Fiscal rule and shock amplification : A stochastic endogenous growth model

Maxime Menuet

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Maxime Menuet[a,1]

[a] *Univ. Orléans, CNRS, LEO, UMR 7322, FA45067, Orléans, France*

Abstract

This paper develops a discrete-time stochastic endogenous growth model to study the amplification role of fiscal rules. In our model, transitory shocks exert permanent effects on the level of variables in equilibrium (*hysteresis*), and can be strongly amplified by the public debt adjustment, leading to a procyclical amplification mechanism (the "public debt accelerator"). This procyclical stance depends on the speed of adjustment of the debt-to-GDP ratio under a fixed-fiscal rule. A cold turkey strategy removes the public debt shock, but at the risk of destabilizing other variables, while a gradualist strategy has a stabilization effect, with detrimental consequences in the long-run. Finally, we show that a flexible-fiscal rule helps smooth aggregate variables by limiting the cuts in productive public spending.

Keywords: Endogenous growth model, Hysteresis, Fiscal Rules

1. Introduction

Almost all countries including both developed and developing ones have run into a massive expansion of public debt from 1970s. This expansion was amplified by fiscal stimuli in response to the Great Recession, but the high level of budget deficit and debt is a persistent and structural characteristic of industrialized economies for forty years.[2]

At the same time, many countries attempted to strengthen their fiscal governance frameworks by introducing fiscal rules (FR).[3] Undoubtedly, the adoption of FR allows

[1]Corresponding author: maxime.menuet@univ-orleans.fr.

[2]Indeed, since the early 1970s until 2000s, namely before the ongoing economic crisis, the average ratio of public deficit in OECD countries comes close to 3%.

[3]Nowadays, the popularity of FR is such that, while less than 10 countries adopted FR in 1985, more than 90 countries around the world enabled at least one type of FR by 2015 (see the IMF's Fiscal Rule dataset 1985-2015).

limiting the deficit bias and may foster fiscal discipline (Alesina and Bayoumi, 1996; Bohn and Inman, 1996; Fatás and Mihov, 2006; Manasse, 2006; Ardagna et al., 2007; Debrun et al., 2008). Nevertheless, the evidence regarding the impact of FR in the cyclical stance of fiscal policy is largely inconclusive: FR can amplify macroeconomic shocks by making fiscal policy pro-cyclic (Von Hagen, 2003; Gali and Perotti, 2003; Wyplosz, 2006), or lead to a stabilization effect through a counter-cyclical fiscal policy (Dabla-Norris et al., 2019; Bova et al., 2014; Guerguil et al., 2017). This controversy may be related to the exceptional heterogeneity of FR (Combes et al., 2017). Indeed, the recent literature provides a clear opposition between *fixed* FR and *flexible* FR. Regarding the former, for example, the Maastricht Treaty requires the deficit-to-GDP ratio to be at most 3%. Regarding the latter, FR may be cyclically adjusted, so that the deficit or the debt target, for example, can varies over the business cycle. Consequently, a major challenge faces by economic theory is to study the impact of FR (both fixed and flexible) on indebted economies.

Our paper revisits the impact of FR (both fixed and flexible) in the cyclical stance of fiscal policy in a theoretical perspective by building a prototypal stochastic endogenous growth model. The model is based on Barro (1990)'s endogenous growth archetype, with productive public expenditures modeled as flows of productive services in a constant-return-to-scale production function, and the public debt-to-GDP ratio evolves according to a standard FR. In a deterministic configuration, many authors have shown that such a model is compatible with the existence of a growing public debt in the long-run.[4] This papers extends these preceding analysis in a stochastic setup.

From a methodological perspective, two major reasons should make it helpful to introduce the persistence of public debt in stochastic real business cycle (RBC) models. The traditional New Synthesis framework (see, e.g., Shapiro and Watson, 1988; Blanchard, 1989; Campbell and Mankiw, 1989; Stock and Watson, 1999, among others) (i) abstract from long-run growing variables, (ii) and suffers from a lack of propagation mechanisms. Indeed, the main factor explaining the persistence in business cycle fluctuation comes from the persistence of exogenous technological shocks itself. Based on the approach of

[4]See, e.g., Minea and Villieu (2010, 2012); Boucekkine et al. (2015a); Nishimura et al. (2015a,b); Menuet et al. (2017).

King et al. (1988) and Stadler (1990), our model can overcome these two shortcomings, because the impact of technology shocks is strongly amplified by the adjustment of public debt.

Our results are threefold.

(i) Our setup provides a new transmission channel based on the dynamics of public debt. Effectively, following a recessive transitionary shock, the government cuts in productive public spending to maintain the debt-to-GDP ratio consistent with the fixed fiscal rule, hence an procyclical amplification of the recessive effect, called the "public debt accelerator".

(ii) The procyclical behavior of fiscal policy crucially depends on the specification of the FR, especially on the speed of adjustment of the debt-to-GDP ratio. A "cold turkey" strategy (a high speed of adjustment) removes the public debt shock, but at the risk of destabilizing other variables, including productive public expenditure. In contrast, a "gradualist" strategy (a low speed) helps smooth variables, with detrimental long-run effects. By analyzing the inter-temporal social welfare, we show that the welfare-maximizing policy following an adverse shock is the gradualist strategy (resp. the cold turkey strategy) if the subjective discount rate is small (resp. high).

(iii) We extend the analysis of FR by modelling a flexible rule in which the speed of adjustment of the debt-to-GDP ratio depends on the business cycle. We show that the degree of flexibility dramatically affects the cyclical behavior of the government spending. Under a flexible rule, the volatility of fluctuations are reduced along the business cycle, and following both a transitionary or a temporary shock, the flexible rule helps smooth variable.

Related literature. Our model addresses two major long-lasting topics in macroeconomics.

(i) The closest body of work is the literature modeling stochastic shocks in endogenous growth models. King et al. (1988) et Stadler (1990) first noticed that disturbances in the production function may lead to persistent fluctuations. Indeed, transitionary deviations add up over time through the constant-return of the production function. Such a framework has been extended in several directions. On the one hand, some studies introduce, in a standard AK setup, a stochastic process affecting technology in discreet (Fatás, 2000; Jones et al., 2000; Menuet and Villieu, 2014) or continuous models (Boucekkine et al., 2015b). On the other hand, Malley et al. (2007) specify a balanced budget rule in a political economy setup. Our main innovation is the introduction of public debt by relaxing the balanced budget rule. The present analysis reveals that a government's budget constraint in which productive public spending are financed by growing public deficits lead to a new endogenous mechanism of transitory shocks amplification.

(ii) Our paper is related to the fast-growing literature on macroeconomic effects of FR. From a normative perspective, Azzimonti et al. (2010) highlight that the balanced budget rule (BBR) could be costly in the short run and beneficial in the long run, since the government has to cut spending and raise taxes. From the inter-temporal view, the authors find that the short run costs are too large to offset for the steady state benefits of a lower debt. Our model extends the Azzimonti et al. (2010)'s analysis in two directions. First, we relax the BBR specification by permitting the accumulation of public deficits in equilibrium. Second, we show that the welfare-maximizing adjustment strategy, following an adverse shock, depends on the time preference of households: if households have short-run (resp. long-run) views, the best policy is the gradualist (resp. the cold turkey) strategy.[5]

From a positive approach, our model echoes the wide range of papers emphasizing that the adoption of *fixed* FR leads to a procyclical behavior of public expenditure[6]. In our model, following an adverse technological shock, the total factor productivity decreases, rising the debt-to-GDP ratio, and in return the government cuts in public spending, hence the procyclical amplification mechanism of the "accelerator of public debt".

Recently, some empirical studies suggest, however, that the implementation of *flexible*

[5]This result is consistent with the Horvath (2011)'s approach. Indeed, in a New Keynesian framework (without economic growth), the author show that the speed of debt reduction consistent with the unconditionally optimal policy is determined by the rate of time preference of agents.

[6]See e.g., Roubini and Sachs (1989); Alt and Lowry (1994); Poterba (1994); Alesina and Bayoumi (1996); Fatás and Mihov (2006); Lane (2003b); Fatás and Mihov (2012), among others.

FR (called the "second generation" rules), such as the use of cyclically-adjusted targets, will-defined escape clauses, together with strong legal and enforcement arrangements, may be associated with less procyclicality or even counter-cyclicallity (Bova et al., 2014; Guerguil et al., 2017; Combes et al., 2017),[7] reduction the volatility of aggregate variables in the short-run. From the long-run perspective, many authors show that these counter-cyclical policies can improve the economic growth (Aghion and Howitt, 2006; Aghion et al., 2007).[8] In our model, in contract, following a transitory shock, the flexible rule exacerbates the long-term loss in economic growth, because public debt having a crowding-out effect on productive public spending in equilibrium. Finally, the present paper offers, to the best of your knowledge, the first theoretical setup modeling a second-generation rule. Therefore, we share the Bova et al. (2014)'s intuition whereby flexible FR help smooth aggregate variables.

The rest of the paper is organized as follows. Section 2 presents the model, section 3 solves the steady-states, and section 4 presents the public debt accelerator. Section 5 studies the effect of the profile adjustment in both deterministic and stochastic environments, section 6 introduces a flexible fiscal rule, and section 5 concludes the paper.

2. The model

We consider a one-sector endogenous growth model with three infinitely-lived agents: a representative household, a representative from and a government. All agents have perfect foresight.

2.1. Firms

Output is produced using a Cobb-Douglas technology with constant returns at the private level but with a public good externality, namely $Y_t = \Phi_t K_t^{\alpha}(\mathrm{L}_t G_t)^{1-\alpha}$, where K_t and G_t respectively stands for private capital and productive public expenditure, and Φ_t

[7] Especially, in high-income countries (Gali and Perotti, 2003; Manasse, 2006; Fatás and Mihov, 2012). Some authors (Bergman and Hutchison, 2015; Combes et al., 2017) find a nonlinear relationship: the fiscal policy will be contra-cyclic below a critical public debt level, and procyclic beyond.

[8] In a stimulating paper, by focusing on US states' fiscal policy, Svec and Kondo (2012) show that (i) the ex-post budget restriction constrains the cyclicality in total spending, and (ii) a US state can increase its growth by 0.4% per year by relaxing the ex-post budget restriction.

is productivity. Population L_t will be normalized to unity, so that all variables are per capita, and we obtain

$$Y_t = \Phi_t K_t^{\alpha} G_t^{1-\alpha}.$$

The elasticity of output to private capital is $\alpha \in (0, 1)$. Following Barro (1990), public expenditure provides "productive services", with an elasticity $1 - \alpha$. This production function displays a decreasing marginal productivity of private capital and constant returns to scale ($\alpha < 1$) in order to generate an endogenous growth path in the long-run.

As usual in the RBC literature, productivity Φ_t is a stochastic function, namely

$$\Phi_t = A \exp(z_t),$$

with $A > 0$ a scale parameter, and z_t a technology shock following an AR(1) process

$$z_t = \psi z_{t-1} + \nu_t, \tag{1}$$

where $\{\nu_t\}_{t\geq 0}$ is a set of i.i.d. shocks with zero mean and variance $\sigma_\nu^2 > 0$, and ψ is the autoregressive coefficient.

The accumulation of private capital comes from investment (I_t), net of depreciation, with $\delta \in (0, 1)$ the rate of capital depreciation per unit of time

$$K_{t+1} = I_t + (1 - \delta) K_t.$$

Thus, the first order conditions for profit maximization is

$$r_t = \alpha(1 - \tau) Y_t / K_t - \delta.$$

2.2. Household

The representative household sets the consumption path $\{C_t\}_{t\geq 0}$ to maximize the following time-separable utility function

$$U = \mathbb{E}_t \left[\sum_{i=0}^{+\infty} \left(\frac{1}{1+\beta} \right)^i u(C_{t+i}) \right], \tag{2}$$

where $\beta > 0$ is the subjective discount rate.

To obtain endogenous growth solutions, we define a constant-elasticity of substitution (CES) utility function

$$u(C_t) = \begin{cases} \frac{S}{S-1}\left[(C_t)^{(S-1)/S} - 1\right], & \text{if } S \neq 1, \\ \log(C_t), & \text{if } S = 1, \end{cases} \tag{3}$$

with $S := u''(C_t)C_t/u'(C_t) > 0$ the elasticity of intertemporal substitution in consumption. Household use their income (Y_t) to consume (C_t), to invest ($\dot{K}_t$), to buy government bonds (B_t), which return the real interest rate r_t, and to pay taxes (τY_t, where $\tau \in (0, 1)$ is a proportional income tax rate). All variables are defined in real terms; hence the following budget constraint

$$B_{t+1} = (1 + r_t)B_t + (1 - \tau)Y_t - C_t - I_t. \tag{4}$$

First order conditions for the maximization of households program give rise to the familiar Keynes-Ramsey relationship

$$(C_t)^{-1/S} = \left(\frac{1 + r_t}{1 + \beta}\right)\mathbb{E}_t\left[(C_{t+1})^{-1/S}\right]. \tag{5}$$

Notice that, as current shocks are known at the beginning of the period, the rate of return on private capital (r_t) is perfectly expected at period t.

2.3. Government

The government provides public expenditures G_t, levies taxes, and borrows from households. Fiscal deficit is financed by issuing debt (B_t); hence, the following budget constraint

$$B_{t+1} = (1 + r_t)B_t + G_t - \tau Y_t. \tag{6}$$

Contrary to standard models that assume a balanced budget rule, endogenous growth models are compatible with the existence of a growing public debt in the long-run.[9]

[9] In such models, output grows continuously along the BGP, and public debt also may grow continuously, thus removing the balanced budget rule hypothesis in the long-run (Minea and Villieu, 2010, 2012; Boucekkine et al., 2015a; Nishimura et al., 2015a,b; Menuet et al., 2017).

Indeed, the only requirement for the transversality condition to be verified is that the rate of growth of public debt must be less than the real interest rate. The balanced budget rule imposed by standard New-Keynesian models in long-run equilibrium would be very inaccurate for studying public debt dynamics, explaining why we consider that a stochastic endogenous growth model is particularly interesting in addressing this issue.

At this stage the model is not closed, because there is one free variable in the government budget constraint (6). To close the model, the government must fix either the public spending or the public debt path. Since, public expenditure is an endogenous variable, we take the debt-to-output (B_t/Y_t) that determines the public debt path as the instrument. This characterizes a great number of countries that adopted deficit rules. To this end, we specify the fiscal rule governing the changes of the debt-to-output ratio as follows[10]

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t} = \mu \left(\theta - \frac{B_t}{Y_t} \right). \tag{7}$$

Thus, the fiscal policy instruments are the flat tax rate (τ), the targeted debt-to-output ratio in the long-run (θ), and the speed of adjustment of current debt to this target (ξ). A low value of the last parameter describes a "gradualist" strategy (i.e. the speed of adjustment is small), and a high value accounts for a "shock therapy" strategy, which gives rise to a faster reduction in the debt-to-output ratio.

Let us introduce the economic growth rate $\gamma_t := Y_{t+1}/Y_t - 1$. From (6) and (7), the productive-public-expenditure to output ratio is

$$\frac{G_t}{Y_t} = \tau - (r_t - \gamma_t)\frac{B_t}{Y_t} + \mu(1+\gamma_t)\left(\theta - \frac{B_t}{Y_t} \right),$$

hence;

$$\frac{d}{d\mu}\left(\frac{G_t}{Y_t} \right) = (1+\gamma_t)\left(\theta - \frac{B_t}{Y_t} \right). \tag{8}$$

From (8), if the current debt-to-output ratio is lower than its long-run value ($\theta >$

[10] Interestingly, such a fiscal rule is consistent to the unconditionally optimal fiscal policy (Horvath, 2011). Indeed, the author show that the first-order condition to minimize the loss of welfare leads to a gradual reduction in public debt to its steady state value, where the speed of debt reduction is determined by the rate of time preference of agents.

B_t/Y_t), any increase of the speed of adjustment reduces the productive expenditure (in % of GDP) all along the transition path. In our simulations below, we will study the economies' properties according to the speed of adjustment.

3. Equilibrium

This section computes endogenous growth solutions. By so doing, growing variables are deflated by the capital stock to obtain long-run stationary ratios, namely: $bk_t := B_t/K_t$, $yk_t := Y_t/K_t$, $ck_t := C_t/K_t$, $gk_t := G_t/K_t$, and the growth of the private capital is $\gamma k_t := K_{t+1}/K_t - 1$. In equilibrium, the dynamics system is given by

i. Keynes-Ramsey relationship

$$(ck_t)^{-1/S} = \left(\frac{1+r_t}{1+\beta}\right)\mathbb{E}_t\left[(ck_{t+1}[1+\gamma k_t])^{-1/S}\right], \tag{9}$$

ii. IS equilibrium

$$\gamma k_t = yk_t - ck_t - gk_t - \delta, \tag{10}$$

iii. The government's budget constraint

$$bk_{t+1} = \frac{(1+r_t)bk_t + gk_t - \tau yk_t}{1+\gamma k_t}, \tag{11}$$

iv. Public debt path (where $by_t := B_t/Y_t$):

$$by_{t+1} = by_t + \mu(\theta - by_t), \tag{12}$$

v. Real interest rate

$$r_t = \alpha(1-\tau)yk_t - \delta, \tag{13}$$

vi. Production function

$$yk_t = \exp(z_t)Agk_t^{1-\alpha}. \tag{14}$$

The set (9)-(14) is a 6-equations systems, which needs to be solved.

3.1. Deterministic steady-state

Let us first characterize determinist solutions ($\nu_t = 0$). In this case, the real interest rate (r_t) and all capital-deflated variables are constant in equilibrium. Therefore, all variables expressed in level grow at the same balanced rate (γ).

To compute the steady-state, we proceed by induction. First, the long-run public debt ratio is: $bk = \theta yk$. Thus, the public spending ratio in equation (11) leads to

$$gk = \tau yk - (r - \gamma)bk = [\tau - (r - \gamma)\theta]yk. \tag{15}$$

From (15), by assuming a balanced budget rule (i.e. $\theta = 0$) we find the Barro (1990)'s solution, namely: $gk = \tau yk$. In contrast, the public spending ratio is lower in the presence of public debt, since the standard transversality condition ensures $r > \gamma$ (see Minea and Villieu, 2012; Menuet et al., 2017). The basic mechanism driving this crowding-out effect is the following. The public deficits generate (i) a permanent flow of new resources ($B_{t+1} - B_t$), and (ii) a permanent flow of new unproductive expenditures (the debt burden $r_t B_t$). In steady state, the standard transversality condition ($r_t > \gamma_t = B_{t+1}/B_t$) means that the latter dominates the former, thereby any rule that permit permanent deficits involves net long-run costs for public finance, irrespective of the precise nature of this rule.[11]

After determining gk, we compute the steady-state output ratio

$$yk = A^{1/\alpha}[\tau - (r - \gamma)\theta]^{(1-\alpha)/\alpha}.$$

As usual, endogenous growth solution is obtain by the crossing-point of two relationships between r and γ. The first one comes from the Keynes-Ramsey relationship (9)

$$\gamma = \gamma c = \left(\frac{1 + r}{1 + \beta}\right)^S - 1. \tag{16}$$

[11] An interesting case can deserve our attention. Using a logarithmic utility function (namely $S = 1$), we have $r - \gamma \approx r - S[r - \beta] = \beta$, and, from (15), it follows that $\frac{dg_k/y_k}{d\theta} = -\beta$. As usual, the subjective discount rate is lower than 5%, so that the crowding-out effect is rather low. Indeed, public debt generates costs (through the debt burden), but provides new receipts (the deficit, in each period).

The second is directly linked to the government's budget constraint

$$\gamma = \gamma b = \frac{1}{\theta}\left[\left(\frac{r+\delta}{\alpha(1-\tau)A^{1/\alpha}}\right)^{\alpha/(1-\alpha)} + r\theta - \tau\right]. \tag{17}$$

We obtain the balanced growth path at the crossing-points of (16)-(17). In the general case, there are two steady-state solutions (SS): a high steady-state, and a low steady-state (see Minea and Villieu, 2012, for a mathematical proof), as depicted in Figure 1.[12] However, in our model, the multiplicity can be removed. Indeed, there is one and only one stable steady-state. Our simulation shows that the high SS is still saddle-path stable, while the low SS is unstable. Consequently, the high SS is the unique equilibrium of the model.

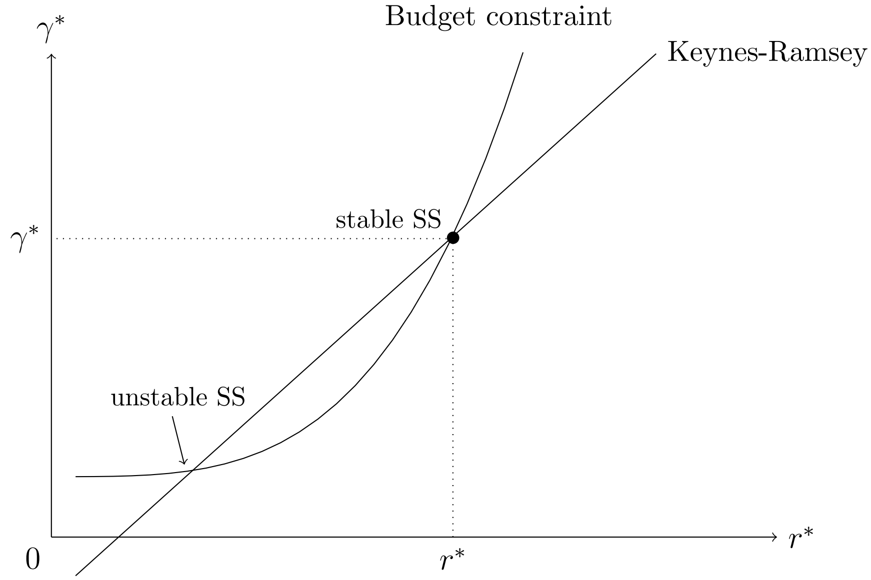


Figure 1: Long-run equilibrium

3.2. A numerical illustration

Our simulations are based on reasonable values for parameters (see Table 1). We choose a usual discount rate $\rho = 0.02$ to match long-run historical data for the risk-free real interest rate. The consumption elasticity of substitution (inverse of the risk-aversion coefficient) is fixed at $S = 1$. As regards the technology, we set $A = 0.5$ to obtain

[12] Eq. (16) describes a decreasing line (for $S = 1$), and Eq. (17) a convex increasing curve (when $\alpha > 1/2$). Therefore, there is a non-empty set of parameters such that two steady-states emerge.

a realistic rate of economic growth, and the capital share in the production function is $\alpha = 0.7$, as in Gomes et al. (2013), close to the value (0.715) used by Gomme et al. (2011). Such a capital share allows reproducing the empirical results of Munnell (1990) on the elasticity of output to productive public spending ($1 - \alpha = 0.3$). The depreciation rate of capital is set at $\delta = 0.05$, which roughly corresponds to the average value of depreciation rates used in Gomme et al. (2011).

PARAMETERS

S	1	Intertemporal elasticity of substitution
β	0.02	Discount rate
A	0.5	Productivity parameter
α	0.7	Capital share in the production function
δ	0.05	Depreciation rate
τ	0.38	Tax rate on income
θ	0.572	Long-run deficit ratio (target value)

TARGET VALUES

	Model	Data	Source
Long-run economic growth	0.034	0.033	BEA, 1950-2015
After-tax return of capital	0.054	0.0516	Gomme et al. (2011)
Investment to capital ratio	0.084	0.088	Gillman and Kejak (2011)
Public debt to GDP	0.572	0.572	BEA, 1950-2015
Public spending to GDP	0.367	0.366	OECD, 1970-2015

Table 1: Baseline Calibration (high BGP)

Despite the highly-stylized nature of our deterministic model, the baseline calibration allows obtaining quite realistic results, close to postwar US annual data. The BGP is characterized by a 3.4% long-run rate of economic growth (3.3% in the data). The calibration well reproduces the after-tax rate of return of capital (5.4%, while the average mean on 1954-2008 in Gomme et al. (2011) is 5.16%), the investment to capital ratio (0.084), which is close to the estimate (0.088) used in Gillman and Kejak (2011), and the public-spending-to-GDP ratio is 36.7% (36.6% in the data).

4. The amplification mechanism

After examining the deterministic setup, we introduce stochastic shock on technology. We consider a positive technological innovation ($z_t > 0$) which persists over time and affects the economy in the neighborhood of the steady-state BGP (defined by our benchmark calibration).[13]

We show that transitory shocks exert a permanent effect on the level of variables along the BGP, characterizing the usual hysteresis feature of endogenous growth models. Figure 2 depicts stochastic simulations of the logarithm of GDP in response to technological shocks. The initial balanced growth path is represented by a deterministic trend (the dashed line), and stochastic trend (the continuous line) highlights the non-stationary nature of GDP.

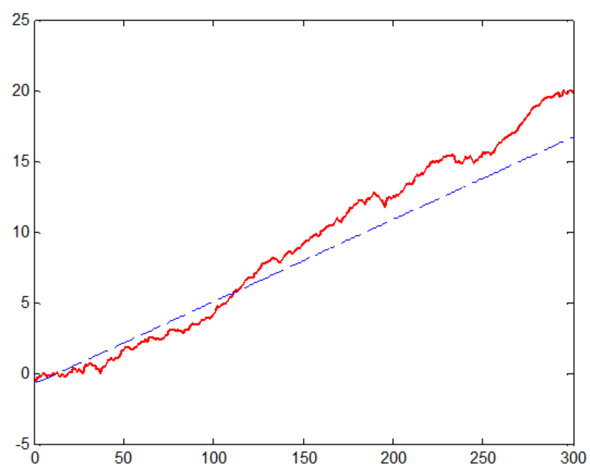


Figure 2: Hysteresis: stochastic simulations of the output (in logs)

In our model, the public debt path can be smooth depending on the speed of adjustment (ξ). This property provides a potential amplification mechanism: if ξ is small (thereafter, under the "gradualist" strategy), the public debt ratio adjusts slowly to shocks, thereby its impact on public investment, output and growth can be very persistent. This is an important finding in light of standard DSGE and RBC models. In these setups, a well-known shortcoming is the weakness of the propagation mechanism.

[13]The law of motion of the technological shock is: $z_t = 0.9z_{t-1} + \nu_t$, where $\nu_t \sim\ N(0, 0.07)$.

Indeed, the channel of transmission is mainly based on the persistence of exogenous technological shocks itself, giving rise to an exogenous theory of fluctuations. Our model can overcome such a difficulty, because the persistence of exogenous technology shocks can be substituted by the endogenous dynamics of public debt. In our framework, the effect of technological shocks is strongly strengthened by the adjustment profile of the public debt, producing an amplification mechanism that we call the "public debt accelerator".

The basic mechanism, described in Figure 3 is as follows. When an adverse technological shock occurs, the total factor productivity and the output decrease, raising the debt-to-GDP ratio. In return, the government will reduce the primary deficit and cuts in public spending to keep the debt ratio consistent with the long-term target. This lack of productive public expenditure leads to a recession effect, and the harmful circle is renewed.

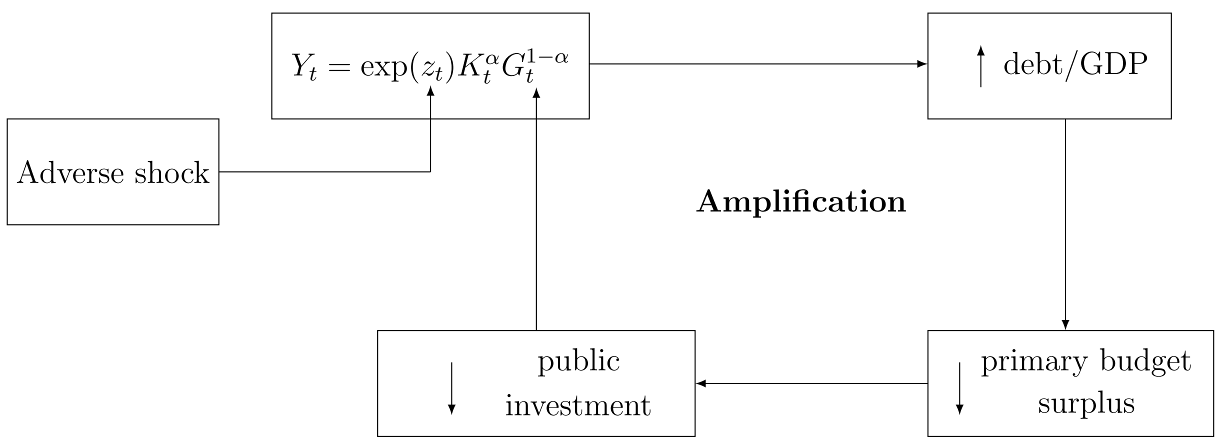


Figure 3: The "public debt accelerator"

To precise some quantitative aspects of this mechanism, we assume a small autoregressive coefficient ($\phi = 0.45$ in the AR(1) process in Eq. (1)). Without public debt, the economy adjusts very quickly to a negative technological innovation, because the dynamics turns off after only 5 quarters (see the continuousline IRFs in Figure 4). With public debt, in contrast, the response of the variables is more persistent: dashed-line IRFs on Figure 4 (computed for $\mu = 0.05$) highlights that the variables not adjusted to their steady-state values after 40 quarters (except for economic growth).

Thus, the procyclical feature of the "public debt accelerator" may provide an efficient transmission mechanism of shocks, as concerns both their quantitative impact and their persistence all along the business cycle.

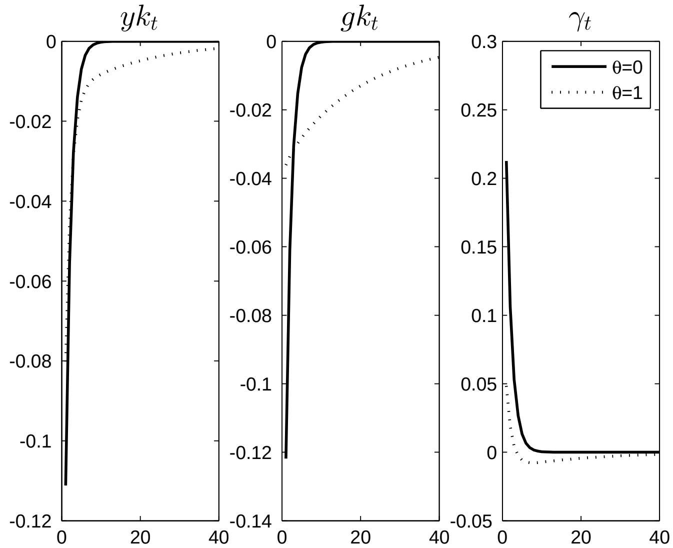


Figure 4: Amplification mechanism

5. Gradualist strategy *versus* shock therapy

Based on our baseline calibration, this section studies how the adjustment strategy (ξ) affects the economy in both deterministic and stochastic environments. To this end, we distinguish two adjustment strategies: a "gradualist" strategy with a low speed of adjustment ($\mu = 0.05$), and a "cold turkey" strategy ($\mu = 0.25$) accounting for a faster reduction in the debt ratio.

5.1. Permanent shock in a deterministic framework

Let us first examine the effect of both strategies on the adjustment of variables following a decrease in the long-term debt target (from $\theta = 100\%$ to $\theta = 50\%$) in a deterministic framework. These adjustments are shown in Figure 5 (the continuous line for a gradualist strategy, and the dashed line for the cold turkey strategy).

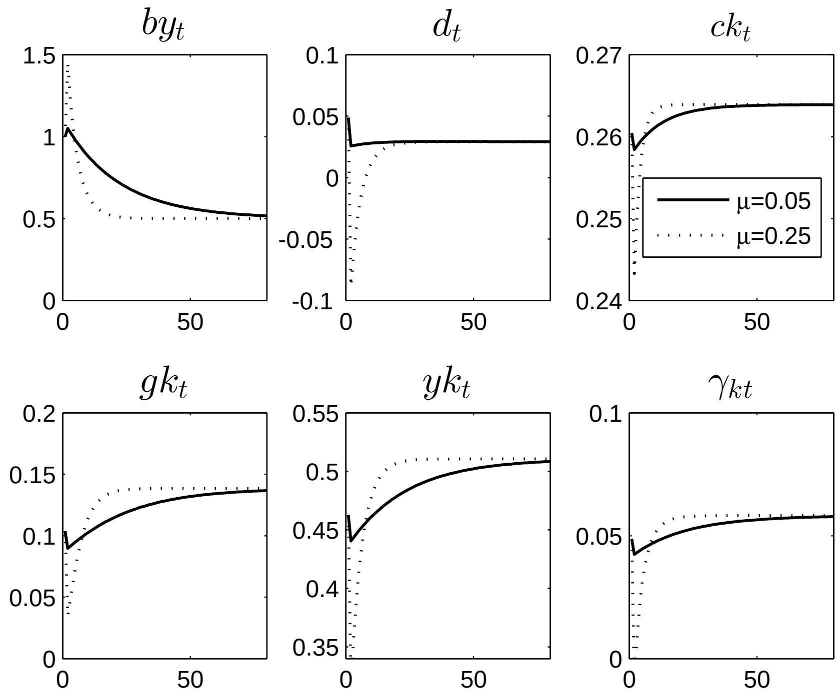


Figure 5: Transitory dynamics following a permanent reduction in the public debt ratio

In the long-run, the rate of economic growth increases, following the rise in productive public spending. In the short-run, the impact of the change in the debt target crucially depends on the speed of adjustment.

As shows Figure 5, the debt adjustment (in % of GDP) differs between the two strategies. Under the gradualist strategy, the debt adjustment is very slow and the neighborhood of the new target ($\theta = 50\%$) is reached after more than 50 quarters (if we interpret a period as a quarter). In contrast, the speed of adjustment is high under the cold turkey strategy (less than 10 quarters), and all variables adjust more quickly to their steady-state values.

However, the cold turkey strategy is, in the short run, a costly policy. As the public deficit immediately decreases, the economy lacks productive public spending, hence a decline of output and consumption. During the transition path, these initial costs are reduced, because the debt burden and the crowing out effects decrease. Finally, in the long-run, the output ratio, the consumption ratio and the public investment ratio are higher than initially.

Figure 5 simulations show that the higher the speed of adjustment of public debt, the higher the initial cost of the correction of the fiscal stance. In particular, the cold turkey strategy leads not only to a lower fiscal deficit, but gives rise to fiscal surpluses: from an initial deficit close to 5% of GDP, the deficit shrinks to zero under the gradualist strategy, but becomes a surplus close to 10% of GDP under the cold turkey strategy. As a result, the latter strategy gives rise to very drastic reductions in public spending, consumption and output ratios.

5.2. Transitory stochastic shocks

This subsection introduces stochastic shocks on technology. Figures 6 computes the impulse response functions (IRF) for the ratios of output, consumption, public spending, the economic growth rate, and the debt-to-output ratio to a negative shock on technology.

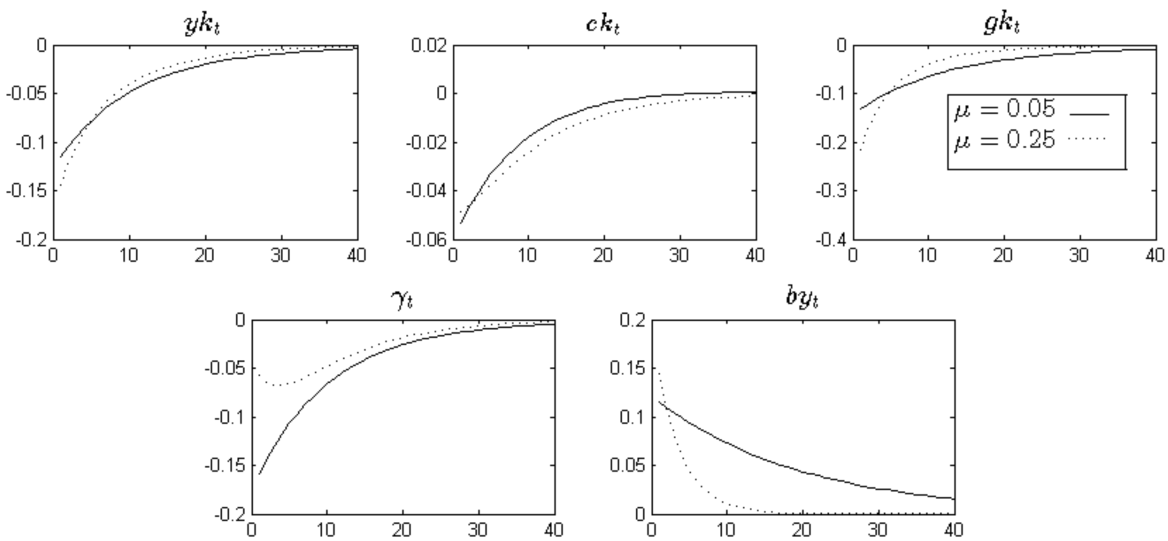


Figure 6: Adjustment to a negative technology shock: deviation from steady-state (in %)

Under the gradualist strategy, all variables adjust quite slowly to the transitory shock since the half-life of deviations from steady-state is between 12 and 20 quarters. On the contrary, adjustment is much more rapid under the cold turkey strategy, but this strategy produces unstable response on some variables, like short-run economic growth. Contrary to our results with permanent changes in the debt target, transitory deviations of the consumption ratio are very similar under both strategies.

In our model, hysteresis means that the choice between the two corrective policies during the transition path will be of crucial importance for the level of variables in the long-run and not only for their changes during the transition path. Gradualist versus cold turkey strategies will exert very long-lasting effects on the business cycle and will permanently deform the equilibrium trend of GDP, consumption and public investment.

5.3. Social welfare

Intuitively, following permanent or transitory shocks, the best policy would be the gradualist strategy in the short run, but the cold turkey strategy in the long run. This subsection determines the first-best strategy form the intertemporal welfare perspective.

Our approach is as follows. We first simulate an adverse technological shock, then we compare the present value of the intertemporal gain of welfare associated with two trajectories: one linked to a gradualist strategy (denoted by G), and the other to a cold turkey strategy (C).

Under the strategy $s \in \{C, G\}$, the consumption level C_t^s is given by

$$C_t^s = c_t^s K_0 \Pi_{u=0}^{t} (1 + \gamma_{k,u}^s)^u,$$

where c_t^s is the consumption-to-capital ratio, γ_{kt}^s the growth rate of the private capital, and K_0 the initial (predetermined) capital level. We compute the set of trajectories ($\{c_t^s\}_{t \geq 0}$, $s \in \{C, G\}$) and ($\{\gamma_{k,t}^s\}_{t \geq 0}$, $s \in \{C, G\}$), and the present value of the intertemporal welfare is

$$W^s = \sum_{i=0}^{+\infty} \left(\frac{1}{1+\beta} \right)^i u(C_i^s).$$

We compute W^C (in the third line of Table 2) and W^G (in the second line) for different values of the subjective discount rate (β). The last line computes the differences of welfare (in %). Therefore, the best fiscal policy crucially depends on the households' discount rate. If households have short-sighted views (β is high), the loss of consumption in the short run will dramatically impact the intertemporal welfare, and the best policy is the gradualist strategy. In contrast, if β is small, the long run losses of welfare are minimized under the cold turkey strategy.

β	0.001	0.01	0.02	0.05	0.1	0.5	
$\mu = 0.05$	**2.1036**	**1.9776**	1.6514	1.601	0.7284	-1.7013	$*10^6$
$\mu = 0.25$	2.1001	1.9737	**1.6542**	**1.6045**	**0.7342**	$\mathbf{-1.5889}$	$*10^6$
Loss of welfare (in %)	0.17	0.20	-0.17	-0.22	-0.79	-6.60	

Table 2: Intertemporal welfare in function of the subjective discount rate

In summing up, a gradualist strategy helps smooth variables. Such a strategy limits the cut of productive spending, but delays the adjustment of the variables to their equilibrium levels. In contrast, a cold turkey strategy risks destabilizing the economy, but brings a long-term gain. However, our analysis cannot provide a clear conclusion: for realistic values of the discount rate ($\beta \in (0; 0.05)$), both strategies must be implemented.[14]

So far, we have considered an exogenous speed of adjustment. However, the choice of the adjustment strategy could depend on the business cycle. For example, in economic downturn, the government could adopt a gradualist strategy, and a cold turkey strategy in expansionary periods, thereby the speed of adjustment would lead to a counter-cyclical public spending behaviour. The following section addresses this issue and extends our model by introducing an endogenous speed of adjustment.

6. A "second generation" fiscal rule

Up to now, our fiscal rule is fixed, and does not depend on business cycles. However, recent studies conclude that fiscal rules can make fiscal policy less pro-cyclic, especially in high-income countries, provided their design allowed for flexibility, including proper escape clause or cyclically-adjusted targets.[15] For Guerguil et al. (2017), such flexible rules, called "second generation" rules, are associated with counter-cyclical changes in overall spending.

In this section, we provide the first theoretical analysis that focuses on the stabilization effect of flexible fiscal rules. To this end, we model an endogenous speed of adjustment

[14] This finding is consistent with the result of Horvath (2011), whereby the speed of adjustment depends on the time preference of agents under the unconditional optimal policy in a New Keynesian economy.

[15] Frankel et al. (2013) concluded that, in 2000s, a third of high-income countries have experimented countercyclical fiscal policy. This finding is consistent with the results of Bova et al. (2014), Fatás and Mihov (2012), and Combes et al. (2017).

that negatively depends on the gap between the long-run economic growth rate (γ^*) and the current growth rate (γ_t). The fiscal rule (7) writes now

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t} = \mu_t \left(\theta - \frac{B_t}{Y_t}\right); \quad \mu_t = \mu_0 - \xi(\gamma^* - \gamma_t), \tag{18}$$

where $\gamma_t := (Y_{t+1} - Y_t)/Y_t$ is the economic growth rate, and $\xi > 0$ reflects *the flexibility degree* of the fiscal rule. If $\xi = 0$, we have a fixed fiscal rule (*first generation*), as previously detailed. If $\xi > 0$, in contrast, the fiscal rule is flexible (*second generation*), and the speed of adjustment depends on the business cycle (through the gap $\gamma^* - \gamma_t$)[16].

In long run, if $\gamma_t = \gamma^*$, the speed of adjustment is simply μ_0. This is no long the case in the short run. If $\gamma_t < \gamma^*$ (resp. $\gamma_t > \gamma^*$), the speed of adjustment is lower (resp. higher) than its long-run level, namely $\mu_t < \mu_0$ (resp. $\mu_t > \mu_0$).

From Eqs. (6) and (18), the public-spending-to-GDP ratio becomes

$$\frac{G_t}{Y_t} = \tau - (R_t - \gamma_t)\frac{B_t}{Y_t} + [\mu_0 - \xi(\gamma^* - \gamma_t)](1 + \gamma_t)\left(\theta - \frac{B_t}{Y_t}\right),$$

hence;

$$\frac{d}{d\xi}\left(\frac{G_t}{Y_t}\right) = -(1 + \gamma_t)\left(\gamma^* - \gamma_t\right)\left(\theta - \frac{B_t}{Y_t}\right).$$

If an adverse technological shock occurs, the public-spending-to-GDP ratio is higher than its long-run target ($\theta < B_t/Y_t$), exhibiting a counter-cyclical behavior: G_t/Y_t increases when the current growth is lower than the potential level ($\gamma_t < \gamma^*$), and decreases in the opposite case ($\gamma_t > \gamma^*$).[17].

Formally, steady-states are still determined by a 6-equations system. The only change is that the public debt path (12) evolves according to

$$by_{t+1} = by_t + [\mu_0 - \xi(\gamma^* - \gamma k_t)](\theta - by_t). \tag{19}$$

Following the preceding section, we can show that the new system (9)-(10)-(11)-(13)–

[16]To ensure $\mu_t > 0$, we assume that μ_0 is high enough.

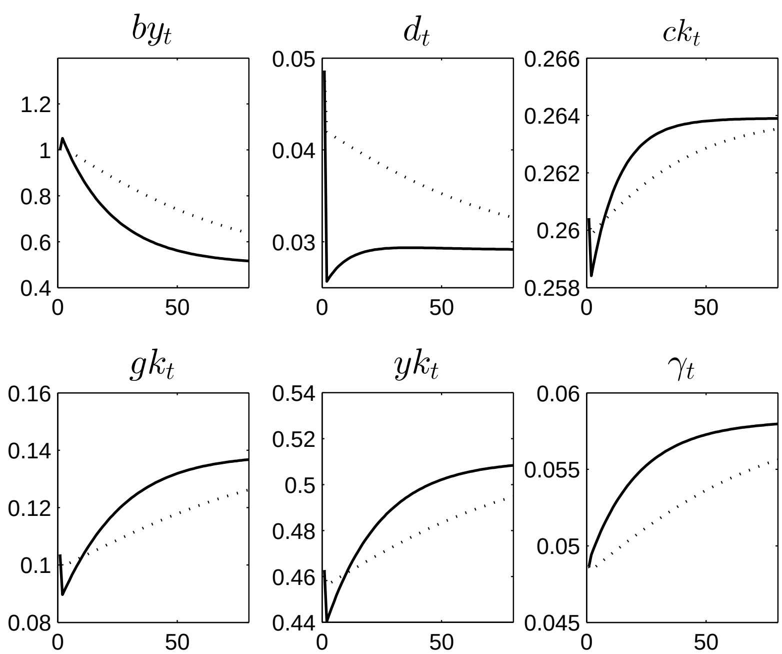
[17]When a positive shock occurs (namely, $\theta > B_t/Y_t$), the public spending ratio exhibits a procyclical behavior, i.e. G_t/Y_t increases if and only if $\gamma_t > \gamma^*$. In this section, as below, we focus only on adverse shocks.

(14)-(19) determines two steady-states (SS), but only the high SS is saddle-path stable, defining the unique equilibrium of the model. Notice that the level of this BGP does not change with our specification (19), since the public-debt-to-output ratio still equals θ in the long run. Consequently, the flexibility degree (ξ) of the fiscal rule (18) only affects the transitory dynamics, as we will see in our simulations below.[18]

6.1. A permanent shock

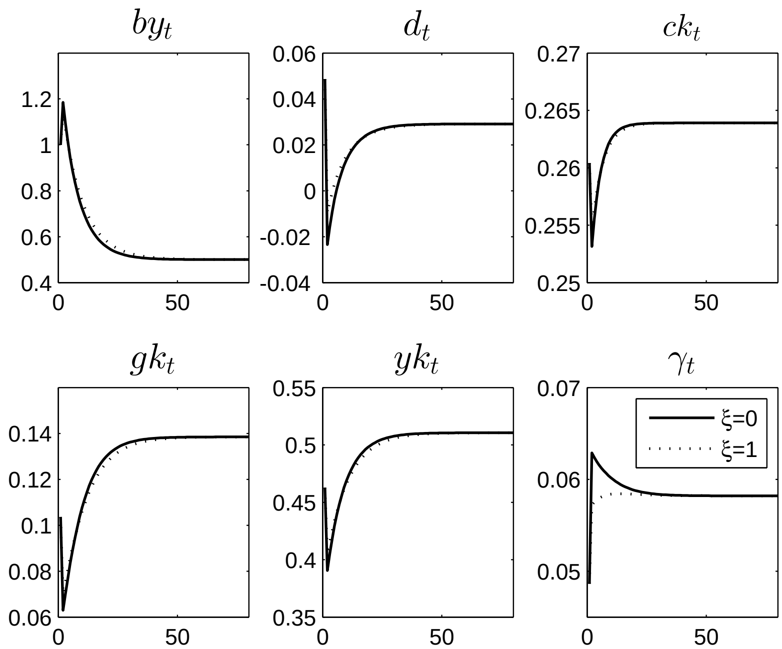
Let us studying our flexible fiscal rule by analyzing the adjustment of variables following a drop in the debt target (from 100% to 50% of GDP) in a deterministic framework.

Figures 7a-b depict the IRF of the debt-to-GDP ratio (by_t), the deficit-to-GDP (d_t), the ratios of consumption (ck_t), public expenditure (gk_t), output (yk_t), per unit of capital, and the growth rate (γ_t) under a flexible rule (dashed lines), and under a fixed rule (continuous lines).



a. $\mu_0 = 0.05$

[18]The topological nature of steady-states does not change with the fiscal rule (18). Indeed, in equilibrium, we have, from (19): $\partial by_{t+1}/\partial \gamma k_t = -\xi(\theta - by_t) = 0$, and $\partial by_{t+1}/\partial by_t = 1 + \xi(\gamma^* - \gamma k_t) = 1$, as in the preceding fixed fiscal rule.



b. $\mu_0 = 0.15$

Figure 7: Transitory dynamics following a reduction of the long-run debt target

In Figure 7a ($\mu_0 = 0.05$), all the adjustments are slower under a flexible rule. Without taking into account the business cycle ($\xi = 0$), the variables come back to equilibrium after 50 quarters, and after 150 quarters under a flexible rule ($\xi = 1$). From the short-term perspective, the initial drop in consumption, public spending and output is less prominent under a flexible rule. Therefore, such a rule results in a stabilizing effect by reducing the volatility of the aggregate variables along the business cycle. Figure 6.12b ($\mu_0 = 0.15$) clearly emphasizes the stabilization of the economic growth rate.

6.2. Transitory shocks

Let us now analyze the adjustment of variables following an adverse technological shock.[19] In Figure 8 (for a autoregressive coefficient $\phi = 0.9$), we represent the fluctuations in economic growth and in speed of adjustment, as deviations from their long-run values (in %).

[19] We assume $\mu_0 = 0.25$ to ensure than $\mu_t > 0$, for any $t \geq 0$.

The main message is that a flexible fiscal rule allows reducing the volatility of the economic activity. Indeed, the variance of the output (in ratio to the steady-state) is 0.57 for $\xi = 0$, and 0.29 for $\xi = 1$. This stabilizing effect comes from the endogenous speed of adjustment: following an adverse technological shock, the speed of adjustment is lower than the long-run level that reduces the cuts in productive spending, smoothing the recessive effects. In contrast, under a fixed fiscal rule ($\xi = 0$), the speed of adjustment equals the constant long-run level ($\mu_t = \mu_0, \forall t$), and the "accelerator of public debt" amplifies the transitionary shocks.

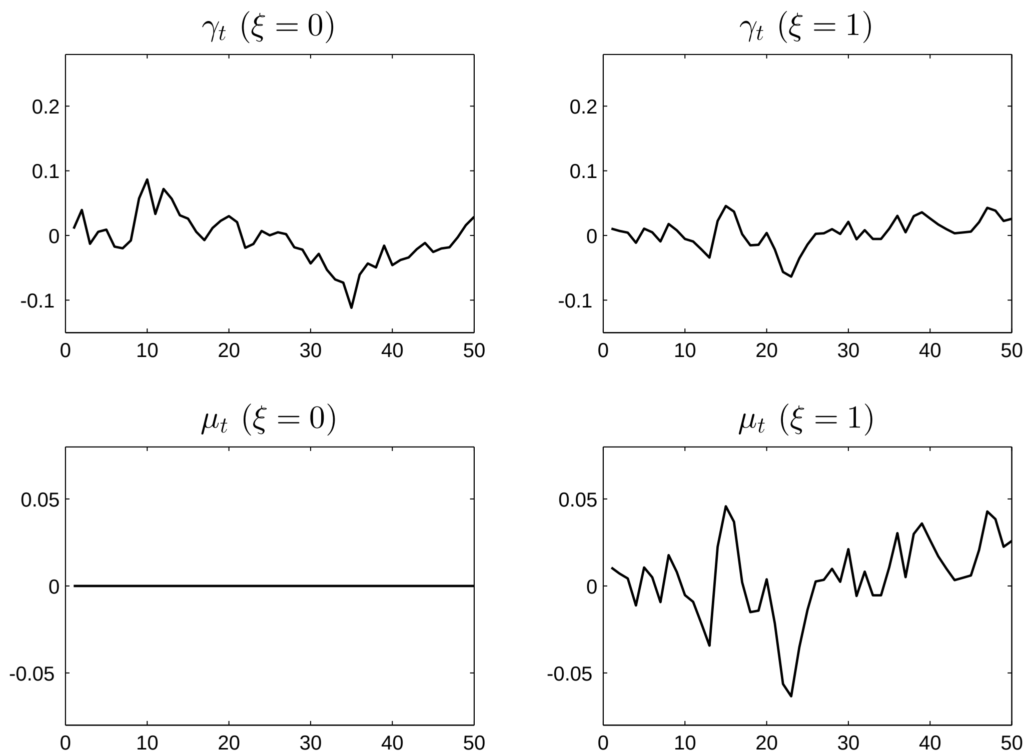


Figure 8: Adjustment following an adverse technological shock: deviation from steady-state (in %)

Consequently a flexible fiscal rule help smooths the economy activity, at the cost of a decline in the long-run value. As shown the previous section, the choice of the flexible degree of the fiscal rule depends on the subjective discount rate: if households are characterized by short-term views, the welfare-maximizing strategy will be the flexible rule. Finally, we show that a second generation rule reduces fluctuations in growth and brutal cuts in public expenditure following a recessive shock. Thus, our theoretical framework shares the empirical results of Lane (2003a), Bova et al. (2014), Guerguil et al. (2017) and Fatás and Mihov (2012) whereby a flexible fiscal rule allows smoothing the trajectory of expenditure public, and stabilizes the economy.

7. Conclusion

Our paper builds a stochastic endogenous growth model to study the impact of FR on indebted countries, and provides three methodological innovations.

First, as public spending exert a productive externality, an adverse technological shock hits an endogenous total productivity factor. Therefore, our setup is consistent with numerus works emphasizing the crucial role of productive public spending in slowing productivity in industrial economies (see, for example, Aschauer, 1989). Second, our model exhibits an amplification shocks mechanism ("the public debt accelerator"). When an adverse transitory shock occurs, the government cuts in productive public spending to maintain the debt-to-GDP ratio consistent with the long-run target. This results in a procyclical behavior of public expenditure. The procyclical stance depends on the degree of flexibility of the debt rule: the welfare-maximizing policy is the "gradualist" strategy in the short-run, and the "cold turkey" strategy in the long-run. Third, although all the existing theoretical literature only focuses on fixed fiscal rules, we develop a flexible rule, such that the speed of adjustment of the debt-to-GDP ratio depends on the business cycle. We reveal that such a rule helps smooth aggregate variables, and leads to a stabilization effect by limiting the lack of productive public spending.

An interesting extension of our analysis would consist in providing a political economy mechanism to endogenize the policy instruments (as the long-run debt target, or the speed of adjustment). In this regard, the government's inter-temporal payoff would depend on popularity, so that the choice of policy instrument can be viewed as the outcome of a coordination scheme (possibly through a voting process) or a conflict between different groups about income distribution, for example. Besides, further works could attempt to build a general model connecting DSGE models with an endogenous growth framework. The introduction of nominal rigidities, and the specification of policymakers' preferences could open the way for a laboratory model encompasses both long-run economic growth and business cycles. These two possible directions are left for future research.

References

Aghion, P., Howitt, P., 2006. Appropriate Growth Policy, Schumpeter Lecture. Journal of the European Economic Association 4, 269314.

Aghion, P., Marinescu, I., Caballero, R.J., Kashyap, A.K., 2007. Cyclical budgetary policy and economic growth: What do we learn from OECD panel data? NBER Macroeconomics annual 22, 251–297.

Alesina, A., Bayoumi, T., 1996. The costs and benefits of fiscal rules: evidence from US states. NBER Working Papers No. w5614 .

Alt, J., Lowry, R., 1994. Divided government, fiscal institutions, and deficits: Evidence from the States. American Political Science Review 88.

Ardagna, S., Caselli, F., Lane, T., 2007. Fiscal discipline and the cost of public debt service: some estimates for OECD countries. The B.E. Journal of Macroeconomics 7, 1935–1690.

Aschauer, D., 1989. Is Public Expenditure Productive? Journal of Monetary Economics 23, 177–200.

Azzimonti, M., Battaglini, M., Coate., S., 2010. On the case for a balanced budget amendment to the u.s. constitution. MPRA Paper 25935 .

Barro, R., 1990. Government Spending in a Simple Model of Economic Growth. Journal of Political Economy 98, S103–S125.

Bergman, U.M., Hutchison, M., 2015. Economic stabilization in the post-crisis world: Are fiscal rules the answer? Journal of International Money and Finance 52, 82–101.

Blanchard, O., 1989. A traditional interpretation of macroeconomic fluctuations. The American Economic Review 79, 1146–1164.

Bohn, H., Inman, R.P., 1996. Balanced budget rules and public deficits: Evidence from the U.S. States. NBER Working Paper No. 5533 .

Boucekkine, R., Nishimura, K., Venditti, A., 2015a. Introduction to financial frictions and debt constraints. Journal of Mathematical Economics 61, 271–275.

Boucekkine, R., Pintus, P., Zou, B., 2015b. Stochastic stability of endogenous growth: Theory and applications. WP AMSE 32 .

Bova, E., Carcenac, N., Guerguil, M., 2014. Fiscal rules and the procyclicality of fiscal policy in the developing world. IMF Working Paper WP/14/122 .

Campbell, J.Y., Mankiw, N.G., 1989. International evidence on the persistence of economic fluctuations. Journal of Monetary Economics 23, 319–333.

Combes, J.L., Minea, A., Sow, M., 2017. Is fiscal policy always counter-(pro-) cyclical? The role of public debt and fiscal rules. Economic Modeling 65, 138–146.

Dabla-Norris, E., Allen, R., Zannaa, L., Prakash, T., Kvintradze, E., Lledo, V., Yackovlev, I., Gollwitzer, S., 2019. Budget institutions and fiscal performance in low-income countries. IMF Working Paper 10/80 .

Debrun, X., Moulin, L., Turrini, A., i Casals, J.A., Kumar, M.S., Drazen, A., Fuest, C., 2008. Tied to the mast? National fiscal rules in the European Union. Economic Policy 23, 297–362.

Fatás, A., 2000. Endogenous growth and stochastic trends. Journal of Monetary Economics 45, 107–128.

Fatás, A., Mihov, I., 2006. The macroeconomic effects of fiscal rules in the US States. Journal of Public Economics 90, 101–117.

Fatás, A., Mihov, I., 2012. Fiscal policy as a stabilization tool. The B.E. Journal of Macroeconomics 12, 1–68.

Frankel, J.A., Végh, C.A., Vuletin, G., 2013. On graduation from fiscal procyclicality. Journal of Development Economics 100, 32–47.

Gali, J., Perotti, R., 2003. Fiscal policy and monetary integration in europe. NBER Working Papers No. 9773 .

Gillman, M., Kejak, M., 2011. Inflation, Investment and Growth: a Money and Banking Approach. Economica 78, 260–282.

Gomes, F., Michaelides, A., Polkovnichenko, V., 2013. Fiscal Policy and Asset Prices with Incomplete Markets. The Review of Financial Studies 26, 531–566.

Gomme, P., Ravikumar, B., Rupert, P., 2011. The Return to Capital and the Business Cycle. Review of Economic Dynamics 14, 262–278.

Guerguil, M., Mandon, P., Tapsoba, R., 2017. Flexible fiscal rules and countercyclical fiscal policy. Journal of Macroeconomics 52, 189–220.

Horvath, M., 2011. Alternative perspectives on optimal public debt adjustment. The B.E. Journal of Macroeconomics, 11, 1–22.

Jones, L.E., Manuelli, R.E., Stacchetti, E., 2000. Technology (and policy) shocks in models of endogenous growth. Federal Reserve Bank of Minneapolis Research Department Staff Report 281 .

King, R.G., Plosser, C.I., Rebelo, S.T., 1988. Production, growth and business cycles: Ii. New directions. Journal of Monetary Economics 21, 309–341.

Lane, P., 2003a. Business cycles and macroeconomic policy in emerging market economies. International Finance 6, 89–108.

Lane, P., 2003b. The cyclical behaviour of fiscal policy: evidence from the OECD. Journal of Public Economics 87, 2661–2675.

Malley, J., Philippopoulos, A., Woitek, U., 2007. Electoral uncertainty, fiscal policy and macroeconomic fluctuations. Journal of Economic Dynamics and Control 31, 1051–1080.

Manasse, P., 2006. Procyclical fiscal policy: Shocks, rules, and institutions. IMF Working Paper WP/06/27 .

Menuet, M., Minea, A., Villieu, P., 2017. Deficit, Monetization, and Economic Growth: A Case for Multiplicity and Indeterminacy. Economic Theory forthcoming.

Menuet, M., Villieu, P., 2014. Public debt adjustment in a simple model of stochastic endogenous growth. Revue d'Economie Politique 124, 991 – 1012.

Minea, A., Villieu, P., 2010. Endogenous Growth, Government Debt and Budgetary Regimes: A Corrigendum. Journal of Macroeconomics 32, 709–711.

Minea, A., Villieu, P., 2012. Persistent Deficit, Growth, and Indeterminacy. Macroeconomic Dynamics 16, 267–283.

Munnell, A., 1990. Why has Productivity Declined? Productivity and Public Investment. New England Economic Review, Federal Reserve Bank of Boston 6, 3–22.

Nishimura, K., Nourry, C., Seegmuller, T., Venditti, A., 2015a. Growth and Public Debt: What are the Relevant Tradeoffs? AMSE Working Paper 1606 .

Nishimura, K., Seegmuller, T., Venditti, A., 2015b. Fiscal Policy, Debt Constraint and Expectations-Driven Volatility. Journal of Mathematical Economics 61, 305–316.

Poterba, J.M., 1994. State responses to fiscal crises: The effects of budgetary institutions and politics. Journal of Political Economy 102, 799–821.

Roubini, N., Sachs, J., 1989. Political and Economic Determinants of Budget Deficits in the Industrial Democracies. European Economic Review 33, 903–938.

Shapiro, M.D., Watson, M.W., 1988. Sources of business cycle fluctuations. NBER Macroeconomic Annual 3, 111–148.

Stadler, G.W., 1990. Business cycles models with endogenous technology. The American Economic Review 80, 763–778.

Stock, J.H., Watson, M.W., 1999. Business cycle fluctuations in us macroeconomic time series. Handbook of macroeconomics , 3–64.

Svec, J., Kondo, A., 2012. Fiscal policy cyclicality and growth within the US states. The B.E. Journal of Macroeconomics 12, 1935–1690.

Von Hagen, J., 2003. Fiscal discipline and growth in Euroland: experiences with the stability and growth pact. Working Paper No. B06, ZEI, Bonn .

Wyplosz, C., 2006. European Monetary Union: the dark side of a major success. Economic Policy 21, 208–261.