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Scaling approach to itinerant quantum critical points

Catherine Pépin [1], Jérome Rech [1,3] and Revaz Ramazashvili[2]
[1] *SPhT, L'Orme des Merisiers, CEA-Saclay, 91191 Gif-sur-Yvette, France*
[2] *Materials Science Division, Argonne National Laboratory, Argonne, IL 60439, USA*
[3] *Center for Materials Theory, Rutgers University, Piscataway, NJ 08855, USA*

Based on phase space arguments, we develop a simple approach to metallic quantum critical points, designed to study the problem without integrating the fermions out of the partition function. The method is applied to the spin-fermion model of a T=0 ferromagnetic transition. Stability criteria for the conduction and the spin fluids are derived by scaling at the tree level. We conclude that anomalous exponents may be generated for the fermion self-energy and the spin-spin correlation functions below $d = 3$, in spite of the spin fluid being above its upper critical dimension.

78.20.Ls, 47.25.Gz, 76.50+b, 72.15.Gd

One of the explanations advanced for the breakdown of the Fermi liquid theory in the normal state of high temperature superconductors is the proximity to a quantum critical point (QCP), hidden under the superconducting dome. The nature of this zero temperature transition remains controversial. However, in several heavy fermion materials, the Fermi liquid state was experimentally shown to break down near a well characterized $T = 0$ antiferromagnetic instability [1–4]. A detailed renormalization group study of QCPs in itinerant magnets was first undertaken by Hertz [5], and later augmented by Millis [6]. The key observation [5] was that spin fluctuations relax critically in time, with a dynamic exponent z relating the time and the length scales as $\tau \sim \xi^z$. Thus a d-dimensional system can be viewed as having effective dimensionality $d + z$. For an antiferromagnetic QCP, one finds $z = 2$, while for a ferromagnetic QCP, $z = 3$. After integrating the fermions out of the partition function, the authors of [5,6] argued that, in $d = 2$ or 3 (the cases of interest for heavy fermions as well as high-T$_c$ superconductors), $d + z \geq 4$. Thus the effective Ginzburg-Landau theory for the spin fluid falls above its upper critical dimension, and has Gaussian critical behaviour.

Recently, the validity of integrating out the fermions has been questioned [7,8], since gapless fermions may lead to singular coefficients in the Ginzburg-Landau expansion. Another outstanding question is whether the Fermi liquid theory may break down in two or three dimensions, i.e. whether the quasiparticles may become ill-defined while the magnetic fluctuations are only innocuously critical, being described by a Gaussian theory. In this case, the conduction and the magnetic fluids would behave as if decoupled, each having its own upper critical dimension. Finally, integrating out the fermions to describe a QCP in a metal is conceptually unsatisfactory, as it greatly complicates a consistent account of electron transport.

In this paper, we introdice a simple scaling approach, designed to study a spin-fermion model at a QCP without integrating the fermions out of the partition function. Already at the tree level, it reveals that the critical behavior is controlled by several couplings, rather than by the single fermion-boson coupling constant g, expected naively. At a ferromagnetic quantum critical point, the coupling g becomes relevant below one spatial dimension, analogously to the four-boson coupling constant at a ferromagnetic QCP [5]. At the same time, the four-fermion coupling u_f, mediated by the bosons, controls the breakdown of the Fermi liquid theory, and becomes relevant below three spatial dimensions.

We will illustrate the idea on the spin-fermion model of a ferromagnetic quantum critical point, comprising three parts: a conduction electron term S_f, a boson term S_b, describing the critical magnetic modes, and S_i, representing interaction between the fermions and the bosons:

$$
\begin{aligned}
S &= S_f + S_b + S_i \qquad (1)\\
S_f &= \int d\omega \; d^d\mathbf{k} \; \psi^\dagger_{\mathbf{k}} \; \left(i\omega - \xi_{\mathbf{k}}\right) \; \psi_{\mathbf{k}} \\
S_b &= \int d\omega \; d^d\mathbf{q} \; \bar{\varphi}_{\mathbf{q}} \; \left(\frac{|\omega|}{q} + q^2\right)\varphi_{\mathbf{q}} \\
S_i &= g \int d\omega_1 d\omega_2 d^d\mathbf{k}_1 d^d\mathbf{k}_2 \left[\varphi_{\mathbf{k}_1-\mathbf{k}_2}\psi^\dagger_{\mathbf{k}_1}\psi_{\mathbf{k}_2} + h.c.\right] \; .
\end{aligned}
$$

Here ψ (φ) are the fermion (boson) fields, and $\xi_{\mathbf{k}}$ is the quasiparticle energy counted from the chemical potential. For simplicity, we consider a spherical Fermi surface.

We first perform a Benfatto-Galavotti-Shankar [9,10] renormalization at the tree level, removing the high energy degrees of freedom, and retaining only their contribution to the low energy effective action. At each step we eliminate a shell $\Lambda/s \leq \omega \leq \Lambda$, where Λ is the cut-off and $s > 1$. The energies and momenta are then rescaled to restore the original cut-offs and, lastly, the fields are rescaled to leave the quadratic part of the action intact.

However, implementation of this program poses two difficulties. The first one is of geometric origin: the fermion momenta are restricted to a thin shell around the Fermi surface, while the boson momenta are confined to a sphere. The second difficulty stems from the different dynamic exponents z of the fermion and boson fluids: in the Fermi liquid, $z_f = 1$, while the magnetic modes are characterized by $z_b = 3$. To perform simultaneous mode elimination for the two species, we rescale the energies and the momenta as per

$$\omega' = s\omega;\ k'_\perp = s^{1/z_f} k_\perp = s k_\perp;\ q' = s^{1/z_b} q = s^{1/3} q, \quad (2)$$

while the fields rescale as per

$$\psi' = s^{-3/2}\psi; \quad \varphi' = s^{-(d+z_b+2)/2z_b}\varphi \quad (3)$$

where $k_\perp$ is defined by $\xi_{\mathbf{k}} = v_F k_\perp$.

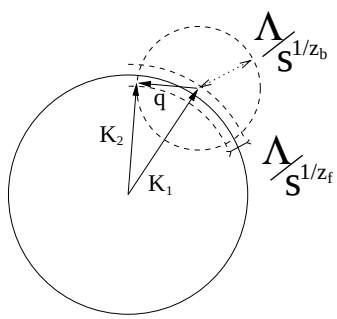

FIG. 1. An illustration of the phase space restriction for the fermion scattering off magnetic modes, here for a ferromagnet. The angle between $\mathbf{k}_2$ and $\mathbf{k}_1$ is restricted by the spin fluctuation cut-off that scales as $\Lambda s^{-1/z_b}$.

Before proceeding with the scaling of different quantities, let us make several observations. First, notice an important consequence of the different rescaling of boson and fermion momenta: since $z_b = 3 > z_f = 1$, the cut-off for bosons reduces slower than for fermions and, near each point at the Fermi surface, the fermion scattering processes become restricted to a thin cylindrical slab, whose thickness scales as $\Lambda s^{-1/z_f} = \Lambda s^{-1}$, while its base radius scales as $\Lambda s^{-1/z_b} = \Lambda s^{-1/3}$, as illustrated in Fig. 1.

Hence the fermion momentum transfer occurs predominantly parallel to the base of the slab. As the initial fermion momentum $\mathbf{k}_1$ spans the shell of width Λ around the Fermi surface, the solid angle of the final fermion momentum $\mathbf{k}_2$ with respect to $\mathbf{k}_1$ is restricted by the magnetic modes (see Fig. 1). Thus, to deal with the scattering vertex, it is convenient to separate the two fermion momenta $\mathbf{k}_1, \mathbf{k}_2$ into $\mathbf{k} \equiv (\mathbf{k}_1 + \mathbf{k}_2)/2$ and the transferred boson momentum $\mathbf{q} \equiv \mathbf{k}_1 - \mathbf{k}_2$. Then, we divide both $\mathbf{k}$ and $\mathbf{q}$ into the one-dimensional components $k_\perp$ and $q_\perp$ normal to the Fermi surface, and the $(d-1)$-dimensional components $\mathbf{k}_\parallel$ and $\mathbf{q}_\parallel$, locally tangential to the Fermi surface. Note that, according to the above, these components scale differently under the RG transformation:

$$k'_\perp = s\ k_\perp\ ;\ \ d^{d-1}\mathbf{k}'_\parallel = d^{d-1}\mathbf{k}_\parallel\ ; \quad (4)$$

$$q'_\perp = s\ q_\perp\ ;\ \ d^{d-1}\mathbf{q}'_\parallel = s^{(d-1)/z_b}\ d^{d-1}\mathbf{q}_\parallel \quad (5)$$

The physical reason behind $d^{d-1}\mathbf{k}_\parallel$ not rescaling is that the size of the Fermi surface does not change upon rescaling, and that $\mathbf{k}$ spans the entire Fermi surface.

Finally, let us make an observation regarding consistency between our rescaling procedure and momentum conservation in a single boson scattering process. The two fermion momenta $\mathbf{k}_{1,2} = k_F \hat{k}_{1,2}(1 + \eta_{1,2})$, where $|\eta_{1,2}| \ll 1$, are related by $\mathbf{k}_1 - \mathbf{k}_2 = \mathbf{q}$. Since, near a ferromagnetic QCP, the boson momenta are much smaller than k_F, to first order in $\eta_{1,2}$ one finds

$$\theta^2\left[1 + \eta_1 - \eta_2\right] = \left(\frac{\mathbf{q}}{k_F}\right)^2,$$

where $\cos\theta \equiv (\hat{k}_1 \cdot \hat{k}_2)$, and $\theta \ll 1$. From this, two points follow: (i) rescaling of the boson momentum has to be accompanied by rescaling of the scattering angle θ betwen the two fermion momenta, as in the procedure we adopted; (ii) asymptotycally close to the Fermi surface ($\eta_{1,2} \to 0$), rescaling fermion and boson momenta differently is consistent with momentum conservation.

Now we are in a position to find the scaling properties of various vertices. To obtain the rescaling of g, rewrite S_i as

$$S_i = g \int d\omega_1 d\omega_2 dk_\perp d^{d-1}k_\parallel dq_\perp d^{d-1}q_\parallel (\varphi_{\mathbf{q}} \psi^\dagger_{\mathbf{k}_1} \psi_{\mathbf{k}_2} + h.c.).$$

Using the scaling properties of the fields (3) and of the different components of momenta (4,5), one arrives at the sought scaling relation:

$$g' = s^{-\frac{d+z_b-4}{2z_b}}\ g\ . \quad (6)$$

We find that g is irrelevant for $d > 1$, similarly to the four-boson coupling constant in the φ^4 theory ($z_b = 3$). [5,6]

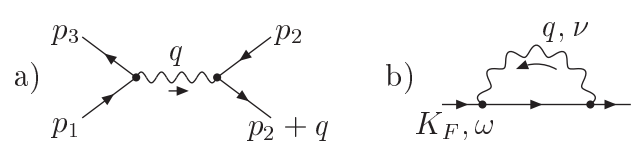

FIG. 2. Feynman diagrams associated with a) the fermi self-energy and b) the boson-mediated four-fermi interaction. Solid lines are fermion and wavy lines are boson propagators.

One may inquire about the relation between the coupling constant u_f of the boson-mediated four-fermion interaction (Fig. 2 a) and the spin-fermion scattering vertex g. Naively, one would expect u_f to scale as g^2 and, hence, become irrelevant in one spatial dimension or less; we will show that this is not the case: at a ferromagnetic

quantum critical point, u_f becomes relevant already in $d < 3$ spatial dimensions, possibly leading to an instability of the Fermi liquid ground state. Notice that, of the three independent momenta ($\mathbf{p_1}$, $\mathbf{p_2}$, $\mathbf{p_3}$) in the diagram a) in Fig. 2, only the momentum transfer $\mathbf{q} \equiv \mathbf{p_1} - \mathbf{p_3}$ is subject to the phase space restriction discussed above, whereas $\mathbf{p} \equiv (\mathbf{p_1} + \mathbf{p_3})/\mathbf{2}$ and $\mathbf{p_2}$ independently span the entire Fermi surface. Hence the four-fermion term in the action can be re-written to make explicit the scaling properties of the various components of momenta:

$$S_{4f} \to u_f \int \frac{d\omega_1}{s}\frac{d\omega_2}{s}\frac{d\omega_3}{s}\frac{dp_\perp}{s} d^{d-1}\mathbf{p}_\parallel \frac{dq_\perp}{s}\frac{d^{d-1}\mathbf{q}_\parallel}{s^{(d-1)/z_b}} \times \quad (7)$$

$$\times \frac{d\mathbf{p}_{2\perp}}{s} d^{d-1}\mathbf{p}_{2\parallel}\; s^6 \psi^\dagger_{\mathbf{p}-\mathbf{q}/2}\; \psi_{\mathbf{p}+\mathbf{q}/2}\; \psi^\dagger_{\mathbf{p}_2+\mathbf{q}}\; \psi_{\mathbf{p_2}} \times \quad (8)$$

$$\times \frac{s^{2/z_b}}{\frac{|\omega_1-\omega_3|}{|\mathbf{q}|} + \mathbf{q}^2}, \quad (9)$$

from which we read off

$$u_f \;\to\; u_f s^{(3-d)/z_b}.$$

Which means that, already at the tree level, u_f becomes relevant below three spatial dimensions, while g is relevant only below $d = 1$. This indicates that the Fermi liquid state may break down below three dimensions, where the naive φ^4 theory would be still above its upper critical dimension.

Finally, let us illustrate how one can use (4,5) to find the scaling of the fermion self-energy $\Sigma(\omega)$ in the lowest order in g. It can be found by power counting of the diagram b) in figure 2; its contribution to the self energy is

$$\Sigma(\omega) \simeq g^2 \int d\nu dq_\perp d^{d-1}\mathbf{q}_\parallel G_\psi(\omega-\nu;\mathbf{p}-\mathbf{q})\; \mathbf{G}_\varphi(\nu;\mathbf{q}).$$

Since $G_\psi^{-1}(\omega;\mathbf{p}) = i\omega - v_F p_\perp$ and $G_\varphi^{-1}(\omega,\mathbf{q}) = |\omega|/\mathbf{q} + \mathbf{q}^2$, one finds, again using (3,4,5):

$$\Sigma(s\omega) = \Sigma(\omega) s^{-d/z_b} \quad (10)$$

This is in agreement with previous work [7,8,11], and points to a possible non-Fermi behavior in $d \leq 3$, as previously observed in the context of gauge theories [11] and recently noted in the context of a ferromagnet [12].

To summarize, we introduced an RG scheme in the spirit of the Shankar approach [9], which allows to treat the fermion and the boson degrees of freedom in the spin-fermion models on an equal footing. We showed that, already at the tree level, the boson-mediated four-fermion coupling is relevant below three spatial dimensions, even though the fermion-boson coupling constant g is relevant only above one dimension. Our approach is general and can be applied to the theory of an antiferromagnetic QCP – as well as to other situations where fermions interact with critical modes, e.g. those involving different dimensionalities of fermions and spin fluctuations. The phase space restriction associated with the fermion scattering off the critical modes is the key ingredient of the approach. A one loop RG treatment in $d = 3 - \epsilon$, including transport properties and thermodynamics, would be a natural extension of this work [14].

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