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Discrete predictor-based event-triggered control of networked control systems

Vincent Léchappé^a, Emmanuel Moulay^b, Franck Plestan^c, Qing-Long Han^d

^a*INSA de Lyon – Laboratoire Ampère, UMR CNRS 5005, Villeurbanne, France*

^b*XLIM, UMR CNRS 7252, Université de Poitiers, Poitiers, France*

^c*Ecole Centrale de Nantes – LS2N, UMR CNRS 6004, Nantes, France*

^d*School of Software and Electrical Engineering, Swinburne University of Technology, Melbourne, Vic 3122, Australia*

Abstract

A predictor-based controller combined with two event-triggering mechanisms is proposed in order to control an LTI system over a network. The controller is designed in the discrete-time domain which allows to deal with a long sampling period. Similarly large input and output delays can be compensated thanks to the use of a predictor-based method. Two event-triggering mechanisms, in the sensor-to-controller and controller-to-actuator channels are introduced in order to limit the number of packets sent over the network while preserving the ultimate boundedness of the solutions. The effect of input and output quantization introduced by the network is considered in the stability analysis. The results are illustrated by simulation.

Key words: Event-triggered control; predictor-based control; networked control system.

1 Introduction

The control of systems through a network, named Networked Control Systems (NCS), has received a lot of attention from the control theory community [3], [29]. Indeed, the communication of the information over a network brings new challenges such as input/output delays, sampled measurement, hold feedback and quantization among others. The majority of the results on NCSs deals with some of the constraints mentioned above. In the next paragraphs a literature review is given on these different topics.

One of the methods to deal with input and output (I/O) delays is to use predictor-based control since it allows to compensate for large delays [24]. The results on this topic are mainly focused on continuous-time systems [1]. In order to extend the result from continuous-time systems with I/O delays to sampled-data systems with I/O delays, the emulation method has been used. In [16], a continuous-discrete observer and a predictor-based con-

troller are combined to stabilize sampled-data system with a sufficiently small sampling period and an arbitrarily large delay. In [22] and [23], an event-triggered predictor-based controller is used to stabilize a sampled-data system with both input and output delays.

Few results use the discrete-time domain approach to design a predictor-based controller with both input and output delays. In [14], a predictive controller based on a discrete-time version of the plant is designed. It is shown that it is robust to small variations of the delay and the sampling period. As a difference with the current result, the delay is a multiple of the sampling period and there is no output delay. In [18], a reduction approach for nonlinear sampled-data systems with an input delay is proposed. Note that in [6], a prediction is used in order to improve the event-triggering mechanism but not to compensate for delays.

The first results on event-triggered control were mainly focused on delay-free systems with continuous measurement [25]. With the development of distributed control, some articles have extended the event-triggered control to deal with periodic measurement and delays [7]. However, to the best knowledge of the authors, only few results have combined predictor-based control with event-

Email addresses: vincent.lechappe@insa-lyon.fr (Vincent Léchappé), emmanuel.moulay@univ-poitiers.fr (Emmanuel Moulay), franck.plestan@ec-nantes.fr (Franck Plestan), qhan@swin.edu.au (Qing-Long Han).

triggering mechanisms [22], [23]. Note that none of the results mentioned above study the effect of quantization on the stability. Note also that these latter results [22] and [23] consider non-uniform sampling and delay uncertainties. As a consequence, the stability is guaranteed only for a sufficiently small sampling period because it is difficult to use the discrete-time approach in this context.

Seminal results about quantization are focused on the delay-free systems [12]. More recently, the combination of delay, event-triggering mechanism and quantization has been partially addressed. The control of systems with delay and quantization was tackled in [4] (only output quantization) and [28] (both channels quantization). In [26], event-triggering mechanisms and quantization are considered in both communication channels; however, no delay is taken into account.

The first contribution of the present work is to use the discrete-time approach to estimate the retarded output and to compute a predictor-based controller which ensure that the system is ultimately bounded even for a large sampling period. The second contribution is the study of quantization effect on the stability of the closed-loop system in presence of event-triggering mechanisms in both channels. Finally, the last contribution is to propose a tuning method based on the Artstein reduction method [1] in order to help the gain computation along.

2 Problem statement

In this paper, we deal with an LTI system controlled over a communication network (see Figure 1). The system is represented by the following equations

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ and matrices A , B and C are constant and have appropriate dimensions. The initial condition of the system is $x(\theta) = x_0$, for all $\theta \in [-\tau_y, 0]$ where τ_y is a constant and known output delay introduced by the network. The input applied to system (1) is piecewise constant

$$\begin{cases} u(t) = u(\xi_k) \quad \forall t \in [t_k, t_{k+1}), \\ u(t) = 0 \quad \forall t < t_0 \end{cases} \quad (2)$$

with $t_k = \xi_k + \tau_u$ where τ_u is a constant and known input delay introduced by the network. The t_k 's represent the sampling instants on the actuator side. Note that the ξ_k 's define the sampling instants on the controller side and are defined by $\xi_k = s_k + \tau_y$ with s_k the sampling instants on the sensor side. The s_k 's are defined as follows $s_{k+1} - s_k = \Delta$, $s_0 = 0$ with $\Delta > 0$ the constant sampling period. This implies that $t_0 = \tau_u + \tau_y$. These different instants are represented on Figure 2 and all the notations are reminded in Table 1 for clarity. In

addition to the input and output constant delays τ_u and τ_y , it is also considered that the network induces quantization on both channels because of the limited number of bytes that can be transmitted in a packet. In order to model the quantization phenomenon, one defines, as in [12], the following functions q^i such that for all v of appropriate dimension

$$\begin{cases} \|q^i(v) - v\| \leq m_i & \text{if } \|v\| \leq M_i \\ \|q^i(v) - v\| > M_i - m_i & \text{else} \end{cases} \quad (3)$$

with $M_i, m_i > 0$ for $i \in \{y, u\}$. As a result, on the sensor-to-controller channel, the value received by the controller is

$$y_k = q^y(\tilde{y}(s_k)) \quad (4)$$

where $\tilde{y}(s_k)$ denotes the output of an event-triggering mechanism that will be defined in Section 3. Similarly, the control value received by the actuator is

$$u(\xi_k) = q^u(\tilde{u}_k) \quad (5)$$

where \tilde{u}_k denotes the output of an event-triggering mechanism that will be defined in Section 4. Note that a saturation is underlying in the quantization definition (3). This saturation level is defined by M_u and M_y for the input and the output respectively. Similarly the quantization levels are defined by m_u and m_y for the input and the output respectively. Note that the saturation level has to be larger than the quantization level, i.e. $M_u > m_u$ and $M_y > m_y$. Before stating the assumptions, as in [2], note that there exists an integer $h \geq 0$ such that

$$h\Delta \leq \tau = \tau_y + \tau_u < (h+1)\Delta. \quad (6)$$

Assumption 1 *The pair (\bar{A}, \bar{B}) is controllable where $\bar{A} = e^{A\Delta}$, $\bar{B} = A^{-h}\bar{B}_1 + A^{-h-1}\bar{B}_2$ with $\bar{B}_1 = e^{(h+1)\Delta-\tau} \int_0^{\tau-h\Delta} e^{As} ds B$ and $\bar{B}_2 = e^{A((h+1)\Delta-\tau)} \int_0^{\tau-h\Delta} e^{As} ds B$.*

Assumption 2 *The pair (A, C) is observable.*

Denoting by $\sigma(A)$ the spectrum of the matrix A , we state the following assumption.

Assumption 3 *For any $\lambda_i, \lambda_k \in \sigma(A) \cap CRHP^1$,*

$$\lambda_i \neq \lambda_k + jl \frac{2\pi}{\Delta}, \quad l = \pm 1, \pm 2, \dots \quad (7)$$

Assumption 1 arises because of the presence of the delays that are not necessarily multiples of the sampling period Δ . A similar assumption is used in [9] and [13]. Assumption 3 is necessary in order to avoid ‘‘pathological’’ cases detailed in [17] for which the controllability and observability are lost because of the sampling. Note that

$$u(t) = u(\xi_k) = u(t_k - \tau_u) \quad \forall t \in [t_k, t_{k+1}) \quad (8)$$

¹ Closed Right Half of the complex Plane.

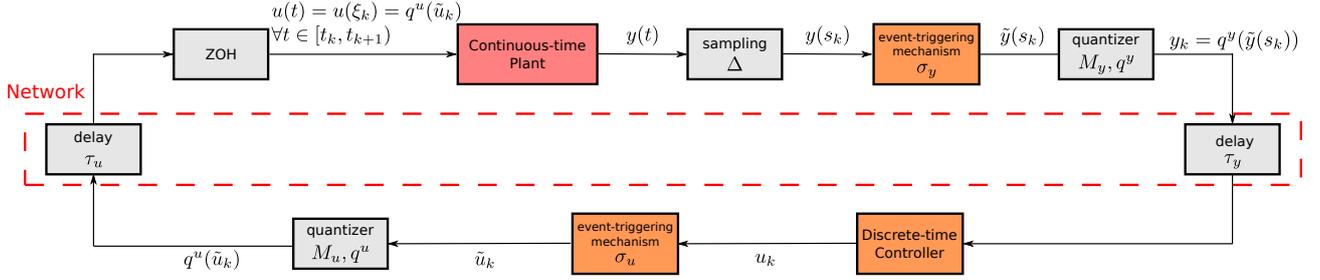


Fig. 1. Networked control system with delay, quantization and event-triggering mechanisms on both communication channels.

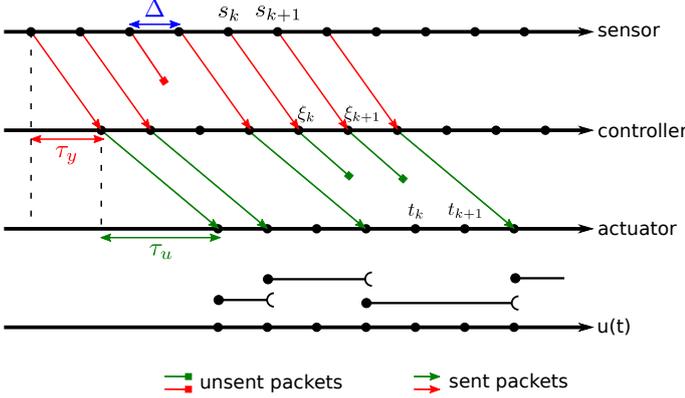


Fig. 2. Transmitted signals and event-triggering mechanism

so system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + Bu(t_k - \tau_u) \quad \forall t \in [t_k, t_{k+1}). \quad (9)$$

Similarly, one has

$$y(s_k) = y(\xi_k - \tau_y). \quad (10)$$

From (9) and (10), it is clear that the system and the communication channels can be seen as a whole system with input and output delays. In the following sections, event-triggering mechanisms will be introduced in both channels in order to limit the network congestion by sending as few packets as possible while preserving the stability.

The different variables and sampling instants are summed up on Figures 1 and 2 respectively. Note that since τ_u and τ_y are constant, one has $\xi_{k+1} = \xi_k + \Delta$ and $t_{k+1} = t_k + \Delta$.

Remark 1 Note that the quantizer blocks are indirect effect of the network but are not generated by the network itself. Indeed, the data is formatted to the desired number of bytes before being sent over the network. Note also that the order of the “event-trigger” and “quantizer” blocks could be switched without affecting the form of the stability conditions given in the section below. Our preference to present the case with the “event-trigger” block before the “quantizer” block is because one can consider that the quantization effect comes from conversions due to hardware architecture like a conversion for wireless transmission.

In the next sections, the discrete-time method [8] is used in order to design a discrete-time observer (Section 3) and a discrete-time predictor-based controller (Section 4).

Table 1
Notations used in this paper

Variables	Meaning
τ_u	constant input delay
τ_y	constant output delay
τ	round trip delay: $\tau = \tau_u + \tau_y$
h	constant such that $h\Delta \leq \tau < (h+1)\Delta$
Δ	constant sampling period
s_k	sensor sampling instants: $s_k = k\Delta$
ξ_k	controller sampling instants: $\xi_k = s_k + \tau_y$
t_k	actuator sampling instants: $t_k = \xi_k + \tau_u$

3 Discrete-time observer design

Since only a part of the state is available, a state observer is needed to estimate the whole state. From (1), one gets $\dot{x}(t - \tau_y) = Ax(t - \tau_y) + Bu(t - \tau_y)$ so denoting $\bar{x}(t) = x(t - \tau_y)$, one has $\dot{\bar{x}}(t) = A\bar{x}(t) + B\bar{u}(t - \tau_y)$. This leads to the discrete-time system

$$\bar{x}(\xi_k + \Delta) = \bar{A}\bar{x}(\xi_k) + \bar{B}_1 u(\xi_k - h\Delta) + \bar{B}_2 u(\xi_k - (h+1)\Delta) \quad (11)$$

with \bar{A} , \bar{B}_1 and \bar{B}_2 defined in Assumption 1, computations details can be found in [2]. Note that the design of the observer and controller will mainly rely on discrete-time system (11). In order to limit the number of packets sent over the network, a periodic event-triggering mechanism [20] between the sensor and the controller is implemented as follows:

$$\tilde{y}(s_k) = \begin{cases} y(s_k) & \text{if } \|\tilde{y}(s_{k-1}) - y(s_k)\| > \sigma_y \|y(s_k)\| \\ \tilde{y}(s_{k-1}) & \text{if } \|\tilde{y}(s_{k-1}) - y(s_k)\| \leq \sigma_y \|y(s_k)\| \end{cases} \quad (12)$$

with $\sigma_y \geq 0$ to be determined later.

Remark 2 Note that the minimal inter-event time is equal to the sampling period Δ so Zeno behaviour cannot occur with periodic event-trigger [21].

Defining the event-trigger error for the variable y by $e^y(s_k) = \tilde{y}(s_k) - y(s_k)$, it follows from (12) that

$$\|e^y(s_k)\| \leq \sigma_y \|y(s_k)\|. \quad (13)$$

We can now design the discrete-time system

$$\begin{aligned} \hat{x}(\xi_k + \Delta) = & \bar{A}\hat{x}(\xi_k) + \bar{B}_1 u(\xi_k - h\Delta) + \bar{B}_2 u(\xi_k - (h+1)\Delta) \\ & + L[C\hat{x}(\xi_k) - y_k]. \end{aligned} \quad (14)$$

Reminding that $y_k = q^y(\tilde{y}(s_k))$ and $y(s_k) = Cx(s_k) = Cx(\xi_k - \tau_y) = C\bar{x}(\xi_k)$, the dynamics of the error denoted by

$$e(\xi_k) = \hat{x}(\xi_k) - \bar{x}(\xi_k) \quad (15)$$

has the dynamics

$$\begin{aligned} e(\xi_k + \Delta) = & (\bar{A} + LC)e(\xi_k) \\ & \underbrace{-Le^y(s_k)}_{\text{event-trigger error}} + \underbrace{L[\tilde{y}(s_k) - q^y(\tilde{y}(s_k))]}_{\text{quantization error}}. \end{aligned} \quad (16)$$

If L is chosen such that $\bar{A} + LC$ is Schur then the error converges in a ball around zero for any sampling period whatever the delay size so (14) is an observer of (11). This means that $\hat{x}(\xi_k)$ tends to a neighborhood of $x(\xi_k - \tau_y)$ which size depends on the event-trigger error $e^y(s_k)$ and the quantization error $\tilde{y}(s_k) - q^y(\tilde{y}(s_k))$.

Remark 3 Note that when Assumptions 2 and 3 are true, it is always possible to choose L such that $\bar{A} + LC$ is Schur [10].

4 Discrete predictor-based controller design

In this section, a predictor-based controller is designed using the estimated state from the above section. The design is based on an augmented system, as in [2], [13], [14], [15] obtained from the discrete system (11). In order to get rid of the delays $h\Delta$ and $(h+1)\Delta$ in (11), an extended state $X(\xi_k) = [\bar{x}(\xi_k)^T, u(\xi_k - \Delta)^T, \dots, u(\xi_k - h\Delta)^T, u(\xi_k - (h+1)\Delta)^T]^T \in \mathbb{R}^{(n+(h+1)m) \times 1}$ is introduced where h is defined by (6). This leads to the extended system

$$X(\xi_k + \Delta) = A_{ext}X(\xi_k) + B_{ext}u(\xi_k) \quad (17)$$

for $h > 0$ with

$$A_{ext} = \begin{bmatrix} \bar{A} & 0 & 0 & \dots & \bar{B}_1 & \bar{B}_2 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & I_m & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & I_m & 0 & 0 \\ 0 & \dots & \dots & 0 & I_m & 0 \end{bmatrix}, B_{ext} = \begin{bmatrix} 0 \\ I_m \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

where I_m is the identity matrix of order m .

Remark 4 The case $h = 0$ can be treated similarly with slightly different matrices A_{ext} and B_{ext} . In the sequel we treat the case where $h > 0$ but all the following results hold for $h = 0$ as well.

Remark 5 From Assumption 1, one can conclude that the discrete-time delayed system (11) is controllable [19].

It follows that extended system (17) is also controllable since it is just a different way of writing the same system (11). Assuming that (A, B) is controllable and that Assumption 3 is true would probably be enough to guarantee, in most of the cases, that the sampled system with delay (11) is controllable but this would require a rigorous analysis which is not the purpose of this paper.

In order to limit the number of packets sent on the network, a second event-triggering mechanism is implemented on the controller-to-actuator side as follows $u(\xi_k) = q^u(\tilde{u}_k)$ where

$$\tilde{u}_k = \begin{cases} u_k & \text{if } \|\tilde{u}_{k-1} - u_k\| > \sigma_u \|u_k\| \\ \tilde{u}_{k-1} & \text{if } \|\tilde{u}_{k-1} - u_k\| \leq \sigma_u \|u_k\| \end{cases} \quad (18)$$

with $\sigma_u \geq 0$ that will be chosen in the sequel and where

$$u_k = K\hat{X}(\xi_k) \quad (19)$$

with $\hat{X}(\xi_k) = [\hat{x}(\xi_k)^T, u(\xi_k - \Delta)^T, \dots, u(\xi_k - h\Delta)^T, u(\xi_k - (h+1)\Delta)^T]^T \in \mathbb{R}^{(n+(h+1)m) \times 1}$. Defining the event-trigger error for the variable u_k by $e^u(\xi_k) = \tilde{u}_k - u_k$, one can deduce from (18) that

$$\|e^u(\xi_k)\| \leq \sigma_u \|u_k\|. \quad (20)$$

Then (17) becomes

$$\begin{aligned} X(\xi_k + \Delta) = & (A_{ext} + B_{ext}K)X(\xi_k) + \underbrace{B_{ext}e^u(\xi_k)}_{\text{event-trigger error}} \\ & + \underbrace{B_{ext}[q^u(\tilde{u}_k) - \tilde{u}_k]}_{\text{quantization error}} + \underbrace{B_{ext}KGe(\xi_k)}_{\text{observation error}} \end{aligned} \quad (21)$$

with $G = [I_n, 0, \dots, 0]^T \in \mathbb{R}^{(n+(h+1)m) \times n}$ and $e(\xi_k)$ is the observation error defined in (15). Rewriting (16) and (21) as a single extended system gives

$$\chi(\xi_k + \Delta) = \mathcal{A}\chi(\xi_k) + \phi(\xi_k) \quad (22)$$

with $\chi(\xi_k) = [X(\xi_k)^T, e(\xi_k)^T]^T$ and

$$\mathcal{A} = \begin{bmatrix} A_{ext} + B_{ext}K & B_{ext}KG \\ 0 & \bar{A} + LC \end{bmatrix},$$

$$\phi(\xi_k) = \begin{bmatrix} B_{ext}e^u(\xi_k) + B_{ext}[q^u(\tilde{u}_k) - \tilde{u}_k] \\ -Le^y(s_k) + L[\tilde{y}(s_k) - q^y(\tilde{y}(s_k))] \end{bmatrix}.$$

From Assumptions 1, 2, 3, it is possible to choose K and L such that \mathcal{A} is Schur so there exists $P > 0$ such that

$$\mathcal{A}^T P \mathcal{A} - P = -I_{2n+(h+1)m}. \quad (23)$$

5 Main result

Before stating the main result, it is reminded that usually event-triggering mechanisms are determined in order to preserve the asymptotic stability of a system by maintaining the time-derivative of a Lyapunov function negative. Here a similar idea is used but with the concept of uniform ultimate boundedness².

² See definition 4.6 in [11].

Theorem 6 Assume that (23) is verified and that σ_y and σ_u are small enough so that

$$1 - 2c_1\|\mathcal{A}^T P\| - c_1^2\|P\| > 0 \quad (24)$$

with $c_1 = \sigma_u\|K\| + \sigma_y\|L\|\|C\|$ and that M_y and M_u are large enough compared to m_y and m_u respectively so that

$$\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}c_6 > \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}c_5 \quad (25)$$

with

$$\begin{aligned} c_2 &= m_u + \|L\|m_y, c_3 = 1 - 2c_1\|\mathcal{A}^T P\| - c_1^2\|P\| \\ c_4 &= \|\mathcal{A}^T P\| + c_1\|P\|, c_5 = c_2 \frac{c_4(1+\varepsilon) + \sqrt{c_4^2(1+\varepsilon)^2 + c_3\|P\|}}{c_3}, \\ c_6 &= \min\left(\frac{M_y}{(1+\sigma_y)\|C\|}, \frac{M_u}{(1+\sigma_u)\|K\|}\right) \end{aligned} \quad (26)$$

for any $\varepsilon > 0$. Then, the solutions of the extended system (22) are uniformly ultimately bounded which guarantees that there exists an integer T such that for all initial conditions

$$\left\| \begin{bmatrix} x(0) \\ \hat{x}(\tau_y) - x(0) \end{bmatrix} \right\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}c_6, \quad (27)$$

the solutions of system (1) satisfy

$$\|x(s_k)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}c_5 \quad (28)$$

for all $k \geq T$.

PROOF. Defining $V_k(\xi_k) = \chi(\xi_k)^T P \chi(\xi_k)$, one obtains

$$\begin{aligned} V_{k+1}(\xi_{k+1}) - V_k(\xi_k) &= -\|\chi(\xi_k)\|^2 + 2\chi(\xi_k)^T \mathcal{A}^T P \phi(\xi_k) \\ &\quad + \phi(\xi_k)^T P \phi(\xi_k). \end{aligned} \quad (29)$$

Note that $y(s_k) = C\bar{x}(\xi_k)$ so

$$\|y(s_k)\| \leq \|C\|\|\bar{x}(\xi_k)\| \leq \|C\|\|\chi(\xi_k)\|. \quad (30)$$

Then from (13) one gets

$$\|e^y(s_k)\| \leq \sigma_y\|C\|\|\chi(\xi_k)\|. \quad (31)$$

In addition, from (19) and (20), one obtains

$$\|e^u(\xi_k)\| \leq \sigma_u\|K\|\|X(\xi_k)\| \leq \sigma_u\|K\|\|\chi(\xi_k)\|. \quad (32)$$

Reminding that for $\|\tilde{u}_k\| \leq M_u$ and $\|\tilde{y}(s_k)\| \leq M_y$, one has

$$\|q^u(\tilde{u}_k) - \tilde{u}_k\| \leq m_u \quad (33)$$

and

$$\|q^y(\tilde{y}(s_k)) - \tilde{y}(s_k)\| \leq m_y, \quad (34)$$

then one gets $\|\phi(\xi_k)\| \leq c_1\|\chi(\xi_k)\| + c_2$ with $c_1 = \sigma_u\|K\| + \sigma_y\|L\|\|C\|$ and $c_2 = m_u + \|L\|m_y$. As a consequence, one has $V_{k+1}(\xi_{k+1}) - V_k(\xi_k) \leq -c_3\|\chi(\xi_k)\|^2 + 2c_2c_4\|\chi(\xi_k)\| + c_2^2\|P\|$ with $c_3 = 1 - 2c_1\|\mathcal{A}^T P\| - c_1^2\|P\|$ and $c_4 = \|\mathcal{A}^T P\| + c_1\|P\|$. From the above inequality we can deduce that choosing σ_y and σ_u sufficiently small guarantees that $c_3 > 0$. Thus, if $c_5 \leq \|\chi(\xi_k)\| \leq c_6$ with

c_5 and c_6 defined in Theorem 6, one gets

$$V_{k+1}(\xi_{k+1}) - V_k(\xi_k) \leq -2c_2c_4\varepsilon\|\chi(\xi_k)\| \quad (35)$$

for any $\varepsilon > 0$. Note that for

$$\|\chi(\xi_k)\| \leq \min\left(\frac{M_y}{(1+\sigma_y)\|C\|}, \frac{M_u}{(1+\sigma_u)\|K\|}\right),$$

one has

$$\|\chi(\xi_k)\| \leq \frac{M_u}{(1+\sigma_u)\|K\|} \quad (36)$$

so

$$\|\tilde{u}_k\| \leq \|e^u(\xi_k)\| + \|u_k\| \leq (1+\sigma_u)\|K\|\|\chi(\xi_k)\| \leq M_u \quad (37)$$

then (33) holds. Similarly since

$$\|\chi(\xi_k)\| \leq \min\left(\frac{M_y}{(1+\sigma_y)\|C\|}, \frac{M_u}{(1+\sigma_u)\|K\|}\right),$$

one has

$$\|\chi(\xi_k)\| \leq \frac{M_y}{(1+\sigma_y)\|C\|}. \quad (38)$$

Since $\|\tilde{y}(s_k)\| \leq \|e^y(s_k)\| + \|y(s_k)\|$ then from (30), (31) and (38), it can be deduced that

$$\|\tilde{y}(s_k)\| \leq (1+\sigma_y)\|C\|\|\chi(\xi_k)\| \leq M_y \quad (39)$$

then (34) holds. Note that (25) implies

$$\sqrt{\lambda_{\min}(P)}c_6 > \sqrt{\lambda_{\max}(P)}c_5. \quad (40)$$

Since

$$\lambda_{\min}(P)\|\chi(\xi_k)\|^2 \leq V_k \leq \lambda_{\max}(P)\|\chi(\xi_k)\|^2 \quad (41)$$

and (35) and (40) hold, the assumptions of Theorem 4.18³ in [11] are verified and one can conclude on the uniform ultimate boundedness of the solutions of (22). That is to say, for every initial conditions $\|\chi(\xi_0)\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}c_6$ there is an integer T such that

$$\|\chi(\xi_k)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}c_5, \quad \forall k \geq T. \quad (42)$$

Since $u(t) = 0$ for all $t \leq 0$, one has

$$\|[x(s_0)^T, e(\xi_0)^T]^T\| = \|\bar{x}(\xi_0)^T, e(\xi_0)^T\|^T = \|\chi(\xi_0)\|. \quad (43)$$

As a result if (27) is verified then (42) holds and since $\|x(s_k)\| = \|\bar{x}(\xi_k)\| \leq \|\chi(\xi_k)\|$, one obtains (28). This ends the proof. \square

Note that for any value of the delays τ_y, τ_u and the sampling period Δ , it is always possible to find m_u, σ_u, m_y and σ_y small enough and M_u, M_y large enough such that conditions (24), (25) and (27) are verified. As far as the quantization is concerned, it is taken into account in the stability analysis but no particular method is used to limit its effect. However, this is an interesting topic for future development. The reader is also referred to [26]

³ The theorem is given in [11] for continuous-time systems but the discrete-time version of the result is straightforward.

where a dynamic quantizer is used with event-triggering mechanisms and an output feedback. Note that the values of the delays and the sampling period appears in conditions (24), (25) and in the bound (28) through the matrix \mathcal{A} .

Remark 7 *Uniform ultimate boundedness is obtained instead of asymptotic stability because of the input and output quantizations.*

Let us analyze some particular cases of the previous result:

- If the quantizers have an infinite number of levels ($M_y \rightarrow \infty, M_u \rightarrow \infty$) then (25) is always verified because c_5 is fixed and $c_6 \rightarrow \infty$. This means that the state converges to a neighborhood of the origin for any initial conditions on the system and on the observer.
- If $m_y \rightarrow 0$ and $m_u \rightarrow 0$ (no quantization) then $c_2 \rightarrow 0$ and $c_5 \rightarrow 0$ so (25) is always verified and the state converges exactly to the origin.
- If $\sigma_y = \sigma_u = 0$ (time-trigger) then $c_1 = 0$, c_3 is maximum and c_4 is minimum so c_5 is minimum which means that the more packets are sent, the smaller the convergence neighborhood around the origin will be.

Compare to simulated values, $\sqrt{\frac{\lambda_{min}(P)}{\lambda_{max}(P)}}c_6$ is underestimated and $\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}c_5$ is overestimated because of the conservatism of the stability analysis. The optimization of these values is out of the scope of this paper and the reader is referred to [27] for additional details on this topic. Note that from condition (25), one can say that all the solutions of (1) that start in ball $B_0 = \left\{ \|x\| \leq \sqrt{\frac{\lambda_{min}(P)}{\lambda_{max}(P)}}c_6 \right\}$ enter the smaller ball $B_f = \left\{ \|x\| \leq \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}c_5 \right\}$ after a finite time T . Finally, note that escape in finite is not possible because the system is linear. Consequently, the system cannot escape in finite time between sampling instants. More precisely, the relation between $x(t)$ and $x(s_k)$ is $x(t) = e^{A(t-s_k)}x(s_k) + \int_{s_k}^t e^{A(t-s)}Bu(s)ds$ for all $t \in [s_k, s_{k+1}[$. Since u is bounded by M_u , one gets the following maximization

$$\|x(t)\| \leq e^{\|A\|\Delta} \|x(s_k)\| + \frac{e^{\|A\|\Delta} - 1}{\|A\|} \|B\| M_u$$

so from (28) one gets

$$\|x(t)\| \leq e^{\|A\|\Delta} \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}c_5 + \frac{e^{\|A\|\Delta} - 1}{\|A\|} \|B\| M_u.$$

6 Computation of the gain K

The advantage of the extended system representation (17) is that a discrete-time system with delays can be turned into a discrete-time system without delay. However, a drawback of this representation is that the size of

the extended system can be very large especially if the delay is large with respect to the sampling period. For example, if $\tau_u + \tau_y = 0.6$ s and $\Delta = 0.001$ s then the extended system will be of size 600. This can make the computation of gain K difficult since it relies on a pole placement on the extended system⁴ (17). A less computationally expensive computing strategy inspired by the Artstein transformation for discrete-time systems [5] is presented below.

First define the variable

$$z_k = \hat{x}_k + \sum_{j=0}^{h-1} \bar{A}^{-j-1} \bar{B}_1 u_{k-h+j} + \sum_{j=0}^h \bar{A}^{-j-1} \bar{B}_2 u_{k-h-1+j} \quad (44)$$

where h is defined in (6) and \bar{A} , \bar{B}_1 and \bar{B}_2 are defined after (11). The auxiliary state z_k verifies the discrete dynamics

$$z_{k+1} = \bar{A}z_k + \bar{B}u_k + LCe_k - Le_k^y + L[\tilde{y}_k - q^y(\tilde{y}_k)] \quad (45)$$

with \bar{B} defined in Assumption 1. Similarly to system (17), system (45) is delay free but it has the same dimension as the original system.

Remark 8 *Note that in the above section, the extended system approach has been used instead of the Artstein reduction method because of the combination with the event-trigger scheme. Indeed, the relation $\|\bar{x}_k\| \leq \|X_k\| \leq \|\chi_k\|$ used to get (30) was not transposable directly to z_k because z_k depends on u .*

Remark 9 *Note that negative powers of \bar{A} are well defined since $\bar{A} = e^{A\Delta}$.*

The reduced system (45) is delay free so we can apply the controller

$$u_k = K_z z_k \quad (46)$$

to stabilize it where K_z is chosen such that $\bar{A} + \bar{B}K_z$ has eigenvalues inside the unit circle. Note that this is possible since from Assumption 1, system (45) is controllable.

Remark 10 *As in Remark 5, this is usually possible to choose K_z such that $\bar{A} + \bar{B}K_z$ is Schur provided that Assumption 1 is true and the sampling is not "pathological".*

Feedbacks (19) and (46) both depend on the same variables \hat{x}_k and $u_{k-1}, \dots, u_{k-h-1}$ so using the notation $K = [K_0, \dots, K_{h+1}]$ and taking

$$\begin{aligned} K_0 &= K_z, \\ K_j &= K_z(\bar{A}^{-j}\bar{B}_1 + \bar{A}^{-j-1}\bar{B}_2), \quad j = 1, \dots, h \\ K_{h+1} &= K_z\bar{A}^{-1}\bar{B}_2 \end{aligned} \quad (47)$$

lead to $K\hat{X}(\xi_k) = K_z z_k$. As a consequence, one can use a pole placement method to compute K_z in system (45) and then use relations (47) to compute K . This method

⁴ The use of matlab function "place" is not well adapted for large systems.

avoid a pole placement method on the extended system which size is $n + (m + 1)h$.

7 Simulation results

The second order system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -0.5 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad (48)$$

with $y(t) = [1 \ 0]x(t)$ is chosen to illustrate the design presented in the above sections. The communication delays τ_u and τ_y will be defined later. Note that the system is open-loop unstable since the eigenvalues of A are $0.5 \pm 0.5i$. The initial condition of the observer is $\hat{x}(0) = [0, 0]^T$. Note that the initial condition of the system will vary in the different simulations. The eigenvalues of $\bar{A} + LC$ are fixed to $\lambda_{\bar{A}+LC} = \{e^{-4\Delta}, e^{-5\Delta}\}$. and the design method presented in Section 6 is used to compute the gain K where K_z is chosen such that $\lambda_{\bar{A}+BK_z} = \{e^{-1\Delta}, e^{-2\Delta}\}$.

7.1 Simulations 1 and 2: delay and sampling case

On Figure 3a, simulations are run with different values of the sampling period Δ . It can be observed that the system always converges to the origin whatever the sampling period. This is coherent with the result of Theorem 6 which states that there is no restriction on the sampling size.

On Figure 3b, simulations are run with different values of the delays τ_u and τ_y (incremented by 0.5 s). It can be observed that the system always converges to the origin whatever the delay values. This is consistent with the result of Theorem 6 which states that the stability of the closed-loop system can be achieved for any delay values. Note that the initial value of the input is 0 and since the system is open-loop unstable there is a large transient that is amplified with the delay size. This is inherent to the presence of the delay. Note that the choice of the delay values and the difference between τ_u and τ_y is arbitrary and has no influence on the stability. Note that since $m_y = m_u = 0$, $c_2 = 0$ so $c_5 = 0$ that is why, in both Figures 3a and 3b, the state converges exactly to the origin.

7.2 Simulation 3: delay, sampling, quantization case

On Figure 4, the phase portrait x_2 vs. x_1 where $x = [x_1, x_2]^T$ is plotted for different initial conditions of the system. One can see that if the initial condition is too far from the origin then the state diverges (black curve). This is because of the saturations M_y and M_u . When the initial conditions are close enough to the origin then the trajectories converge to a neighborhood of the origin. Balls B_0 and B_f defined at the end of Section 5 are

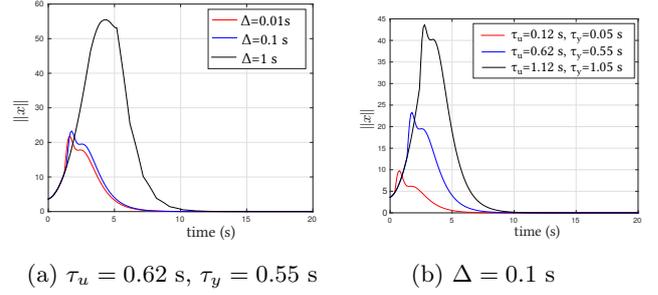


Fig. 3. $\sigma_y = \sigma_u = 0$, $m_y = m_u = 0$, $M_y = M_u = +\infty$, $x(0) = [2, -3]^T$.

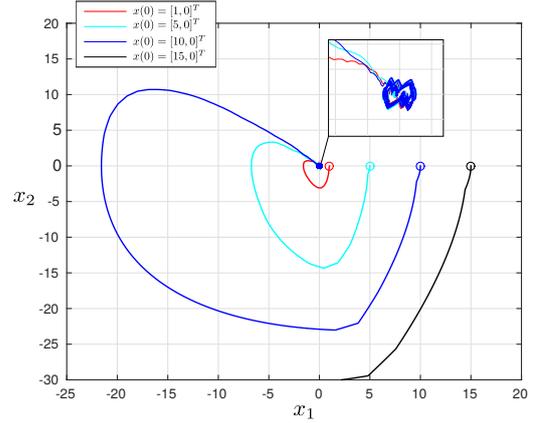


Fig. 4. $\Delta = 0.1$ s, $\tau_u = 0.12$ s, $\tau_y = 0.05$ s, $\sigma_y = \sigma_u = 0$, $m_y = m_u = 0.1$, $M_y = M_u = 25$.

not plotted because their radius values are very conservative as mentioned previously. Indeed, condition (25) is verified when M_y and M_u have an order of magnitude of 10^3 and m_y and m_u of 10^{-3} .

7.3 Simulation 4: delay, sampling, quantization and event-trigger case

Finally, the event-triggering mechanisms are tested by running simulations with different values of σ_u and σ_y . Results are gathered in the histogram of Figure 5. As expected in Theorem 6, the stability is guaranteed provided that σ_y and σ_u are sufficiently small to verify (25) and one can see that stability can be maintained even if only 25% of the packets are sent. However, one has to keep in mind that reducing the number of packets sent by increasing the event-trigger thresholds also result in a reduction of the admissible set of initial condition B_0 since c_6 is inversely proportional to σ_u and σ_y . Note that without quantization ($m_y = m_u = 0$, $M_y = M_u = +\infty$) it is possible to further reduce the number of packets sent to around 17%.

8 Conclusion

A discrete-time observer is designed in order to estimate the retarded state. Then, a predictor-based controller is

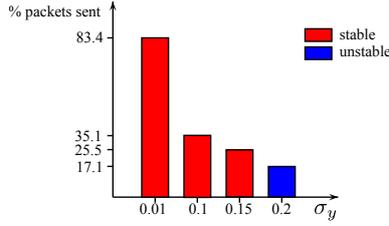


Fig. 5. $\Delta = 0.1$ s, $\tau_u = 0.62$ s, $\tau_y = 0.55$ s, $m_y = m_u = 0.1$, $M_y = M_u = 25$, $x(0) = [2, -3]^T$.

computed using the estimated state. It is shown that this method can deal with large sampling period and can compensate for large input and output delays. Some conditions which consider the effect of the quantization and event-triggering mechanisms are provided in order to guarantee the uniform ultimate boundedness of the solutions. Future works include the extension to time-varying delays which can also be uncertain or unknown.

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