Fractional derivatives: from magnetic spectroscopy to high amplitude dynamic hysteresis in soft ferromagnetism

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Considering harmonic evolution of the magnetic induction field, we establish a link between a time domain, high amplitude, fractional hysteresis model and the frequency domain, weak amplitude, fractional, ferromagnetic Cole-Cole model. A relation between α (the fractional order), τ (the relaxing time) and ρ (the dynamic constant) can be set. Experimental results performed on a typical soft ferromagnetic (Fe-Si) validate the theory. Correct comparison simulations/measures are observed for both situations (weak and high amplitude excitation) conserving the same set of parameters (α, τ, ρ). The dynamic behavior of a ferromagnetic sample can be set under weak amplitude excitation using simple impedance LCR meter characterization and conserved for the simulation of high amplitude dynamic hysteresis cycles.

Index Terms — **Hysteresis, frequency dependence, fractional derivatives, Cole-Cole model.**

The development and design of new electromagnetic devices (electric motors, electric transformers, power converters …), such as the improvement of already existing ones requires precise simulation tools. The highly non-linear frequency dependence of the ferromagnetic materials has led to real simulation challenges for decades. The ferromagnetic power losses have been continuously studied since the first formulation (Steinmetz, [1]). According to Bertotti's theory, in a ferromagnetic material, the magnetic power losses can be decomposed into the sum of a quasi-static (frequency independent) hysteresis contribution, and two dynamic (frequency dependent) contributions [2]: the classical losses and the excess eddy-current losses. The quasi-static contribution is physically linked to the domain wall displacement against the pinning effect. The dynamic contributions are related to the classic eddy current losses and to an additional term (anomalous losses) dependent on the domain nature [2]:

$$\begin{aligned} P &= P_{hyst} + P_{eddy} + P_{exc} \\ P &= k_h\ f\ B + k_e\ f^2 B^2 + k_a\ f^{1.5} B^{1.5} \end{aligned} \qquad (1)$$

Here, P stands for the sum of all the ferromagnetic losses per unit volume. k_h, k_e, k_a are respectively the hysteresis losses, the eddy current and the excess losses coefficients.

For the simulation of electromagnetic devices, the most accurate simulation results can be obtained by coupling Space Discretization Techniques (SDT), Finite Elements Methods (FEM), Finite Differences Method (FDM) …) for the resolution of the Maxwell equation to a precise vector dynamic hysteresis material law. For this magnetic material law, the best solution is to extend a quasi-static (frequency independent) hysteresis model (Preisach model [3], Jiles-Atherton model [4]), in their vector version (2 or 3D) with dynamic (frequency dependent) contributions.

$$\overrightarrow{H_{surf}} = \overrightarrow{H_{static}}(\vec{B}) + \rho.\frac{d\vec{B}}{dt} \qquad (2)$$

H_{static} is the quasi-static contribution and ρ a constant depending on the nature and on the shape of the sample. The simultaneous resolution of the Maxwell equations and the hysteresis frequency dependent material law is done by iterative techniques [5]. Due to the highly non-linear property of hysteresis (saturation, frequency dependence …) uncertain convergences and numerical issues are recurrent [6][7].

Alternative solutions can be found in the literature. One of them consists on replacing the simultaneous resolution material law/SDT by an improved dynamic hysteresis lump model. By instant, for ferromagnetic laminations, the Bertotti's separation losses theory can be indirectly considered through H equivalent contributions and written with a lump-type equation:

$$H_{surf} = H_{static}(B) + \rho_1.\frac{dB}{dt} + \rho_2.\left(\frac{dB}{dt}\right)^{1/2} \qquad (3)$$

H_{static} is the quasi-static contribution, ρ_1 and ρ_2 two constants depending on the nature and the shape of the sample. Another more recent alternative solution consist on conserving the lump approach but instead of separating the dynamic contributions, to use a fractional derivative operator.

$$H_{surf} = H_{static}(B) + \rho.\frac{d^{\alpha}B(t)}{dt^{\alpha}} \qquad (4)$$

This new technique has been first tested with success on ferroelectric materials [9]-[10]. It has also been validated on ferromagnetic materials thanks to the fractional derivative extension of the well-known quasi-static Jiles-Atherton and Preisach hysteresis models [11][12]. Outstanding simulation results have been obtained on large frequency bandwidths in both cases.

Inductance spectroscopy (IS) also known as ferromagnetic spectroscopy (FS) consists on measuring the complex inductance or the complex permeability variations as a function of frequency and under weak amplitude magnetic excitation. It is a remarkable technique to characterize ferro- and ferrimagnetic materials dynamic behavior [13]-[15]. IS and FS rely on the property that all magnetization processes contribute to the total magnetization. They can be resolved by measuring them under different frequencies because they possess different dynamics, or different time-constants. The value of permeability in each frequency range, the frequency at which a process becomes unable to follow the excitation field, and also the dispersion (the manner a magnetization process changes from a process to another one) are significant and contribute to the understanding of dynamical magnetization phenomena. Frequency dependent, fractional magnetic Cole-Cole model is particularly suitable for the simulation of the ferromagnetic spectroscopy.

In the extended version of this article, we will show how assuming harmonic evolution of the magnetic induction field, it is very simple to establish a link between a time domain, high amplitude, fractional, hysteresis model and the frequency domain, fractional, magnetic Cole-Cole model. For both models, α value is conserved and a relation between τ (the relaxing time) and ρ (the material constant) can be set (eq. 5, 6).

$$\begin{cases} \mu^*(\omega) = \mu'(\omega) - j\mu''(\omega) = \mu_\infty + \dfrac{\Delta\mu}{1+(i\omega\tau)^\alpha} & (5) \\ \tau^\alpha = \mu_{static} \cdot \rho \quad (6) \end{cases}$$

Experimental results performed on a typical soft ferromagnetic (Fe-Si) material validate our theory. Correct comparison simulations/measures have been obtained under major hysteresis loop situation. These simulations have allowed to set the dynamic parameters (fig. 1).

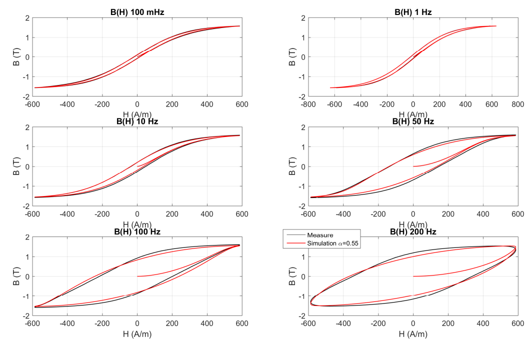

Fig. 1 - Comparison simulation (α = 0.55) / measure on major hysteresis cycles for excitation frequency varying from 100 mHz to 200 Hz.

Using (5), (6) and the same set of parameters, successful Cole-Cole simulations are obtained (behaviour under weak amplitude, large frequency bandwidth …)(fig. 2 and 3)

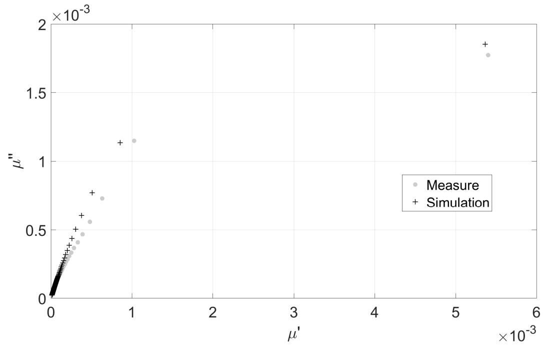

Fig. 2 – Permeability Cole-Cole plot, comparison measure/simulation

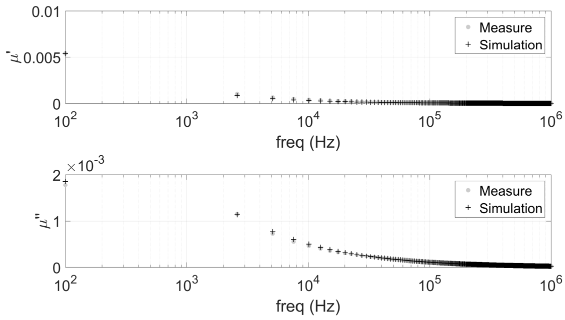

Fig. 3 – Comparison measure/simulation for the evolution of *μ'* and *μ''* as a function of the frequency

These observations leads to the conclusion that the dynamic parameters (fractional order, dynamic constant) can be conserved under weak and large amplitude simulations. From a physical point of view, it means that the relaxations' behaviours due to high frequency magnetizations are similar under weak and high amplitude excitation fields and consequently they can be modelled using similar mathematical operators.

References

[1] C. P. Steinmetz, "On the law of hysteresis", Am. Inst. Electr. Eng. Trans., vol. IX, no. 1, pp. 1–64, Jan. 1892.

[2] G. Bertotti, "General properties of power losses in soft ferromagnetic materials", IEEE Trans. on Mag.., 24, pp. 621-630, 1988.

[3] F. Preisach, "Über die magnetische Nachwirkung". Zeitschrift für Physik, 94: 277-302, 1935.

[4] D. C. Jiles and D. L. Atherton, "Theory of ferromagnetic hysteresis", J. Magn. Magn. Mater., vol. 61, no. 1–2, pp. 48–60, Sep. 1986.

[5] M. Kuczmann, A. Iványi, "The Finite Element Method in Magnetics", Budapest, Academic Press, 2008.

[6] M. A. Raulet, B. Ducharne, J.P. Masson, and G. Bayada, "The magnetic field diffusion equation including dynamic hysteresis: a linear formulation of the problem", IEEE Trans. on Mag., vol. 40, n° 2, pp. 872 – 875, 2004.

[7] B. Gupta, B. Ducharne, G. Sebald, T. Uchimoto, "A space discretized ferromagnetic model for non-destructive eddy current evaluation", IEEE Trans. on. Mag, vol. 54 Iss. 3, 2018.

[8] D. Guyomar, B. Ducharne, G. Sebald, "The use of fractional derivation in modeling ferroelectric dynamic hysteresis behavior over large frequency bandwidth", Journal of Applied Physics, vol. 107, Iss. 11, n° 114108, 2010.

[9] D. Guyomar, B. Ducharne, G. Sebald, "Dynamical hysteresis model of ferroelectric ceramics under electric field using fractional derivatives", Journal of Physics D: Applied Physics, vol. 40, Iss. 19, pp. 6048-6054, 2007.

[10] D. Guyomar, B. Ducharne, G. Sebald, "Time fractional derivatives for voltage creep in ferroelectric materials: theory and experiment", Journal of Physics D: Applied Physics, Vol. 41, Iss. 12, 125410, 2008.

[11] B. Zhang, B. Gupta, B. Ducharne, G. Sebald, T. Uchimoto, "Preisach's model extended with dynamic fractional derivation contribution", IEEE Trans. on Mag., vol. 54 iss. 3, 2017.

[12] B. Zhang, B. Gupta, B. Ducharne, G. Sebald, T. Uchimoto, "Dynamic Magnetic scalar hysteresis lump model based on Jiles-Atherton quasi-static hysteresis model extended with dynamic fractional derivative contribution", IEEE Trans. on Mag., iss. 99, pp. 1-5, 2018.

[13] R. Valenzuela, H. Montiel, M.P. Gutiérrez, I. Betancourt, "Characterization of soft ferromagnetic materials by inductance spectroscopy and magnetoimpedance", Journal of Magnetism and Magnetic Materials, vol. 294, pp. 239-244, 2005.

[14] R. Valenzuela, "Low-frequency magnetoimpedance: domain wall magnetization processes", Physica B, vol. 299, pp. 280-285, 2001.

[15] M. Vàzquez, J.P. Sinnecker, G.V. Kurlyandskaya, "Hysteretic behavior and anisotropy fields in the magneto-impedance effect", Materials science forum, Vol. 302-303, pp. 209-218, 1999.