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On the observation of distant objects in general relativity and its implications in cosmology

Théophile Caby,

CMUP, Departamento de Matemática, Faculdade de Ciências, Universidade do Porto, Rua do Campo Alegre s/n, 4169007 Porto, Portugal. tcaby@fc.up.pt

Abstract

We carry out a careful analysis of the notion of observation of distant objects in a curved static universe. To this end, we introduce an observer-based representation referential endowed with a Euclidean structure, and postulate that all measurement are performed with respect to this reference frame. For a universe of constant positive spatial curvature, this causes a distortion of metric quantities which grows with the distance to the observer. In this context, the redshift of cosmological objects is not caused by their recessions, but instead by the spatial curvature along the trajectory of photons from the emitting source to the observer. Applying these considerations to the Einstein's static universe, we obtain a redshift/distance relationship that is consistent with cosmological measurements. We show that this effect also explains the existence of a Cosmic Microwave Background whose characteristics agree with the observations of the Planck mission. Furthermore, this model predicts the abundance of well-formed galactic structures in the high redshift universe, which were recently detected by the JWST.

1 Introduction

In 1917, Albert Einstein proposed a finite static solution to its equations of general relativity as a representation of the universe. He envisioned a universe with constant spherical curvature that required the introduction of a cosmological constant to prevent it from collapsing due to gravitational effects. However, this view of the universe soon faced two major challenges. Firstly, in 1930, Eddington raised concerns about the stability of the Einstein world under small perturbations and demonstrated that it is unstable in certain situations [2]. But it is the discovery, by Hubble, of a linear relation between the redshift of galaxies and their distance [4] that convinced Einstein, at the beginning reluctant, to finally change his mind and accept the growing consensus on models based on expansion of space [5, 7, 8]. Such models suited quite well the small amount of observational data yet available, although the existence of an initial spacetime singularity it requires raised important physical and philosophical questions, that still remain unanswered today. The discovery of a cosmic microwave background (CMB) made by Penzias and Wilson in 1964 [9] became a strong support for this theory, since its characteristics are compatible with the radiation of a dense primordial plasma. In 1998, an acceleration of the expansion of the universe was detected and surprised cosmologists [10]. This led to the introduction in the models of a dark energy that acts as a repulsive gravitational force, whose origin remains very speculative. Efforts have been made to combine all these considerations and the most accomplished result is the standard model of cosmology, so called Λ CDM.

Alternative theories have been developed to explain the redshift of cosmological objects, such as tired light theory [11], but errors have been pointed out and the theory has never been commonly accepted [12]. In the current understanding of general relativity, if we neglect

hypothetical physical processes that steal energy from photons, the only possible explanation for this redshift is a mechanism that acts on the metric of spacetime. As for now, there is growing evidence that this standard cosmological model has fundamental flaws. Thanks to a new generation of telescopes, and to an enhanced precision of measurements, some tensions in the standard model have become increasingly apparent [13, 14].

We here propose an alternative explanation for this redshift. We postulate that the wavelength of photons (as well as their frequency) is affected by the radial curvature of space. In the Einstein's static universe, this produces a redshift for incoming photons that grows with the distance of the light source, in a way that is consistent with observations. In the proposed theory, this effect originates from the observer's perception of its environment as being flat, affecting the nature of incoming light. We start by motivating this idea and by introducing a suitable referential in which we postulate that all observations of our physical environment are performed. We then show the agreement of our model with observations.

2 The perception of distant objects in a curved universe

In his book *Wholeness and the implicate order* [15], David Bohm points out a fundamental duality of the physical reality: a world, hardly accessible to our senses, home to unfamiliar processes that he calls the *implicate order*, reveals itself in an unfolded, *explicate order*, in which measurements can be performed, and which corresponds to our everyday experience of the physical environment. Originally thought to account for the puzzling properties of quantum mechanics, these views also find an echo in general relativity. In the latter, the fundamental object is a curved 4-dimensional pseudo-Riemannian manifold in which space and time are entangled in a profound interlace. Yet in our way of representing this environment, space and time are fundamentally distinct objects, for otherwise no measurement would be possible. We will refer to this fact as the *separation principle*.

In this explicate world of observation, the environment is also perceived as being flat, as we are living in a locally flat portion of the universe and our brain is equipped to make linear representations of our surroundings. To this extent, Albert Einstein writes in one of his famous 1905 paper [16]:

"If a material point is at rest relatively to a system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Euclidean geometry, and can be expressed in Cartesian co-ordinates."

A few years later, the development of general relativity challenged this view and introduced more sophisticated geometrical tools to describe the physical world. Let us now take the point of view of an observer living in a universe whose geometry is described by Einstein's field equations (possibly with a cosmological constant). In order to measure quantities such as lengths and durations associated to events around him, the observer can directly compare them with different measuring devices, such as a ruler or a chronometer. In Minkowskian spacetime and for objects in uniform linear motion with respect to the observer, direct comparison is not possible and a procedure was proposed by Einstein to determine the length of such an object, based on the constancy of the speed of light c. This procedure turns out to give a value in the direction of motion that is different from the length one would measure directly by comparing the object with a ruler. This is the core of special relativity. A similar procedure allows to determine the distance between the observer and an object at rest: one can send a light ray in the direction of the object at time t_0 , and wait for the light ray to be reflected back at time t_1 . The distance d from the event s is then defined in the following way:

$$d(s) = c \frac{t_1 - t_0}{2}.$$
 (1)

This definition of distance extends to more general spacetime geometries and is called *proper* distance in the context of a static universe. It can be thought of as the length of the

shortest spatial geodesic connecting the observer to the object.

Now that he is able to determine the distance of an object, the observer may ask the following question: Is there a procedure to assign lengths and durations to events situated at large distances, and how to do so? In a flat spacetime, and if the object is at rest, the answer is trivial: it can be done by using of the tools of Euclidean geometry. To answer this question in the more general set up of curved spacetime geometry, we have to introduce a *representation referential* R_o , according to which the observer creates a consistent representation of its physical environment.

Consider an observer living in a spatial universe U, that we suppose static. The position of the observer is denoted by $o \in U$. We denote R_o the tangent space of U at o. Let $x \in U$ be a point at proper distance d_U from o. Denote $T(x) : U \to R_o$ the representation map, which transforms a point $x \in U$ to the point T(x) in R_o situated at distance $d_R = d_U$ from o and such that the vector $\overrightarrow{oT}(x)$ has the same initial direction as that of the minimal geodesic connecting o to x in U. Suppose for now that that T is invertible. In that case, any object $X \subset U$ has a unique representation T(X) in R_o . T^{-1} is called the exponential map in the mathematical literature. In R_o , the trajectory of a photon traveling from x to oin U is represented as a straight line. We now impose a linear geometrical structure in R_o , by endowing it with the Euclidean metric. Note that T does not constitute a mere change of the coordinates describing the spatial geometry, since it has a different metric.

In this reference frame, physical objects are sometimes represented stretched, deformed, eventually split and present at different locations, which corresponds exactly to the way we perceive them. There would be no way for the observer to recover the spatial geometry in U from its observations in R_o without further knowledge of the constitution of its physical environment. For this reason, we will postulate that all the measures performed by the observer are performed with respect to R_o , and are therefore consistent with the rules of Euclidean geometry. In particular, radiations should behave as in a flat space and the photons, once emitted, keep a constant wavelength.

Now, in curved spatial geometries, although the proper distance of an object is preserved by T, the metric quantities associated to it in R_o differ from the ones in the object's natural referential, even if at rest with respect to o. To quantify this distortion, we introduce for each point x in U a scalar a(x), the *scaling factor*, which can be obtained in the following way: a bundle of light rays with infinitesimal solid angle $\delta\Omega$ is sent in the direction of x, and we compute

$$a(x) = \sqrt{\frac{\delta S}{\delta S_o}},$$

where δS is the area intersected by the bundle in the neighborhood of x in the natural local referential at x and δS_o is the area that intersects the bundle in R_o . Note that $\delta S_0 = d^2 \delta \Omega^2$ for a point at proper distance d from o. The obtained scaling field is a priori not continuous, can contain singularities and domains where it is not defined (in a black hole region for example). In order to compute it in a general set up, one needs to study the metric induced on geodesic balls. To the author's knowledge, no general treatment of this problem has been performed in the mathematical literature. It is obvious that if U is the Euclidean space, then a(x) = 1 for all x. We will, in the following section, compute this quantity explicitly in the case when U has spherical geometry.

We make the additional assumption that the speed of light is constant equal to c everywhere in R_o . For that, we must suppose that the scaling factor a(x) also applies to durations associated to events happening at x. Such time dilations has been observed for distant Supernovae [17].

3 Cosmological considerations

3.1 Einstein's world

Let us from now on place ourselves in the Einstein's static universe of radius of curvature R. It has closed spatial geometry and is the only static solution to the field equations, assuming a fined-tuned homogenous distribution of matter/energy and a positive cosmological constant [1]. The spatial component of the metric is the one induced by the ambient Euclidean metric in \mathbb{R}^4 on the manifold

$$U = S^{3} = \{ (x_{1}, x_{2}, x_{3}, x_{4}) \in \mathbb{R}^{4} : \sum_{i=1}^{4} x_{i}^{2} = R^{2} \}.$$

The time component of the metric is set equal to -1 and allows the possibility of a cyclic time. In this eternal return scenario, contemplated by many cosmogonies, the age of astronomical objects would be bounded, as observed for nearby stars.

3.2 Scaling factor and redshift

The procedure described in the preceding section gives for a point at distance d from the observer the following scaling factor:

$$a = \sqrt{\frac{\delta S}{\delta S_o}} = \frac{R \sin(\frac{d}{R})}{d}.$$
 (2)

In Fig. 1, we give a visual representation of this distortion effect in dimension 2.



Figure 1: 2-dimensional representation of the distortion effect in the Einstein universe.

As observed in Fig. 2a, the scaling factor decreases with the distance d until degeneration when the antipode A is reached, which corresponds to the distance $D = \pi R$. From the observer's perspective, space seems to vanish at this point. An infinitesimally small object located at A appears stretched over the whole celestial vault. As postulated in the previous section, incoming photons, which act as a messenger between the emitting source and the observer, keep track of the orthogonal dilation of the metric quantities. The resulting redshift z can be computed by comparing the measured wavelength λ_{ob} of an incoming photon with its wavelength at emission λ_{em} . We have by definition:

$$z = \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}.$$
(3)



Figure 2: Evolution with proper distance of the scaling factor (left) and the redshift (right) in the Einstein universe

Since

$$a = \frac{\lambda_{em}}{\lambda_{ob}},\tag{4}$$

we get that:

$$z = \frac{d}{Rsin(\frac{d}{R})} - 1.$$
(5)

A graphical representation of Eq. 5 is shown in Fig. 2b. The derivative of the scaling factor, that in big bang theories is interpreted as the rate of expansion of the universe, is given by



 $a'(d) = \frac{\cos(\frac{d}{R})}{d} - \frac{R}{d^2}\sin(\frac{d}{R}).$ (6)

Figure 3: Evolution of the derivative of the scaling factor with proper distance

As seen in Fig. 3, a' changes monotony at a certain distance d_0 . Observations of type Ia Supernovae have confirmed this behavior [6], and actual cosmological models attribute it to an acceleration of the expansion of the universe that started a some billions of years ago. In the present model, we do not have to refer to dark energy to explain this behavior. It is a simple consequence of the distortion effect.

3.3 On the relation between different distances in our model

The proper distance of celestial bodies is difficult to estimate from observation. Instead, two redshift-independent methods are commonly used to assess their distance. Both require some knowledge about the observed object. The luminosity distance d_L is computed by comparing the flux of incoming light with the known luminosity of the source. The estimation of the angular diameter distance d_A requires for its part some knowledge about the size of the observed object.

 d_L is defined implicitly in the relation:

$$F_{obs} = \frac{L}{4\pi d_L^2}.$$
(7)

Here, F_{obs} is the flux measured by the observer and L is the known luminosity of the object in its local referential. Let us consider an object situated at proper distance d from the observer, that emits n photons of energy $h\nu$ per unit of surface per unit of time, where all these quantities are expressed in the natural referential of the object. In R_o , the situation is equivalent to the following: in a flat universe, an object at proper distance d is emitting n photons of energy $h\nu/(z + 1)$ per $(z + 1)^2$ units of surface per (z + 1) units of time. Therefore, the luminosity of the object for the observer is

$$L_{obs} = L/(z+1)^4$$

In the flat space R_o , the measured flux follows the inverse square law, so that

$$F_{obs} = \frac{L_{obs}}{4\pi d^2} = \frac{L}{4\pi d^2 (z+1)^4}.$$
(8)

Combining 7 and 8, we get that

$$d_L = d(z+1)^2. (9)$$

In a space of spherical geometry, the angular diameter distance d_A is for its part given by

$$d_A = R\sin(\frac{d}{R}) = \frac{d}{z+1},\tag{10}$$

so that

$$d_L = d_A (z+1)^3. (11)$$

Note that the Etherington-Ellis reciprocity theorem does not hold in this theory.

3.4 Tests

In Fig. 4a, we test our theory by showing its best χ_2 fit for the Supernovae 1A data of the Union 2 catalog [18], performed on its 257 most distant galaxies. The same general trend is observed between the theoretical curve and the observational data, although the fit for small values of z is not satisfying. We stress however that these data are model dependent and therefore systematic bias may be at stake [19]. The best fit is obtained for $R \approx 2.67$ Gly, for which the antipodal region is at proper distance $D = R\pi = 8.4$ Gly. That is of the same order of magnitude as estimates of the distance of the cosmological horizon in the current standard model of cosmology [6].

In the right hand side of Fig. 4, we plotted 4 curves for the angular diameter distance/redshift relationship given by Eq. 3.3 for different values of R, together with the observational data taken from the hydrostatic equilibrium scenario in [20]. The large error bars for these data render a proper fit irrelevant. The estimates of R seem consistent for both approaches, although a more precise analysis of this relation, using model independent data needs to be performed. We stress that in both cases, our fits involve only one parameter: the radius of curvature R of the universe.



Figure 4: Left: Best fit of the model using the 257 most distant supernovae of the catalog. The best χ^2 fit is obtained for R = 2.67 Gly. Right: Comparison between theoretical curves and angular diameter distance data, for different values of R.

4 On the CMB

The Cosmic Microwave Background is a radiation detectable in all regions of the universe which has the property of being very regular in all directions in space, although having small anisotropies distributed with privileged angle scales [9]. The observed spectrum is one of a black body that has a maximum of emission at $\Lambda \approx 2$ mm. The existence and the characteristics of the CMB are thought to be a strong evidence for a big bang scenario since it fits very well the predictions of Λ CDM [21], although several anomalies have been pointed out [22]. The aim of this section is to show that the presented theory may also be compatible with these observations.

4.1 Over the horizon

Until here, our study has been limited to distances smaller that the maximal geodesic distance $D = \pi R$. Although it corresponds to an apparent horizon, the observer can see (ghost) images from objects at distances larger than D. We can extend the definition of proper distance d to objects over this horizon, so that it corresponds to the length of the trajectory of a photon from the object to the observer. Applying the arguments developed in the previous sections, we get that the scaling factor is given by:

$$a(d) = \left| \frac{R\sin(\frac{d}{R})}{d} \right|.$$

The observed redshift is then:

$$z(d) = \left| \frac{d}{R \sin(\frac{d}{R})} \right| - 1.$$



Figure 5: Evolution of the redshift with distance

4.2 Thermal radiation

In the context of a static universe, we cannot explain the CMB by an extremely hot black body that emitted light in a primordial state of the universe. These photons must emanate from standard cosmological objects. By looking at Fig. 5, we can characterize two classes of sources that produce highly redshifted light, which could contribute to the CMB:

- 1. Very old objects whose radiation has travelled around the universe many times before arriving to us, at huge distance from us. This type of sources will be refered as class 1.
- 2. Objects that emitted the perceived light in the region of one pole of the 3-sphere (our current position or its antipode), whose distance is near a multiple of *D*. This type of objects will be referred as class 2.

We assume that the contribution of class 2 objects does not make up the essential of the microwave background, their proportion being negligible compared to type 1 objects. If we assume a homogenous and isotropic universe, the global contribution in light of the infinity of highly redshifted class 1 objects would create a highly regular background radiation, with no privileged direction in the sky. This very ancient (and therefore highly redshifted) light has suffered scattering and absorption, so that it becomes difficult for the observer to trace back its source. Overall, the resulting radiation should be completely randomized and resemble thermal radiation. We propose that the CMB is constituted by these photons.

4.3 Anisotropies

We must now explain the different levels of primary anisotropies of the radiation. We stated in the previous section that the contribution of class 1 objects should make up a very regular background, leaving no place for irregularities. These small perturbations could be explained by the presence of type 2 objects (mostly galaxies) in the pole regions. It is a well established fact that the distribution of the anisotropies in the CMB is consistent with the distribution of galactic structures in the universe. The radiation of these objects is highly redshifted and leaves some prints on the CMB. Each pole, at different times, contributes to one level of anisotropy and explains the peaks in the angular power spectrum of the CMB temperature anisotropy.

To formalize this idea, we will first assume that the emission spectrum of a galaxy is reduced to its strongest wavelength λ_0 which we assume to be the same for every galaxy. We will also assume that the power spectrum of the CMB is centered around its strongest wavelength Λ . These strong restrictions should still provide a decent estimate of the location of the peaks in the power spectrum.

To add to the regular background of wavelength Λ , the received light must also have observed wavelength Λ , so the source must be at a distance d that satisfies the equation

$$\frac{\lambda_0}{a(d)} = \Lambda,$$

that is:

$$\frac{\lambda_0}{\Lambda}d = \mid R\sin(\frac{d}{R}) \mid.$$
(12)

The solutions of the above equation are typically close to the pole at distance $k\pi R$ from us, that we will call the pole k. For each k, there are two associated solutions d_k and d'_k , which correspond to distances from the pole k of $l_k = k\pi R - d_k$ and $l'_k = d'_k - k\pi R$. Replacing in equation 12, we obtain:

$$\begin{cases} \frac{\lambda_0}{\Lambda} (k\pi R - l_k) = |R\sin(\frac{l_k}{R})|,\\ \frac{\lambda_0}{\Lambda} (k\pi R + l'_k) = |R\sin(\frac{l'_k}{R})|. \end{cases}$$
(13)

As l_k and l'_k are typically small compared to R, we get

$$\begin{cases} \frac{\lambda_0}{\Lambda} (k\pi R - l_k) \approx l_k \\ \frac{\lambda_0}{\Lambda} (k\pi R + l'_k) \approx l'_k, \end{cases}$$
(14)

so that

$$\begin{cases} l_k = \frac{k\pi R}{\frac{\Lambda}{\lambda_0} + 1} \\ l'_k = \frac{k\pi R}{\frac{\Lambda}{\lambda_0} - 1}. \end{cases}$$
(15)

Since $\frac{\Lambda}{\lambda_0}$ has a big order of magnitude ($\approx 10^4$), l_k and l'_k correspond roughly to the same distance to the pole k, that we will call again l_k :

$$l_k = \frac{\lambda_0 k \pi R}{\Lambda} \tag{16}$$

Now that we have found the typical distance of these galaxies to the pole k, we can estimate the contribution of the pole k to the anisotropy power spectrum, by computing the number of galaxies near the pole k that will imprint their mark on the CMB. To constitute an anisotropy, their image must have an observed wavelength between $\Lambda - \varepsilon$ and $\Lambda + \varepsilon$, being ε of the order of the width of the CMB spectrum.

From equation (16), these galaxies are at a distance from the pole k between

$$l_1^k = \frac{\lambda_0 k \pi R}{\Lambda + \varepsilon}$$
$$l_2^k = \frac{\lambda_0 k \pi R}{\Lambda - \varepsilon}.$$

and

Let ρ be the mean density of galaxies in the universe (we assume a homogenous repartition of galaxies). In S^3 , the volume of space between l_1^k and l_2^k is given by

$$V = \pi R^2 \left(2l_2^k - 2l_1^k + R\sin(\frac{2l_2^k}{R}) - \sin(\frac{2l_1^k}{R}) \right) \\ \approx 4\pi R^2 \left(l_2^k - l_1^k \right), \tag{17}$$

so that the number of galaxies at distances between l_1^k and l_2^k to the pole k is

$$N = k\pi\rho 4\pi R^{3}\lambda_{0}\left(\frac{1}{\Lambda-\varepsilon} - \frac{1}{\Lambda+\varepsilon}\right)$$

$$\approx \frac{8\pi^{2}R^{3}\rho k\lambda_{0}\epsilon}{\Lambda^{2}}.$$
(18)

The number of galaxies associated to the pole k whose images are imprinted in the CMB is then 2N, since the galaxies at distance d'_k also leave the same characteristic prints. If we assume equirepartition of the galaxies and independence between the images of the galaxies at distances d_k and d'_k , these galaxies are equally spaced in the sky and their number corresponds to the multipole moment P_k for which we have a peak in the power spectrum:

$$P_k = \frac{16\pi^2 R^3 \rho \lambda_0 \varepsilon k}{\Lambda^2} = C_\varepsilon k.$$
⁽¹⁹⁾

Equation (19) shows a linear relation between P_k and k. We also predict that the strength of these peaks decreases with k, mostly due to the extinction effects endured by distant light. Overall, these results seem compatible with the measurements of the Planck collaboration [21], although we are presently not able to provide a more detailed analysis of the matter. We have made some approximations in our computations and quite strong restrictions to arrive at this result: in reality, the spectra of both galaxies and CMB are composed of a large band of wavelengths. Moreover, the density of galaxies ρ may differ for the different pole regions, in particular due to the lacunary structure of the large-scale distribution of galaxies.

5 Conclusion and comments

We have performed an analysis of the notions of perception in general relativity and introduced a representation referential in which measurements are performed. This theory turns out to be consistent with large-scale observations, such as redshift/distance diagrams and the existence and characteristics of the CMB. Our model also predicts the existence of a gravitational wave background, whose origin is analog to that of the CMB.

This simple theory allows to clear away questions raised by the existence of an initial spacetime singularity, and a dark energy that derives the galaxies away. It also accounts for the existence of well-formed galactic structure in the high-redshift universe.

More complete and model-independent data-based studies are required to bring support to this theory. A careful analysis of the CMB characteristics in the present context still needs to be performed. Several complementary predictions could be verified experimentally on the short and longer run. The theory predict for instance the existence of ghost images of galaxies situated in the pole regions. More difficult to check at our temporal scale, the temperature of the CMB should not vary in time, and its anisotropies should not be fixed, but move at characteristic velocities.

The fact that lengths and durations are not absolute properties of observed events but depend on the referential in which they are measured is well established since the development of relativity. The present theory constitutes an extension of the relativity principle to distant objects in curved spacetime geometries. As in quantum mechanics, it implies a fundamental duality between the physical world and its manifestation to the observer, which needs to be understood at a deep level.

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