



HAL
open science

Thin Shell with Fictitious Motions

Hou Yau

► **To cite this version:**

Hou Yau. Thin Shell with Fictitious Motions. C. Duston; M. Holman Spacetime Physics 1907 - 2017. Selected peer-reviewed papers presented at the First Hermann Minkowski Meeting on the Foundations of Spacetime Physics, 15-18 May 2017, Albena, Bulgaria (Minkowski Institute Press, Montreal 2019), 2019. ⟨hal-02153076v2⟩

HAL Id: hal-02153076

<https://hal.science/hal-02153076v2>

Submitted on 27 Sep 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Thin Shell with Fictitious Motions

Hou Yau

FDNL Research, Daly City, California, USA
hyau@fdnresearch.us

August 23, 2019

Abstract

We show that a spherical thin shell with fictitious radial motions can mimic the gravitational effects of a massive spherical thin shell. These fictitious radial motions (either with a uniform velocity or oscillate about the thin shell) are not physical motions of the thin shell. Instead, we use their hypothetical effects on time and distance measurements to define the spacetime metric on the surface of the timelike thin shell. The system with fictitious motions is spherically symmetric with time translational and time reflection symmetries. The external spacetime outside the thin shell is static and satisfies the Schwarzschild solution for the gravitational field of a spherically symmetric mass.

1 Introduction

The Schwarzschild metric has a remarkably simple form. Its diagonal time component, g_{tt} , is inversely proportional to its diagonal radial component, g_{rr} [1]. Because of this simple form, many attempts have been made to reconcile the metric by invoking the reciprocity of time dilation and length contraction in special relativity [2]. For instance, the Schwarzschild metric can be written as,

$$ds^2 = \left[1 - \frac{v(r)^2}{c^2}\right] dt^2 - \left[1 - \frac{v(r)^2}{c^2}\right]^{-1} dr^2 - r^2 d\Omega^2, \quad (1)$$

where $v(r) = -(2GM/r)^{1/2}$. It happens that $v(r)$ is the free falling velocity of a particle with zero initial velocity at infinity. The diagonal time and radial metric components can then be expressed in terms of the time dilation and length contraction observed for a moving object with velocity $v(r)$. As advocated in refs. [3–8], the Schwarzschild metric is obtained from the study of a radially accelerated free falling frame by insinuating the concepts of time dilation, length contraction, equivalence principle and Newtonian gravity in the derivation. These results have led to a false belief that it is possible to derive the Schwarzschild metric without any reference to the field equations of general relativity.

The result that Schwarzschild metric can be obtained by combining special relativity, equivalence principle and Newtonian gravity is a mere coincidence. As demonstrated by Gruber et al. [9], two gravitational effects are necessary to specify the two *a priori* diagonal components $g_{tt}(r)$ and $g_{rr}(r)$. The first piece

of gravitational information they considered is the free falling radial inward velocity $v(r)$ of a particle measured by an observer stationary at a coordinate r . With the knowledge of this velocity, one can obtain the total mass-energy of a free falling particle measured by the same stationary observer. The diagonal component $g_{tt}(r)$ can then be directly inferred, for example, from a gedanken experiment with photons (Appendix in ref. [9]).

To obtain the diagonal radial component $g_{rr}(r)$, Gruber et al. chose to study the gravitational force law, $f(r) = -d^2r/d\tau^2$, where τ is the proper time measured by the free falling observer. This is ‘a plausible analogy of Newtonian radial acceleration that is at the same time relativistically palatable’. With both $v(r)$ and $f(r)$ known, $g_{rr}(r)$ can be inferred. For example, in the Newtonian gravitational theory, $v(r) = -(2GM/r)^{1/2}$ and $f(r) = GM/r^2$ for a gravitating body of mass M . The diagonal metric components obtained are $g_{tt} = 1 - 2GM/rc^2$ and $g_{rr} = [1 - 2GM/rc^2]^{-1}$, which happen to be the exact solutions we are looking for. However, when we extend our considerations to relativistic gravity, we expect $v(r)$ and $f(r)$ will have to be corrected to capture the relativistic gravitational effects. This can be done by writing $v(r) = -(2GM/r)^{1/2}U(GM/rc^2)$ and $f(r) = (GM/r^2)W(GM/rc^2)$, where $U(GM/rc^2)$ and $W(GM/rc^2)$ are functions with expansions of the dimensionless quantity GM/rc^2 . (Note that $U = W = 1$ in Newtonian gravity.) The relativistic corrections are supposed to be captured in the dimensionless functions U and W . Therefore, in order to determine $g_{tt}(r)$ and $g_{rr}(r)$, information from experiments or theory other than the Newtonian gravity must be specified. This is where the Einstein’s field equations are needed.

The assumption that all of the higher power of GM/rc^2 in functions U and W are zero is unjustified without the Einstein’s field equations, albeit it may be true by accident that the particular metric components are already given exactly in the non-relativistic limit. Using only special relativity, equivalence principle and Newtonian gravity, it is not sufficient to determine the components of the Schwarzschild metric when relativistic gravitational effects are significant. These arguments, thus, falsify the claims proposed in refs. [3–8] that the Schwarzschild metric can be obtained without the explicit use of general relativity.

The aforementioned inverse proportionality of the diagonal coefficients is a source of continuing confusions. Based on various reasonings, Schild [10], Rindler [11], Sacks and Ball [12], Kassner [13] have arrived at the same conclusion that there is no simple derivation of the Schwarzschild metric. Although it is unreasonable to believe that the Schwarzschild metric can be derived from time dilation and length contraction without general relativity, the erroneous attributions of these features do come up with an answer that looks like the correct one. If so, can there be other constructive ways to use these concepts in the gravitational theory? In this paper, we show that the time dilation and length contraction for the ‘fictitious motions’ (either a uniform fictitious radial velocity or fictitious oscillations) of a thin shell can be applied to mimic the gravitational properties of a massive thin shell.

In quantum theory, we learned that a particle is an oscillator. However, its amplitude has only a probabilistic interpretation based on Born’s postulate [14]. In refs. [15–17], we demonstrate a possibility that, apart from the classical description of mass [18,19], a particle can have an intrinsic oscillation in proper time. By allowing matter to vibrate in the time direction, we can reconcile the quantum properties of a matter field. The scalar field describing the vibrations

of matter in time satisfies the Klein-Gordon equation and Schrödinger equation. The energy in this system is quantized under the constraint that the energy of mass is on shell. Apart from the proper time oscillation, we can identify another intrinsic property for a particle. The fictitious radial oscillation introduced in this paper has effects that can curve spacetime. A thin shell with fictitious radial oscillations can be contracted to infinitesimal radius that resembles a point mass. In addition, the spacetime around this thin shell with infinitesimal radius is the Schwarzschild field. The fictitious radial oscillation is another intrinsic property that a particle can hypothetically possess.

This paper is organized as follows. Section 2 outlines the properties of a thin shell with a 'uniform fictitious radial velocity'. Time dilation and length contraction derived from this fictitious velocity can be used to define the spacetime metric on the surface of a thin shell. Section 3 shows that a thin shell with 'fictitious radial oscillations' can also produce the same effects obtained in Section 2 based on a time translational symmetry. Section 4 demonstrates that the external spacetime outside the thin shell with fictitious motions is static and satisfies the Schwarzschild solution for the gravitational field of a spherically symmetric mass. Section 5 is reserved for discussions. In the following analysis, we will adopt the units $c = G = 1$.

2 Thin Shell with a Uniform Fictitious Radial Velocity

The way Eq. (1) is written has created a deceptive impression that the Schwarzschild metric can be derived from time dilation and length contraction. As it appears, an observer O' stationary at a particular radial coordinate r is subject to a velocity $v(r)$. However, this velocity is not physical motion of the observer in spacetime. In fact, observer O' is considered stationary in the spatial coordinates. This "fictitious velocity" is what is responsible for the apparent time dilation and length contraction, i.e. $(dt')^2 = [1 - v(r)^2]dt^2$ and $(dr')^2 = [1 - v(r)^2]^{-1}dr^2$. As discussed earlier, it is unreasonable to assume that we can apply these concepts at every point in spacetime to obtain the Schwarzschild metric. Here, we will limit their applications only on the surface of a timelike hypersurface. Our goal is not to derive the Schwarzschild metric but to show that the gravitational properties derived from a spherical thin shell with fictitious radial motions is compatible with those for a spherical thin shell with mass M .

Let us consider a coordinate system (t, r, θ, ϕ) . The coordinate time t is measured by the clock of a stationary observer O located at spatial infinity. The radial coordinate r is defined as the circumference, divided by 2π , of a sphere centered around the spherical thin shell to be investigated. The angular coordinates θ and ϕ are the usual polar spherical angular coordinates. This coordinate system is the same adopted for the conventional Schwarzschild field.

We will assume an infinitesimally thin spherical shell Σ with radius \check{R} is centered at the origin of this coordinate system. A stationary observer \check{O} on this thin shell will be subject to a uniform fictitious radial velocity v_{fm} as if it is constantly moving relative to the observer O at spatial infinity. However, the thin shell itself has no physical movement in regular spacetime. The uniform fictitious radial velocity v_{fm} has effects on the time and distance measured at

$r = \check{R}$ while observer \check{O} remains stationary on the thin shell. Outside this thin shell, the spacetime is a vacuum with no fictitious radial motion. The hypothetical effects of the fictitious velocity on time and distance measurements are used to define the spacetime metric on the surface of the timelike thin shell.

In a Minkowski spacetime, the clock of a stationary observer at any location shall be synchronized with the clock of observer O at spatial infinity. However, this is not the case for observer \check{O} stationary on the thin shell. For the purpose of our discussions, it will be easier if we adopt the use of a fictitious frame \check{Q} . In this fictitious frame, it is the clock of the fictitious observer \check{Q} that synchronizes with the clock of O . In addition, the two observers measure the same length for the same object. We shall consider the effects of the fictitious velocity at $r = \check{R}$ as if \check{O} is moving in the fictitious frame of \check{Q} . An observer \check{O} on the thin shell will have a uniform fictitious velocity v_{fm} moving relative to the fictitious observer \check{Q} . Therefore, \check{O} is under the constant effects of a uniform fictitious velocity while remaining at rest relative to O at spatial infinity.

Let us consider two events in frame \check{O} . We will assume the fictitious velocity is $|v_{fm}| < 1$. The infinitesimal $d\check{t}$ and $d\check{r}$ are the differences in the time and radial coordinates between the two events. They can be related to the coordinate increments dt and dr for the same two events observed by O ,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} \Upsilon_{\check{t}}^t & \Upsilon_{\check{r}}^t \\ \Upsilon_{\check{t}}^r & \Upsilon_{\check{r}}^r \end{bmatrix} \begin{bmatrix} d\check{t} \\ d\check{r} \end{bmatrix}. \quad (2)$$

In the local frames of O and \check{O} , the respective basis vectors in the time and radial directions are orthogonal, i.e. $\vec{e}_t \cdot \vec{e}_r = 0$ and $\vec{e}_{\check{t}} \cdot \vec{e}_{\check{r}} = 0$. On the other hand, \check{O} is stationary relative to O . The time and radial basis vectors in frame O are parallel to their counterparts in frame \check{O} , i.e. $\vec{e}_{\check{t}} \parallel \vec{e}_t$, and $\vec{e}_{\check{r}} \parallel \vec{e}_r$. Under these conditions, the transformation matrix Υ is diagonal,

$$\Upsilon_{\check{r}}^t = \Upsilon_{\check{t}}^r = 0. \quad (3)$$

When $d\check{r} = 0$, $d\check{t}$ is a proper time measured by the clock carried by \check{O} . This timelike interval can be Lorentz transformed to the fictitious frame of \check{Q} ,

$$d\underline{t} = \gamma d\check{t}, \quad (4)$$

$$d\underline{r} = \gamma v_{fm} d\check{t}, \quad (5)$$

where $\gamma = [1 - (v_{fm})^2]^{-1/2}$. In the fictitious frame, \check{O} travels a distance $d\underline{r}$ over a time $d\underline{t}$. On the other hand, the clocks of O and \check{Q} are synchronized. O shall measure the same time as \check{Q} ,

$$dt = d\underline{t} = \gamma d\check{t}. \quad (6)$$

However, O is physically stationary relative to \check{O} , i.e.

$$dr = 0. \quad (7)$$

The underlined quantity in Eq. (5) is a fictitious infinitesimal displacement that appears only in the fictitious frame of \check{Q} . Under the effect of this fictitious velocity, time measured by the clock of O is dilated but there is no physical movement between O and \check{O} . From Eq. (6),

$$\Upsilon_{\check{t}}^t = \gamma = [1 - (v_{fm})^2]^{-1/2}. \quad (8)$$

Next, we will consider a measuring rod with length $d\check{r}$ carried by \check{O} . This spacelike interval can be expressed as two events measured at the endpoints of the rod simultaneously, $d\check{t} = 0$. Again, we can Lorentz transform these two events to the fictitious frame of \underline{O} ,

$$d\underline{t} = \gamma \underline{v}_{fm} d\check{r}, \quad (9)$$

$$d\underline{r} = \gamma d\check{r}. \quad (10)$$

From the viewpoint of \underline{O} , the rod carried by \check{O} is moving at a velocity \underline{v}_{fm} . To obtain the moving length $d\underline{l}$ of the rod, we shall subtract $d\underline{r}$ by the distance traveled by the rod during $d\underline{t}$,

$$d\underline{l} = d\underline{r} - \underline{v}_{fm} d\underline{t} = \gamma^{-1} d\check{r}. \quad (11)$$

Since the rod is of the same length as measured by O and \underline{O} ,

$$dr = d\underline{l} = \gamma^{-1} d\check{r}. \quad (12)$$

On the other hand, the rod carried by \check{O} is stationary relative to O . The underlined quantities in Eqs. (9) and (11) are fictitious temporal and spatial infinitesimal displacements that only appear in the fictitious frame of \underline{O} . Their effects shorten the rod observed in frame O but there is no physical movement between O and \check{O} . The spacelike interval dr representing the length of the rod in frame O is measured simultaneously at the endpoints,

$$dt = 0. \quad (13)$$

From Eq. (12),

$$\Upsilon^r_{\check{r}} = \gamma^{-1} = [1 - (\underline{v}_{fm})^2]^{1/2}. \quad (14)$$

Therefore, Eq. (2) becomes,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} [1 - (\underline{v}_{fm})^2]^{-1/2} & 0 \\ 0 & [1 - (\underline{v}_{fm})^2]^{1/2} \end{bmatrix} \begin{bmatrix} d\check{t} \\ d\check{r} \end{bmatrix}, \quad (15)$$

which is based on the results from Eqs. (3), (8) and (14).

3 Thin Shell with Fictitious Oscillations

In this section, we will show that the same transformation matrix Υ from Eq. (15) can be obtained by considering a thin shell that has fictitious oscillations in the radial direction. Let us again consider the infinitesimally thin spherical shell Σ with radius \check{R} . We will introduce fictitious radial oscillations on the surface of this thin shell,

$$\check{r}_f(\underline{t}) = -\check{\mathfrak{R}} \cos(\omega_0 \underline{t}), \quad (16)$$

$$\check{v}_f(\underline{t}) = \frac{\partial \check{r}_f(\underline{t})}{\partial \underline{t}} = \check{\mathfrak{R}} \omega_0 \sin(\omega_0 \underline{t}), \quad (17)$$

where $\check{\mathfrak{R}}$ and ω_0 are the amplitude and angular frequency of the fictitious oscillations respectively. As we have discussed in the last section, we will adopt the

use of a fictitious frame \check{O} . In this fictitious frame, it is the clock of a fictitious observer \check{O} oscillating about $r = \check{R}$ that synchronizes with the clock of observer O at spatial infinity, i.e. $t = \check{t}$. In addition, the two observers measure the same length for the same object, i.e. $l = \check{l}$. Although an observer \check{O} on the thin shell has no physical motion relative to observer O at spatial infinity, \check{O} is subject to a fictitious oscillation. At a particular time, \check{O} is displaced a distance r_f with an instantaneous velocity v_f relative to \check{O} in the fictitious frame. The fictitious oscillation does not carry an observer through regular spacetime. There is no physical motion of matter. Instead, the fictitious oscillation is used to define the geometry of spacetime at $r = \check{R}$. Furthermore, the system is spherically symmetric with fictitious oscillations in the radial direction only. The region outside the thin shell is a vacuum which is source free. There is no fictitious vibration in the vacuum spacetime.

From Eqs. (16) and (17), the fictitious displacement and instantaneous velocity at $\check{t} = \check{t}_m = \pi/(2\omega_0)$ are,

$$r_f(\check{t}_m) = 0, \quad (18)$$

and

$$v_f(\check{t}_m) = v_{fm} = \mathfrak{R}\omega_0, \quad (19)$$

where we will assume $v_{fm} = \mathfrak{R}\omega_0 < 1$ in this section. \check{O} is traveling with a velocity v_{fm} in the fictitious frame with no displacement relative to \check{O} . (Note that we equate the peak fictitious velocity $v_f(\check{t}_m)$ with the uniform fictitious velocity v_{fm} developed in the previous section to show later that both the use of fictitious oscillations and the uniform fictitious velocity can produce the same gravitational effects.) As we shall recall, the properties at $r = \check{R}$ with fictitious velocity v_{fm} are already discussed in Section 2. However, in the fictitious oscillating system, apart from the instantaneous velocity v_f , \check{O} also has a displacement r_f relative to the fictitious observer \check{O} . As a simple harmonic oscillating system, we expect both the fictitious displacement and its instantaneous velocity can have effects on \check{O} .

Let us rewrite the transformation matrix Υ from Eq. (15) in terms of a constant \check{I} for the thin shell with fictitious radial oscillations, i.e.

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} (1 - \check{I})^{-1/2} & 0 \\ 0 & (1 - \check{I})^{1/2} \end{bmatrix} \begin{bmatrix} d\check{t} \\ d\check{r} \end{bmatrix}, \quad (20)$$

where

$$\check{I} = \omega_0^2 [r_f(\check{t})]^2 + [v_f(\check{t})]^2 = v_{fm}^2 = \mathfrak{R}^2 \omega_0^2. \quad (21)$$

At $\check{t} = \check{t}_m = \pi/(2\omega_0)$, $\check{I} = v_{fm}^2$. The oscillation has an instantaneous velocity v_{fm} with no displacement relative to the fictitious observer \check{O} as discussed earlier. On the other hand, the constant fictitious velocity from Section 2 is replaced by a fictitious oscillation in Eqs. (20) and (21). The constant \check{I} is the summation of two parts analogous to the ‘‘potential’’ and ‘‘kinetic’’ components of a classical oscillating system. At a particular instant, the fictitious displacement $r_f(\check{t})$ and the fictitious instantaneous velocity $v_f(\check{t})$ satisfy Eq. (21) which is a typical solution for a simple harmonic oscillating system. In addition, Υ is a constant matrix. The effects of the fictitious oscillation on \check{O} at any instant is equivalent to those produced by the fictitious velocity v_{fm} at $\check{t} = \pi/(2\omega_0)$.

The same transformation matrix Υ from Eq. (15) can be obtained by replacing the constant fictitious radial velocity with fictitious oscillation in the radial direction.

As we shall recall, the total energy of a simple harmonic oscillating system is typically conserved. This is what we expect for the system with fictitious oscillations. Based on the Noether's theorem, the system we are considering shall have a time translational symmetry. This fact will be confirmed below. As a result, the combined effects from the fictitious displacement r_f and fictitious velocity v_f on observer \check{O} shall be a constant over time. Therefore, there is no surprise that Υ is a constant matrix for the thin shell with fictitious radial oscillations.

The fictitious oscillations in the system are entirely radial. It is spherically symmetric and there is no rotational motion. Thus, the line element at $r = \check{R}$ can be written as [20],

$$ds^2 = g_{tt}(\check{R})dt^2 + 2g_{tr}(\check{R})dtdr + g_{rr}(\check{R})dr^2 - \check{R}^2 d\Omega^2, \quad (22)$$

where Ω is the metric induced on each 2-sphere using the radial coordinates of our reference system.

The temporal (radial) coordinate in frame O , and its counterpart in frame \check{O} , are of different scale. From Eq. (20), we can relate the basis vectors in frames O and \check{O} ,

$$\vec{e}_{\check{t}} = \vec{e}_t(1 - \check{I})^{1/2}, \quad (23)$$

$$\vec{e}_{\check{r}} = \vec{e}_r(1 - \check{I})^{-1/2}. \quad (24)$$

As a result, the metrics at O and \check{O} are different. From Eqs. (23) and (24),

$$g_{tt}(\check{R}) = \vec{e}_{\check{t}} \cdot \vec{e}_{\check{t}} = (1 - \check{I})\vec{e}_t \cdot \vec{e}_t = 1 - \check{I}, \quad (25)$$

$$g_{rr}(\check{R}) = \vec{e}_{\check{r}} \cdot \vec{e}_{\check{r}} = (1 - \check{I})^{-1}\vec{e}_r \cdot \vec{e}_r = -(1 - \check{I})^{-1}, \quad (26)$$

$$g_{tr}(\check{R}) = g_{rt}(\check{R}) = \vec{e}_{\check{t}} \cdot \vec{e}_{\check{r}} = \vec{e}_t \cdot \vec{e}_r = 0, \quad (27)$$

where $\vec{e}_t \cdot \vec{e}_t = 1$, $\vec{e}_r \cdot \vec{e}_r = -1$, and $\vec{e}_t \cdot \vec{e}_r = 0$. Therefore, the line element at $r = \check{R}$ is,

$$ds^2 = [1 - \check{I}]dt^2 - [1 - \check{I}]^{-1}dr^2 - \check{R}^2 d\Omega^2. \quad (28)$$

Apart from the time translational symmetry, the spacetime at $r = \check{R}$ is also invariant under time reflection symmetry ($t \rightarrow -t$).

4 Schwarzschild Spacetime

From Eq. (21), both the uniform fictitious radial velocity v_{fm} and the fictitious oscillation with amplitude \mathfrak{R} can give rise to the same line element in Eq. (28). Using these fictitious radial motions (either a uniform fictitious radial velocity or fictitious oscillations) to define the geometry of spacetime at $r = \check{R}$, the metric derived on the surface of a thin shell is different from the assumed flat spacetime at spatial infinity. The geometry of spacetime at these two distant locations are different. If the spacetime manifold outside the thin shell is smooth and continuous, its structures cannot be flat. The fictitious motions have effects that can curve spacetime.

Eq. (28) is the line element of the Schwarzschild metric on the surface of a thin shell with total mass M if we set

$$\check{I} = \frac{2M}{\check{R}}, \quad (29)$$

or

$$M = \frac{\check{R}\check{\mathfrak{R}}^2\omega_0^2}{2} = \frac{\check{R}v_{fm}^2}{2}. \quad (30)$$

From general relativity, the vacuum space-time v^+ outside this spherical thin shell Σ (a time-like hypersurface) is the Schwarzschild spacetime, i.e.

$$ds^2 = \left[1 - \frac{\check{R}\check{\mathfrak{R}}^2\omega_0^2}{r}\right]dt^2 - \left[1 - \frac{\check{R}\check{\mathfrak{R}}^2\omega_0^2}{r}\right]^{-1}dr^2 - r^2d\Omega^2, \quad (31)$$

or

$$ds^2 = \left[1 - \frac{\check{R}v_{fm}^2}{r}\right]dt^2 - \left[1 - \frac{\check{R}v_{fm}^2}{r}\right]^{-1}dr^2 - r^2d\Omega^2. \quad (32)$$

The spacetime around the thin shell with fictitious motions is static with time translational and time reflection symmetries.

The Birkhoff's theorem [21, 22] states that the gravitational field of any spherically symmetric vacuum region is necessarily static, and its metric is that of the Schwarzschild spacetime. This applies to the external field of any non-rotating, spherical, uniform thin shell whether the shell is static, fluctuating or collapsing. Applying the same principle, the time-like hypersurface Σ can be expanded (or contracted) by 'carrying' the fictitious oscillations along geodesics orthogonal to the original surface to a new sphere Σ' . As long as mass M given in Eq. (30) is remaining constant during this transformation, the metric and curvature of the external field will not be affected. Under this condition, the amplitude of the radial oscillation is,

$$\check{\mathfrak{R}} = \sqrt{\frac{2M}{\check{R}\omega_0^2}}, \quad (33)$$

or the uniform fictitious radial velocity is,

$$v_{fm} = \sqrt{\frac{2M}{\check{R}}}. \quad (34)$$

The metric from Eqs. (31) and (32) will encounter a coordinate singularity when the shell is contracted to a radius $\check{R} = 2M$ (the event horizon). Although the fictitious instantaneous velocity on the shell at event horizon can reach the speed of light (i.e. $v_{fm} = 1$ from Eqs. (19), (33) and (34) when $\check{R} = 2M$), it is not physical motion of matter. As we shall recall, the fictitious motions have effects on the time and distance measurements of a stationary observer. The spacetime metric at $r = \check{R}$ can be inferred from these fictitious motions. However, there is no physical movement by the stationary observer or the thin shell. The fictitious motions can curve spacetime but will not produce superluminal transfer of energy even for a shell that is inside the event horizon with $v_{fm} > 1$.

The amplitude $\check{\mathfrak{R}}$ from Eq. (33), the uniform fictitious velocity v_{fm} from Eq. (34), and the related spacetime curvature tensors derived from the metric

(e.g. the coordinate independent Kretschmann invariant [23], $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} = 48M^2/r^6 = 12\check{R}^2\check{\mathfrak{R}}^4\omega_0^4/r^6$), are well defined as the shell is contracted until it reaches a radius $\check{R} \rightarrow 0$ where it meets a true singularity [24, 25]. At this point, the shell is infinitely small but has infinitely large amplitude of oscillations, $\check{\mathfrak{R}} \rightarrow \infty$, or infinitely large fictitious radial velocity, $v_{fm} \rightarrow \infty$. As predicted by Birkhoff's theorem, the metric around this infinitely small shell is the Schwarzschild spacetime. An infinitesimally small thin shell with fictitious motions can mimic the gravitational properties of a point mass in relativity.

Before we conclude this section, let us address a question about the fictitious oscillations. From Eq. (34), the uniform fictitious radial velocity v_{fm} can be established directly from the mass M . Therefore, there is no ambiguity about what value v_{fm} shall be used when applying to a thin shell with mass M . However, this is not so trivial for the amplitude of fictitious oscillations $\check{\mathfrak{R}}$. In order to determine $\check{\mathfrak{R}}$, we also need to specify the angular frequency ω_0 in Eq. (33). This is ambiguous when we apply the fictitious oscillations for a macroscopic mass since there is no known 'distinguished' angular frequency that we can associate with a macroscopic object at rest. The angular frequency can therefore be assigned arbitrarily. On the contrary, there is one reasonable choice when the application is for a quantum particle. As de Broglie has conjectured in his 1924 thesis [26], a particle possesses an internal clock. This hypothesis has been recently tested, and the experimental data obtained are found to be compatible with the theory [27]. Considering the particle as a point mass, the angular frequency ω_0 specified in our formulation can be taken as the frequency of the de Broglie's internal clock. This frequency is unique for the particle. However, it is unreasonable to extend this idea of an internal clock for a macroscopic object. Quantum effects are supposed to be negligible in the classical energy level and there is no analogy of an internal clock for a macroscopic object. Therefore, to avoid ambiguity, we will limit our application of the fictitious oscillations for quantum systems only.

5 Conclusions and Discussions

The misconception that time dilation and length contraction can be applied to derive the Schwarzschild metric without general relativity is a source of continuing confusions. Repeated attempts have been made despite this idea has been refuted many times by others. Yet the simple form of this concept makes its application very appealing. In this paper, based on a better understanding of what limits its applications, we borrow the idea and apply it to derive the metric on the surface of a thin shell. One major difference from the previous attempts is that the outside spacetime is maintained as vacuum without fictitious motions in our study. Our goal is not to derive the Schwarzschild metric without general relativity. Instead, this application helps to paint a simple picture on how matter can be connected to spacetime. A massive thin shell exerts fictitious motions on its surface. This alters the spacetime metric on the surface of the thin shell and curves the surrounding external spacetime. In turn, the curved spacetime tells other matters how to react in the presence of the thin shell. The previously unsuccessful idea can have another application in the gravitational theory.

The fictitious radial oscillation is different from the proper time oscillation

of a particle studied in refs. [15–17]. In these referenced papers, we show that a particle can have an intrinsic oscillation in proper time. As a part of spacetime, this oscillation in time shall have effects on its surrounding spacetime. Here, we demonstrate another intrinsic property possible for a particle. The fictitious radial oscillations with de Broglie’s frequency on a thin shell with infinitesimal radius can give rise to the Schwarzschild spacetime. Unlike the proper time oscillation of a particle, the fictitious radial oscillations are not motions of matter in the time direction (nor in the spatial direction). Instead, their effects can curve spacetime, These fictitious radial oscillations can be what we need to connect the proper time oscillation of a particle with the external spacetime. The proper time oscillator is the source of the particle’s mass-energy which can be the driving force for the fictitious radial oscillations. It is not implausible that a particle can have additional intrinsic properties that are linked more directly with spacetime.

References

- [1] T. Jacobson, "When is $g_{tt}g_{rr} = -1$?", *Class. Quant. Grav.* **24**, 5717 (2007).
- [2] K. Kassner, "A physics-first approach to the Schwarzschild metric", *Advanced Studies in Theoretical Physics* **11**, 179 (2017) - A brief summary of the attempts to reconcile the Schwarzschild metric from special relativity, equivalence principle and Newtonian gravity can be found in the Introduction of this paper.
- [3] W. Lenz, 1944 unpublished work cited in A. Sommerfeld, *Electrodynamics (Lectures on Theoretical Physics Vol. 3)*, (Academic Press, New York, 1967).
- [4] L. Schiff, "On experimental tests of the general theory of relativity", *Am. J. Phys.* **28**, 340 (1960).
- [5] M. Harwit, *Astrophysical Concepts*, (Wiley, New York, 1973).
- [6] P. Rowlands, "A simple approach to the experimental consequences of general relativity", *Physics Education* **32**, 49 (1997).
- [7] J. Czerniawski, "The possibility of a simple derivation of the Schwarzschild metric", *Preprint gr-qc/0611104*.
- [8] R. Cuzinatto, B. Pimental, and P. Pompeia, "Schwarzschild and de Sitter solution from the argument by Lenz and Sommerfeld", *Am. J. Phys.* **79**, 662 (2011).
- [9] R. Gruber, R. Price, S. Matthews, W. Cordwell, and L. Wagner, "The impossibility of a simple derivation of the Schwarzschild metric", *Am. J. Phys.* **56**, 265 (1988).
- [10] A. Schild, "Equivalence Principle and Red-Shift Measurements", *Am. J. Phys.* **28**, 778 (1960).
- [11] W. Rindler, "Counterexample to the Lenz-Schiff Argument", *Am. J. Phys.* **36**, 540 (1968).

- [12] W. Sacks and J. Ball, "Simple derivations of the Schwarzschild metric", *Am. J. Phys.* **36**, 240 (1968).
- [13] K. Kassner, "Classroom reconstruction of the Schwarzschild metric", *Eur. J. Phys.* **36**, 065031 (2015).
- [14] M. Born, "On the Quantum Theory of the Electromagnetic Field", *Proc. Roy. Soc. Lond. A* **143**, 410 (1934).
- [15] H. Y. Yau, "Emergent quantum field of a deterministic system with vibrations in space and time", *AIP Conf. Proc.* **1508**, 514 (2012).
- [16] H. Y. Yau, "Probabilistic nature of a field with time as a dynamical variable", *Lect. Notes Comp. Sci.* **10106**, 33 (2016).
- [17] H. Y. Yau, "Temporal vibrations in a quantized field", in *Quantum Foundations, Probability and Information* pg. 269, ed. A. Khrennikov and B. Toni, (Springer, Verlag, 2018).
- [18] M. Jammer, *Concepts of Mass in Contemporary Physics and Philosophy*, (Princeton University Press, Princeton, 2009).
- [19] L. Okun, *Energy and Mass in Relativity Theory*, (World Scientific, Singapore, 2009).
- [20] R. Wald, *General Relativity*, (University of Chicago Press, Chicago, 1984).
- [21] G. Birkhoff, *Relativity and Modern Physics*, (Harvard University Press, Cambridge-Massachusetts, 1923).
- [22] H. Schmidt, "The tetralogy of Birkhoff theorems", *Gen. Rel. Grav.* **45**, 395 (2013).
- [23] C. Cherubini, D. Bini, S. Capozziello, and R. Ruffini, "Second order scalar invariants of the Riemann tensor: applications to black hole spacetimes", *Int. J. Mod. Phys. D* **11**, 827 (2002).
- [24] S. Hawking and R. Penrose, "The singularities of gravitational collapse and cosmology", *Proc. Roy. Soc. Lond. A* **314**, 529 (1970).
- [25] S. Hawking and G. Ellis, *The Large-Scale Structure of Space-Time*, (Cambridge University Press, Cambridge, 1973).
- [26] L. de Broglie, Ph. D. thesis, Universite De Paris (1924).
- [27] P. Catillon et al., "A search for the de Broglie particle internal clock by means of electron channeling", *Found. Phys.* **38**, 659 (2008).