Information, Reputation and Imitative Choice. A Simple Bayesian Model

Sylvain Marsat

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Sylvain MARSAT*

Faculté de Sciences Economiques et de Gestion
41, Boulevard François Mitterrand
63002 Clermont Ferrand Cedex
and
Centre de Recherches en Sciences de Gestion (CRSG)
Sylvain.marsat@u-clermont1.fr

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ABSTRACT. How does an agent choose an investment when his private information and the behaviour of other preceding actors are opposed? What are the factors encouraging him to act independently of the behaviour of others, or, on the contrary, to imitate them? We propose an extension of the Bayesian model of Scharfstein and Stein (1990), in order to introduce the informational aspect developed by Bikhcandani, Hirshleifer and Welch (1992) and Orléan (1989, 1990, 1992). Agent B can be induced to imitate an agent or a group of agents A preceding him (i) because the information held by A is more reliable than his own information, (ii) because agent B a priori relies more on agent A than on its own abilities or (iii) because he doesn't want to deviate from A, in order to preserve his reputation. We thus seek to synthesize the informational and reputational approaches in order to better understand their respective importance and relationships in imitative investment choices.

KEYWORDS: Herding Behaviour, Bayesian Modelling, Decision Making, Investment Choice, Reputation, Informational Cascade

* Université d'Auvergne – Clermont Ferrand 1, Faculté de Sciences Economiques et de Gestion, 41 Boulevard François Mitterrand, 63 002 Clermont-Ferrand Cedex, Centre de Recherches en Sciences de Gestion (CRSG), E-Mail : sylvain.marsat@ecogestion.u-clermont1.fr. I am grateful for helpful comments and advices from Lorraine April, Solange Gannat, André Orléan, Jean-Pierre Védrine, Benjamin Williams, the participants at the ACSEG 12th Annual Meeting in Aix-en-Provence, and two anonymous referees who helped me to improve the paper with their numerous suggestions. All remaining errors are my own.

Introduction

« Most people are other people. Their thoughts are
someone else's opinions, their lives
a mimicry,their passions a quotation. »

Oscar Wilde (1854 - 1900), *De Profundis*, 1905

From teenagers' fashions to financial investments, sports in vogue to medical practices, social influence has a significant impact in the majority of human activities (Shiller [1984]). In a great number of their decisions, individuals take the choices and opinions of others into account. For the social psychologist Bandura (1986), observation of others is an essential element in the human learning process.

According to Moschetto (1998), the emergence of the concept of imitation in economy and management was not an easy thing. Indeed, the economic agent in the classical theory is rational, independent of the other individuals, as Katona (1953) mentions it. Mimicry is often attached, probably because of the ethologic origin of the concept, with gregarious behaviours, inevitably animal and irrational.

However, based on a keynesian approach, Orléan (1989) shows that rationality and imitation are not inevitably antinomic. Indeed, if an agent does not know anything about the outcome of a decision, he may find it beneficial to imitate another agent who is at least as informed as himself. In a more general way, the behaviour of the other agents partly reveals information that they hold on the situation and, within this framework, imitation makes possible to optimize a decision in a fully rational manner.

Two major mainstreams used a Bayesian approach to explain imitation in investment decisions. Seminal work from Bikhchandani, Hirshleifer and Welch (hereafter referred to as BHW, 1992) models a mimetic chain based on the use of others' behaviour, revealing their own information. In this framework, information signals have the same precision among individuals. If only two individuals made the same decision, the third one, if he acts in a rational Bayesian way, must follow the others whatever his private information. An informational cascade is engaged and every agent who arrives then rationally follows.

Scharfstein and Stein (1990) propose an alternative approach by linking imitation and reputation (Holmström [1982]). The agent does not seek to imitate in order to profit from the information transmitted by other agents' behaviours, but because if he deviates from the preceding actors, he will be judged by the principal as being not qualified enough in the task assigned to him. In fact, it is assumed that good informed managers act in the same way in response to "good identical information". As the informative signals of two good informed people are correlated, the agent seeks through imitation to be considered as a good informed agent, and thus to preserve his reputation.

This paper jointly analyzes some of the various motivations which induce an agent to imitate another one, and their relationships. We propose an extension of the model of Scharfstein and Stein (1990), in order to introduce the informational aspect approached in particular by Bikhchandani, Hirshleifer and Welch (1992) and Orléan (1989, 1990, and 1992). After a presentation of the stakes and assumptions of this simple model (1), the impact of the reliability of information is analysed (2) as well as *a priori* confidence (3), and reputation (4).

1 Assumptions

1.1 Objectives and settings

The aim of this paper is to develop a clearer understanding of the choice of information for an agent, B, thanks to a Bayesian model. We seek to identify some of the forces that can lead to imitation, individuals neglecting their personal information to join the group. This analysis applies generally to any decision of investment including the framework of the decisions of portfolio managers on financial markets[1].

Scharfstein and Stein (1990) rely on reputation to explain imitative behaviour. This work is analysed in detail by the famous comment of Ottaviani and Sorensen (2000), who develop it and show how this model can be close to the BHW (1990) cascade model. The main difference between the two models is in the correlation of signals used by Schafstein and Stein (1992), which can be released partially (Graham [1999]) or totally (Ottaviani and Sorensen [2000]).

Most of the theoretical models postulate signal independence in their settings. This allows multi agent settings' convenient analysis. Chamley (2004, p.228) rejects the idea of signal correlation, which he considers as "irrelevant and confuses the issue",

[1] As Scharfstein and Stein do (1990, p.477), considering a perfectly elastic supply.

postulating that the decision model of each agent is perfectly known by the principal. However, if not needed for herding, this correlation can be interesting insofar as it introduces another payoff for the agent, rewarding agents acting like previous ones. Scharfstein and Stein (1990) show that in uncertainty, principals are based to update their beliefs on two pieces of evidence: (1) whether the investment was profitable and (2) whether the agent's behaviour was similar to other agents. Since systematically unpredictable components affect the profitability, competent agents can receive misleading signals. The fact that all made the same choice can be viewed as a proof of ability. Reputation is then considered to be higher if agents are acting together, which is another constraint and payoff for an agent.

In the following formalization, we consider a principal, delegating his portfolio, with bounded rationality, since he does take into account the fact that the agent could act strategically and fool him. This assumption could seem too simplistic but is consistent with clients who have a clear preference for relative performance, without interest on the possible effects of this evaluation on the manager, or accepting them. This effect has been highlight by Maug and Naik (1996): principals often "prefer the insurance possibilities of a relative performance contract to the higher returns available if fund managers do not herd ".

We present a simple binary signal and dichotomous choice model based on Scharfstein and Stein (1990). The independence signal setting will be used to present basic informational conclusions (sections 2 and 3), adding the *a priori* ability of agents. Hence, section 4 will introduce the judgement of a principal. If this principal has the same information set, the decision of the agent will not be affected at all. An asymmetry of beliefs is then postulated to introduce the relative performance bias: the principal will estimate the ability of the agent on both the probability of making a good decision and the similarity of behaviour with the preceding agent.

1.2 Theoretical assumptions

In order to introduce the concept of reputation, we rely on the work of Scharfstein and Stein (1990) and Ottaviani and Sorensen (2000). We consider two states of the world, {R=P} or {R=N}. {R=P} corresponds to the positive outcome of an investment, and {R=N} corresponds to the negative outcome of the same value[2]. We consider the state

[2] Agent's utility is considered to be symmetric, assuming $x_l + x_h = 0$ in Scharfstein et Stein (1990) framework, as it is assumed in Birkhchandani, Hirshleifer et Welch (1992).

of the world R has a prior probability of ω to be positive, and $(1 - \omega)$ to be negative. In order to simplify the model[3], we set $\omega = ½$. The price is exogenously fixed.

In this very simple framework, two agents A and B act sequentially. Agent A can be regarded as one or more agents[4]. In order to make a decision, agent B receives a private signal which can take value in {-; +}. A {+} signal is a buying signal, i.e. indicates that profitability is positive. Conversely, a {-} signal indicates to the agent that he must sell to avoid a negative profitability. On the stock market, this signal comes from his own fundamental interpretation of the value of the stock and we therefore call it the fundamental signal, Sf. Agents A and B have to optimize their portfolio in executing a binary choice: to buy if they estimate that the future profitability of the action will be positive, and to sell otherwise.

The interpretation of this signal is complex insofar as there are two types of agents on the market. The first type corresponds to rational, informed agents called "smart". Conversely, "dumb" is the category of the agents acting in an irrational way[5]. Agent B, just like agent A, is unaware of the category in which he belongs, "smart" or "dumb", whose prior probabilities are:

$$P(A{=}smart) = P(B{=}smart) = \theta$$

$$P(A{=}dumb) = P(B{=}dumb) = (1-\theta)$$

If he belongs in the "smart" category, the fundamental signal received by agent B is informative, which means that the signal {Sf=+} has more probability to occur in the {R=P} state than in the {R=N}state:

$$P(Sf{=}+/R{=}P,\ B{=}smart) = p = P(Sf{=}-/R{=}N,\ B{=}smart)$$

$$P(Sf{=}+/R{=}N,\ B{=}smart) = 1-p = P(Sf{=}-/R{=}P,\ B{=}smart)$$

With $p > ½$

[3] This assumption "eliminates any incentive for manager 1 to signal his ability by deviating from the efficient outcome" (Avery and Chevalier [1999]). Agent A knowing that ω is close to 0 is incited to sell to be seen as an informed agent. With ω= ½, agent A has no *a priori* incentive to sell or buy, and uses only her private information signal as a cutoff.

[4] If agent A is a group of agents, we consider this group to be globally perceived by agent B, and have only aggregated characteristics.

[5] They can be qualified of "noise traders" by the literature see for instance De Long et al. (1990). Noise traders can however also be motivated by liquidity constraints, which is not the case in this framework.

On the contrary, if agent B belongs to the category "dumb", which occurs with a probability (1- θ), he receives completely uninformative fundamental signals and has as much probability of receiving the signal {Sf=+} than the signal {Sf =-}.

$$P(Sf=+/R=P, B=dumb) = P(Sf=+/R=N, B=dumb) = ½$$

We assume that agent B receives also a signal called "Sm", the signal of agent A, who acted before him. This "mimetic" signal thus corresponds for agent B to the action of the preceding agent. The reliability q of the mimetic signal depends on the category of agent A. If he is "smart", this signal is informative with a reliability q:

$$P(Sm=+/R=P, A=smart) = q = P(Sm=-/R=N, A=smart)$$

$$P(Sm=+/R=N, A=smart) = 1-q = P(Sm=-/R=P, A=smart)$$

With $q > ½$

On the other hand, an agent A "dumb" does not transmit information through his decisions because he also receives uninformative signals:

$$P(Sm=+/R=P, A=dumb) = P(Sm=+/R=N, A=dumb) = ½$$

2 Simple informational Bayesian choice

2.1 Choice of agent B with a private signal

Let us suppose that agent B takes his decision starting from only one signal of information: his private, fundamental signal. One can easily calculate the probabilities of realization of the events according to the received signal with P(R=P)= ½ and P(Sf=+)= ½[6] :

$$P(R=P/Sf=+)= P(R=P,B=smart/Sf=+)+P(R=P,B=dumb/Sf=+) =$$
$$P(Sf=+/R=P,B=smart).P(B=smart)+P(Sf=+/R=P,B=dumb).P(B=dumb) \quad (2.1.1)$$

$$P(R=P/Sf=+)= ½ + \theta\,(p- ½) \qquad (2.1.2)$$

In the same way,

$$P(R=P/Sf=-)= ½ + \theta\,(½ - p) \qquad (2.1.3)$$

$$P(R=N/Sf=-)=½ + \theta\,(p- ½) \qquad (2.1.4)$$

$$P(R=N/Sf=+)=½ + \theta\,(½ - p) \qquad (2.1.5)$$

[6] See proof in appendix.

The choice of agent B depends on two parameters: θ and p. These two variables enable him to increase his probability of making a sound decision, compared to a random choice, of probability ½. The reliability of the signal makes it possible to improve the choice, insofar as this signal is informative[7]. The confidence of agent B in his ability, i.e. the probability that the actor considers himself as "smart" plays a multiplying role of this reliability of the signal.

Following Scharfstein and Stein (1990), we assume that the investment is attractive if a positive signal was received and on the other hand that it is not attractive if the signal is negative. Insofar the fundamental signal is informative, $p \in]0.5;1[$ then if $\theta>0$[8]:

$$P(R=P/Sf=+)=P(R=N/Sf=-)>\tfrac{1}{2}$$

$$P(R=P/Sf=-)=P(R=N/Sf=+)<\tfrac{1}{2}$$

According to this result, agent B follows his fundamental signal whatever it is, since the state of the world {R=P} (respectively {R=N}) has a greater probability of being realized if the received fundamental signal is positive: {Sf=+} (resp. negative: {Sf =-}).

2.2 Choice of agent B on private signal and information on A's behaviour

Let us suppose a sequential investment. In a first period, agent A decides to invest or not. During the second period, agent B chooses, with his own fundamental signal and the behaviour of agent A. The signal transmitted by the action of A will constitute for agent B a "mimetic" signal, Sm, with reliability q and as the first mover, his action reveals his signal. Sf and Sm are considered as independent.

It is impossible to distinguish an imitative behaviour if two information signals induce the same choice. The description of an informational choice thus requires a divergence of the mimetic and fundamental behaviours. We will study the situations for which agent B must deal with two contradictory signals, which correspond to the events {Sf=+, Sm=-} and {Sf=-, Sm=+}. From these two information signals, B must make a binary choice: to follow his fundamental signal, or the mimetic signal, according to the buying and selling signals received. If the second agent only acts on the basis of the two

[7] i.e. when p > ½. Indeed, if p = ½, the signal does not inform more the agent B which is facing a random choice because $P(R=P/Sf=+)=P(R=P/Sf=-)=\tfrac{1}{2}$.

[8] If $\theta=0$, agent B does not believe in his own ability and the choice is no more that a random one. If p= ½ the choice amounts to a random choice too.

independent signals which are transmitted to him, one can calculate the probability of the event {R=P}, given the signals {Sf=+, Sm = -}[9]:

P(R=P/Sf=+, Sm = -) =

$$\frac{P(Sf=+/R=P).P(Sm=-/R=P)}{P(Sf=+/R=P).P(Sm=-/R=P)+P(Sf=+/R=N).P(Sm=-/R=N)} \quad (2.2.1)$$

And thus[10]:

$$P\,(R=P/Sf=+,\ Sm=-) = \frac{[\frac{1}{2}+\theta(p-\frac{1}{2})].[\frac{1}{2}+\theta(\frac{1}{2}-q)]}{[\frac{1}{2}+\theta(p-\frac{1}{2})].[\frac{1}{2}+\theta(\frac{1}{2}-q)]+[(\frac{1}{2}+\theta(\frac{1}{2}-p)].[\frac{1}{2}+\theta(q-\frac{1}{2})]} \quad (2.2.2)$$

The rule of decision making according to the information signals is as follows[11]:

- if *P (R=P/Sf=+, Sm = -)* > ½, the probability that profitability is positive is higher than ½, agent B buys;
- if *P (R=P/Sf=+, Sm = -)* < ½, the probability that profitability is positive is lower than ½, agent B sells.

As in agent B's decision having only the fundamental signal, θ plays an amplifying role of the reliability of information[12]. In this configuration nevertheless, this amplification takes place in the same way for the fundamental signal and the mimetic one, which cancels its effect. In other words, the probability for agent A of being "smart" is equal to that of the agent B to be "smart". Under these conditions, parameters p and q, the reliability of the signals received by the agents, make the difference. Agent B chooses the decision whose reliability is higher: he buys if $p>q$, sell if $p<q$. If $p=q$, he chooses one or the other with probability ½.

[9] We now consider the {Sf=+, Sm = -} signals. These results can be extended to {Sf=-, Sm = +} given the symmetry of assumptions.
[10] See proof in appendix.
[11] *P(R=P/Sf=+, Sm = -)* can be seen as a *relative* a posteriori confidence between A and B
[12] If θ =1, the reliability of the signal p is equal to the posterior probability *P(Sf = +/R=P) = P(Sf = -/R=N).* If p=q, this model converges with that of Birkhchandani, Hirshleifer and Welch (1992). Within our framework of analysis, the reliability of mimetic signal is not a direct function of the fundamental signal (q=f(p)), as it is the case in the mimetic sequence suggested by these authors, who postulate that q increases with the number of actors who chose the same action in such a way that as soon as two succeeding people acted in concert, q is systematically higher than p, therefore which it is informationnally rational to imitate. We do not study the mimetic mechanism of sequence and we restrict with a model with two actors, in the line of Scharfstein and Stein (1990).

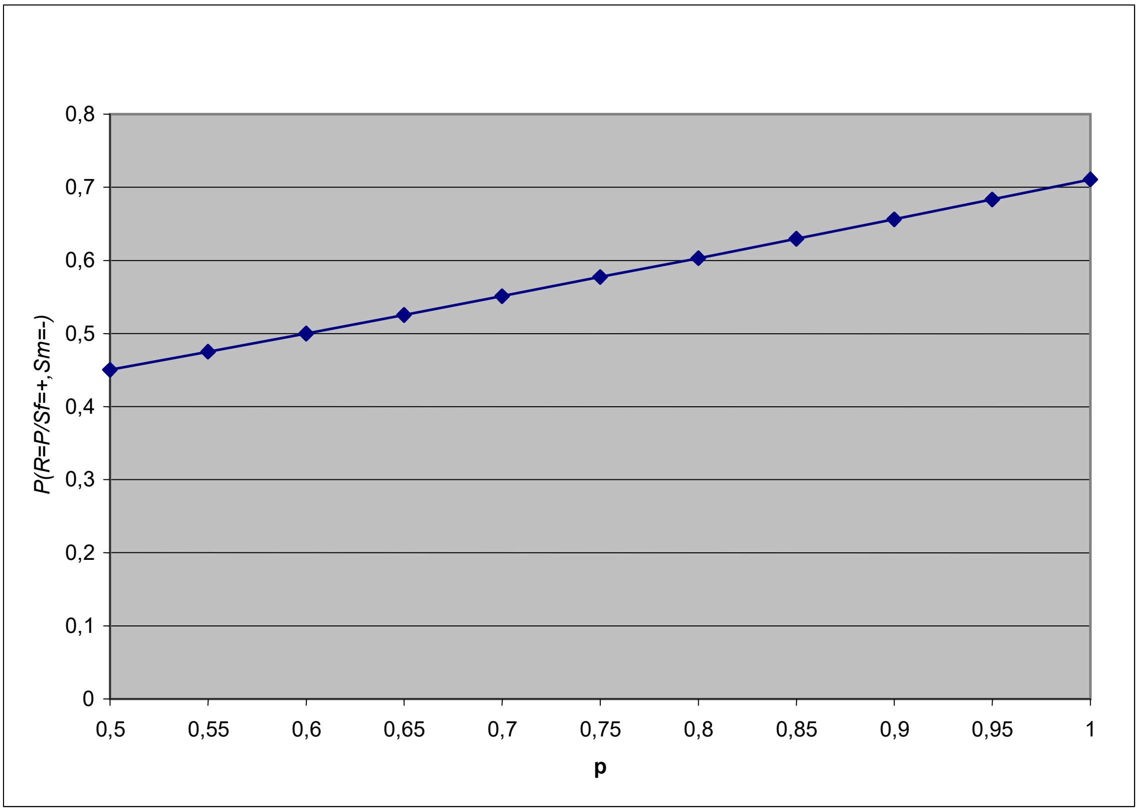

Figure 1. Variation of *P(R=P/Sf=+, Sm=-)*

Parameters: q=0.6; $\theta_B=\theta_A=½$

As an example, let q= 0.6 and θ = ½. In this special case, *P(R=P/Sf=+, Sm = -)* vary according to p on figure 1. This probability is lower than ½ when p<0.6, and that on the contrary, when p>0.6, is higher than ½. Agent B which has a mimetic signal of a reliability of 0.6 will thus imitate A when the reliability of the mimetic signal q is higher than the reliability of the fundamental signal, p.

Let us suppose nevertheless that the actor receives a highly reliable fundamental signal, but that his preferences encourage it to act according to the mimetic signal. The experiment of Asch (1951) presents a striking illustration. An individual subjected to these tests of length recognition has a negligible probability to be mistaken. However, in more than one third of the cases, he chooses to act as the group. Asch mentions that some subjects hesitate, lack confidence in themselves and thus feel a strong tendency to join in the majority. This dimension[13] can be integrated by differentiating the capacities

[13] Of course, overconfidence is a well documented bias in the behavioural literature. Asch's experiment is invocated to motivate that people have a *relative*-low confidence in their ability. An agent can be overconfident ($\theta_{B\text{"overconfident"}}$) and however less confident in his ability than in agent A ($\theta_{B\text{"overconfident"}} < \theta_A$).

of actors A and B. Beyond the reliability of the signal, the *a priori* confidence of the ability of actors can explain some herding behaviours.

3 Informational influence and *a priori* confidence in the source

3.1 Distinction of *a priori* confidence and consequences

Let us suppose now that the ability of the two actors have different probabilities for agent B. As Avery and Chevalier (1999), we assume that agents have information about their ability[14]. There are therefore two distinct prior probabilities: the *a priori* probability of ability of agent B himself and of the first agent A[15].

$$P(A{=}smart) = \theta_A\, ;\; P(A{=}dumb) = (1- \theta_A)$$

And $$P(B{=}smart) = \theta_B\, ;\; P(B{=}dumb) = (1- \theta_B)$$

Agent B does not know to which category he belongs, but estimates the probabilities of agent A and himself to be "smart". The choice does not then depend any more on the only reliability of information signals, but also on the *a priori* confidence the actor grants to the agents. In this case, the equation (2.2.2) becomes:

$$P\,(R{=}P/Sf{=}+,\, Sm{=}-) = \frac{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]}{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]+[(\frac{1}{2}+\theta_B(\frac{1}{2}-p)].[\frac{1}{2}+\theta_A(q-\frac{1}{2})]} \quad (3.1.1)$$

The decision rule remains unchanged: agent B invests when the probability of a profitability knowing the two signals is positive, otherwise he sells. In this new configuration, the prior probabilities play a differentiated role on p and q. When $\theta_A = \theta_B$, one goes back to the preceding case of an informational Bayesian choice only based on the reliability of information. If $\theta_A > \theta_B$, agent B grants a greater *a priori* confidence in the ability of agent A than in his own ability and this induces an amplification of the mimetic signal q more significant than that of the fundamental signal: agent B propensity to imitate is thus more important. An individual who does not feel capable, and lacks confidence in his abilities, will have a low θ_B. Consequently, his future

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Overconfidence can explain why people are experimentally less prone to follow the herd (with $\theta_{B\text{"overconfident"}} > \theta_B$,) which is consistent with Kramer, Nörth and Weber (2006).

[14] Differing from Avery and Chevalier (1999), this information is not private since these probabilities are assumed to be common knowledge.

[15] Graham (1999, p.241) uses the term "initial reputation" to define this probability.

decision will be affected by minimizing the impact of the reliability of individual information. Otherwise, if $\theta_A < \theta_B$, agent B relies more on his own ability and he will attach *a priori* more importance to the reliability of fundamental signal, p. His propensity to imitate is thus less significant.

Let us once again take an example with q=0.6 and θ_A = 0.7 while varying *a priori* confidence of agent B in his ability. As shown in figure 2, the more θ_B is weak, i.e. the less the agent believes in his own capacities, the more p must be significant so that he can adopt a fundamentalist behaviour. If θ_B is very weak (for example θ_B = 0.1), the agent B imitates whatever the reliability of the private information is. On the other hand, if agent B relies a lot on his capacities (θ_B = 0.9), then a less reliable signal (p<0.6) can nevertheless enable him to adopt a fundamentalist behaviour.

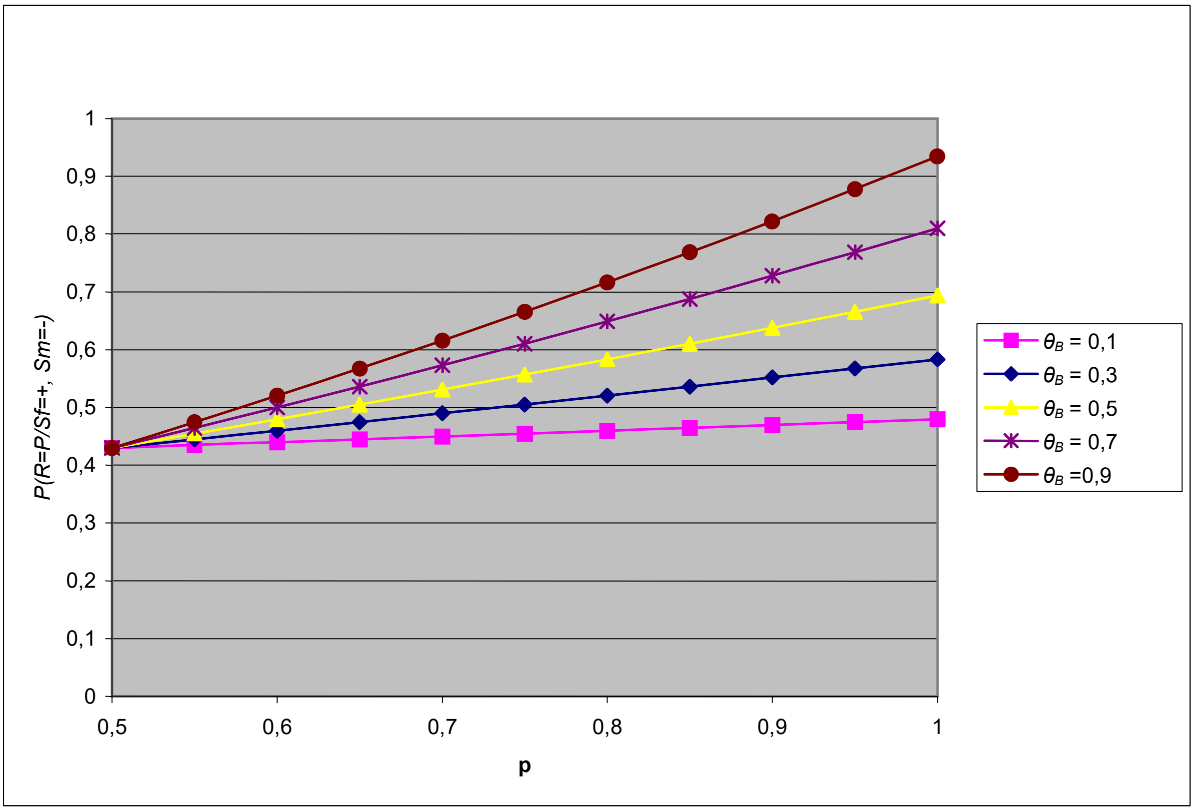

Figure 2. Variation of *P(R=P/Sf=+, Sm=-)*

Parameters: q=0.6; θ_A= 0.7

3.2 Reliability of information signal and credibility of the source

The interest of this concept of credibility is to distinguish for the actor (i) the information objectively received and his interpretation (*p* and *q*), of (ii) the beliefs or the preferences on their source, which also plays a role, whatever this information signal is. One could thus differentiate in the precision of the signal two different components:

- the acquisition of information and interpretation in buying and selling signals of a more or less great reliability (*p* and *q*);
- its weighting according to personal criteria with the agent, relating to the ability of the agents (θ_A and θ_B).

Most of the models stemming from BHW (1992) elude this difference, assuming that all agents receive signals of the same probability (p=q) and that all have a perfect confidence in their ability (θ_A = θ_B =1). Differentiating these two parameters can be interesting since in an informational efficient market with no informational asymmetry (p=q), the difference of ability can be a factor explaining herding behaviour for a second agent, less confident in his *a priori* ability. This is consistent with several empirical studies, such as Lamont (1995), Hong and al. (2000), or Chevalier and Ellison (1999) who find that younger decision-makers herd more than their older counterparts, more experienced and therefore having more prior probability of being "smart"[16]. An interesting experiment of Cote and Sanders (1997) propose the same information to evaluate a firm: subjects who were less confident in their ability were proved to be more influenced by consensus. Signal precision seems then to have an endogenous component, depending on the subject and not the only information, captured by this prior confidence.

4 Reputational aspects

Asch (1951) notes, in order to explain the conformism observed during his experiment, the assumption according to which subjects proceed in a "distortion of the action". Even if they think they are right, they are afraid that their "true judgement" isolate them, and to be badly perceived by the group. They do not give up their judgement which they believe true, but modify the action and act in order to conform to others' judgement. Thus, they do not adopt the judgement of the group -which would be a "distortion of judgement"- but only its action -from there this concept of "distortion of action"-.

We consider an agent B sensitive to the evaluation of his behaviour by the principal. The agent thus worries about the signal transmitted by his behaviour. If this signal is

[16] i.e. with E($\theta_{youngerDMSr}$) < E($\theta_{olderDMSr}$), DMS for decision makers.

different to agent A's choice, he may have problems with his reputation. As Keynes (1936, p.158) mentions it, "*worldly wisdom teaches that it is better for the reputation to fail conventionally than to succeed unconventionally*".

In the absolute, agents should only seek to maximize their profit expectation concerning the investment, and thus to use information indicating that to invest has a positive expectation, as we saw previously. However, two smart managers could receive two misleading signals because of systematically unpredictable components. Therefore, similarity in A and B behaviours is considered as an important piece of information for the principal, estimating the ability of agent B.

Scharfstein and Stein (1990) decide to allow external observers[17], customers, employers or peers, to update their beliefs on agents' abilities. $\hat{\theta}$ is their revised estimate of the probability that a manager is "smart". Taking into account the competition between managers, the level of wages is regarded as directly related: a manager perceived as being more qualified will have higher wages. Insofar as their wages increase in a linear way, the managers are incited to maximize the expected value of $\hat{\theta}$, rather than to invest in an efficient way.

We postulate that the principal has a bias towards relative performance but does not take into account that the agent may act strategically. The principal either (i) delegated the decision and do not want to spend time or resources on it, or (ii) is conscious of the possible strategic action of agent B but clearly prefers a relative performance than a higher return, implying a deviant choice, as pinpointed by Maug and Naik (1996). This is consistent with the study of Arnwald (2001) who shows this relative performance concern in a survey among German portfolio managers: their first objective is to beat the benchmark, their first risk, under-performance. Likewise, evaluation criteria are relative to benchmarks and results of comparable funds, by far more than absolute return or risk-adjusted measures.

4.1 Correlation of signals

Acting like others can be considered for the principal as a proof of ability, and relative performance insurance, since investment decision contains systematically unpredictable components. Introducing this aspect in the model, an asymmetry of beliefs between

[17] We will now refer to the principal, estimating the ability of the agent B who has access to the same set of information than agent B but assumes the correlation of signals between two smart agents.

agent B and the principal is needed[18]. In this agency relationship, the principal updates the ability of agent B according to the probability of signals, and assuming they are correlated if both are "smart". Agent B, who believes in an independent signal setting, knows the principal estimation function as well as this signal correlation, and will try to optimize his reputation.

Correlation[19] of signals between "smart" agents has been introduced by Scharfstein and Stein (1990). Two agents are assumed to observe exactly the same signal if they are "smart". "Smart" agents receive perfectly correlated signals since they are assumed to share the same vision of part of the "truth", whereas the signals of "dumb" agents are not correlated at all, since they observe a noise which has nothing to do with fundamentals[20]. In other words, if two agents receive different signals, at least one of them must be dumb. Then, the probability does not follow an independent draw relating to the initial distributions from the principal standpoint.

Since for the principal the signals of "smart" agents are perfectly correlated, agent B tends to make the same decisions as A, indicating they have received the same information, and thus belong to the "smart" category. If the choice of agent B deviates from that of agent A, this means that the signal Sf received is different from Sm, therefore that his signal is different from that of the group. In this case, either agent B is "dumb", or agent A is "dumb", or both are "dumb". On the other hand, if agent B does not use his information and follows agent A, the principal estimation function will judge that the two agents have a strong probability of being "smart" insofar as, apparently, the two agents seem to act according to same information[21].

4.2 Estimation fonction $\hat{\theta}$

Let us examine now when the second manager, B, seeks to maximize the expected value $\hat{\theta}_B$ in the event *{Sf=+,Sm=-,R=P}*. $\hat{\theta}_B$ *(Sf=+,Sm=-, R=P)* is the revision of the estimate of the principal i.e. the posterior probability that the manager B is "smart"

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[18] If the principal perfectly knows the agent problem and constraints, agent B has no stronger incentive to follow agent A and he his placed in a simple informative decision as in the previous sections. Without signal correlation, the estimation function is not biased and correctly estimates the choice of the agent according to an efficient Bayesian choice.

[19] Here, only perfect correlation is analysed. This could be extended to partially correlated signals (see Graham [1999])

[20] Thus, the probability that the two agents observe {Sf=+, Sm=+ } when the state of the world is {R=P} is q.

[21] Scharfstein and Stein (1990) assume that two agents making the same error « share the blame », i.e. that if both made a bad investment, it was because of an unpredictable factor and not because of their ability.

given these events, assuming the correlation of signals. According to the definition of the estimation function, the probability agent B to be "smart" with {Sf=+, Sm = -} signal set is[22] :

$$\hat{\theta}_B\ (Sf{=}+,\ Sm{=}-,\ R{=}P) = \frac{2\theta_B p(1-\theta_A)}{2\theta_B p(1-\theta_A)+2(1-\theta_B)\theta_A(1-q)+(1-\theta_B)(1-\theta_A)} \qquad (4.2.1)$$

In the same way, the probability that the agent is "smart" if {R=N} is:

$$\hat{\theta}_B\ (Sf{=}+,\ Sm{=}-,\ R{=}N) = \frac{2\theta_B(1-p)(1-\theta_A)}{2\theta_B(1-p)(1-\theta_A)+2(1-\theta_B)\theta_A q+(1-\theta_B)(1-\theta_A)} \qquad (4.2.2)$$

And the *a posteriori* expectation of reputation while acting according to the signals he acquires is :

$$\Theta_B\ (+,-) = \hat{\theta}_B\ (Sf{=}+,\ Sm{=}-,\ R{=}P)\ P(R{=}P/\ Sf{=}+,\ Sm{=}-)$$
$$+\ \hat{\theta}_B\ (Sf{=}+,\ Sm{=}-,\ R{=}N)\ P(R{=}N/\ Sf{=}+,\ Sm{=}-) \qquad (4.2.3)$$

This expectation can easily be calculated thanks to equations (3.1.1), (4.2.1) and (4.2.2)[23].

4.3 Agent B hides his private information

If agent B decides to imitate A, he transmits to the market a signal on his ability. He shows that his private or fundamental signal is coherent with the signal transmitted by the agent or the group of agents A. Thus let us suppose that the individual chooses to imitate, i.e. that he deliberately chooses to transmit a signal different from his private information. This signal transmitted thus means that the private signal of information is coherent with the mimetic signal, which is not the case since both signals are opposed. In this case, one obtains the following probabilities of his estimation by the principal:

$$\hat{\theta}_B\ (Sf{=}-,\ Sm{=}-,\ R{=}P) = \frac{2\theta_B(1-p)(1-\theta_A)+4\theta_A\theta_B(1-q)}{2\theta_B(1-p)(1-\theta_A)+4\theta_A\theta_B(1-q)+2\theta_A(1-\theta_B)(1-q)+(1-\theta_A)(1-\theta_B)} \qquad (4.3.1)$$

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[22] On numerator is the probability of agent B being smart with a {Sf=+} signal. Since signals are correlated, A and B being smart is not possible with different signals. On the denominator, all the possible cases: B smart and A dumb, A smart and B dumb, and both dumb, according to {Sf=+} and {Sm=-} signals.

[23] See the detailed equation in appendix.

and

$$\hat{\theta}_B\ (Sf{=}-,\ Sm{=}-,\ R{=}N) = \frac{4\theta_A\theta_B q+2\theta_B p(1-\theta_A)}{4\theta_A\theta_B q+2\theta_B p(1-\theta_A)+2(1-\theta_B)\theta_A q+(1-\theta_A)(1-\theta_B)} \qquad (4.3.2)$$

With equations (3.1.1), (4.3.1) and (4.3.2) the expectation of *a posteriori* reputation is:

$$\Theta_B\ (-,-) = \hat{\theta}_B\ (Sf{=}-,\ Sm{=}-,\ R{=}P)\ P(R{=}P/\ Sf{=}+,\ Sm{=}-)$$
$$+\ \hat{\theta}_B\ (Sf{=}-,\ Sm{=}-,\ R{=}N)\ P(R{=}N/\ Sf{=}+,\ Sm{=}-) \qquad (4.3.3)$$

4.4 Optimal choice with reputation

According to the simplifications of their model, the choice of signal is not informational any more[24] and Scharfstein and Stein (1990) show that an optimal behaviour is systematically mimetic. In this framework, herding behaviour depends on the parameters p, q, θ_A and θ_B. An optimal decision for agent B is then to have the highest expectation of *a posteriori* reputation:

- if $\Theta_B\ (-,-) < \Theta_B\ (+,-)$, agent B acts according to his private signal;
- if $\Theta_B\ (-,-) > \Theta_B\ (+,-)$, agent B hides his private signal and follows agent A.

When agent B hides his private information not to appear inadequate towards others actors, because of the correlation of the signals, he has the possibility to increase the probability of being perceived like "smart". In fact, he has by far a more significant propensity to hide his private information compared to an informational choice as we saw in sections 2 to 4.

By adding the dimension of reputation, the conditions by which the agent B acts in a fundamental, non imitative way are much more restrictive: he must receive a fundamental signal of a great reliability p, have a great confidence in this signal θ_B but also a weak confidence θ_A in the ability of agent A, of which the reliability of the signal q must be low (close to 0.5). Let us take once more the example where q is 0.6 and $\theta_A = \theta_B = ½$. For agent B, which is his estimation by the principal when he shows or when he hides his private information?

[24] i.e. P(R=P/Sf=+, Sm = -) = ½.

If $\theta_A = \theta_B = ½$, *P(R=P/Sf=+, Sm=-)* is higher than ½ when *p* exceeds *q*, i.e. when *p*>0.6. By taking into account his reputation, agent B puts aside this reliability of information to stick to the maximization of his perceived ability by the principal. One can see that this constraint of maximization means that he must systematically hide his information ($\Theta_B(-,-) > \Theta_B(+,-)$) whatever the reliability of his private information may be.

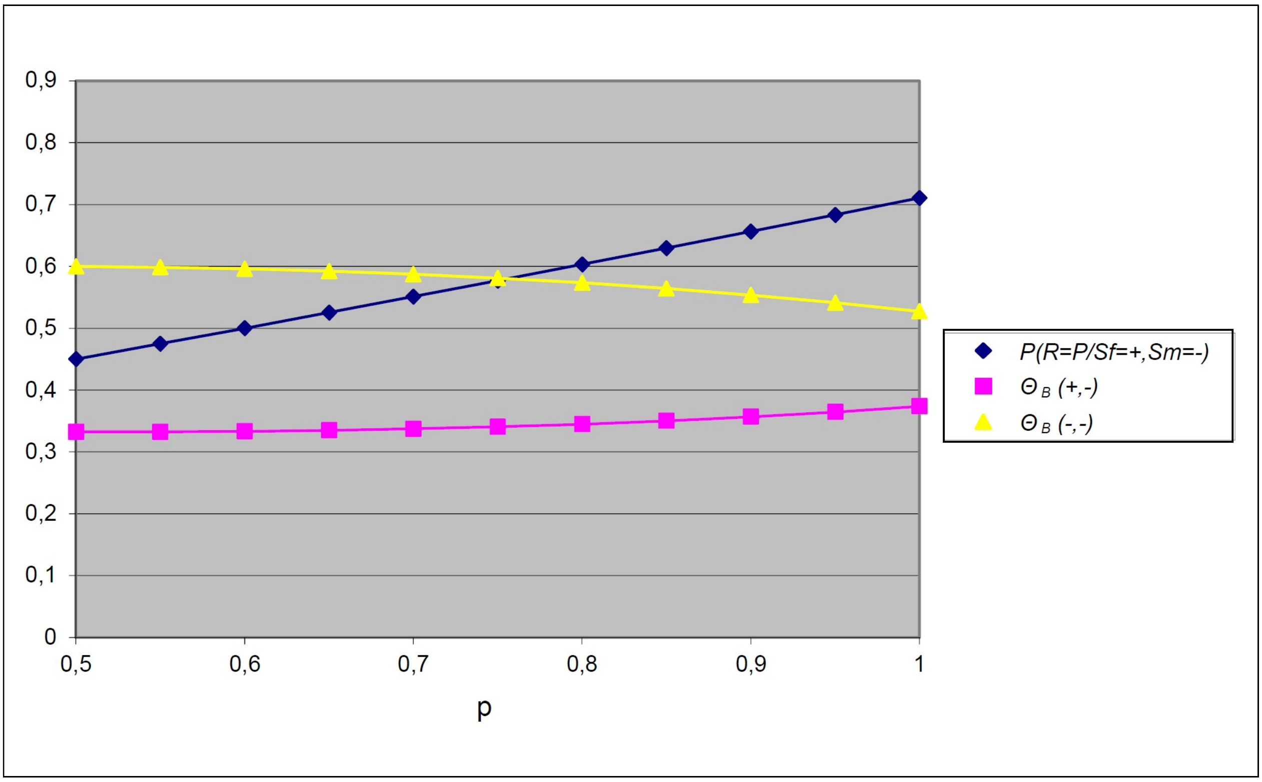

Figure 3. Variation of *P(R=P/Sf=+, Sm=-)*, $\Theta_B(-,-)$ and $\Theta_B(+,-)$)

Parameters: q=0.6; $\theta_A = \theta_B = 0.5$

What does occur if one modifies the values of θ_A and θ_B? Let us take for example a higher *a priori* confidence of agent B in his ability than those of agent A, so that θ_A=0.3 and θ_B=0.7 (see figure 4 below).

The modification of *a priori* confidence changes estimation fonctions $\Theta_B(-,-)$ and $\Theta_B(+,-)$. Because *a priori* confidence in the ability of A is less significant, and confidence in his own capacities greater, agent B can be found in situations in which it is not optimal to imitate. These situations are nevertheless very restrictive. Indeed, the agent is rationally induced to act according to his signal ($\Theta_B(+,-) > \Theta_B(-,-)$) only when this signal is highly reliable and almost equal to one. His fear of being regarded as "dumb" if he uses his own information leads him to a very great prudence. The concept of reputation developed by Scharfstein and Stein (1990) thus adds to the simple quality of information a dimension of the pressure of the group, outside informational maximization. Agent B may find it beneficial to change his decision, in order to transmit, by his action, a signal which identifies him by the principal as being a "smart" agent.

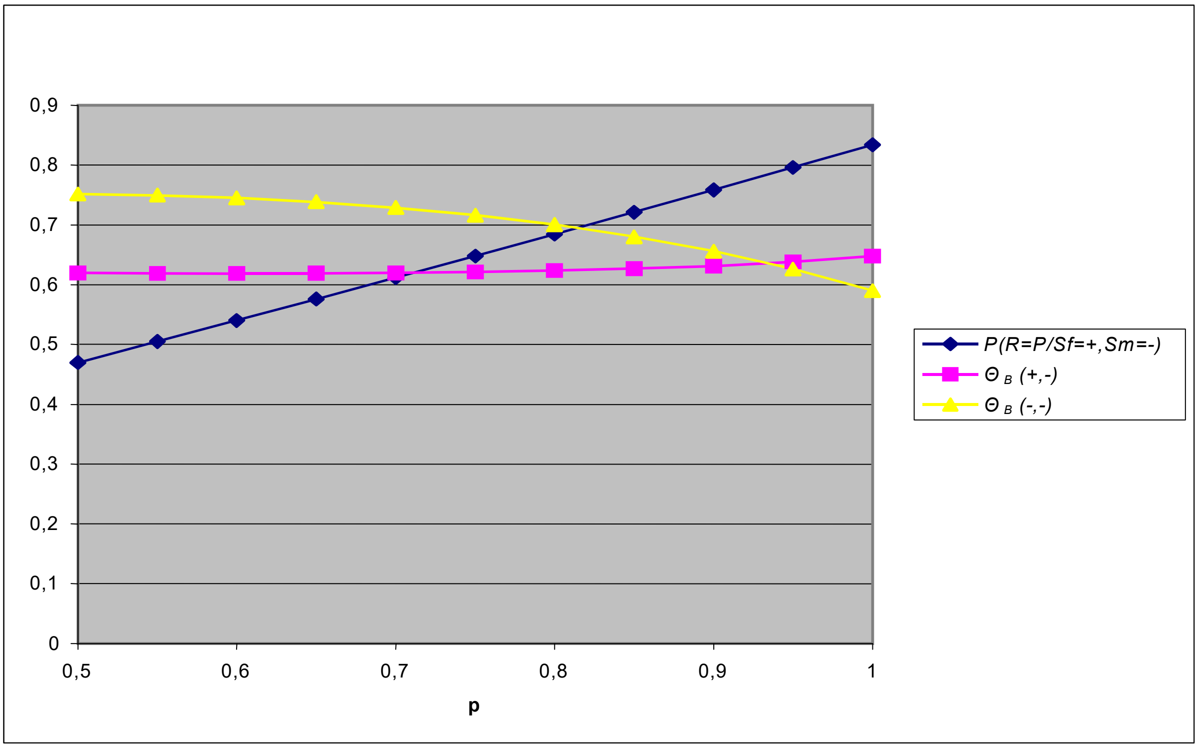

Figure 4. Variation of *P(R=P/Sf*=+, *Sm*=-), *Θ B (-,-)* and *Θ B (+,-)*)

Parameters: q=0.6; θ_A=0.3; θ_B =0.7

If agent B acts isolated, it is necessary for him to have a lot of confidence in his ability, a highly reliable private information, and at the same time a low reliability and credibility in agent A. Otherwise, if he has uncertainties about his private information, agent B seeks to reduce its idiosyncratic risk: he imitates, for fear of feeling set aside. With the requirement of reputation maximization, agent B thus has a tendency to focus his attention more on the action of agent A and less on his private signal compared to an informationally efficient decision.

Conclusion

Through a principal-agent asymmetry of beliefs, this simple model allows the analysis of the decision of an agent B facing two kinds of payoffs: an informational one and a reputational one. As in traditional informational models[25], signal precision is an important component in the decision. This model introduces *a priori* confidence in ability. Low ability introduces noise and affects the precision of signal, which is consistent with some empirical works[26] showing that more experienced agents are less prone to herd.

The principal's signal correlation belief modifies the pure informational decision of agent B, since the estimation function of ability rewards taking the same behaviour than agent A. Aware of this agency problem, agent B then tries to fool the principal in acting like the previous agent. Reputation is analysed as a constraint fostering imitation, already present for informational reasons.

This framework allows informational and reputational factors to be studied together. The most important results are the view of imitation in a multiple factor analysis, building a bridge between informational and reputational works. It is particularly interesting to analyze the impact of traditional "informational" parameters like signal reliability and prior ability on reputation. An agent B with high reliability signal and a high *a priori* confidence will not herd neither for informational reasons nor for reputational ones, since the principal's estimation is then based more on these parameters than on the similarity of behaviour between A and B. This result was not possible in Scharfstein and Stein (1990) framework in which signals were not informative.

This simple model proposes a few concrete hypotheses on imitation. A natural further task would be to develop it with partial correlation, a dynamic approach and the introduction of a price mechanism. Besides, testing the theoretical hypotheses through an experimental setting, and finding empirical evidence of the different factors involved in herding behaviour in investment decisions seems to be a fertile area for future research.

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[25] These models tem from BHW (1992).
[26] e.g. Chevalier and Ellison (1999).

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Appendix

A. List of notation

Notation	Meaning
R	State of the world
P	Profit
N	Loss
ω	*P(R=P), a priori* probability of a profit (set to ½)
Sm	Signal from the choice of agent(s) A
Sf	Private signal
+	Buying signal
-	Selling signal
p	*P(Sf=+ /R=P, B=smart)* Reliability of Sf, when B is « smart »
q	*P(Sm=+ /R=P, A=smart)* Reliability of Sf, when A is « smart »
θ_A	*P(A=smart)* *ex ante* probability that agent A is "smart"
θ_B	*P(B=smart)* *ex ante* probability that agent B is "smart"
$\hat{\theta}_B$	*a posteriori* probability that agent B is "smart"
Θ_B	*a posteriori* expectation of reputation of l'agent B

B. Proof that P(Sf=+)= ½

P(Sf=+) = P(Sf=+/B=smart).P(B=smart)+P(Sf=+/B=dumb).P(B=dumb)
= P(Sf=+/B=smart, R=P).P(B=smart).P(R=P)+ P(Sf=+/B=dumb, R=P).P(B=dumb).P(R=P)
+ P(Sf=+/B=smart, R=N).P(B=smart).P(R=N)+ P(Sf=+/B=dumb, R=N).P(B=dumb).P(R=N)

And from the hypotheses:

P(Sf=+) = p. θ. ½ + ¼.(1- θ)+ ½.(1-p). θ+ ¼.(1- θ)

Then*: P(Sf=+) = ½, Q.E.D.*

C. Equation (2.2.2) - Proof

$$P(R=P/Sf=+, Sm=-) = \frac{P(Sf=+/R=P).P(Sm=-/R=P)}{P(Sf=+/R=P).P(Sm=-/R=P)+P(Sf=+/R=N).P(Sm=-/R=N)} \quad (2.2.1)$$

With:

P(Sf =+/R=P)= P(Sf=+ /R=P, B=smart) . P(B=smart) + P(Sf=+/R=P, B=dumb). P(B=dumb)
= ½ + θ (p- ½)= P(Sf=-/R=N)
P(Sf=- /R=P) =P(Sf=- /R=P, B=smart) . P(B=smart) + P(Sf=- /R=P,B=dumb). P(B=dumb)
= ½ + θ (½ - p)= P(Sf=+/R=N)

And in a similar manner:
P(Sm=+ /R=P) = ½ + θ (q-½) = P(Sm=- /R=N)
P(Sm=- /R=P) = ½ + θ (½ -q) = P(Sm=+ /R=N)
Thus:

$$P(R=P/Sf=+, Sm=-) = \frac{[\frac{1}{2}+\theta(p-\frac{1}{2})].[\frac{1}{2}+\theta(\frac{1}{2}-q)]}{[\frac{1}{2}+\theta(p-\frac{1}{2})].[\frac{1}{2}+\theta(\frac{1}{2}-q)]+[(\frac{1}{2}+\theta(\frac{1}{2}-p)].[\frac{1}{2}+\theta(q-\frac{1}{2})]} \quad (2.2.2)\ Q.E.D.$$

D. Full report of equation (4.2.3)

$$\Theta_B(+,-)= \hat{\theta}_B(Sf=+, Sm=-, R=P)\, P(R=P/ Sf=+, Sm=-) + \hat{\theta}_B(Sf=+, Sm=-, R=N)\, P(R=N/ Sf=+, Sm=-) \quad (4.2.3)$$

Given equations: (3.1.1) (4.2.1) (4.2.2), then (4.2.3):

$$\Theta_B(+,-)=$$

$$\frac{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]}{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]+[(\frac{1}{2}+\theta_B(\frac{1}{2}-p)].[\frac{1}{2}+\theta_A(q-\frac{1}{2})]}\cdot\frac{2\theta_B p(1-\theta_A)}{2\theta_B p(1-\theta_A)+2(1-\theta_B)\theta_A(1-q)+(1-\theta_B)(1-\theta_A)}$$

$$+\left(1-\frac{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]}{[\frac{1}{2}+\theta_B(p-\frac{1}{2})].[\frac{1}{2}+\theta_A(\frac{1}{2}-q)]+[(\frac{1}{2}+\theta_B(\frac{1}{2}-p)].[\frac{1}{2}+\theta_A(q-\frac{1}{2})]}\right)\cdot\frac{2\theta_B(1-p)(1-\theta_A)}{2\theta_B(1-p)(1-\theta_A)+2(1-\theta_B)\theta_A q+(1-\theta_B)(1-\theta_A)}$$