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Species separation of a binary mixture under acoustic streaming*

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Abstract. An analytical and numerical study of the action of ultrasonic waves on species separation within a rectangular cavity filled with a binary fluid (water-ethanol mixture) is presented. An ultrasonic wave was emitted on a portion of one of its vertical walls, while the opposite wall was perfectly absorbent. The two horizontal walls were differentially heated. A progressive acoustic wave was used to generate, at a large scale, a stationary flow of the viscous binary fluid (Eckart Streaming) within the cavity. The authors analytically determined the temperature T, mass fraction C and velocity fields under the parallel flow hypothesis used for cells with high aspect ratio $B \gg 1$, in the presence of the gravitational field. From the analysis of the velocity fields the authors concluded that the associated flow is either unicellular or consists of three counter-rotating cells superimposed in the horizontal direction of the cavity. They also found the variation domains of the physical parameters leading to one or the other of these two types of flows. The algebraic equation allowing the calculation of the mass fraction gradient and hence the species separation between the vertical walls of the cavity was determined. The variation of the dimensional mass fraction gradient, for the water-ethanol mixture, as a function of the two control parameters of the problem, namely the acoustic parameter A and the temperature difference ΔT imposed on the two horizontal walls was studied.

1 Introduction

It is well known that the propagation of an ultrasonic wave in a fluid may induce a stationary flow at large scale. This phenomenon, called acoustic streaming, was respectively described by Faraday [1], Eckart [2], Lighthill [3], Nyborg [4] as a coupling between acoustic propagation and fluid motion. A review of the most pertinent papers of the last decade on acoustic streaming can be found in the recent papers cited below. Lei *et al.* [5] used 3D numerical simulations in order to analyse the natural convection in an enclosure subjected to a horizontal temperature gradient and a longitudinal sound field. The authors considered the Rayleigh streaming due to the stresses in the Stokes shear-wave layers that form at the solid boundaries.

Dridi *et al.* [6,7] studied the effect of Eckart streaming respectively in a three-dimensional side-heated parallelepipedic cavity and in a fluid layer confined between two infinite horizontal walls submitted to a horizontal temperature gradient and an ultrasound beam. In the first study the flows induced by acoustic streaming and their stability were studied. In the second study, the velocity of the basic flow was determined analytically in the case of an ultrasound beam centered on one side of the cavity and in the case of the beam not centered. The authors studied the linear stability of Eckart streaming flows in an isothermal or laterally heated mono-constituent fluid layer in the gravity field using a spectral method. For the isothermal case, the critical value of the acoustic parameter A_c leading to a Hopf bifurcation, was calculated as a function of the normalized width ε of the beam. For a centered beam A_c is minimum for $\varepsilon = 0.32$. Moudjed *et al.* [8] carried out a numerical and experimental study of isothermal flows in water under the action of ultrasonic waves in the near field of a circular plane ultrasonic transducer. The parallelepipedic cavity with a free upper surface used for their experimentations was delimited by absorbent walls to avoid acoustic reflections. Flow velocities were measured by particle image velocimetry (P.I.V.). Recently Lyubimova *et al.* [9] studied a linear stability of stationary flow in horizontal cavity containing a binary fluid and submitted to a longitudinal acoustic wave. They determined the domain of parameters for which acoustic waves had a stabilizing effect on finite wavelength disturbances.

In the present paper the interaction between the Eckart streaming and the Soret effect in a binary fluid is studied. Let us consider some important results about the

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Soret effect. A thermal gradient applied to a homogeneous binary solution generates a mass fraction gradient: this effect is called thermo-diffusion or Soret effect. Under gravity field, the coupling between thermodiffusion and convection may lead to important species separation. Thermo-gravitational separation was first carried out by Clusius and Dickel [10]. The authors performed the separation of gas mixtures in a vertical cavity heated from the side (thermo-gravitational column (TGC)). The optimum separation between the two horizontal ends of the cavity was obtained for an appropriate convective velocity and mass diffusion time, and for cells with very small thickness. Furry, Jones and Onsager [11] developed a theory to interpret the experimental processes of isotope separation. In order to increase the separation Lorenz and Emery [12] introduced a porous medium inside the TGC columns. In the past decades a lot of studies were carried out in order to increase the separation either in TGC columns or in horizontal parallelipidic cells submitted to a temperature gradient parallel to the gravity field ([13–18]). In all these configurations the thermal gradient applied on the walls induced simultaneously the diffusion phenomena and the natural convective flow. The species separation process in a binary fluid mixture by decoupling the thermal gradient from the convective velocity was performed by Khouzam *et al.* [19]. The unicellular flow in the cavity was generated by a constant velocity sliding of the horizontal upper wall of the cavity regardless of the applied thermal gradient. To our knowledge, only one contribution by Charrier-Mojtabi *et al.* [20] was carried out with the use of acoustic streaming to increase the species separation of the binary mixture in microgravity and for constant acoustic intensity A and in a rectangular cavity.

Therefore a new configuration is considered in this paper. The species separation is studied in a parallepipedic horizontal cell, differentially heated and submitted to an acoustic horizontal wave, emitted from a transducer positioned on a portion of one of its vertical walls. In microgravity, the flow was unicellular for a properly placed acoustic source. It was generated only by the action of the acoustic wave [20]. By cons, in the gravity field, the flow structure was much more complex. If the thermodiffusion coefficient D_T is positive the heavy component migrates toward the cold wall and the light component to the hot wall and conversely if D_T is negative. The flow generated also depends on whether the cavity is heated from above or below. For given values of the Rayleigh number, Ra and dimensionless acoustic parameter A, the authors notice that the flow inside the cavity is the superposition of the flow generated by the ultrasonic wave and a convective flow of opposite direction.

2 Mathematical formulation

We consider a rectangular cavity of large aspect ratio $B = L/H$, where H is the height of the cavity along the y-axis and L is the length along the z-axis. The cavity is filled with a binary fluid mixture of density ρ and dynamic viscosity μ. The two walls $z = 0$ and $z = L$ are adiabatic and impermeable. The two other walls $y = 0$ and $y = H$ are kept at a constant uniform temperature T_1 for $y = 0$ and T_2 for $y = H$ (fig. 1).

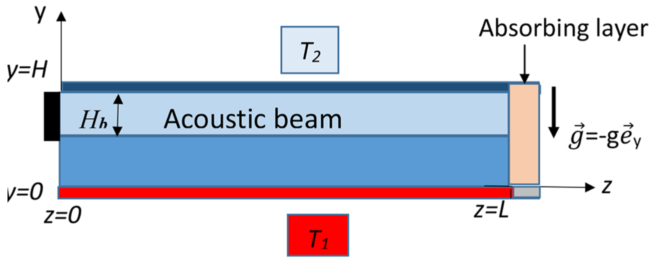

Fig. 1. Geometrical configuration of the cell submitted to an ultrasound beam.

On the vertical walls, a piezoelectric transducer producing an acoustic beam of height H_B is positioned at the top of the cell to induce a unicellular flow (necessary condition to obtain an important species separation between the vertical walls ($z = 0$ and $z = L$)). The Boussinesq approximation is assumed valid, thus, the thermo-physical properties of the binary fluid are constant except the density in the buoyancy term which varies linearly with the local temperature T and the local mass fraction C of the denser component:

$$\rho = \rho_0(1 - \beta_T(T - T_{ref}) - \beta_C(C - C_{ref})), \quad (1)$$

where β_T and β_C are, respectively, the thermal and mass expansion coefficients of the binary fluid and ρ_0, T_{ref}, C_{ref} are, respectively, the mixture density, the temperature, the mass fraction at reference state.

The acoustic streaming is a phenomenon which cannot be studied from the linear acoustic theory. The contributions of Lighthill [3] and Nyborg [4] lead to a formulation for the streaming flow velocity that obeys the Navier-Stokes equation in which an averaged acoustic force f_{ac} appears.

The dimensionless mathematical formulation of the problem is given by

$$\begin{cases} \nabla \cdot \vec{V} = 0, \\ \dfrac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla P + A\delta \vec{e}_z + RaPr(T + \Psi C)\vec{e}_y \\ \quad +Pr\nabla^2 \vec{V}, \\ \dfrac{\partial T}{\partial t} + \vec{V} \cdot \nabla T = \nabla^2 T, \\ \dfrac{\partial C}{\partial t} + \vec{V} \cdot \nabla C = \dfrac{1}{Le}(\nabla^2 C - \nabla^2 T), \end{cases} \quad (2)$$

where the parameter δ in the system (2), is defined by $\delta = 1$ for $y \in [1 - \varepsilon, 1[$ and $\delta = 0$ for $y \in [0, 1 - \varepsilon[$ and $\varepsilon = H_B/H$ denotes the normalized acoustic beam width.

The reference scales are H for the length, α/H for the velocity, α is the thermal diffusivity of the binary fluid, H^2/α for the time, $\Delta T = T_1 - T_2$ for the temperature, and $\Delta C = -\Delta T C_0(1 - C_0)D_T/D$ for the mass fraction, where D_T and D are, respectively, the thermo-diffusion and mass-diffusion coefficient of the denser component. The problem under consideration depends on seven non-dimensional parameters: the thermal Rayleigh number, $Ra = g\beta \Delta T H^3/\alpha\nu$, the Prandtl number, $Pr = \nu/\alpha$, the acoustic parameter A, which is the dimensionless acoustic force, $A = f_{ac}H^3/\rho_0\alpha^2$, the Lewis number $Le = \alpha/D$, the separation ratio $\Psi = -\beta_C C_{ref}(1 - C_{ref})D_T/\beta_T D$, where C_{ref} is the reference mass fraction of the denser component and the aspect ratio of the cell $B = L/H$.

The corresponding dimensionless boundary conditions are

$$y = 0, 1: \qquad \vec{V} = 0: \qquad \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y}; \qquad z = 0,$$

$$B = \frac{L}{H}: \qquad \vec{V} = 0, \qquad \frac{\partial T}{\partial z} = \frac{\partial C}{\partial z} = 0. \tag{3}$$

3 Analytical solution in the case of a shallow cavity

In the rest of the study, the acoustic parameter A is assumed to be uniform.

In the case of a shallow cavity $B \gg 1$, in order to obtain an analytical solution, the parallel flow approximation, used by many previous authors [15], is considered. The solution corresponding to the unicellular flow is given as follows:

$$\vec{U_0} = U_0(y)\vec{e}_z, \qquad T_0 = h(y), \qquad C_0 = mz + f(y), \tag{4}$$

where m is the mass fraction gradient along the horizontal z-axis.

We suppose that the acoustic wave does not interact with the wall $y = 1$.

With these assumptions and for the steady state, the system of eqs. (2) with the boundary conditions (3) is reduced to a set of the following equations solved using the Maple software:

$$\begin{cases} Pr\dfrac{\mathrm{d}^2U_0}{\mathrm{d}y^2} - \dfrac{\partial P_0}{\partial Z} + A\delta = 0, & \text{(a)} \\ \dfrac{\partial P_0}{\partial y} + RaPr(T_0 + \Psi C_0) = 0, & \text{(b)} \\ \dfrac{\partial^2 T_0}{\partial y^2} = 0, & \text{(c)} \\ mLeU_0 - \dfrac{\partial^2 C_0}{\partial y^2} = 0. & \text{(d)} \end{cases} \tag{5}$$

To solve the system (5), the following assumptions are considered:

- continuity of the velocity, the stress constraint, the mass fraction and temperature at the interface of the acoustic beam for $z = 1 - \varepsilon$;
- the mass flow is zero through any section vertical on the cavity: $\int_0^1 U_0 \mathrm{d}y = 0$;
- the mass flow rate through any cross section perpendicular to the z-axis is equal to zero: $\int_0^1 (U_0C_0 - \frac{1}{Le}(\frac{\partial C_0}{\partial z} - \frac{\partial T_0}{\partial z}))\mathrm{d}y = 0$;
- the conservation of the solute mass in the cavity at every moment: $\int_0^B (\int_0^1 C_0 \mathrm{d}y)\mathrm{d}z = 0$.

Thus the expression of the temperature, the velocity and the mass fraction field are given by the following polynomial expressions:

$$T_0 = 1 - y, \tag{6}$$

$$U_{01}(y) = -\frac{Ky^3}{6Pr} + \frac{(K - 4A(1-\varepsilon)^2(2\varepsilon + 1))y^2}{4Pr} + \frac{(-K + A(1-\varepsilon)^2(1+\varepsilon))y}{12Pr} - \frac{A(1-\varepsilon)^2}{2Pr} \tag{7}$$

for $0 \le y \le 1 - \epsilon$, where $K = Ra\, Pr\, \psi\, m$

$$U_{02}(y) = -\frac{Ky^3}{6Pr} + \frac{(K + 2\varepsilon^2 A(3 - 2\varepsilon))y^2}{4Pr} - \frac{(K + 12\varepsilon^2 A(1-\varepsilon))y}{12Pr} \tag{8}$$

for $1 - \epsilon \le y \le 1$.

The 1D velocity profile is found under the form of the 3rd polynomial expression in y (eqs. (7) and (8)):

$$C_{01}(y,z) = mz + f_1(y, Ra, Le, Pr, \psi, A, B, \varepsilon, m) + y + f_2(Ra, Le, Pr, \psi, A, B, \varepsilon, m) \tag{9}$$

for $0 \le y \le 1 - \epsilon$ and for $1 - \epsilon \le y \le 1$:

$$C_{02}(y,z) = C_{01}(y,z) - \frac{ALem(y + \epsilon - 1)^4}{24Pr}, \tag{10}$$

where f_1 is fifth-order polynomial function in y and f_2 depends only on dimensionless parameters. Their expressions are given by

$$f_1 = -\frac{mLe}{12Pr}\left(\frac{Ky^5}{10} + \frac{(-3K + 12A\varepsilon^3 - 18A\varepsilon^2)y^4}{12} + \frac{(K - 12A\varepsilon^3 - 12A\varepsilon^2)y^3}{6}\right),$$

$$f_2 = \frac{KLem}{1440Pr} + \frac{ALem(\varepsilon^5 - 5\varepsilon^4 + 2\varepsilon^3)}{120Pr} - \frac{Bm + 1}{2}, \tag{11}$$

The separation S is defined as the difference of the mass fraction of the denser species between the two ends of the cell, $z = 0$ and $z = B$, and its expression is $S = mB$, with m the solution of a third degree algebraic equation:

$$Q^2m^3 + 6LeAQ(3\varepsilon^4 - 6\varepsilon^3 - \varepsilon^2 + 4\varepsilon + 9)(\varepsilon - \varepsilon^2)^2m^2 - (504Pr(Q - 720Pr) + 288(ALe)^2\varepsilon^4(1-\varepsilon)^4(\varepsilon^2 - \varepsilon - 3))m - 15120ALePr(\varepsilon - \varepsilon^2)^2 = 0, \tag{12}$$

where $Q = Le\, Ra\, Pr\, \psi$.

Equation (12) is obtained by writing that the mass flow rate through any cross section perpendicular to the z-axis is equal to zero:

$$\int_0^{1-\varepsilon} \left(U_{01}C_{01} - \frac{m}{Le}\right)\mathrm{d}y + \int_{1-\varepsilon}^1 \left(U_{02}C_{02} - \frac{m}{Le}\right)\mathrm{d}y = 0. \tag{13}$$

Table 1. Different flows inside the cell function of A, Ra for $\Psi > 0$.

Cell heated from above, $Ra < 0$, $\Psi > 0$		
$A = 0$	$A > 0$	$A > 0$, $(C(0, y) - C(B, y)) > \Delta C_c$
Equilibrium solution, the water goes to the bottom and ethanol to the top plates, respectively. $\forall\ Ra < 0$.	Acoustic convective flow leads to $C(z = 0, y) > C(B, y)$: the flow is unicellular.	Superposition of mass convection in opposite direction to the acoustic convection. The flow is tri-cellular if the condition (15) is satisfied.
Cell heated from below, $Ra > 0$, $\Psi > 0$		
$A = 0$	$A > 0$	$A > 0$
The equilibrium solution leads to thermal convective rolls rotating in a clockwise or opposite direction for $Ra > Ra_c$.	For moderate $Ra > Ra_c$, if the convective thermal velocity is in the same direction as the acoustic velocity, the flow is unicellular.	For $Ra > Ra_c$, if the convective thermal velocity is in the opposite direction as the acoustic velocity, the flow is tri-cellular if the condition (15) is satisfied.

To obtain the maximum value of m according to the geometric parameter ε for fixed control parameters Ra and A, we write that the partial derivative of eq. (12) with respect to ε is equal to 0. The simplified equation obtained is equal to 0 for $\varepsilon = 0, 1$ and $\frac{1}{2}$. Only the physical solution $\varepsilon = \frac{1}{2}$ was maintained throughout the rest of the study. We check that the algebraic equation of the third degree admits a real root and two complex conjugate roots. This real root of eq. (11) is a function of Ra, Pr, Le, Ψ, A.

4 Physical interpretation of analytical results in optimized geometry ($\varepsilon = \frac{1}{2}$)

4.1 Velocity field

The intensity of the velocity U_0:

a) increases as A increases for a fixed value of $K = Ra\, Pr\, \psi\, m$;
b) increases as function of K for a fixed value of A.

The velocity U_0 is a third degree function of the variable y.

For A going to 0 the velocity U_0 is equal to

$$U_0 = -\frac{mRa\psi y(2y-1)(y-1)}{12}. \qquad (14)$$

For a mono-component fluid, Ben Hadid *et al.* [21] find a similar result.

In microgravity ($Ra = 0$), the intensity of the velocity U_0 is proportional only to $\frac{A}{Pr}$ and is a two-degree function of the variable y. In this case the velocity $U_0(y)$ vanishes only for $y = 0$, $y = 1/2$ and $y = 0$, thus leading to a unicellular flow. In the presence of gravity, the velocity $U_0(y)$ vanishes in $y = 0$, $y = 1/2$ and $y = (1 + 3A/2K)$ on the segment $[0, 1/2]$. On the segment $[1/2, 1]$ it vanishes for $y = 1/2$, $y = -3A/2K$ and $y = 1$. The flow is no longer unicellular if, on the one hand, $0 < (1 + 3A/2K) < 1/2$ and if, on the other hand, $1/2 < -3A/2K < 1$.

The first condition being included in the latter; the flow is then formed of three superimposed cells along the z-axis if the following two conditions are verified for $A > 0$:

$$Ra\Psi m < 0 \quad \text{and} \quad \|Ra\Psi m\| < 3A/Pr < \|2Ra\Psi m\|. \qquad (15)$$

In the presence of gravity and for a cell heated from above ($Ra < 0$), and for a positive separation ratio ($\psi > 0$) the heaviest component migrates towards the cold bottom wall leading to a stable thermal situation. The unicellular flow inside the cavity is initially generated by the acoustic wave. This unicellular flow leads initially to the species separation between the two walls, $z = 0$ and $z = B$. For high Rayleigh numbers, the horizontal mass gradient becomes sufficient to generate a convective flow of mass origin in the same direction or in the opposite direction of the one generated by the acoustic wave. For given values of A and the product $Ra\psi m$, the superposition of these two types of flows can lead either to a unicellular flow or, if the conditions (15) are verified, to a flow with three superimposed cells in the y-direction.

For a cell heated from below, $Ra > 0$, the flow generated by the acoustic streaming is superimposed to the thermal convective flow whenever the Rayleigh number Ra is greater than the linear critical Rayleigh number Ra_c and whatever the sign of the separation factor Ψ and the A value of the ultrasonic wave. If $\Psi > 0$ and $m > 0$ or $\Psi < 0$ and $m < 0$, one of the conditions (15) is no longer verified then the flow is unicellular. If, on the other hand, $\Psi > 0$ and $m < 0$ or $\Psi < 0$ and $m > 0$ the flow is constituted by three superimposed cells if the conditions (15) are verified. The mass fraction gradient m is a function of the non-dimensional parameters Ra, Pr, Le, Ψ, A and ε.

To illustrate our study, we will restrict ourselves in what follows for the presentation of the results to the water-ethanol binary solution previously studied by Platten *et al.* [13].

We summarize in table 1 these results in the case of the binary water-ethanol mixture for positive separation factor, $\Psi = 0.20$, where C is the mass fraction of water (table 2).

Table 2. Properties for a water (60.88 wt%) – ethanol (39.12 wt%) mixture at a mean temperature of 22 °C.

D ($\mathrm{m^2\,s^{-1}}$)	D_T ($\mathrm{m^2\,s^{-1}\,K^{-1}}$)	β_c	β_T (1/K)	α ($\mathrm{m^2\,s^{-1}}$)	ρ_0 ($\mathrm{kg\,m^{-3}}$)	ν ($\mathrm{m^2\,s^{-1}}$)
$4.32\ 10^{-10}$	$1.37\ 10^{-12}$	-0.212	$7.86\ 10^{-4}$	10^{-7}	935.17	$2.716\ 10^{-6}$

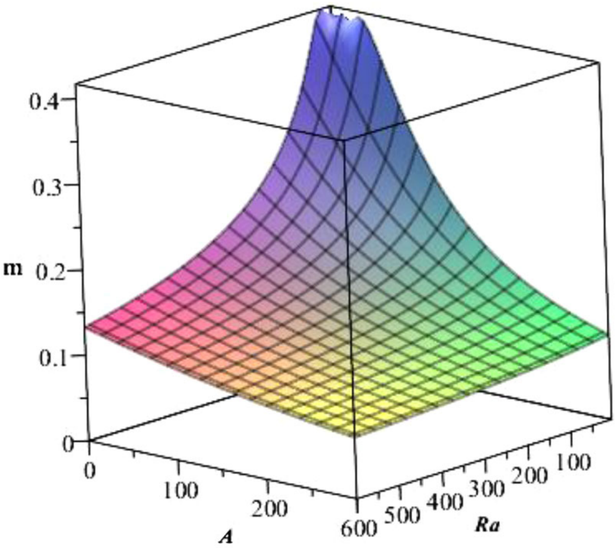

Fig. 2. The mass fraction gradient m *versus* A and $Ra > 0$.

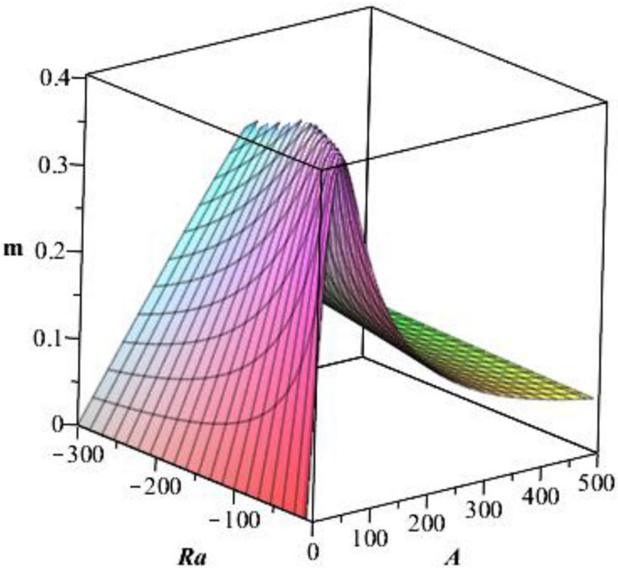

Fig. 3. The mass fraction gradient m *versus* A and $Ra < 0$.

The values of the thermophysical properties of this binary solution at the average temperature $T = 22.5\,^\circ\mathrm{C}$ are given in table 2.

From table 2, we deduce the values of the Prandtl number ($Pr = 27.16$), Lewis number ($Le = 231.5$) and separation factor ($\Psi = 0.20$) for this mixture. For the water-ethanol binary solution, where C denotes the mass fraction of water, the separation factor Ψ has the same

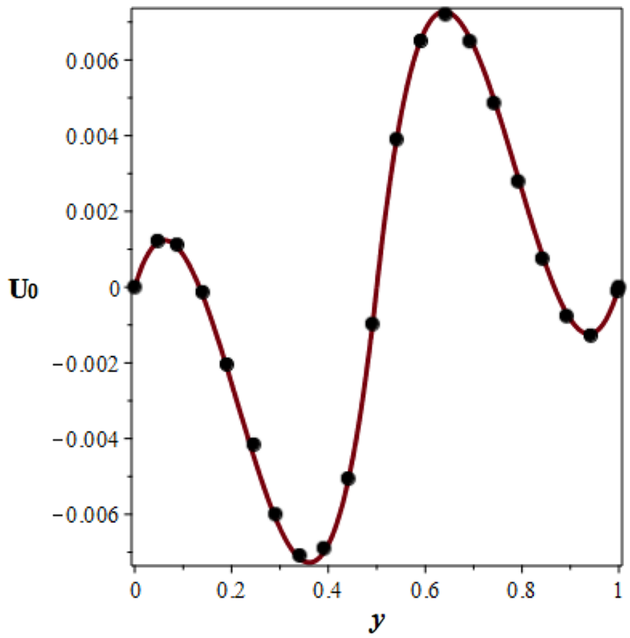

Fig. 4. Velocity profile U_0 (eqs. (7) and (8)) for water (60.88 wt%) – ethanol, obtained analytically (continuous line) and using a numerical simulation (dotted line) for $A = 60$, $Ra = -200$.

sign as the thermodiffusion coefficient D_T. By replacing Pr, Le, Ψ by their value in the case of the water-ethanol binary mixture in eq. (12) with $\varepsilon = 1/2$ we obtain

$$Ra^2m^3 + 0.70ARam^2 + (0.12A^2 - 10.89Ra + 169.31)m - 3.76A = 0. \tag{16}$$

The plot (fig. 2), of the surface associated with the real solution of eq. (16) shows that for $Ra > 0$, m is positive and for a given value of m there exists a single pair (Ra, A) of values. In this case the condition (15) is not verified since $Ra\Psi m < 0$ and the flow for $Ra > 0$ is always unicellular for moderate values of A and Ra. On the other hand, for $Ra < 0$, m is positive and for each value of $m < 0.41$ two pairs of values of Ra and A are associated (fig. 3).

In the latter case, if we take for example $A = 60$, $Ra = -200$ we find $m = 0.096$; the flow consists in three superimposed cells since the 2 conditions (13) are verified, $Ra\Psi m < 0$,

$$\|Ra\Psi m\| = 3.83 < \frac{3A}{Pr} = 6.62 < \|2Ra\Psi m\| = 7.67.$$

In fig. 4 we present the velocity profile U_0, for water (60.88 wt%) – ethanol obtained analytically and using a finite element method (Comsol Multiphysics) to solve the system (2) with associated boundary conditions for

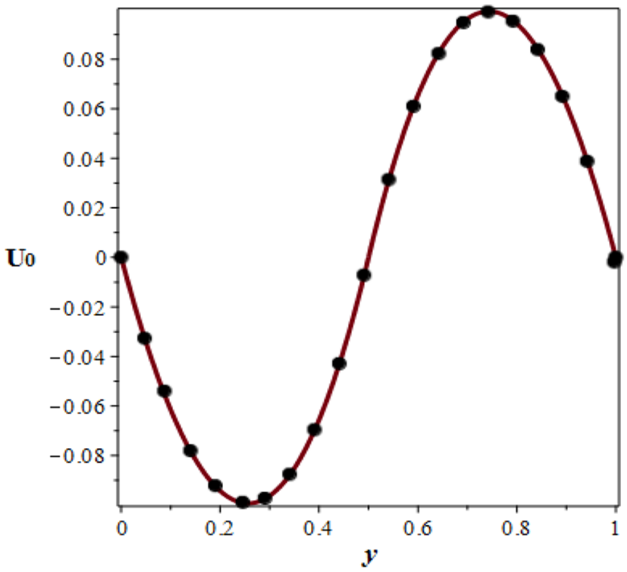

Fig. 5. Velocity profile U_0 (eqs. (7) and (8)) for water-ethanol obtained analytically (continuous line) and using a numerical simulation (dotted line) for $A = 200$, $Ra = -60$.

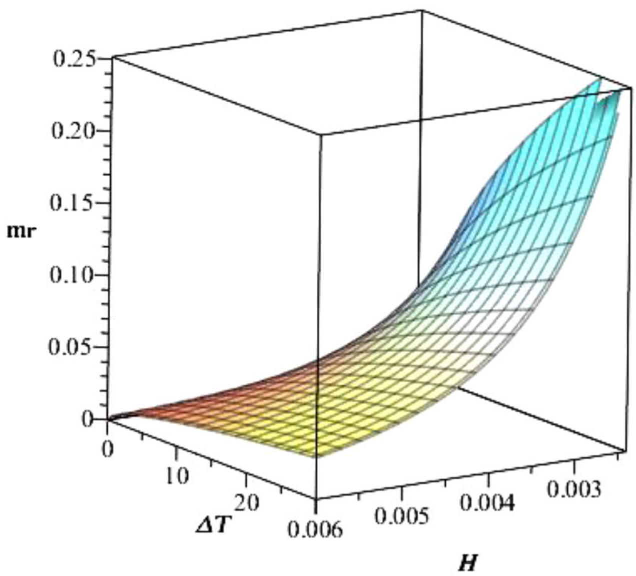

Fig. 6. Dimensional mass fraction gradient m^r as a function of $\Delta T = T_1 - T_2$ and A.

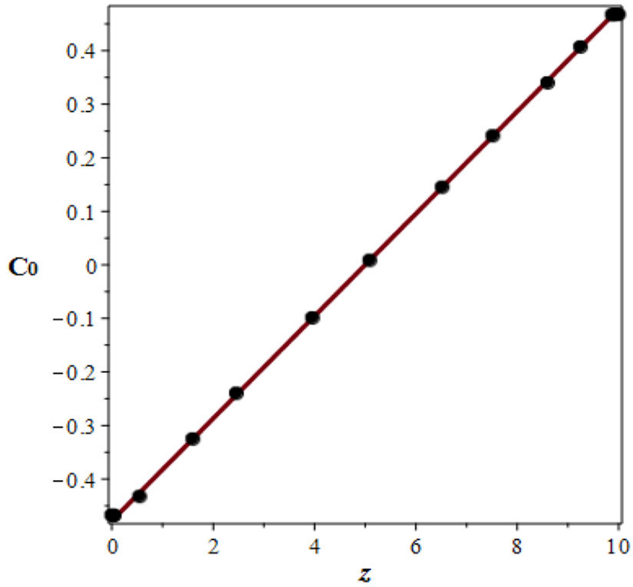

Fig. 7. Mass fraction *versus* z for water-ethanol obtained analytically (continuous line) and using a numerical simulation (dotted line) for $A = 60$, $Ra = -200$.

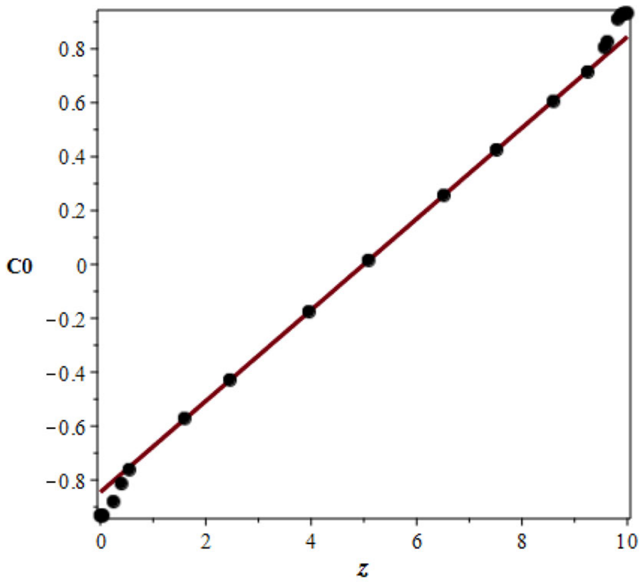

Fig. 8. Mass fraction *versus* z for water-ethanol obtained analytically (continuous line) and using a numerical simulation (dotted line) for $A = 200$, $Ra = -60$.

$A = 60$, $Ra = -200$. There is a perfect agreement between the analytical results and those obtained using direct numerical simulation.

If we take $A = 200$ and $Ra = -60$, we obtain $m = 0.169$ and the flow is unicellular since only the condition $Ra\Psi m < 0$ is verified:

$$\|Ra\Psi m\| = 2.02 < 3A/Pr = 22.06 > \|2Ra\Psi m\| = 4.04.$$

In fig. 5 we present the velocity profile U_0 for water (60.88 wt%) – ethanol obtained for $A = 200$, $Ra = -60$ and $m = 0.169$. The results of the numerical simulation are in good agreement with the analytical one.

4.2 The dimensional separation gradient

The non-dimensional separation, $S = mB$, is defined as the difference in the mass fractions of the denser component in the vicinity of left and right vertical walls of the horizontal cell. The dimensional value of the separation between the two ends of the cavity is given by

$$S^r = m^r L = C_0(1 - C_0)D_T \Delta T m L/DH, \qquad (17)$$

where m is the real root of eq. (16) function of Ra and A for the water (60.88 wt%) – ethanol binary mixture defined

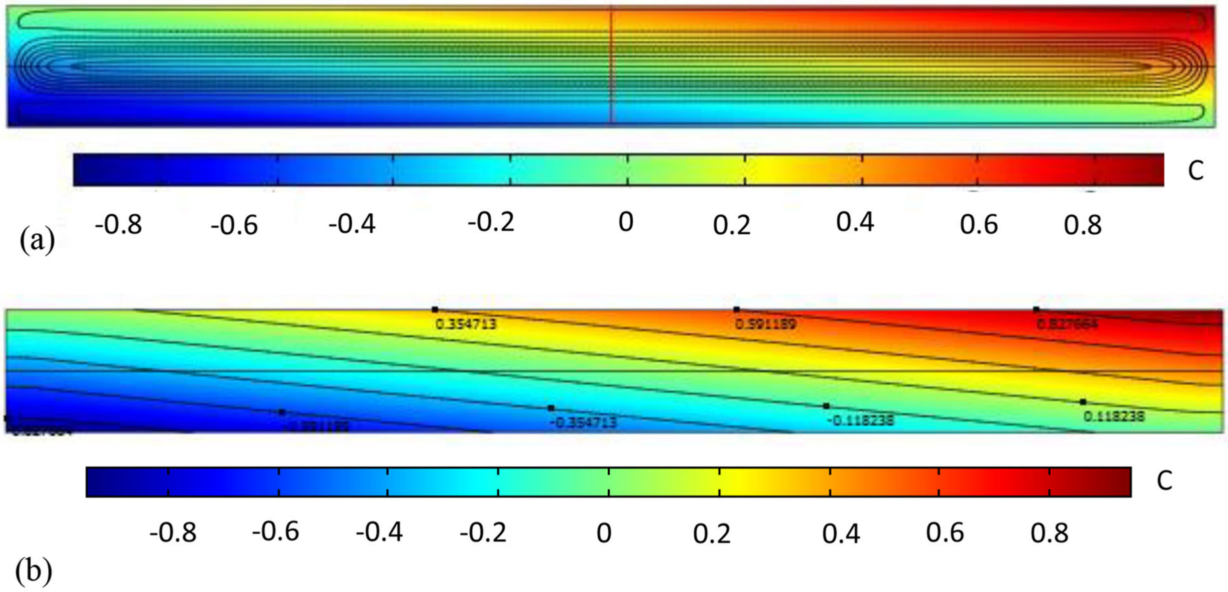

Fig. 9. Streamlines (a) and iso-mass fraction lines (b) obtained for $A = 60$, $Ra = -200$.

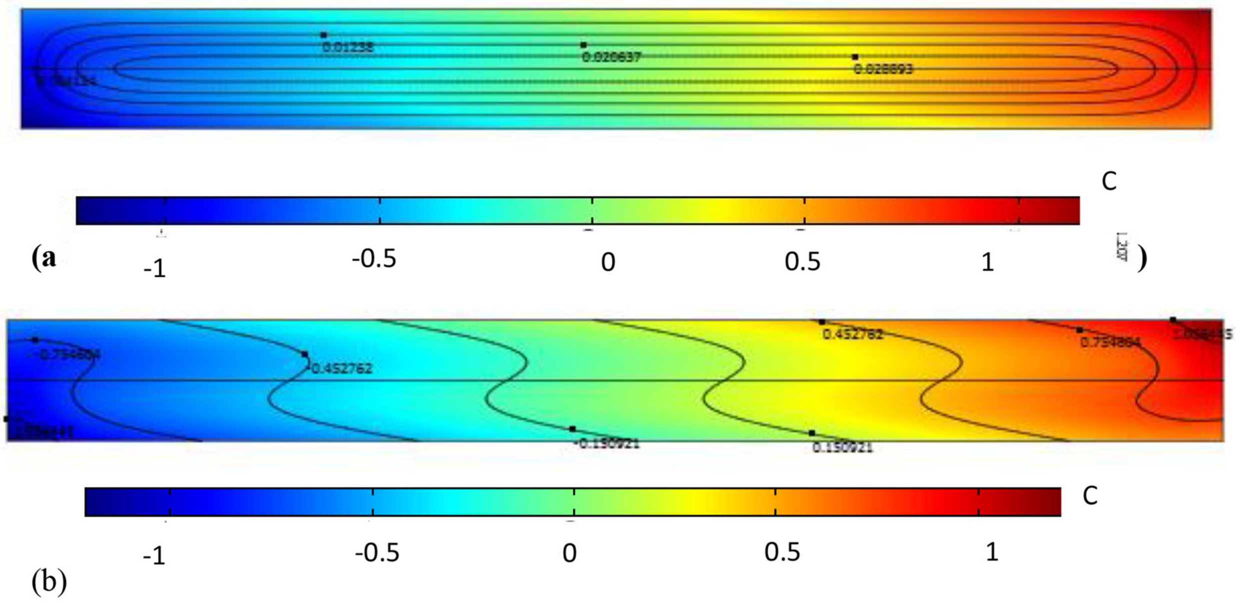

Fig. 10. Streamlines (a) and iso-mass fraction lines (b) obtained for $A = 200$, $Ra = -60$.

in table 1. By replacing in eq. (16) Ra by $Ra = 2.84\ 10^{10} \Delta T H^3$, we obtain the m^r function only of H and ΔT.

In fig. 6 are reported the analytical results obtained for the real mass fraction gradient m^r *versus* the acoustic streaming parameter A and the temperature difference $\Delta T = T_1 - T_2$.

5 Numerical simulations

The system of eq. (2) associated to the boundary conditions (3) is solved numerically using a finite element code (Comsol industrial code) with a rectangular grid, better suited to the rectangular configuration. For the computations, an aspect ratio $B = 10$ is considered. The quadrangle spatial resolution is 200×50 or 200×60 and even for high values of the Rayleigh number leads to the same result. In order to eliminate the vertical walls effect, to determine the mass fraction gradient, the curve $C = C(y, z)$ is plotted at a given value of $y (y = 0.5)$ in fig. 7 for $A = 60$ and $Ra = -200$ and in fig. 8 for $A = 200$ and $Ra = -60$. The results of direct numerical simulations are in good agreement with the analytical ones. However, there is a slight influence of the vertical walls of the profile of the mass fraction which is no longer linear along z due to the confinement of the cavity. The analytical resolution is obtained for a cavity of infinite horizontal extension.

The streamlines and iso-mass fraction lines associated to the three-cell flow superimposed along the z-axis obtained for $A = 60$ and $Ra = -200$ and those associated to the unicellular flow obtained for $A = 200$ and $Ra = -60$, in the case of the binary mixture water (60.88 wt%) – ethanol, are presented, respectively, in fig. 9(a) and (b) and in fig. 10(a) and (b).

6 Conclusion

In this paper, the influence of the acoustic streaming on species separation in a rectangular cavity, filled with a binary fluid was presented in the gravity field and in the limit case of weightlessness. A new experimental configuration was considered in order to obtain an important species separation. To the authors' knowledge, no work has yet been presented on this topic.

In the presence of the gravity field, for moderate values of the acoustic parameter A and the temperature difference $\Delta T > \Delta Tc > 0$, the flow obtained can be either unicellular or can consist of three counter-rotating cells superimposed along the vertical axis. In this case, the flow inside the cavity is the superposition of the flow generated by the ultrasonic wave and a convective flow of thermal origin, in the opposite direction.

In the case where the cavity is heated from above and for $D_T > 0$, the heavy component migrates towards the cold wall at the bottom of the cell. At first the flow is generated by the ultrasonic wave since the imposed thermal gradient has a stabilizing effect. The heaviest component accumulates gradually near the vertical wall $z = 0$, while the lightest one accumulates near the vertical wall $z = B$. The presence of a horizontal mass gradient in the cavity then generates a flow in the opposite direction of the one due to the acoustic wave. The superposition of these two flows leads to a flow with three counter-rotating cells one above the other one. This configuration is very unusual in natural convection. The analytical results were corroborated by numerical simulations using finite elements Navier-Stokes solver.

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Author contribution statement

All authors discussed the results and contributed to the final manuscript. AM highlighted the physical interpretation of the results obtained.

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