_{ij}}} D_{z_0}$ and $C_z(\hat{\alpha}_{ij}) = \sqrt{\hat{\alpha}_{ij}} C_{z_0}$.

Pros: Restrains $\|u_{ij}^*(t)\|_\infty$, abiding to Eq. (8).

Cons: Same as Ap. v. Needs *LMI* solver for the offline design procedure

4.7 Approach vii: Heuristic Fault-Dependent Weights

Goal: The *LQ*-based *LMI* solution can be adapted to cope directly with both the control performances (10) and fault events.

Structure: Take $z^{ij}(t) = \text{col}\{\ddot{z}_{s_{ij}}, \ddot{z}_{us_{ij}}, u_{ij}^*(t)\}^T$, solving:

$$\min_{u_{ij}(t)} = \int_0^{+\infty} [z_{s_{ij}}^{\ddot{\cdot}}{}^T(t) q_{s_{ij}}^{ij} z_{s_{ij}}^{\ddot{\cdot}}(t) + z_{us_{ij}}^{\ddot{\cdot}}{}^T(t) q_{us_{ij}}^{ij} z_{us_{ij}}^{\ddot{\cdot}}(t) + (u_{ij}^*)^T(t) q_u^{ij} (u_{ij}^*)(t)] dt, \quad (14)$$

which can be done by taking matrices C_z and D_z as:

$$C_z = \begin{bmatrix} \frac{-\sqrt{q_s^{ij}} k_{ij}}{m_s} & \frac{-\sqrt{q_s^{ij}} c_{0_{ij}}}{m_{s_{ij}}} & \frac{\sqrt{q_s^{ij}} k_{ij}}{m_{s_{ij}}} & \frac{\sqrt{q_s^{ij}} c_{0_{ij}}}{m_{s_{ij}}} \\ \frac{\sqrt{q_{us}^{ij}} k_{ij}}{m_{us_{ij}}} & \frac{\sqrt{q_{us}^{ij}} c_{0_{ij}}}{m_{us_{ij}}} & \frac{-\sqrt{q_{us}^{ij}} (k_{ij} + k_{t_{ij}})}{m_{us_{ij}}} & \frac{-\sqrt{q_{us}^{ij}} c_{0_{ij}}}{m_{us_{ij}}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

$$D_z = \begin{bmatrix} \sqrt{q_s^{ij}} & -\sqrt{q_{us}^{ij}} & \sqrt{q_u^{ij}} & \sqrt{q_u^{ij}} \end{bmatrix}^T \quad (16)$$

Adjusting $q_{s_0}^{ij}$, $q_{u_{s_0}}^{ij}$ and $q_{u_0}^{ij}$ for nominal conditions, the fault re-configuration can be done simply by taking:

$$q_s^{ij}(\hat{\alpha}_{ij}) = \hat{\alpha}_{ij} q_{s_0}^{ij}, q_{u_s}^{ij}(\hat{\alpha}_{ij}) = \hat{\alpha}_{ij} q_{u_{s_0}}^{ij}, q_u^{ij}(\hat{\alpha}_{ij}) = \frac{q_{u_0}^{ij}}{\hat{\alpha}_{ij}}.$$

Pros: Coherent with the vehicle suspension dynamics and faulty situations.

Cons: Case dependent.

Remark 3. It is important to notice that approaches *iv-vi* only require the **offline** computation of matrices $K^{ij}(\hat{\alpha}_{ij})$ gridded according to a number of points $\hat{\alpha}_{ij}^k$ inside the fixed interval $(0, 1)$. The online implemented control loop is a simple gridding (look-up table) that finds K^{ij} according to $\hat{\alpha}_{ij}$.

5. RESULTS

Simulation results are presented to assess the behaviour of the considered *FT SF* control laws. The following results are obtained with the aid of softwares packages *Matlab*, *Yalmip* and *SDPT3*. Herein, a 1/5-scaled vehicle model equipped with 4 *SA* dampers is considered⁷, and its full nonlinear (7-*DOF*) vertical model is used for simulation. Model parameters are: $m_{s_{ij}} = 2.27$ kg; $m_{u_{s_{ij}}} = 0.32$ kg; $k_{t_{ij}} = 18097$ N/m; $k_{ij} = 1396$ N/m. High-frequency measurement noise is added to each measured output (y^{ij}) in order to mimic realistic conditions. From here onwards, the results depict the front-left corner of the vehicle⁸.

The chosen road profile (input disturbance, $w^{ij}(t)$) represents a car running in a straight line on a dry road, when it encounters ($t' = 0.5$ s) sequence of five 5 mm bumps on all its wheels, exciting a bouncing motion. An oil leakage fault that leads to 50% loss of damping effectiveness ($\alpha_{fl} = 0.5$) occurs from $t'' = 1$ s, which shrinks the amount of available set by half, as illustrates Figure 1. Both $w_{fl}(t)$ and $\alpha_{fl}(t)$ (and estimation) are given in Figure 3. In respect to Eq. (10) (control objective), a_1 and a_2 are taken, respectively, as 0.95 and 0.05 such that the chassis acceleration is most minimized, aiming to isolate the passengers from the road bumping.

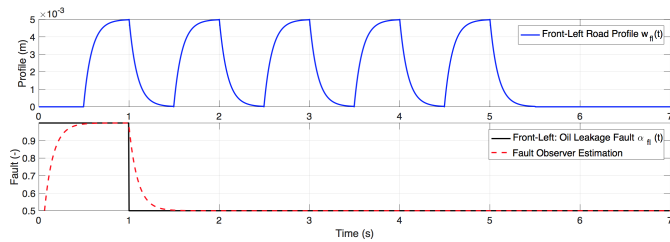


Fig. 3. (Front-Left) Simulation Scenario

All seven (clipped) *FT SF* control policies are tested with this scenario. Table 1 sums up the obtained results in terms of fault-tolerance, dissipativity constraints abidance (excessive use of clipping action and maximal unclipped control value, $\|u_{fl}^*(t)\|_\infty$) and chassis acceleration minimization (comfort performances, J_{SF}^{ij}). Clearly, approaches *vi* and *vii* achieve the best results. Approaches

⁷ Refer to <http://www.gipsa-lab.fr/projet/inove/>.

⁸ Similar results were obtained for the other corners and are not shown due to lack of space.

i-iii did not abide to the dissipativity constraints when the region shrank due to faults, which implied in excessive clipping action and thereby becoming, basically, min. /max. strategies.

Fig. 4 depicts the evolution of the system states $x^{fl}(t)$ (with Ap. *vii*) and their estimation by the used *LPV* observer. Fig. 5 compares the comfort performance ($\ddot{z}_s(t)$) with the seven approaches, where it becomes evident that approaches *v-vii* yield smoother performances, altogether with fault-tolerance, providing a more comfortable ride for the vehicle passenger. Finally, Fig. 6 compares the control action of approaches *i, iii, v* and *vii*. Therein, it becomes clear that approach *i* has a great peak at the moment of the fault event, which is not coherent with *FT*, as discussed in the Introduction. Approaches *iii* and *v* use excessive control action when bumps occur, whereas approach *vii* provides the most graceful performance.

Clearly, the control action tries to isolate the effect of the disturbances $w_{fl}(t)$ caused upon the wheel $z_{u_{s_{fl}}}(t)$ from the chassis corner $z_{s_{fl}}(t)$, in such a way that comfort is enhanced. The improvements between the approaches are subtle because this is a reduced (small) vehicle, and, thus, small changes in $z_{s_{fl}}$ and $\dot{z}_{s_{fl}}$ (and, consequently in $\ddot{z}_{s_{fl}}$) do influence the passenger's comfort. Using a larger vehicle model, the order of magnitude of $z_{s_{fl}}$ and $\dot{z}_{s_{fl}}$ would also enlarge. Also, remark that the effect of faults would be much more degrading when there is no fault detection or controller reconfiguration.

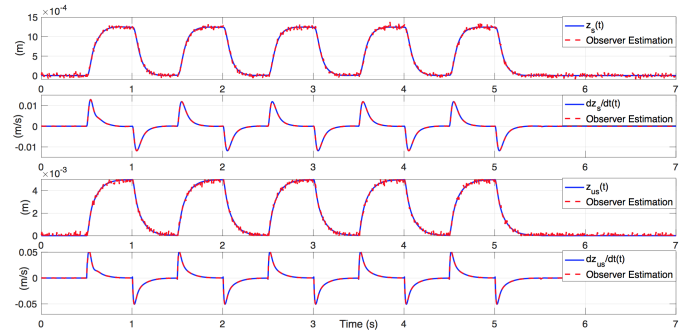


Fig. 4. (Front-Left) States

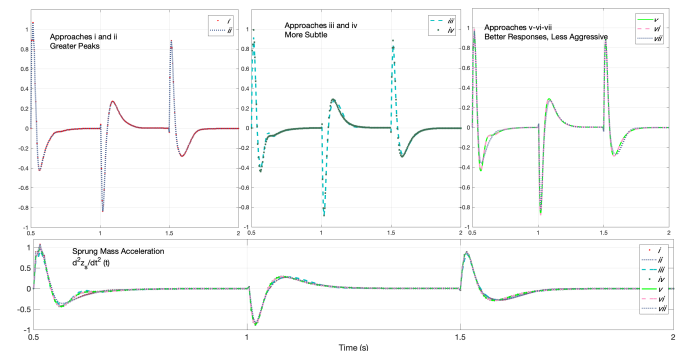


Fig. 5. (Front-Left) Sprung Acceleration $\ddot{z}_s(t)$ (Comfort Performances)

Moreover, the vehicle-oriented *LQR* method (Ap. *vii*) is the most coherent strategy, given that the feasibility region D_{fl} (available damping) was used with larger spread.

Table 1. Simulation Results: Performance of SF Policies

Ap.	$RMS(J_{SF}^{fl})$	$\ \ddot{z}_{s_{ij}}(t)\ _2$	$\ \ddot{z}_{u_{s_{ij}}}(t)\ _2$	$\ u_{fl}^*(t)\ _\infty$	$u_{fl}^* \in \mathcal{D}_{fl}, FT ?$	Clipping	Comments
<i>i</i>	3.05	4.54	17.56	31.67	No	Excessive	Biggest peak on u_{fl}^*
<i>ii</i>	3.05	4.54	17.56	31.36	No	Excessive	Too Conservative
<i>iii</i>	2.97	4.52	17.57	30.90	No	Excessive	Too Conservative
<i>iv</i>	2.94	4.49	17.56	4.68	Yes	Small	Restrains $\ u_{ij}(t)\ _\infty$
<i>v</i>	3.02	4.49	17.57	4.67	No	Average	Restrains $\ u_{ij}(t)\ _\infty$
<i>vi</i>	2.94	4.49	17.55	4.66	Yes	None	Coherent with FT , Small $u_{ij}(t)$
<i>vii</i>	2.94	4.50	17.55	4.60	Yes	None	Best Option: FT + Performance

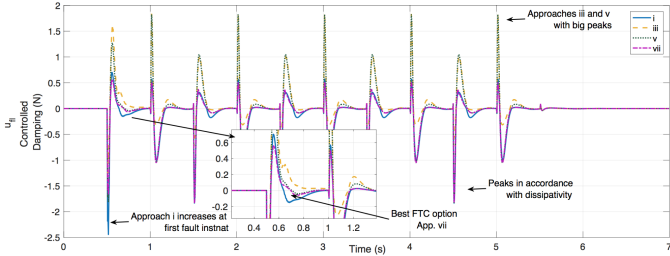


Fig. 6. (Front-Left) Control Action

This is clear in Figure 7, where the dissipativity region is presented⁹, comparing Ap. *i* (basically a max/min policy when faulty, due to excessive clipping) and *vii*.

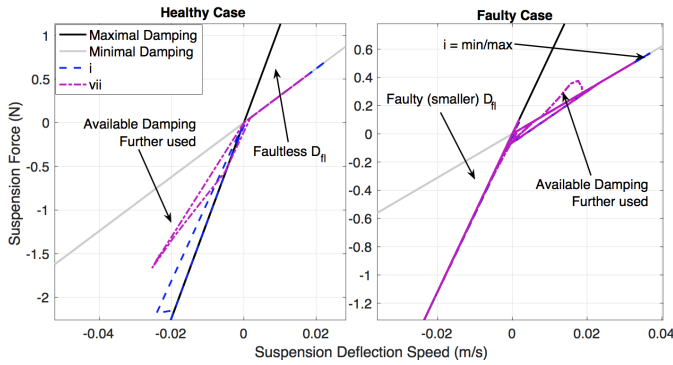


Fig. 7. (Front-Left) Damper Dissipativity Constraints

6. CONCLUSIONS

This paper presented the issue of controlling a full SA suspension, subject to damper faults, via direct, simple-to-implement fault-tolerant SF policies. Also, parallel extended observers are designed to provide accurate informations on faults and system states. Seven different autonomously-adjusting SF policies were tested through realistic simulation¹⁰ and results enlighten the interest of heuristic LQR to enhance the comfort of passengers. For further works, the analysis of badly estimated faults and states in terms of robustness will be made. Comparisons with a centralized full car FTC controller will also be investigated.

REFERENCES

Aubouet, S., Dugard, L., and Sename, O. (2010). Experimental results of an h_∞ observer for an industrial semi-active suspension. *IFAC Proceedings Volumes*, 43(7), 318–323.

⁹ \mathcal{D}_{fl} is used with wider spread with Ap. *vii*.

¹⁰ Codes available at: <https://drive.google.com/drive/folders/18ZvurwlCHnkg1T2hpoWsIeBtl5ctILA?usp=sharing>.

Blanke, M., Izadi-Zamanabadi, R., Bogh, S., and Lunau, C. (1997). Fault-tolerant control systems - a holistic view.

Hernandez-Alcantara, D., Tudon-Martinez, J.C., Amézquita-Brooks, L., Vivas-Lopez, C.A., and Morales-Menéndez, R. (2016). Modeling, diagnosis and estimation of actuator faults in vehicle suspensions. *Control Engineering Practice*, 49, 173–186.

Kiencke, U. and Nielsen, L. (2005). *Automotive control systems: for engine, driveline, and vehicle*. Springer Science & Business Media.

Lu, J. and DePoyster, M. (2002). Multiobjective optimal suspension control to achieve integrated ride and handling performance. *IEEE Transactions on Control Systems Technology*, 10(6), 807–821.

Morato, M.M., Sename, O., Dugard, L., and Nguyen, M.Q. (2019a). Fault estimation for automotive electro-rheological dampers: LPV-based observer approach. *Control Engineering Practice*, 85, 11–22.

Morato, M.M., Nguyen, M.Q., Sename, O., and Dugard, L. (2019b). Design of a fast real-time LPV model predictive control system for semi-active suspension control of a full vehicle. *Journal of the Franklin Institute*, 356(3), 1196–1224.

Nguyen, M.Q., Sename, O., and Dugard, L. (2015). An LPV fault tolerant control for semi-active suspension-scheduled by fault estimation. *IFAC-PapersOnLine*, 48(21), 42–47.

Savarese, S.M., Poussot-Vassal, C., Spelta, C., Sename, O., and Dugard, L. (2010). *Semi-active suspension control design for vehicles*. Butterworth-Heinemann. Elsevier.

Seron, M.M. and De Doná, J.A. (2015). Robust fault estimation and compensation for LPV systems under actuator and sensor faults. *Automatica*, 52, 294–301.

Tseng, H.E. and Hrovat, D. (2015). State of the art survey: active and semi-active suspension control. *Vehicle system dynamics*, 53(7), 1034–1062.

Tudon-Martinez, J.C., Varrier, S., Sename, O., Morales-Menendez, R., Martinez, J.J., and Dugard, L. (2013). Fault tolerant strategy for semi-active suspensions with LPV accommodation. In *2nd International Conference on Control and Fault-Tolerant Systems (SysTol)*, 631–636. IEEE.

Unger, A., Schimmack, F., Lohmann, B., and Schwarz, R. (2013). Application of LQ-based semi-active suspension control in a vehicle. *Control Engineering Practice*, 21(12), 1841–1850.