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# Optimal pump excitation frequency for improvement of damage detection by nonlinear vibro acoustic modulation method in a multiple scattering sample

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## Abstract

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We present a method to systematically optimize nonlinear damage detection in multiple scattering media by the nonlinear Vibro-Acoustic Modulation (VAM) technique. The latter consists here of exciting a medium simultaneously with a high frequency ultrasonic sinusoidal burst and with a low frequency continuous sinusoidal wave. Modulation of the high frequency (probe) by the low frequency (pump) is made possible by the presence of nonlinear scatterers, *i.e.* cracks, defects. A signal processing technique consisting of a closed loop system drives the automatic adaptation of the pumping frequency, yielding to the optimization of the nonlinear modulation (NM) of the output probing coda signal without *a priori* information on the medium and the scatterers. The correlation coefficient between a reference output probe signal without the pumping wave and an output modulated probe signal with a pumping wave was considered as our cost function. A multiple scattering solid beam where nonlinear scatterers can be controllably added or removed is designed and tested. The first step of this study is an empirical search of the correlation coefficient dependency on the pumping frequency to verify the performances of the proposed method. Then the implemented optimization algorithm based on genetic algorithm (GA) is used to find automatically the optimal pumping frequency. The obtained optimization results show a good agreement with the empirical study. Moreover, the genetic algorithm allowed to find the optimal pump frequency adapted to each configuration of nonlinear scatterers. This relatively fast search of the optimal nonlinear response could be extended to nonlinear scatterer imaging applications using the information on the resonant modes spatial shapes together with the associated optimal response.

*Keywords:* Optimal command, Genetic algorithm, Nonlinear Vibro-Acoustic Modulation, Crack detection.

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## 1 **1. Introduction**

2 Nonlinear ultrasonic nondestructive testing has known a great develop-  
3 ment during the last decades. Numerous studies have shown the possibility  
4 to detect damages at earlier stages than with linear ultrasonic methods.  
5 The integrity of the structure can be easily preserved by an earlier mainte-  
6 nance. The presence of a nonlinear defect, such as a crack or a delamination,  
7 can give rise to nonlinear phenomena, among which are the generation of  
8 higher harmonics, subharmonics, wave modulation or resonance frequency  
9 shift. **Several methods based on the detection of these phenomena have been**  
10 **proposed [1, 2, 3, 4, 5, 6, 7].** One of the most widely studied methods is the  
11 Vibro-Acoustic Modulation (VAM) [8, 9] or the Nonlinear Wave Modulation  
12 Spectroscopy (NWMS) [10], which belongs to the class of nonlinear modu-  
13 lation or nonlinear mixing methods. With the VAM, micro-damage can be  
14 detected by following the amplitude modulation induced on a probe signal  
15 (*e.g.* high frequency ultrasound) by a pump signal (*e.g.* low frequency vibra-  
16 tion). Nonlinear modulation techniques have been widely used on samples  
17 with simple geometries in which coherent waves propagate [11, 12]. How-  
18 ever, the use of these methods has been poorly studied in environments with  
19 complex geometries, which lead to multiple scattering of the waves. Among  
20 the few reported results in such case, Zhang et al. have demonstrated that  
21 a global inspection of multiple scattering materials is possible by combining  
22 the Coda Wave Interferometry (CWI) technique with a nonlinear modulation

23 method [13, 14].

24 Generally, the pump frequency is chosen to correspond to one of the vibra-  
25 tion modes of the studied sample in order to amplify the vibration response  
26 [9, 15, 16, 17, 18]. A preliminary modal analysis is, then, necessary for iden-  
27 tifying and selecting the resonance frequencies of the sample. It is important  
28 to underline that such a procedure requires a further experimental setup and  
29 time. Furthermore, a possible drawback of a modal excitation is that the  
30 crack or the damaged zone can be located at a strain node of the pump or  
31 probe wave, which compromises the generation of the nonlinear modulation.  
32 To overcome this problem, a frequency swept pump and/or probe signals  
33 have been used [13, 17, 19, 20, 21, 18, 22]. As such, several resonance modes  
34 are excited and an averaged effect over a wide frequency range is detected,  
35 without low sensitivity zones. *Dunn et al.* suggest that selecting the res-  
36 onant frequencies as a pumping frequency is not ideal, because the system  
37 nonlinearities are maximum at those frequencies and the resonant peaks shift  
38 in frequency with changing amplitude [23].

39 In order to optimize and to refine the sensitivity of the nonlinear mod-  
40 ulation method to detect damage, various signal processing techniques have  
41 been adopted. These methods make it possible to extract the nonlinear  
42 modulation information in the frequency domain [20, 21, 24, 25, 26, 27, 28].  
43 The VAM was also successfully associated with the time reversal technique  
44 for localizing and imaging damage in materials [29, 30, 31]. All these post-  
45 processing techniques aim for improving the crack detection with fixed ex-  
46 citations, while it has been proved that the effect of excitation parameters,  
47 such as the frequency, can improve the VAM sensitivity [18, 22]. Pieczonka et

48 al. have used a swept sine chirp probe excitation and modal frequency pump  
49 excitation associated with an advanced signal processing technique to find  
50 the optimal probe frequency enhancing the VAM sensitivity [17]. **Recently, a**  
51 **novel method in which the pump frequency is omitted and the high frequency**  
52 **is amplitude- modulated and contains 3 frequencies that interact in the pres-**  
53 **ence of nonlinearity has been reported [32].** However, to our knowledge, no  
54 study using VAM has tried to find the optimal excitation that improve the  
55 sensitivity of the damage detection automatically by a closed loop system.

56 Previous results using the optimal command principle for nonlinear sys-  
57 tems were initiated by Ménigot et al. [33, 34] in medical ultrasound imaging.  
58 A first attempt in NDT was applied in [35] **then in [36]**. The originality of  
59 these results is to search for the optimal input excitation parameter/shape  
60 without any *a priori* knowledge on the studied system, by using known op-  
61 timization algorithms. The key point of this method lies in the choice of the  
62 cost function which must best write the optimization purpose.

63 The present study constitutes an extension to the NDT domain, espe-  
64 cially, to the VAM technique, of the method demonstrated in [33], in which  
65 the optimal command was applied to medical ultrasonic imaging. A conven-  
66 tional VAM system is, then, replaced by a closed loop VAM system permit-  
67 ting a real time optimization of input excitations parameters. We focused,  
68 here, on the pump frequency parameter since the nonlinear modulation inten-  
69 sity increase with the pump amplitude [18, 37, 38]. The aim of our study is  
70 therefore to find automatically the best pumping frequency which maximizes  
71 the nonlinear modulation occurring between the pump and the probe signals  
72 without *a priori* information. The proposed method requires no preliminary

73 modal analysis and use a simple optimization genetic algorithm. In a first  
74 time, the cost function must be adapted to the VAM. An empirical search of  
75 the cost function behavior as a function of the optimization parameter was  
76 achieved in order to show the relevance of our choices.

77 The remainder of the paper is organized as follow, in the Section 2, we  
78 present our closed loop optimization system for the VAM technique. Section  
79 3 describes the experiment including the studied sample and the setup. The  
80 Section 4 is devoted to the experimental results of the empirical optimiza-  
81 tion, a modal analysis for a comparison purpose, and the genetic algorithm  
82 optimization. Finally, a discussion and conclusion are given in Section 5.

## 83 **2. Closed Loop Pump Frequency Optimization**

84 The proposed method is an optimal command method using a closed loop,  
85 in order to optimize the VAM sensitivity of crack detection. Indeed, it makes  
86 it possible to find the pump frequency which optimizes the nonlinear mod-  
87 ulation effects. The cost function and its parameters need to be adequately  
88 chosen. Here, only an iterative optimization procedure is implemented. The  
89 conventional open loop system is replaced by a closed loop system in which  
90 the transmitted pump frequency is modified by adding a feedback, ensuring  
91 the optimization of the cost function. The closed loop system of the VAM  
92 optimization is described in Fig. 1.

### 93 *2.1. Cost function*

94 As mentioned above, the goal of our study is to find automatically the  
95 best pump frequency  $f_p^*$ , which maximizes the Nonlinear modulation (NM)

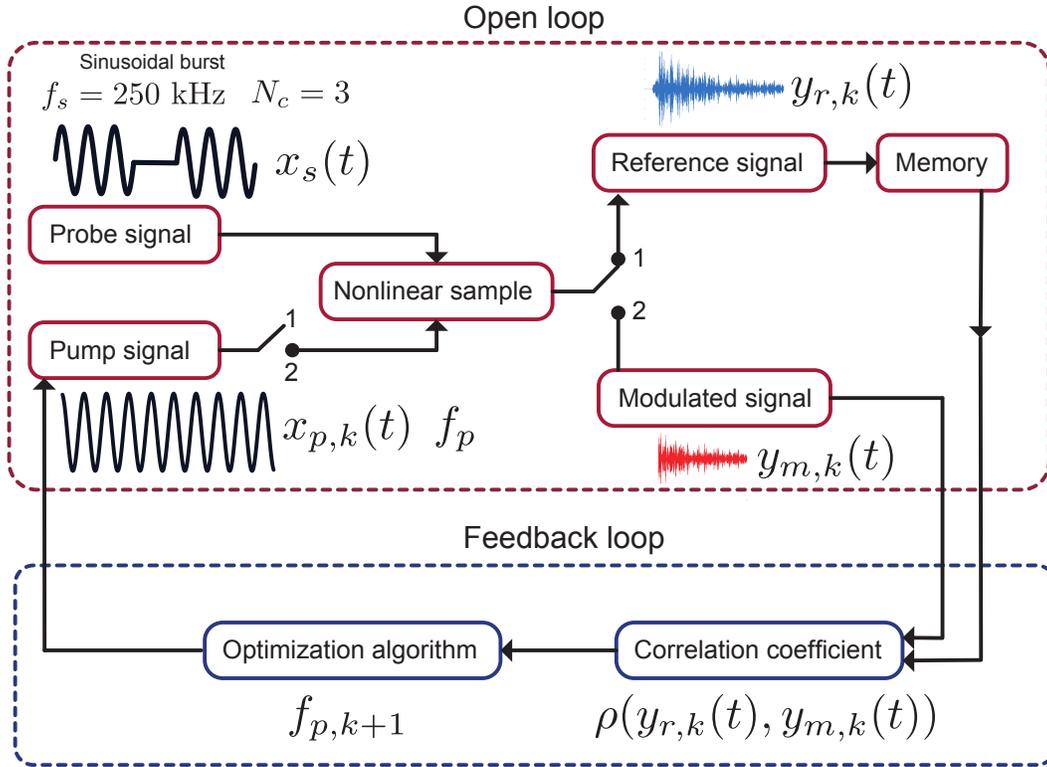


Figure 1: Bloc diagram of closed loop optimization of pump frequency for the VAM technique. The same probe excitation signal  $x_s(t)$  is transmitted to the sample without the pumping excitation. The received reference probe coda signal  $y_{r,k}(t)$  is recorded for a next use( switch position 1). In a second time (switch position 2), a pumping excitation  $x_{p,k}(t)$  at a frequency  $f_{p,k}$  is transmitted simultaneously with the probe excitation to the medium. The modulated coda probe signal  $y_{m,k}(t)$  is also recorded. The added feedback consists of evaluating and minimizing the correlation coefficient  $\rho$  between  $y_{r,k}(t)$  and  $y_{m,k}(t)$ . The optimization algorithm allowed to find a new pumping frequency  $f_{p,k+1}$ .

96 effects. Usually, to quantify the resulting NM effects in the context of mul-  
 97 tiple scattering media, the Coda Wave Interferometry (CWI) in the time  
 98 domain is used [13, 14, 39, 40]. We have chosen to proceed similarly, and  
 99 extract the nonlinear modulation information in the time domain as it seems

100 to be more accessible. Indeed, the frequency response is very complex in  
 101 multiple scattering media, and the modulation sidebands are difficult to see  
 102 (see section 3.3).

103 The correlation coefficient  $\rho$  between the received probe coda signal with-  
 104 out pumping and with the pump excitation constitutes our cost function.  
 105 With the presence of a nonlinear scatterer in the sample, the probe signal is  
 106 expected to be modulated by the pump excitation which induces to a decor-  
 107 relation between the two signals. A correlation coefficient equal to unity  
 108 means that there is no influence of the pump on the probe wave, *i.e.*, no non-  
 109 linear modulation effect in the sample. On the contrary, for the same pump  
 110 amplitude value, a deviation of  $\rho$  from 1 indicates the presence of nonlinear  
 111 damage leading to a nonlinear modulation effect. In a theoretical point of  
 112 view, the problem consists in calculating:

$$f_p^* = \arg \min_{f_p} (\rho(f_p)). \quad (1)$$

113 During the optimization process, for each iteration  $k$ , the same probe  
 114 excitation signal  $x_s(t)$  is transmitted to the sample without the pumping  
 115 signal (see Fig.1 switch position 1), and the received reference probe coda  
 116 signal  $y_{r,k}(t)$  is recorded for a next use. In a second time (switch position 2), a  
 117 pump excitation  $x_{p,k}(t)$  at a frequency  $f_{p,k}$  is transmitted simultaneously with  
 118 the probe excitation to the medium. The modulated coda probe signal  $y_{m,k}(t)$   
 119 is also recorded. The added feedback consists in evaluating and optimizing  
 120 a cost function, which is, in our case, the correlation coefficient between  
 121 the two recorded received signals  $\rho(y_{r,k}(t), y_{m,k}(t))$  within the time interval

122  $[t_1, t_2]$  such as:

$$\rho(y_{r,k}(t), y_{m,k}(t)) = \frac{\int_{t_1}^{t_2} (y_{r,k}(t) - \bar{y}_{r,k}(t)) (y_{m,k}(t) - \bar{y}_{m,k}(t)) dt}{\sqrt{\int_{t_1}^{t_2} (y_{r,k}(t) - \bar{y}_{r,k}(t))^2 dt \int_{t_1}^{t_2} (y_{m,k}(t) - \bar{y}_{m,k}(t))^2 dt}} \quad (2)$$

123 where  $\bar{y}_{r,k}(t)$  is the average of the reference signal within the interval  $[t_1, t_2]$ ,  
 124 and  $\bar{y}_{m,k}(t)$  that of the modulated signal. This coefficient quantifies the re-  
 125 semblance between the reference probe signal and the modulated probe signal  
 126 by the pump signal. The higher the nonlinear modulation between the probe  
 127 and the pump signals is, the lower the value of the correlation coefficient  
 128 is. An optimization algorithm is required to find a new pumping frequency  
 129  $f_{p,k+1}$ , at the iteration  $k+1$ , which maximizes the NM effect and therefore  
 130 minimizes the correlation coefficient. The frequency is then modified and  
 131 all the process described above is reiterated until the algorithm converges  
 132 toward the best solution. In our study, the genetic algorithm was applied.

### 133 2.2. Genetic Algorithm

134 To find iteratively the pumping frequency giving the global minimum of  
 135 the correlation coefficient, the genetic algorithm is used [41]. It is a search  
 136 optimization technique based on the principles of genetics and natural se-  
 137 lection. The genetic algorithm allows a population composed of a set of  
 138 pumping frequencies to evolve under specified selection rules to a state that  
 139 maximizes the "fitness" (*i.e.*, minimizes the correlation coefficient)[34, 42].  
 140 The first step (called generation 1) consists of choosing randomly N pumping  
 141 frequencies from a uniform distribution on a given frequencies interval. In  
 142 our case, we have chosen  $10\text{Hz} \leq f_p \leq 900\text{Hz}$ . This choice is directly related

143 to the empirical optimization for a comparison purpose. Indeed, a largest  
 144 initial population could be chosen. The correlation coefficient is evaluated  
 145 for each pumping frequency value and sorted in descending order. To prepare  
 146 the next step, the  $N/2$  best pumping frequencies that minimize the corre-  
 147 lation coefficient are kept for the next generation  $k+1$  and become parents.  
 148  $N/2$  new pumping frequencies named offspring are generated following the  
 149 expressions [41, 42]:

$$\begin{aligned} \mathbf{offspring1} &= f_{pm} - \beta[f_{pm} - f_{pd}] \\ \mathbf{offspring2} &= f_{pm} + \beta[f_{pm} - f_{pd}] \end{aligned} \quad (3)$$

150 where  $\beta$  is a random value between 0 and 1. the subscripts  $m$  and  $d$  discrim-  
 151 inates between the *mum* and the *dad* pumping frequencies. A percentage  
 152 of the samples is mutated to obtain a robust optimization. There are some  
 153 algorithm parameters that must be chosen such the population size  $N$  and  
 154 the mutation rate  $R$ . In our case,  $N= 12$  and the mutation rate  $R=40\%$  [34].  
 155 Finally, after cost function evaluation, the pumping frequency with the lower  
 156 correlation coefficient is the best solution of generation  $k+1$ . **The genetic**  
 157 **algorithm is adapted to global optimization problem; it means that it will be**  
 158 **able to find the global minimum even if the function presents local minima.**  
 159 **The GA is good for at identifying promising area of the search space but less**  
 160 **efficient at fine-tuning the approximation to the minimum [43].**

### 161 3. Material

#### 162 3.1. Sample description

163 An aluminium bar (600 mm  $\times$  15 mm  $\times$  3mm), with density  $\rho=2700$   
 164 kg/ $m^3$ , Poisson ratio  $\sigma=0.33$ , and Young modulus  $E=69$  GPa [44], is used as

165 the specimen for our experiments. The probe wave propagation velocity in  
166 the bulk of the bar is estimated to 4838 m/s. 10 tapped holes with a 4 mm  
167 diameter and localized at distances of 10 mm, 30 mm, 50 mm, 100 mm and  
168 200 mm from each sides of the bar center are drilled (see Fig. 2 from S1 to S10  
169 from left to right). Identical screws ( $m=4.42$  g) can be placed in the tapped  
170 holes to mimic nonlinear solid contacts (cracks) such as in [45, 39, 40, 46].  
171 These holes constitute linear scatterers when no screw is present. According  
172 to the number of nonlinear scatterers on the bar, different levels of "effective"  
173 damage can be obtained.

174 In the present study, three configurations have been more particularly  
175 studied: the first one (Config 1) corresponds to the case where no screw is  
176 placed in the sample bar; the reference. Nevertheless, one fixation screw is  
177 placed in the middle to insure the link between the shaker and the bar (S0  
178 in Fig. 2). Configuration 2 (Config 2) corresponds to the case where only  
179 2 screws are positioned at S2 and S9, and a third configuration (Config 3)  
180 is the case where all screws are placed (at S1, S2, S3, S4, S5, S6, S7, S8,  
181 S9, and S10). Note that the nonlinearity level in the sample depends on the  
182 screw number and the nuts tightening.

### 183 *3.2. Experimental Setup*

184 The closed loop pump frequency optimization process requires the exper-  
185 imental setup depicted in Fig. 3. Two **broadband** piezoelectric transducers  
186 with **250kHz** central frequencies for transmitting and receiving the probe sig-  
187 nal are glued to the ends of the sample. For the transmission of the probe sig-  
188 nal, a 100 mVpp sinusoidal burst of 3 periods of 250 kHz frequency, repeated  
189 every 20 ms, was emitted by a function generator (AFG3022, Tektronix,

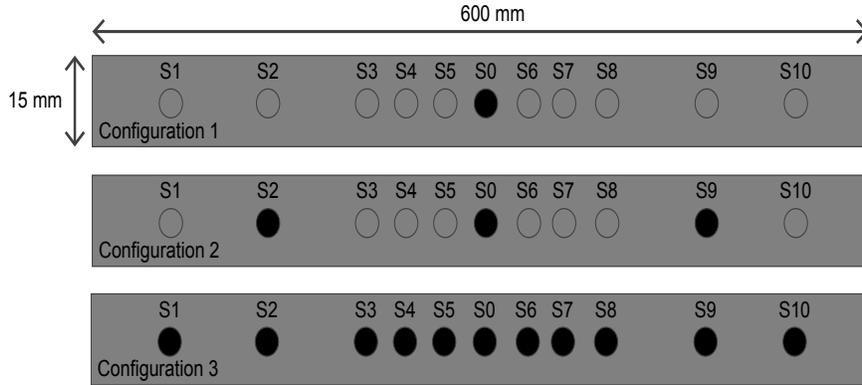


Figure 2: Specimen schemes of the three configurations. One Aluminium bar (600 mm  $\times$  15 mm  $\times$  3mm) containing 10 tapped holes. Config 1: no screws are placed. Config 2: 2 screws are placed at S2 and S9. Config 3: all the screws are placed on the bar.

190 Beaverton, Oregon, USA) and amplified to 60 dB (100V<sub>pp</sub>) by a power am-  
 191 plifier (Type 2713, Bruël & Kjær, Nærum, Denmark). Simultaneously, a  
 192 lower frequency continuous sine pump signal is generated by the computer-  
 193 controlled function generator (Tektronix, AFG3021C, Beaverton, Oregon,  
 194 USA) to change the excitation frequency during the closed-loop optimiza-  
 195 tion process. The pump signal is amplified by a power amplifier (PA100E,  
 196 Bruël & Kjær, Nærum, Denmark), and transmitted to the shaker (LDS  
 197 V406, Bruël & Kjær, Nærum, Denmark) which is connected to the center  
 198 of the sample by a screw. The coda probe signal is detected by the receiving  
 199 transducer and amplified by a preamplifier (Ciprian, Saint ISMIER, France),  
 200 then, transmitted to an oscilloscope (LT 264ML, Lecroy, Chestnut Ridge,  
 201 NY, USA). In order to improve the signal to noise ratio, an average of 300  
 202 successive acquisitions is carried out, and a coda averaged signal is recorded.  
 203 Each measurement lasts about 10s. Both the function generator and the  
 204 oscilloscope are controlled by MATLAB (Mathworks, Natick, MA, USA).

205 The pump signal is desynchronized from the probe so that the nonlinear ef-  
 206 fects are correctly distributed over the successive acquisitions and averaged  
 over the acquired signal. By comparison, An experimental modal analysis

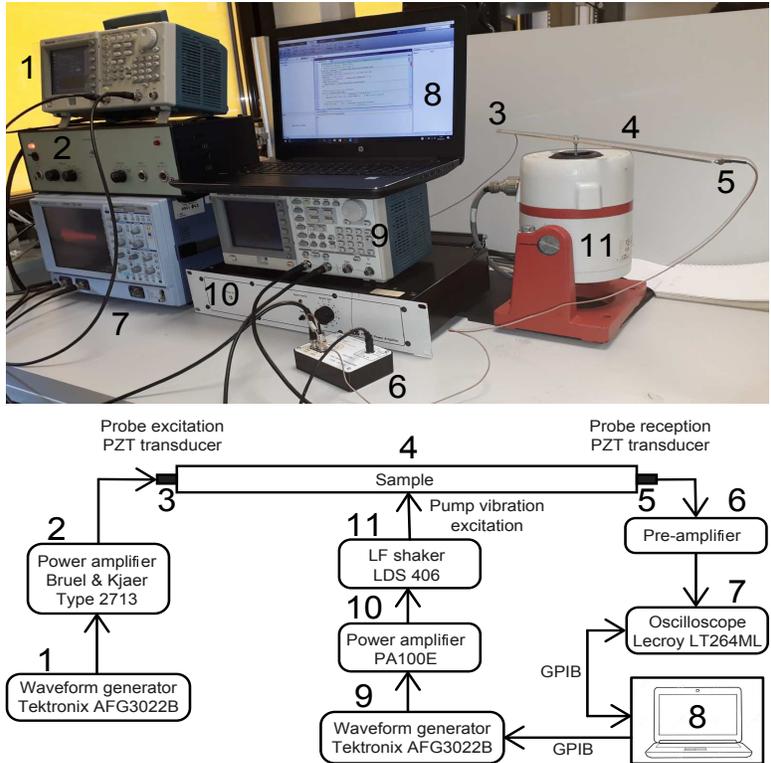


Figure 3: Experimental setup for the closed loop Vibro-Acoustic Modulation (VAM) technique.

207

208 is achieved to identify the frequencies of the first structural resonances of  
 209 the sample. For this purpose, a frequency swept sine signal from 5Hz to  
 210 1000Hz is generated using a signal analyzer (SR785, Stanford Research Sys-  
 211 tems Inc., Sunnyvale, CA, USA). The signal is amplified by a PA100E power  
 212 amplifier ,then, transmitted to a LDS406 shaker. The data are acquired by  
 213 an accelerometer (352C23, PCB Piezotronics Inc, Depew, NY, USA) then

214 transmitted to the signal analyzer. Both the input excitation and the out-  
215 put response are used to obtain the averaged Frequency Response Function  
216 (FRF) of the sample. The excitation and the acquisition are controlled by a  
217 GPIB connection via MATLAB.

### 218 *3.3. Nonlinear modulation effects on the probe coda signal*

219 In order to show the NM effects on the temporal coda signal, two con-  
220 figurations are considered. The first one corresponds to the case without  
221 screws (Config 1), and the second one corresponds to the Config 2. The  
222 pump frequency is 150 Hz. Fig. 4 shows the received probe coda signal from  
223 the Config 1 (Fig. 4a) and Config 2 (Fig. 4d). For each case, early and late  
224 temporal windows are shown with and without the presence of the pump  
225 excitation (Figs. 4b, c and e, f). The pump presence causes no detectable  
226 change on the probe coda signal obtained from the intact sample, for early  
227 and late windows: there is no nonlinear modulation of the probe by the pump  
228 with a correlation coefficient of 1. On the contrary, for the damaged sample,  
229 it can easily be noted that the two coda signals recorded with and without  
230 the pumping wave are different for the late window  $t=[1,1.05]$  ms (Fig. 4f),  
231 with a correlation coefficient  $\rho = 0.976$ . It indicates that a NM has occurred  
232 between the probe and the pump waves. Moreover, the nonlinear effects are  
233 more visible in the late window because, at this time, the wave has crossed  
234 the medium and interacted with the nonlinear scatterers several times. From  
235 previous implementations of the nonlinear modulation of a coda wave by a  
236 pump wave in a multiple scattering medium [13, 14, 46], the decorrelation is  
237 either due to a localized velocity variation, to an amplitude-dependent dissi-  
238 pation effect or to an amplitude-dependent scattering effect. By introducing

239 internal solid contacts in the medium, between the screws and the holes,  
240 amplitude- dependent clapping, tapping and frictional effects are expected.  
241 These results consolidate our choice of quantifying the nonlinear modulation  
242 effects by calculating the correlation coefficient between the probe coda sig-  
243 nals obtained with and without the presence of the pump excitation, in a  
244 time interval between  $t_1=1\text{ms}$  and  $t_2=1.2\text{ ms}$ . It is important to notice that  
245 the choice of the central time and the width of the temporal window in which  
246 the correlation coefficient is estimated, obeys to some constraints [47]. The  
247 characteristic scattering time  $t^*$  is estimated according to the method pro-  
248 posed in [48], and is find to be around  $3\ \mu\text{s}$  for our specimen. The starting  
249 time of our window  $t_1=1\text{ms}$  satisfies the condition  $t_1 \gg t^*$ . This ensures a  
250 global inspection of the entire studied sample. Moreover the time interval  
251 width is set to  $200\ \mu\text{s}$ , such as it includes enough signal periods (50 periods  
252 of the probe signal at  $250\ \text{kHz}$ ). The choice of the window position is also  
253 limited by the signal quality. Indeed, the more  $t$  is increased, the more the  
254 signal to noise ratio is degraded and the higher the uncertainty on the result  
255 is.

256 In parallel, we illustrate in Fig. 5, the frequency spectra (right) of the  
257 coda probe signals (left), obtained for Config 2, with and without the presence  
258 of the pump wave excitation at  $f_p=150\ \text{Hz}$ . The selected time window is  $\Delta t=$   
259  $[1,1.2]\text{ms}$ . We can notice that both frequency spectra are centered on  $250$   
260  $\text{kHz}$ , and include peaks due to the multiple scattered probe signal. The  
261 modulation sidebands are difficult to see in this case, and the only difference  
262 between the two spectra is an amplitude variation and a slight frequency  
263 shift. Moreover, the frequency resolution is not sufficient to observe the

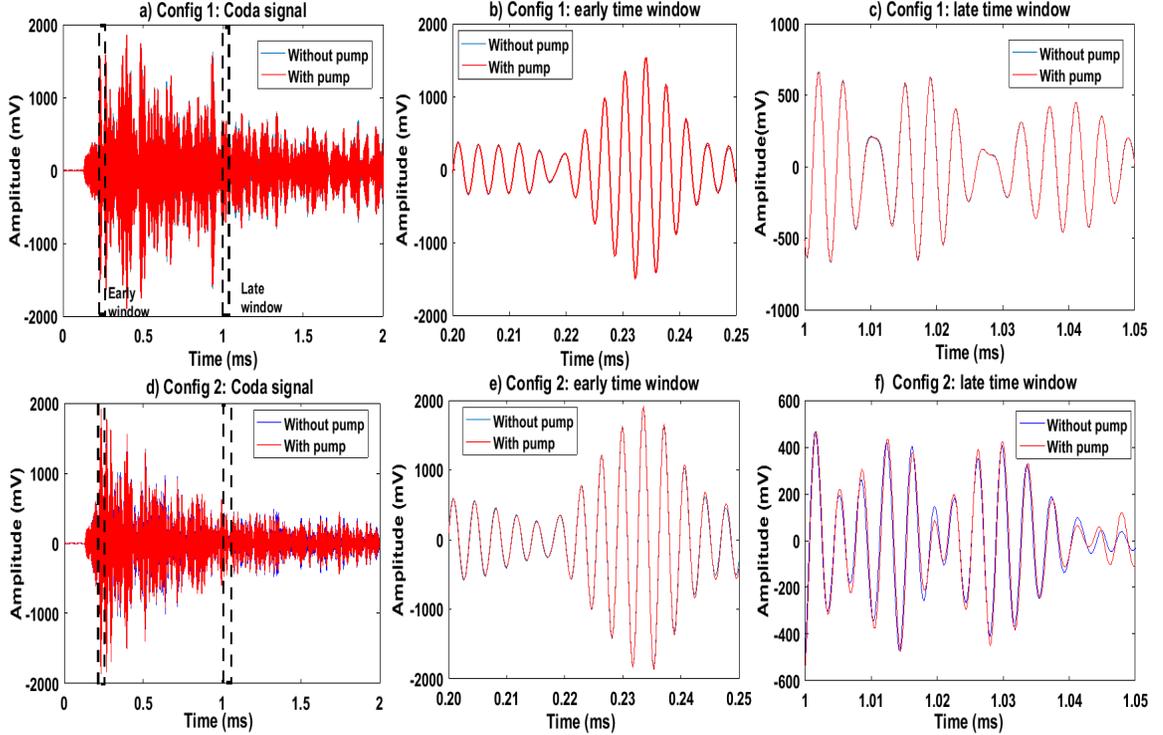


Figure 4: Experimental temporal coda probe signals obtained from the Config 1 (without screws): (a), (b) and (c), and for the Config 2 (with screws): (d), (e) and (f), with and without the pump excitation. The pump excitation is a continuous sine signal at a resonance frequency  $f_p=150$  Hz. (b) and (e) are captures of the coda signals (a) and (d), respectively, during an early time interval between  $[0.2, 0.25]$  ms. (c) and (f) are captures of (a) and (d), respectively, during a late interval between  $[1, 1.05]$  ms.

264 modulation sidebands when  $f_{pump} \ll f_{probe}$ . Therefore, we choose to extract  
 265 to modulation effects information in the time domain.

266 During all this study, it should be noticed that the decorrelation values  
 267 are not observed higher than 2.5%. We recall, here, that we want to test our  
 268 approach in a rather unfavorable configuration, *i.e.*, when the variation of

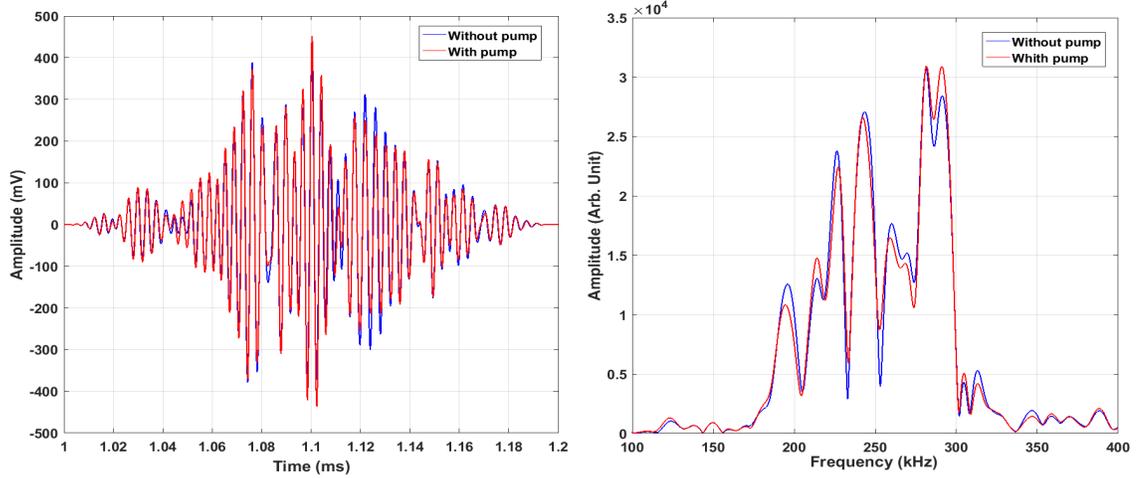


Figure 5: (left) Experimental probe coda signals obtained from the Config 2, with and without the presence of the pump. The pump excitation is a continuous sine wave at 150Hz. The coda signal is filtered with a hanning window for a duration  $\Delta t=[1,1.2]$ ms. (right) The frequency spectra of the coda signals.

269 the correlation coefficient is small. Indeed, a stronger nut tightening and a  
 270 higher pump amplitude would have given higher levels of decorrelation.

#### 271 4. Experimental Results

272 In this section, the main experimental results are presented. First an  
 273 experimental modal analysis is performed in order to obtain the resonance  
 274 modes of the sample for a comparison purpose. Then, an empirical research of  
 275 the correlation coefficient  $\rho$  behavior *versus* the pump frequency is presented  
 276 and compared to the modal analysis results. Finally, a closed loop optimiza-  
 277 tion of the correlation coefficient by the genetic algorithm is performed and  
 278 presented.

279 *4.1. Empirical optimization: Correlation coefficient dependency on the pump*  
280 *frequency*

281 The frequency response function (FRF) is obtained using the experimen-  
282 tal input and output data obtained from the experiment described in the  
283 section 3.2. This preliminary experiment allows identifying the frequency  
284 resonance modes of the sample in the Configs 2 and 3. Figs. 6a and 6c  
285 show the FRFs obtained for Configs 2 and 3, respectively, in a frequency  
286 range from 5Hz to 900 Hz. We can see that the magnitude of the resonance  
287 peaks and the corresponding frequencies are different for the two configura-  
288 tions. The results show that there is a shift in the natural frequencies and  
289 an overall change in the frequency response due to the effect of the added  
290 screws.

291 The first performed experiment is to check the existence of global or lo-  
292 cal minima for the correlation coefficient *versus* the pump frequency. This  
293 step is called "empirical optimization". For time saving, a pump frequency  
294 sine sweeping from 10 Hz to 900 Hz with a step of 10 Hz is achieved. This  
295 allows a global view of the correlation coefficient dependency on the pump  
296 frequency. For this first pass, the prominent observation is the presence of  
297 local minima peaks localized at specific vibration frequencies corresponding  
298 well to the resonances of the bar, and  $\rho \simeq 1$  elsewhere. To refine the results  
299 and to get a better accuracy, a second sweeping was achieved with a finer  
300 frequency step of 1 Hz, over the frequency regions where  $\rho$  deviates from  
301 1 (Figs. 6b and 6d). The procedure cited in the section 2.1 is adopted to  
302 calculate the correlation coefficient for each pump frequency. As illustrated  
303 in Fig. 6, the pump frequencies that give minima peaks of  $\rho$  coincide well

304 with the resonance frequencies of the sample for the Configs 2 and 3. This  
 305 confirms the expected effects that at a resonance frequency of the sample,  
 306 the vibration amplitude is naturally amplified by constructive interferences,  
 307 leading to a more efficient nonlinear modulation effect. Also, we can note a  
 308 frequency shift to the lower frequencies with the increase of the number of  
 309 screws in the plate, as it can be seen in Figs. 6b and 6d. Additionally, we  
 310 can see clearly that for the same resonance mode, the correlation coefficient  
 311 values vary from one configuration to another. In fact, the second minimum  
 312 peak corresponding to  $f_p=150$  Hz is the most important for the Config 2;  
 313  $\rho= 0.975$  (Fig. 6b) while  $\rho= 0.981$  at  $f_p=144$  Hz for the Config 3 (Fig. 6d).  
 314 We can also notice the third  $\rho$  peak is very close to 1 ( $\rho=0.998$ ) for the  
 315 Config 2 (Fig. 6b) and equal to 0.986 for the Config 3. These observations  
 316 confirm the well-known fact that in the pump resonance configuration, the  
 317 NM efficiency on the probe wave by the pump wave depends on the pump-  
 318 ing frequency/mode and on the exact position of the nonlinear scatterers.  
 319 Based on the obtained curves, it can be stated that for each configuration, it  
 320 exists an optimal pump frequency that maximizes the nonlinear modulation  
 321 effects and, thus, the damage detection sensitivity. This optimal frequency  
 322 corresponds to the frequency giving the lowest minimum of  $\rho$ . The empiri-  
 323 cal experiment also shows that the correlation coefficient is a good indicator  
 324 to evaluate the nonlinear wave mixing in the sample. For an easier com-  
 325 parison of results obtained from Fig. 6, table 1 summarizes the correlation  
 326 coefficient minima peaks values and the corresponding pumping frequencies  
 327 obtained from the empirical optimization (EO) and those obtained from the  
 328 FRF for both configurations.

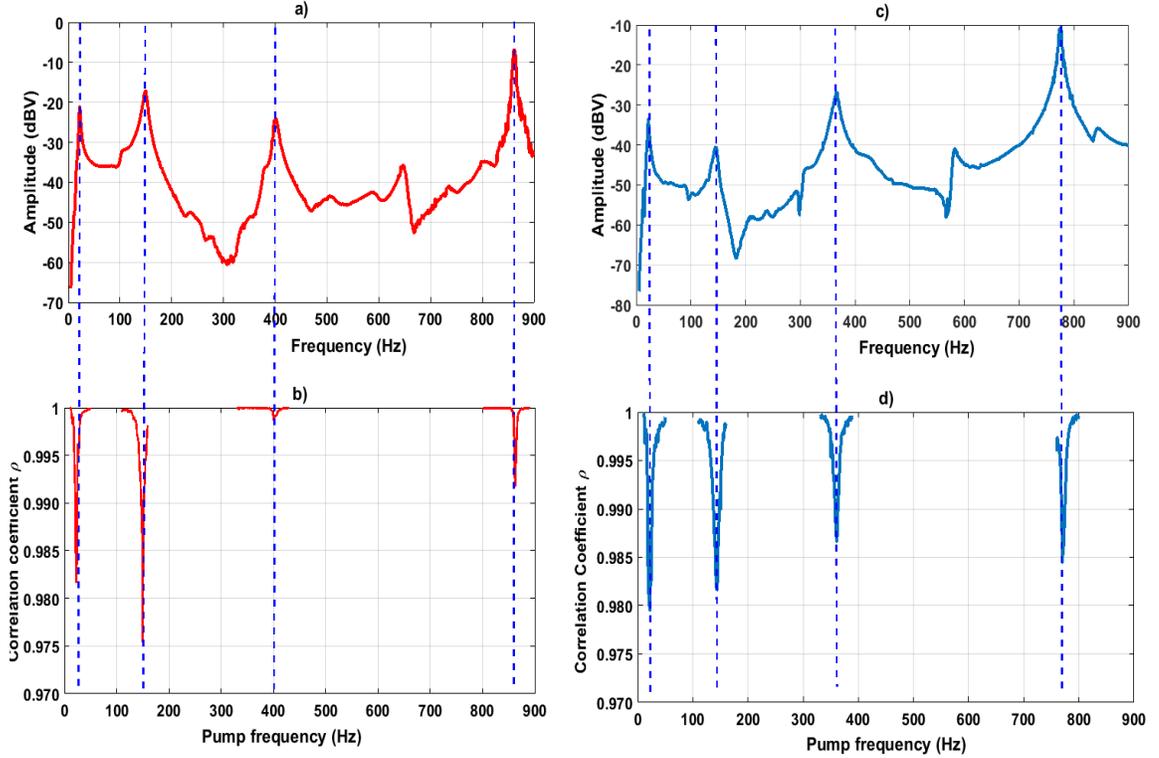


Figure 6: Comparison between the resonance frequencies obtained from the experimental FRF of the Config 2 (a) and the Config 3 (c). Experimental results of correlation coefficient *versus* pump frequency at the sample resonance frequencies areas for the two sample configurations: b) Config 2 and d) Config 3. The FRF peaks coincide with the minimum peaks of the correlation coefficient *versus* pumping frequency.

329 *4.2. Optimal pumping frequency excitation*

330 Fig. 7 shows the results of the pump frequency optimization for the Con-  
 331 fig 2 and Fig. 8 shows those obtained for the Config 3 when using the genetic  
 332 algorithm optimization. For more clarity, the result of the empirical optimiza-  
 333 tion is represented in Figs. 7a and 8a with a zoom on the GA convergence  
 334 area. The initial population was included in a frequency range between 10 Hz

Modes	Config 2			Config 3		
	$\rho_{EO}$	$f_{p,EO}$ (Hz)	$f_{p,FRF}$ (Hz)	$\rho_{EO}$	$f_{p,EO}$ (Hz)	$f_{p,FRF}$ (Hz)
1 <sup>st</sup> Mode	0.981	23	22.1	0.979	22	22.1
2 <sup>nd</sup> Mode	0.975	150	148.7	0.981	144	145.7
3 <sup>rd</sup> Mode	0.998	400	401.0	0.986	361	365.8
4 <sup>th</sup> Mode	0.991	862	861.3	0.984	770	773.9

Table 1: The correlation coefficient  $\rho$  and the  $f_p$  values corresponding to the minimum peaks obtained from the empirical optimization (EO) and the resonance frequencies obtained from the Frequency Response Function (FRF) for Config 2 and 3.

335 and 900 Hz, with a population size of 6 for the Config 2 and 12 for the Config  
336 3. We can see that in both cases, the optimal pump frequency is reached  
337 after 4 generations for the Config 2 with an optimal pump frequency of 153.2  
338 Hz and  $\rho=0.991$  (Figs. 7b and 7c), and after 7 generations for the Config 3  
339 with  $f_p=22.17$  Hz and  $\rho=0.971$  (Figs. 8b and 8c). As it is illustrated in Figs.  
340 7a and 8a the obtained results are quite similar with those obtained by the  
341 empirical optimization. For Config 2, the optimal pump frequency is reached  
342 by the genetic algorithm with a slight discrepancy in the pump frequency and  
343 the corresponding correlation coefficient. Indeed, we can see in Fig. 7a, that  
344 the GA converges nearly to the global minimum at  $f_p=153.2$  Hz, in a pump  
345 frequency range between 10 and 900 Hz, including four (4) local minima of  
346 the cost function, but the accurate value of this minimum, at  $f_p=150$  Hz, is  
347 not reached. We suppose that a greater number of iterations should improve  
348 this result. Moreover, it is important to notice that for the empirical search,  
349 the step between two pump frequencies is chosen empirically, this may be

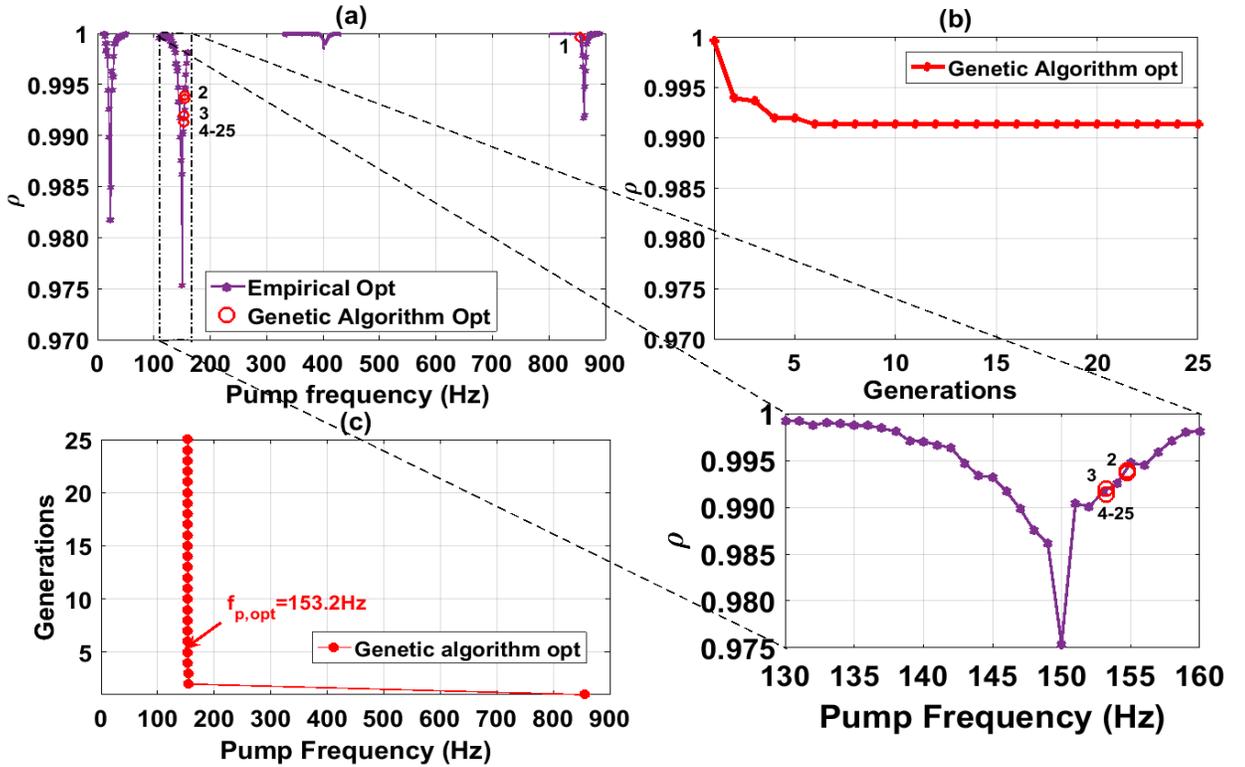


Figure 7: Genetic algorithm (GA) optimization results for the Config 2: (a) Empirical optimization and GA iteration numbers (red circles), a zoom on the GA convergence area is illustrated. (b) The correlation coefficient  $\rho$  versus generations. (c) The corresponding pump frequency versus generations. The GA converges to  $f_{p,opt}=153.2$  Hz after the 4<sup>th</sup> generation.

350 the origin of the discrepancy observed between the empirical and the auto-  
 351 matic optimization. For the Config 3, the optimal pump frequency is nearly  
 352 reached. For both cases, the pump frequency converges while  $\rho$  remains vari-  
 353 able for the same value of the pump frequency. This can be clearly seen in  
 354 Figs. 8b and 8c where the pump frequency converges to  $f_p=22.17$  Hz, and  
 355 the corresponding  $\rho$  still varies slightly. This observation is directly due to

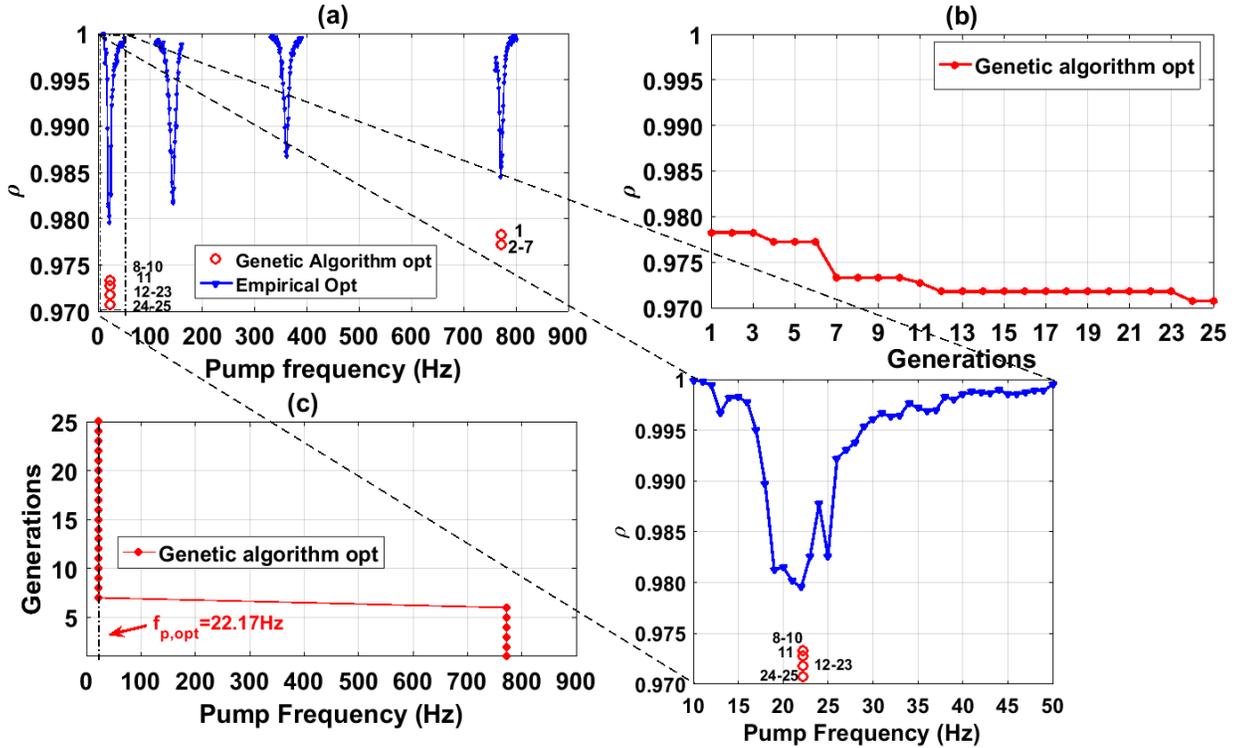


Figure 8: Genetic algorithm (GA) optimization results for the Config 3: (a) Empirical optimization and GA iteration numbers (red circles), a zoom on the GA convergence area is illustrated. (b) The correlation coefficient  $\rho$  versus generations. (c) The corresponding pump frequency versus generations. The GA converges to  $f_{p,opt} = 22.17$  Hz after the 7<sup>th</sup> generation.

356 the experimental conditions since the coda signal is very sensible to small  
 357 temperature changes. Moreover, the correlation coefficient value reaches a  
 358 lower value than that obtained by the empirical optimization (Fig. 8a). **It**  
 359 **is important to mention that the empirical cost function is not a continuous**  
 360 **function, and a measurement is made with a pump frequency step of 1Hz. We**  
 361 **think, that a finer pump frequency step between two successive cost function**

	<b>Config 3</b>		<b>Config 2</b>	
	$\rho$	$f_{p,opt}$ (Hz)	$\rho$	$f_{p,opt}$ (Hz)
Genetic Algorithm	0.971	22.17	0.991	153.2
Empirical optimization	0.979	22.00	0.975	150.0

Table 2: Experimental optimization results of the genetic algorithm convergence and the empirical values of the global minimum obtained by the empirical search, for the Config 2 and 3.

362 measurements should give a better concordance between the empirical and  
363 the automatic search by the GA. Another possible explanation is that for  
364 the empirical optimization, one measurement of the cost function has been  
365 made for each pump frequency value between 10-900Hz, while for the GA  
366 optimization, and after convergence, several measurements have been made  
367 at the same optimal pump frequency (resonance frequency). In fact, exciting  
368 a sample at its resonance frequency during a long time (conditioning), could  
369 gives rise to a greater decorrelation between the reference coda signal and  
370 the modulated coda signal. Indeed, for the modal excitation, the vibration  
371 response is amplified such as the medium does not reach the equilibrium and  
372 the reference signal is still affected by the previous iterations. This may be  
373 linked to "slow dynamics" effects [49].

374 We can also note that the algorithm convergence is faster when the pop-  
375 ulation size is lower. The strong point of our approach is that the algorithm  
376 converged to the global minimum of the correlation coefficient without *a pri-*  
377 *ori* information. For more clarity, table 2 summarizes the results obtained  
378 by both the empirical and the GA optimization.

## 379 5. Discussion and Conclusion

380 In this work, a widely studied nonlinear ultrasonic nondestructive testing  
381 method has been combined with the optimal command method: an open loop  
382 Vibro Acoustic Modulation system has been enriched with a feedback sys-  
383 tem. The main idea was to achieve an optimal damage detection by finding  
384 automatically the best input pump frequency. This input frequency maxi-  
385 mizes the nonlinear modulation effects induced by the presence of nonlinear  
386 scatterers and, therefore, should increase the sensitivity of the VAM method.  
387 The best pump frequency adjusted to each configuration has been success-  
388 fully found by the feedback method. This method is based, principally, on  
389 the good choice of a so called "cost function" and the optimization parame-  
390 ter. Moreover, the experiment setting is easy and not user dependent. The  
391 genetic algorithm was tested on two sample configurations with different lev-  
392 els of nonlinearity to demonstrate the adaptivity of the proposed method. It  
393 is found able to determine automatically the optimal pump frequency over  
394 a wide frequency range including 4 resonance modes. Even if the genetic  
395 algorithm is not always efficient to find the **accurate optimal value**, it guar-  
396 antees to find the global optimum region quite fast since four (4) generations  
397 were sufficient in our case. A preliminary empirical search was achieved to  
398 check for the behavior of the cost function, the correlation coefficient as a  
399 function of the optimization parameter (the pump frequency). The results of  
400 this empirical search, requiring a long blind experimental search, have been  
401 presented in the form of correlation coefficient *versus* pump frequency. The  
402 main outcome is the existence of frequencies, which correspond to some of the  
403 resonance frequencies of the sample, giving a maximum of decorrelation (or

404 a minimum correlation). It constitutes a first suitable result which confirms  
405 our hypothesis of choosing the correlation coefficient of the probe coda sig-  
406 nal, with and without pumping to quantify the NM effects, and the pumping  
407 frequency as an optimization parameter. The quantitative variation of  $\rho$  with  
408 the pump resonance frequencies, for the different configurations is related to  
409 the modal vibration shapes and the associated strain amplitude changes from  
410 a resonance mode to another, at the location of the nonlinear scatterers. For  
411 different arrangements of nonlinear scatterers, we observe differences in the  
412 values of the correlation coefficient, because a given resonant mode does not  
413 necessarily excites differently located nonlinear scatterers with the same effi-  
414 ciency. Similarly, for a given medium configuration, but when comparing two  
415 different resonant modes, the nodes and anti-nodes positions are not located  
416 identically, and they do not excite with the same efficiency a given nonlinear  
417 scatterer. As such, using a single pump frequency leads to the apparition of  
418 blind zones for the VAM, *e.g.* when a single scatterer is at a strain node. For  
419 this reason, a preliminary modal analysis to determine a resonance modes of  
420 the studied medium seems to be not sufficient to ensure an increase of the  
421 VAM sensitivity detection.

422 The proposed method, in a resonance configuration, ensures to avoid the  
423 low sensitivity regions associated with the strain nodes, which could compro-  
424 mise the nonlinear scatterer detection. Moreover, for a fixed pump ampli-  
425 tude, the adjusted pump frequency excitation may bring maximum detection  
426 sensitivity for nonlinear damages, which can be very useful to detect early  
427 stage damage in materials. Furthermore, the advantage of the automatic  
428 optimization is a time gain compared to the empirical optimization which

429 takes about 15 times more then our method.

430 The optimal command can be extended to other NDT methods by defin-  
431 ing the right cost functions adapted to the optimization purpose. Also, as  
432 a perspective, the genetic algorithm outcomes could be used as the initial-  
433 ization for a gradient descent algorithm in order to refine the results. In  
434 addition, information on the local minima of the correlation coefficient may  
435 be relevant for locating the nonlinear scatterers: each resonant mode consti-  
436 tutes a spatial sensitivity kernel for the method and information stacking for  
437 several modes may offer imaging capabilities.

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