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► **To cite this version:**

Maxime Menuet. Fiscal rule and shock amplification: A stochastic endogenous growth model. 2019.  
hal-02153887v2

**HAL Id: hal-02153887**

**<https://hal.science/hal-02153887v2>**

Preprint submitted on 1 Jul 2019

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# Second-generation fiscal rules in a stochastic endogenous growth model

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## Abstract

This paper provides a theoretical basis on the literature that focuses on the “second-generation” fiscal rules. We develop a stochastic endogenous growth model with public debt dynamics in which the speed of adjustment of the debt-to-GDP ratio depends on the business cycle. We show that the degree of flexibility dramatically affects the cyclical behavior of the government spending. The higher the flexibility degree, the lower the volatility of fluctuations along the business cycle. Therefore, following a transitory shock, a flexible-fiscal rule helps smooth aggregate variables by limiting the cuts in productive public spending.

*Keywords:* Endogenous growth model, Hysteresis, Fiscal Rules

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## 1. Introduction

Over the past 30 years, to tackle the massive expansion of public debts, a growing number of countries attempted to strengthen their fiscal governance frameworks by implementing fiscal rules (FR).<sup>2</sup> While the FR undoubtedly reduce the deficit bias and may foster fiscal discipline,<sup>3</sup> the evidence regarding their impact in the cyclical stance of fiscal policy is largely inconclusive. This controversy may be related to the exceptional heterogeneity of FR ([Combes et al., 2017](#)).<sup>4</sup>

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<sup>2</sup>Nowadays, the popularity of FR is such that, while less than 10 countries adopted FR in 1985, more than 90 countries around the world enabled at least one type of FR by 2015 (see the IMF’s Fiscal Rule dataset 1985-2015).

<sup>3</sup>See, e.g., [Fatás and Mihov \(2006\)](#); [Ardagna et al. \(2007\)](#); [Debrun et al. \(2008\)](#).

<sup>4</sup>FR can amplify macroeconomic shocks by making fiscal policy pro-cyclic ([Von Hagen, 2003](#); [Gali and Perotti, 2003](#); [Wyplosz, 2006](#)), or lead to a stabilization effect through a counter-cyclical policy ([Dabla-Norris et al., 2019](#); [Bova et al., 2014](#); [Guerguil et al., 2017](#)).

In the recent empirical literature (see [Eyraud et al., 2018](#)), a clear opposition seems to be emerging between the so-called “first” and “second” generation rules. Such a classification shall be based on the flexible scheme of rules. Regarding the former, FR combined simplicity and little flexibility, as the Maastricht Treaty requiring the deficit-to-GDP ratio to be at most 3%. Regarding the latter, during the post-great-recession period, FR have greatly expanded the flexibility provisions already embedded in first-generation rules, such as the use of cyclically-adjusted targets, will-defined escape clauses, or with strong legal and enforcement arrangements.

The present paper offers, to the best of your knowledge, the first theoretical analysis that takes into account a second-generation rule. We develop a stochastic endogenous growth model based on [Barro \(1990\)](#)’s endogenous growth archetype with productive public expenditures modeled as flows of productive services in a constant-return-to-scale production function.<sup>5</sup> The fiscal rule is such that the speed of adjustment of the debt-to-GDP ratio depends on the current economic activity.

Our results are twofold. (i) We highlight a new transmission channel based on the dynamics of public debt. Following a recessive transitional shock, the government cuts in productive public spending to keep the debt-to-GDP ratio consistent with the fiscal rule, hence a pro-cyclical amplification of the recessive shock. (ii) The flexibility degree of the fiscal rule dramatically affects the magnitude of fluctuations. Under a flexible rule, the volatility of fluctuations are reduced along the business cycle, and following both a transitional or a temporary shock, the flexible rule helps smooth aggregate variables.

From the theoretical perspective, the closest body of work is the literature modeling stochastic shocks in endogenous growth models. [King et al. \(1988\)](#) et [Stadler \(1990\)](#) first noticed that disturbances in the production function may lead to persistent fluctuations. Such a framework has been extended in several directions. On the one hand, some studies introduce, in a standard AK setup, a stochastic process affecting technology in

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<sup>5</sup>In a deterministic configuration, many authors have shown that such endogenous growth models are compatible with the existence of a growing public debt in the long-run; see, e.g., [Minea and Villieu \(2010, 2012\)](#); [Boucekkine et al. \(2015a\)](#); [Nishimura et al. \(2015a,b\)](#); [Menuet et al. \(2018\)](#).

discrete (Fatás, 2000; Jones et al., 2000; Menuet and Villieu, 2014) or continuous models (Boucekkine et al., 2015b). On the other hand, Malley et al. (2007) specify a balanced budget rule in a political economy setup. Our main innovation is to introduce public debt dynamics by relaxing the balanced budget rule.

From the policy perspective, our paper considers a new flexible fiscal rule in indebted economies with a stabilization effect both in case of permanent and transitory stocks. This feature echoes the empirical studies suggesting that the implementation of second-generation rules may be associated with less procyclicality or even counter-cyclicality of the fiscal stance (Bova et al., 2014; Guerguil et al., 2017; Combes et al., 2017)<sup>6</sup>.

The rest of the paper is organized as follows. Section 2 presents the model, section 3 solves the steady-states, section 4 studies the effect of the flexible fiscal rule in both deterministic and stochastic environments, and section 6 concludes the paper.

## 2. The model

We consider a simple model based on a RBC methodology, but implemented in an endogenous growth context. There are three infinitely-lived agents with perfect expectations: a representative firm, a representative household, and a government.

**Firms.** Following Barro (1990), population is normalized to unity, and output is produced using a Cobb-Douglas technology with constant returns, namely  $Y_t = \Phi_t K_t^\alpha G_t^{1-\alpha}$ , where  $K_t$  and  $G_t$  respectively stands for private capital and productive public expenditure,  $\alpha \in (0, 1)$  is the elasticity of output to private capital, and  $\Phi_t$  is productivity.

As usual in RBC literature, productivity  $\Phi_t$  is a stochastic function, namely  $\Phi_t = A \exp(z_t)$ , with  $A > 0$  a scale parameter, and  $z_t$  a technology shock following an AR(1) process

$$z_t = \psi z_{t-1} + \nu_t, \tag{1}$$

where  $\{\nu_t\}_{t \geq 0}$  is a set of i.i.d. shocks with zero mean and variance  $\sigma_\nu^2 > 0$ , and  $\psi > 0$  is the autoregressive coefficient.

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<sup>6</sup>See e.g., Gali and Perotti (2003); Manasse (2006); Fatás and Mihov (2012).

The accumulation of private capital comes from investment ( $I_t$ ) net of depreciation, namely  $K_{t+1} = I_t + (1 - \delta)K_t$ , where  $\delta \in (0, 1)$  is the rate of capital depreciation per unit of time. Thus, the first order conditions for profit maximization is

$$r_t = \alpha(1 - \tau) \frac{Y_t}{K_t} - \delta. \quad (2)$$

Notice that, as current shocks are known at the beginning of the period, the rate of return on private capital ( $r_t$ ) is perfectly expected at period  $t$ .

**Households.** The representative household sets the consumption path  $\{C_t\}_{t \geq 0}$  to maximize the time-separable logarithm utility function

$$U = \mathbb{E}_t \left[ \sum_{i=0}^{+\infty} \left( \frac{1}{1 + \beta} \right)^i \log(C_{t+i}) \right], \quad (3)$$

where  $\beta > 0$  is the subjective discount rate.

Household use their income ( $Y_t$ ) to consume ( $C_t$ ), to invest ( $I_t$ ), to buy government bonds ( $B_t$ ), which return the real interest rate  $r_t$ , and to pay taxes ( $\tau Y_t$ , where  $\tau \in (0, 1)$  is a proportional income tax rate). All variables are defined in real terms; hence the following budget constraint

$$B_{t+1} = (1 + r_t)B_t + (1 - \tau)Y_t - C_t - I_t. \quad (4)$$

First order conditions for the maximization of households' program give rise to the familiar Keynes-Ramsey relationship

$$\frac{1}{C_t} = \left( \frac{1 + r_t}{1 + \beta} \right) \mathbb{E}_t \left[ \frac{1}{C_{t+1}} \right], \quad (5)$$

under the standard transversality condition

$$\lim_{t \rightarrow +\infty} \left( \frac{1}{1 + \beta} \right)^t \frac{K_{t+1}}{C_t} = 0. \quad (6)$$

**Government.** The government provides public expenditures  $G_t$ , levies taxes, and borrows from households. The government deficit is financed by issuing debt ( $B_t$ ); hence, the following budget constraint

$$B_{t+1} = (1 + r_t)B_t + G_t - \tau Y_t. \quad (7)$$

Contrary to standard models that assume a balanced-budget rule, we introduce the possibility of public debt financing in the long-run, so that public deficits may be permanently financed by the debt accumulation. Effectively, many papers show that endogenous growth setups are compatible with the existence of a growing public debt in the long-run (Minea and Villieu, 2010, 2012; Boucekkine et al., 2015a; Nishimura et al., 2015a,b; Menuet et al., 2018).<sup>7</sup> The only requirement for the transversality condition to be verified is that the rate of growth of public debt must be less than the real interest rate.

At this stage the model is not closed, because there is one free variable in the government budget constraint (7). To close the model, the government must fix either the public spending or the public debt path. As public expenditure is an endogenous variable, we take the debt-to-output ( $B_t/Y_t$ ) that determines the public debt path as the instrument. To this end, we specify the fiscal rule governing the changes of the debt-to-output ratio as follows<sup>8</sup>

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t} = \mu_t \left( \theta - \frac{B_t}{Y_t} \right) \quad (8)$$

From (8), the fiscal policy instruments are the flat tax rate ( $\tau$ ) and the targeted debt-to-output ratio in the long-run ( $\theta$ ). If the speed of adjustment of current debt to this target is also an instrument (i.e.  $\mu_t =: \mu_0$ ), the evolution of the debt-to-output ratio does not respond to changes in economic activity and describes a simple gradual process. In this case, the fiscal rule is “fixed” and belongs to the so-called class of first generation rules. Especially, a low value of  $\mu_0$  characterizes a “gradualist” strategy (i.e. the speed of adjustment would be small), and a high value accounts for a “shock therapy” strategy, which gives rise to a faster reduction in the debt ratio.

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<sup>7</sup>In such models, output grows continuously along the balanced-growth-path (BGP), and public debt also may grow continuously, thus removing the balanced-budget-rule hypothesis in the long-run.

<sup>8</sup>Interestingly, such a fiscal rule is consistent to the unconditionally optimal fiscal policy (Horvath, 2011).

In our model, in contrast, we consider a time-varying speed of adjustment. The speed  $\mu_t$  negatively depends on the gap between the long-run economic growth rate ( $\gamma^*$ ) and the current growth rate ( $\gamma_t := (Y_{t+1} - Y_t)/Y_t$ ), namely

$$\mu_t = \mu_0 - \xi(\gamma^* - \gamma_t), \quad (9)$$

where  $\xi > 0$  reflects *the flexibility degree* of the fiscal rule.<sup>9</sup> If  $\xi = 0 \Rightarrow \mu_t = \mu_0$ , the fiscal rule is fixed (*first generation*), as previously discussed. If  $\xi > 0$ , however, the fiscal rule is “flexible” (*second generation*).

In long-run ( $\gamma_t = \gamma^*$ ) the speed of adjustment is simply  $\mu_0$ . This is no long the case in the short-run: at a time of economic downturn ( $\gamma_t < \gamma^*$ ), the speed of adjustment slows down ( $\mu_t < \mu_0$ ). From Eqs. (7), the public-spending-to-output ratio is

$$\frac{G_t}{Y_t} = \tau - (r_t - \gamma_t) \frac{B_t}{Y_t} + [\mu_0 - \xi(\gamma^* - \gamma_t)](1 + \gamma_t) \left( \theta - \frac{B_t}{Y_t} \right),$$

hence;

$$\frac{d}{d\xi} \left( \frac{G_t}{Y_t} \right) = -(1 + \gamma_t) (\gamma^* - \gamma_t) \left( \theta - \frac{B_t}{Y_t} \right).$$

Intuitively, following an adverse technological shock, the public-debt-to-GDP ratio becomes higher than the long-run target ( $\theta < B_t/Y_t$ ), exhibiting a counter-cyclical behavior of the public spending. Indeed, the ratio  $G_t/Y_t$  increases as long as the current growth is lower than the potential level ( $\gamma_t < \gamma^*$ ), and decreases in the opposite case ( $\gamma_t > \gamma^*$ ).<sup>10</sup> This feature is consistent with empirical studies suggesting that flexible fiscal rules are associated with counter-cyclical behavior of fiscal stance (Bova et al., 2014; Guerguil et al., 2017; Combes et al., 2017), notably in occidental countries.

### 3. Equilibrium

**Definition 1.** A competitive equilibrium is a path  $\{C_t, K_t, G_t, B_t, Y_t\}_0^\infty$  which solves Eqs. (2), (5), (7), (8), (9), the transversality condition, and satisfies the IS equilibrium  $K_{t+1} - K_t = Y_t - C_t - G_t - \delta K_t$ .

<sup>9</sup>To ensure  $\mu_t > 0$ , we assume that  $\mu_0$  is high enough.

<sup>10</sup>When a positive shock occurs (namely,  $\theta > B_t/Y_t$ ), the public spending ratio exhibits a procyclical behavior, i.e.  $G_t/Y_t$  increases if and only if  $\gamma_t > \gamma^*$ . In this paper, we focus only on adverse shocks.

**Definition 2.** A balanced-growth path (BGP) is a competitive equilibrium where consumption, capital, public spending, public debt, and output grow at the same (endogenous) rate  $\gamma$ .

To find endogenous growth solutions, we deflate all growing variables by the capital stock to obtain long-run stationary ratios, namely:  $bk_t := B_t/K_t$ ,  $yk_t := Y_t/K_t$ ,  $ck_t := C_t/K_t$ ,  $gk_t := G_t/K_t$ , and the growth of the private capital is  $\gamma k_t := K_{t+1}/K_t - 1$ . The dynamics system is described by the set of following equations.

- i. Keynes-Ramsey relationship

$$\frac{1}{ck_t} = \left( \frac{1+r_t}{1+\beta} \right) \mathbb{E}_t \left[ \frac{1}{ck_{t+1}[1+\gamma k_t]} \right], \quad (10)$$

- ii. IS equilibrium

$$\gamma k_t = yk_t - ck_t - gk_t - \delta, \quad (11)$$

- iii. The government's budget constraint

$$bk_{t+1} = \frac{(1+r_t)bk_t + gk_t - \tau yk_t}{1+\gamma k_t}, \quad (12)$$

- iv. Public debt path (where  $by_t := B_t/Y_t$ ):

$$by_{t+1} = by_t + [\mu_0 - \xi(\gamma^* - \gamma k_t)](\theta - by_t). \quad (13)$$

- v. Real interest rate

$$r_t = \alpha(1-\tau)yk_t - \delta, \quad (14)$$

- vi. Production function

$$yk_t = \exp(z_t)Agk_t^{1-\alpha}. \quad (15)$$

The set (10)-(15) is a 6-equations systems, which needs to be solved.

### 3.1. Deterministic steady-state

Let us first characterize determinist solutions ( $\nu_t = 0$ ). In this case, the real interest rate ( $r_t$ ) and all capital-deflated variables are constant in equilibrium. Therefore, as stated in definition 2, all variables expressed in level grow at the same balanced rate ( $\gamma$ ).



To compute the steady-state(s), we proceed by induction. First, the long-run public debt ratio is:  $bk = \theta yk$ . Thus, the public spending ratio in (12) leads to

$$gk = \tau yk - (r - \gamma)bk = [\tau - (r - \gamma)\theta]yk. \quad (16)$$

From (16), under the balanced budget rule ( $\theta = 0$ ), the public spending just corresponds to the Barro (1990)'s solution (i.e.  $gk = \tau yk$ ). In contrast, public spending are lower in the presence of public debt ( $\theta > 0$ ), since the standard transversality condition ensures  $r > \gamma$  (see Minea and Villieu, 2012; Menuet et al., 2018). The basic mechanism driving this crowding-out effect is the following. The public deficits generate (i) a permanent flow of new resources ( $B_{t+1} - B_t$ ), and (ii) a permanent flow of new unproductive expenditures (the debt burden  $r_t B_t$ ). In steady state, the standard transversality condition ( $r_t > \gamma_t = B_{t+1}/B_t$ ) means that the latter dominates the former, thereby any rule that permit permanent deficits involves net long-run costs for public finance, irrespective of the precise nature of this rule.

At this stage, the steady-state output ratio is  $yk = A^{1/\alpha}[\tau - (r - \gamma)\theta]^{(1-\alpha)/\alpha}$ . As usual, endogenous growth solution is obtain by the crossing-point of two relationships between the real interest rate  $r$  and the economic growth rate  $\gamma$ . The first one comes from the Keynes-Ramsey relationship (10)

$$\gamma = \gamma^c = \left( \frac{1+r}{1+\beta} \right) - 1. \quad (17)$$

The second is directly linked to the government's budget constraint

$$\gamma = \gamma^b = \frac{1}{\theta} \left[ \left( \frac{r + \delta}{\alpha(1 - \tau)A^{1/\alpha}} \right)^{\alpha/(1-\alpha)} + r\theta - \tau \right]. \quad (18)$$

**Proposition 1.** (*Existence and Uniqueness*) *There is a critical discount rate  $\bar{\delta} > 0$ , such that, if  $\delta < \bar{\delta}$ , there is an unique BGP. Especially, in the long run, the targeted debt ratio reduces the economic growth rate.*

Proof: See Appendix.

For a small discount rate ( $\delta < \bar{\delta}$ ), relations (17)-(18) cross once at point  $(r^*, \gamma^*)$ . The intuition is as follows. On the one hand, Eq. (17) describes a linear increasing relationship

through the optimal saving behaviour of households, as depicted in Figure 1. On the other hand, Eq. (18) gives rise to a convex increasing curve between the economic growth rate and the interest rate. Indeed, in the government’s budget constraint, economic growth reduces the public debt burden in the long run, boosts productive public spending, and increases the marginal productivity of private capital in the bond market clearing condition (as  $r_t = \alpha(1 - \tau)g_k^{1-\alpha} - \delta$ ).

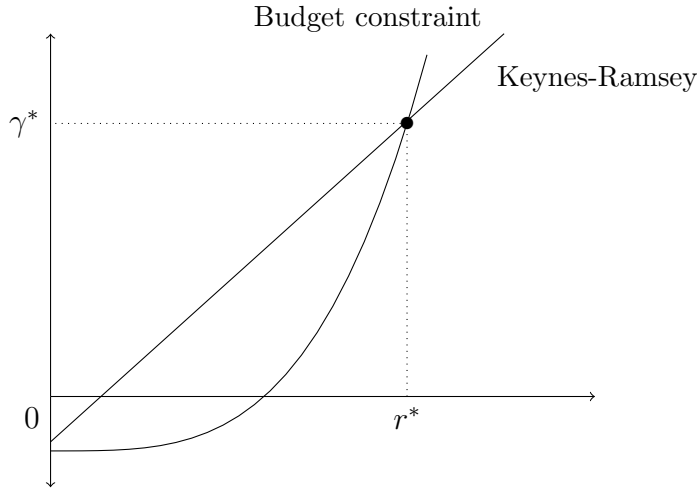


Figure 1: Long-run equilibrium

Our simulation below shows that the unique steady-state is saddle-path stable, ensuring the local and global determinacy of the model. In steady-state, due to the crowding-out on productive public expenditures, the lower the debt ratio ( $\theta$ ), the higher the economic growth rate ( $\gamma$ ).

### 3.2. A numerical illustration

Our simulations are based on reasonable values for parameters (see Table 1). We choose a usual discount rate  $\rho = 0.02$  to match long-run historical data for the risk-free real interest rate. The consumption elasticity of substitution (inverse of the risk-aversion coefficient) is fixed at  $S = 1$ . As regards the technology, we set  $A = 0.5$  to obtain a realistic rate of economic growth, and the capital share in the production function is  $\alpha = 0.7$ , as in [Gomes et al. \(2013\)](#), close to the value (0.715) used by [Gomme et al. \(2011\)](#). Such a capital share allows reproducing the empirical results of [Munnell \(1990\)](#) on the

elasticity of output to productive public spending ( $1 - \alpha = 0.3$ ). The depreciation rate of capital is set at  $\delta = 0.05$ , which roughly corresponds to the average value of depreciation rates used in [Gomme et al. \(2011\)](#).

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PARAMETERS			
$\beta$	0.02	Discount rate	
$A$	0.5	Productivity parameter	
$\alpha$	0.7	Capital share in the production function	
$\delta$	0.05	Depreciation rate	
$\tau$	0.38	Tax rate on income	
$\theta$	0.572	Long-run deficit ratio (target value)	

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TARGET VALUES			
	Model	Data	Source
Long-run economic growth	0.034	0.033	BEA, 1950-2015
After-tax return of capital	0.054	0.0516	<a href="#">Gomme et al. (2011)</a>
Investment to capital ratio	0.084	0.088	<a href="#">Gillman and Kejak (2011)</a>
Public debt to GDP	0.572	0.572	BEA, 1950-2015
Public spending to GDP	0.367	0.366	OECD, 1970-2015

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Table 1: Baseline Calibration (high BGP)

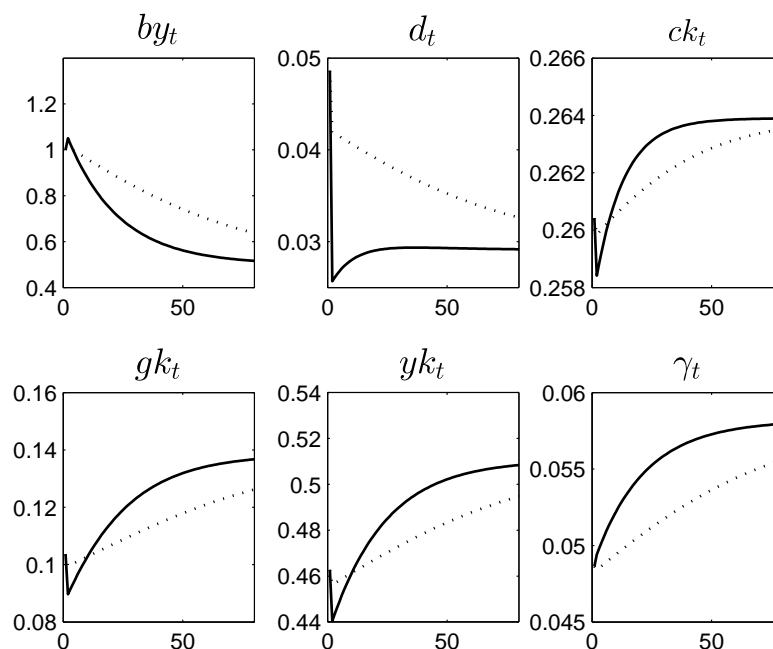
Despite the highly-stylized nature of the deterministic model, the baseline calibration allows obtaining quite realistic results, close to postwar US annual data. The BGP is characterized by a 3.4% long-run rate of economic growth (3.3% in the data). The calibration well reproduces the after-tax rate of return of capital (5.4%, while the average mean on 1954-2008 in [Gomme et al. \(2011\)](#) is 5.16%), the investment to capital ratio (0.084), which is close to the estimate (0.088) used in [Gillman and Kejak \(2011\)](#), and the public-spending-to-GDP ratio is 36.7% (36.6% in the data).

#### 4. The stabilization role of flexible FR

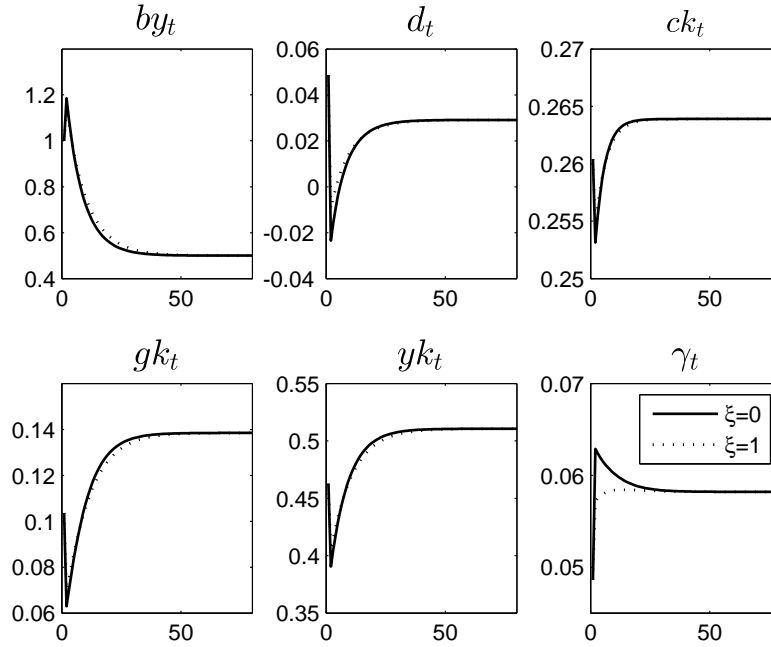
This section provides some simulations to study the effect of the flexibility degree of the fiscal rule on aggregated variables both in case of permanent and transitory shocks, respectively.

4.1. A permanent shock

Let us analyzing the adjustment of variables following a drop in the debt target (from 100% to 50% of GDP) in the deterministic framework. Figures 7a-b depict the IRF of the debt-to-GDP ratio ( $by_t$ ), the deficit-to-GDP ( $d_t$ ), the ratios of consumption ( $ck_t$ ), public expenditure ( $gk_t$ ), output ( $yk_t$ ), per unit of capital, and the growth rate ( $\gamma_t$ ).



a.  $\mu_0 = 0.05$



b.  $\mu_0 = 0.15$

Figure 2: Transitory dynamics following a reduction of the long-run debt target

In the long-run, the rate of economic growth unambiguously increases, following the rise in productive public spending. In the short-run, nevertheless, the impact of the change in the debt target depends on the flexibility degree of the fiscal rule. In what follows, we consider two policies: a “flexible” rule ( $\xi = 1$ , the dashed line), and a “fixe” rule ( $\xi = 0$ , the continuous line).

In Figure 7a ( $\mu_0 = 0.05$ ), all the adjustments are slower under a flexible rule. Without taking into account the business cycle ( $\xi = 0$ ), the variables come back to equilibrium after 50 quarters, and after 150 quarters under a flexible rule ( $\xi = 1$ ).

However, the fixe rule is, in the short run, a costly policy. As the public deficit immediately decreases, the economy lacks productive public spending, hence a decline of output and consumption. During the transition path, these initial costs are reduced, because the debt burden and the crowding out effects decrease. Finally, in the long-run, the output ratio, the consumption ratio and the public investment ratio are higher than initially. Finally, as depicted in Figure 6.12b ( $\mu_0 = 0.15$ ), the flexible rule results in a stabilizing effect of the economic growth rate.

#### 4.2. *Transitory shocks*

Let us now analyze the adjustment of variables following an adverse technological shock.<sup>11</sup> In Figure 8 (for a autoregressive coefficient  $\phi = 0.9$ ), we represent the fluctuations in economic growth and in speed of adjustment, as deviations from their long-run values (in %). Two points deserve particular attention.

First, the transitory shock is strongly amplified during the transition path and gives birth to persistent business fluctuations. Indeed, our model produces a new transmission channel based on the dynamics of public debt. The basic mechanism is as follows. When an adverse technological shock occurs, the total factor productivity and the output decrease, raising the debt-to-GDP ratio. In return, the government will reduce the primary deficit and cuts in public spending to keep the debt ratio consistent with the long-term target. This lack of productive public expenditure leads to a recession effect, and the harmful circle is renewed.

Second, the strength of business cycles crucially depends on the nature of the public debt adjustment. Indeed, the higher the flexibility degree, the lower the magnitudes of fluctuations (the variance of the output, in ratio to the steady-state, is 0.57 for  $\xi = 0$ , and 0.29 for  $\xi = 1$ ). The stabilization effect of the flexible fiscal rule comes from the endogenous speed of adjustment: following an adverse technological shock, the speed of adjustment is lower than the long-run level that reduces the cuts in productive spending, smoothing the recessive effects. In contrast, under a fixe fiscal rule ( $\xi = 0$ ), the speed of adjustment equals the constant long-run level ( $\mu_t = \mu_0, \forall t$ ) that amplifies the transitional shocks.

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<sup>11</sup>We assume  $\mu_0 = 0.25$  to ensure than  $\mu_t > 0$ , for any  $t \geq 0$ .

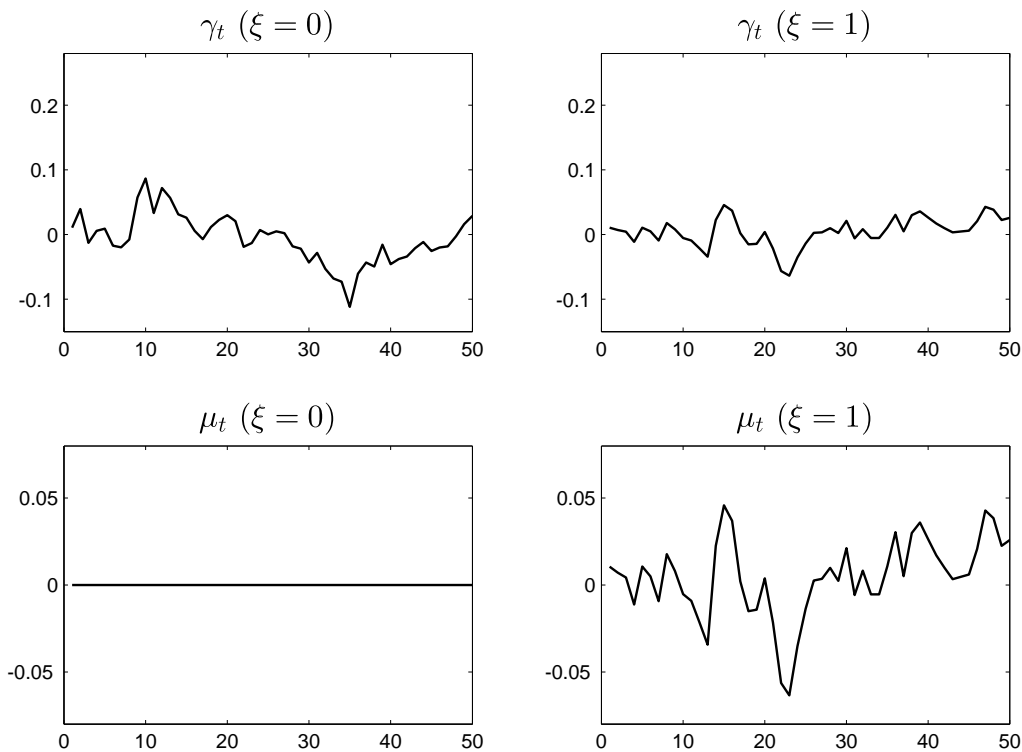


Figure 3: Adjustment following an adverse technological shock: deviation from steady-state (in %)

## 5. Conclusion

Our paper develops a stochastic endogenous growth model. We develop a flexible rule, such that the speed of adjustment of the debt-to-GDP ratio depends on the business cycle. We reveal that such a rule helps smooth aggregate variables, and leads to a stabilization effect by limiting the lack of productive public spending.

An interesting extension of our analysis would consist in providing a political economy mechanism to endogenize the policy instruments (as the long-run debt target, or the speed of adjustment). Besides, further works could attempt to build a general model connecting DSGE models with an endogenous growth framework. The introduction of nominal rigidities, and the specification of policymakers' preferences could open the way for a laboratory model encompasses both long-run economic growth and business cycles. These two possible directions are left for future research.

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## Appendix A. Proof of Proposition 1.

From Eqs. (17)-(18), the interest rate steady-state is given by the implicit relation  $\Psi(r) = 0$ , where

$$\Psi(r) := \left( \frac{1+r}{1+\beta} \right) - 1 - \frac{1}{\theta} \left[ \left( \frac{r+\delta}{\alpha(1-\tau)A^{1/\alpha}} \right)^{\alpha/(1-\alpha)} + r\theta - \tau \right].$$

Clearly  $\Psi \in C^2(\mathbb{R}^+)$ , with

$$\Psi'(r) = -\frac{\beta}{1+\beta} - \frac{1}{\theta\alpha(1-\tau)A^{1/\alpha}} \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{r+\delta}{\alpha(1-\tau)A^{1/\alpha}}\right)^{(2\alpha-1)/(1-\alpha)} < 0.$$

In addition,  $\Psi(+\infty) = -\infty$ , as  $\alpha > 1/2$ , and  $\Psi(0) > 0 \Leftrightarrow \delta < \bar{\delta} := \alpha(1-\tau)A^{1/\alpha}[\tau - \frac{\beta\theta}{1+\beta}]^{(1-\alpha)/\alpha}$ . As  $\tau > \theta$ , it follows that  $\bar{\delta} > 0$ . Consequently, according to the Intermediate Value Theorem, if  $\alpha > 1/2$  and  $\delta < \bar{\delta}$ , there is a unique critical value  $r^* > 0$  such that  $\Psi(r^*) = 0$ . This value defines the real interest rate at the steady-state. The associated level of economic growth is  $\gamma^* = (1+r^*)/(1+\beta) - 1 > 0$ .

Besides, we compute

$$\frac{\partial\Psi(r)}{\partial\theta} = \frac{1}{\theta^2} \left[ \left(\frac{r+\delta}{\alpha(1-\tau)A^{1/\alpha}}\right)^{\alpha/(1-\alpha)} + r\theta - \tau \right] - \frac{r}{\theta}.$$

At the steady-state, it follows that

$$\frac{\partial\Psi(r)}{\partial\theta} \Big|_{r=r^*} = -\frac{\beta(1+r^*)}{\theta(1+\beta)} < 0.$$

Thus, as  $\Psi'(r^*) < 0$ , according to the Implicit Function Theorem, we have, in equilibrium,  $\partial r^*/\partial\theta < 0$  and  $\partial\gamma^*/\partial\theta < 0$ .