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# Behavior of charged and uncharged bubbles in dielectric liquids subjected to electric stress

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This paper is devoted to the calculation of the electric traction  $P_e$  acting at every point on the surface of a charged gas or liquid bubble immersed in an insulating liquid and subjected to an electric field between parallel-plate electrodes. It is shown that the deformation of a bubble charged or situated near the electrodes is not symmetric. This dissymmetry has been verified experimentally. The expression for  $P_e$  used in a previous work [C. G. Garton and Z. Krasucki, Proc. R. Soc. London Ser. A **280**, 211 (1964)] represents a particular case (bubble uncharged or electrically neutral) of the one we have established. In this case, the shape of the distorted bubble is that of a prolate spheroid. This deformation of bubbles also depends on their concentration and positions in the electrode gap.

## I. INTRODUCTION

It is well known that the electric strength of insulating liquids is highly influenced by the presence of gas or liquid bubbles (particularly water droplets). These can either be present right at the beginning or be formed within the body of the liquid. Their behavior and their role in the processes leading to breakdown has been the subject of numerous studies.<sup>1-4</sup>

According to earlier works, a bubble immersed in a liquid medium and subjected to a uniform electric field elongates in the direction of this field and takes the shape of a prolate spheroid. The conditions for this elongated bubble to bridge the electrodes and its instability criteria have been studied on an energy balance basis.<sup>5-7</sup> Previous works have treated the dynamics of the bubble theoretically,<sup>5,8-10</sup> with the assumption that the bubble retained a prolate spheroid shape until it reached the disruptive point.

In this paper, we shall first of all establish the mathematical expression of the electrostatic traction acting at any point of a gas or liquid bubble having a surface charge density  $\sigma$ , and the characteristic equation for the shape of the bubble. Second, we shall discuss the different bubble shapes consequent to our characteristic equation, and in particular the case for which the shape of the elongated bubble could be dissymmetric. We shall conclude with a comparison with experimental results.

## II. CALCULATION OF THE ELECTROSTATIC PRESSURE

Consider a bubble of permittivity  $\epsilon_2$  immersed in a homogeneous and uncharged medium of permittivity  $\epsilon_1$  and subjected to a uniform electric field  $E_1$ , the normal and tangential components of which are, respectively,  $E_{1n}$  and  $E_{1t}$ . Let  $\sigma$  be the density of the surface charge of the bubble and  $E_2$  the electric field within this bubble; the normal and tangential components of  $E_2$  are, respectively,  $E_{2n}$  and  $E_{2t}$ . At any point of the bubble surface, the total electrostatic traction  $P_e$  is

$$P_e = T_1 + T_2, \quad (1)$$

where  $T_1$  and  $T_2$  are the electrostatic tractions corresponding respectively to the electric fields  $E_1$  and  $E_2$  on both sides of the bubble (see Fig. 1). According to the Maxwell tensor,<sup>11</sup>

$$T_1 = \epsilon_1 (E_1 n_1) E_1 - \frac{1}{2} \epsilon_1 E_1^2 n_1 \quad (2)$$

and

$$T_2 = \epsilon_2 (E_2 n_2) E_2 - \frac{1}{2} \epsilon_2 E_2^2 n_2, \quad (3)$$

where

$$\begin{aligned} E_1 &= E_{1n} n_1 + E_{1t} t_1, \\ E_2 &= -E_{2n} n_2 + E_{2t} t_2, \\ E_{1t} &= E_{2t}. \end{aligned} \quad (4)$$

$(n_1, t_1)$  and  $(n_2, t_2)$  are the unit vectors (normal and tangential), respectively, in the liquid and in the bubble at any point of its surface, such that

$$n_1 = -n_2 \quad \text{and} \quad t_1 = t_2.$$

Substitution of (4) in (2) and (3) gives

$$\begin{aligned} T_1 &= \epsilon_1 E_{1n} E_1 - \frac{1}{2} \epsilon_1 E_1^2 n_1, \\ T_2 &= -\epsilon_2 E_{2n} E_2 + \frac{1}{2} \epsilon_2 E_2^2 n_1. \end{aligned}$$

Then

$$\begin{aligned} P_e &= \frac{1}{2} [(\epsilon_1 E_{1n}^2 - \epsilon_2 E_{2n}^2) - (\epsilon_1 - \epsilon_2) E_{1t}^2] n_1 \\ &\quad + (\epsilon_1 E_{1n} - \epsilon_2 E_{2n}) E_{1t} t_1. \end{aligned} \quad (5)$$

On the other hand,  $E_{2t} = E_2 \sin \theta$  and  $E_{2n} = E_2 \cos \theta$ , and since

$$\sigma = \epsilon_1 E_{1n} - \epsilon_2 E_{2n} \quad (6)$$

Eq. (5) becomes after rearrangement

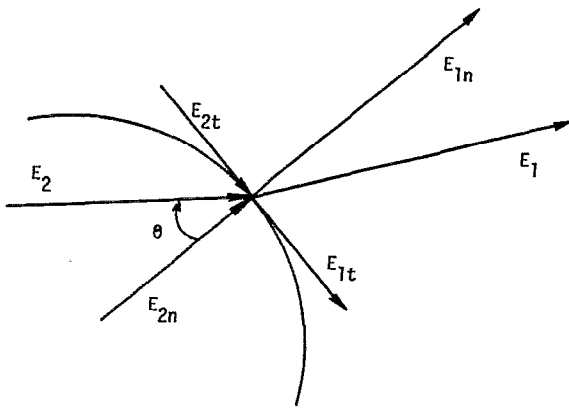


FIG. 1 Schema of surface element of a bubble subjected to internal ( $E_2$ ) and external ( $E_1$ ) electrical fields.

$$P_e = -\frac{1}{2\epsilon_1} \{ (\epsilon_1 - \epsilon_2) [\epsilon_2 + (\epsilon_1 - \epsilon_2) \sin^2 \theta] E_2^2 - 2\epsilon_2 \sigma E_2 \cos \theta - \sigma^2 \} n_1 + \sigma E_2 \sin \theta t_1$$

or

$$P_e = \frac{1}{2\epsilon_1} \{ (\epsilon_1 - \epsilon_2) [\epsilon_2 + (\epsilon_1 - \epsilon_2) \sin^2 \theta] E_2^2 - 2\epsilon_2 \sigma E_2 \cos \theta - \sigma^2 \} n_2 + \sigma E_2 \sin \theta t_2. \quad (7)$$

From this expression, we deduce the normal ( $P_{en}$ ) and tangential ( $P_{et}$ ) components of the electrostatic traction  $P_e$  acting at any point on the surface of the bubble:

$$P_{en} = \frac{1}{2\epsilon_1} \{ (\epsilon_1 - \epsilon_2) [\epsilon_2 + (\epsilon_1 - \epsilon_2) \sin^2 \theta] E_2^2 - 2\epsilon_2 \sigma E_2 \cos \theta - \sigma^2 \} \quad (8)$$

and

$$P_{et} = \sigma E_2 \sin \theta. \quad (9)$$

In the particular case of an uncharged bubble ( $\sigma = 0$ ),  $P_e$  has only a normal component. Thus

$$P_{en}(\sigma=0) = \frac{1}{2\epsilon_1} (\epsilon_1 - \epsilon_2) [\epsilon_2 + (\epsilon_1 - \epsilon_2) \sin^2 \theta] E_2^2$$

and

$$P_{et}(\sigma=0) = 0.$$

These are the expressions of the electrostatic pressure established by Smythe.<sup>12</sup>

### III. EQUATION OF THE SHAPE OF THE BUBBLE

The method consists in setting up and solving an equation for the balance of pressures at any point on the surface of the bubble; hydrodynamic effects are supposed negligible.

After deformation, the equilibrium of applied forces at any point ( $x$ ) on the surface of the bubble is expressed by the relation

$$p_1 + p_{ex} + \sigma_f C_x = p, \quad (10)$$

and at the point  $T$  corresponding to a polar angle of  $\theta = \pi/2$ :

$$p_1 + p_{eT} + \sigma_f C_T = p, \quad (11)$$

where  $\sigma_f$  is the surface tension at any point of the bubble,  $p_1$  is the external pressure;  $p$  is the pressure within the bubble, and  $C_x$  is the curvature of the bubble at the point  $x$ . Subtracting (11) from (10) and substituting in (8) gives

$$C_x - C_T = \frac{1}{2\sigma_f \epsilon_1} [ (\epsilon_1 - \epsilon_2)^2 E_2^2 \cos^2 \theta + 2\epsilon_2 \sigma E_2 \cos \theta + (\sigma^2 - \sigma_e^2) ], \quad (12)$$

where

$$\sigma_e = \sigma(\theta = \pi/2).$$

This equation defines the shape of the bubble in an electric field. It should be noted that the polarity of  $\sigma$  depends on the applied voltage and the conditions  $\gamma_2/\epsilon_2 \gtrless \gamma_1/\epsilon_1$  (where  $\gamma_2$  and  $\gamma_1$  are the conductivities of the bubble and the medium, respectively). Indeed, according to Maxwell's equation,

$$\frac{\delta \sigma}{\delta t} = \gamma_2 E_{2n} - \gamma_1 E_{1n}. \quad (13)$$

Substituting (6) into (13), it becomes

$$\frac{\delta \sigma}{\delta t} + \frac{\gamma_2}{\epsilon_2} \sigma = \left( \frac{\gamma_2}{\epsilon_2} - \frac{\gamma_1}{\epsilon_1} \right) \epsilon_1 E_{1n}. \quad (14)$$

It appears from this equation that in the case of  $\gamma_2/\epsilon_2 = \gamma_1/\epsilon_1$ , there is no accumulation of charges at the interface ( $\sigma = 0$ ).

When the bubble and the medium are both insulating, the deformation of the bubble will be symmetrical. The shape of the elongated bubble is that of a prolate spheroid aligned in the direction of the applied field; this corresponds to the Garton and Krasucki's model,<sup>5</sup> whose equation can be obtained by making  $\sigma = 0$  in (12):

$$C_x - C_T = \frac{1}{2\sigma_f \epsilon_1} (\epsilon_1 - \epsilon_2)^2 E_2^2 \cos^2 \theta. \quad (15)$$

It appears from (12) that an asymmetry is possible with regards to the plane perpendicular to the electric field.

Let  $N$  and  $N'$  represent two points on the surface of the bubble and corresponding, respectively, to  $\theta = 0$  and  $\theta = \pi$ . From (12),

$$C_N - C_{N'} = \frac{s_p}{2\sigma_f \epsilon_1} (2\epsilon_2 E_2 + s_n), \quad (16)$$

where

$$s_p = \sigma(\theta=0) + \sigma(\theta=\pi)$$

and

$$s_n = \sigma(\theta=0) - \sigma(\theta=\pi).$$

The value of  $s_p$  would determine whether the distortion of the bubble will be symmetric or not.

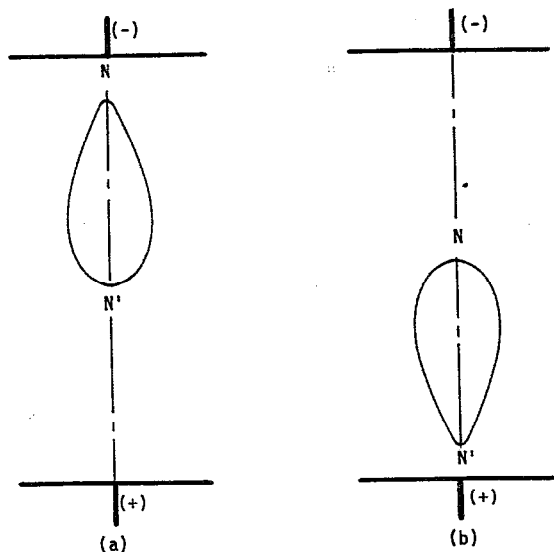


FIG. 2. Different bubble shapes depending on the value of  $s_p$ : (a)  $s_p > 0$ ; (b)  $s_p < 0$ .

#### IV. DEFORMATION OF THE BUBBLE

##### A. Charged bubble or droplet

The deformation of the bubble depends on the value of  $s_p$  and  $s_n$  [i.e., on  $\sigma(\theta=0)$  and  $\sigma(\theta=\pi)$ ]. Thus, when  $s_p > 0$ , the curvature is more important at  $N$  than at  $N'$  [Fig. 2(a)] and when  $s_p < 0$ , it will be the reverse [Fig. 2(b)].

On the other hand, the presence of injection or space charge can induce a global charge on the bubble and non-electroneutrality will be possible, i.e.,  $s_p$  and  $s_n$  will be different from 0. The deformation of the bubble will then be asymmetric. However, we must note that in this case, a term due to this supplementary charge must be taken into account for the calculation of  $P_e$ .

Experimentally, it is very difficult to have a global charge different from zero because the charge carriers sticking to the surface of the bubble under the influence of the field tend to neutralize each other.

##### B. Bubble situated near an electrode

As a result of the effect of the image force, the forces acting at the point nearest to one of the electrodes will be the highest giving rise to a nonsymmetrical charge distribution. Consequently, the shape of the elongated bubble will be asymmetric.

When the field is continuous and if the bubble—or droplet—is near the cathode,  $s_p$  will be positive and its shape will be similar to the one represented in Fig. 2(a). On the other hand, if it is near the anode,  $s_p$  will be negative. The shape of the bubble will be similar to the one represented in Fig. 2(b). When  $s_p = 0$ , the shape of the distorted bubble will be that of a prolate spheroid.

To illustrate the above results, we have undertaken an experimental study considering a water droplet immersed in a silicon oil and situated near the cathode (Fig. 3). The

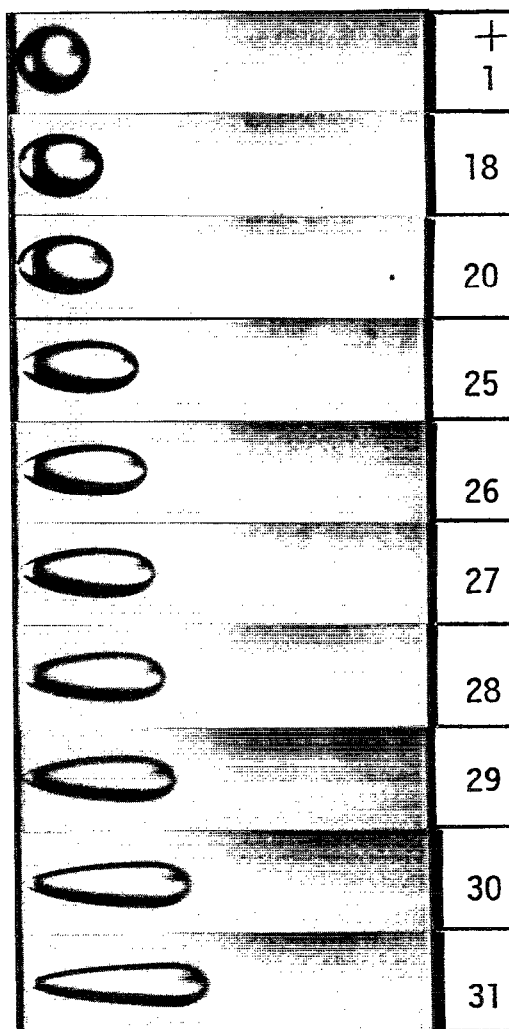


FIG. 3. Deformation of a water droplet within a silicon oil. Electrode gap is 10 mm. Direct voltage is increased from 0 to 10 kV with a duration of 1.5 s. Camera speed is 300 frames/s. The frame number in the film is included on the right-hand side of each photo.

water droplet is first selected and then introduced in the liquid medium between the electrode gap. When the field is applied, we observed that the droplet elongated asymmetrically. The curvature on the side nearer the cathode was higher. We also noticed that for a 0.1-s duration, the elongation of the water droplet was twice its initial diameter.

Let us remark that the equilibrium conditions are applicable to the transient case as long as the bubble has not reached its critical state and as long as the influence of the liquid motion is negligible. In the above experiment, this transient time is about 0.1 s.

Figure 4 gives an example of two water droplets of the same dimensions in positions which (in the electrode gap) are not symmetric. We observe that the curvature is more accentuated at the pole of the droplet nearest the electrode. Consequently, the field (thus the electrical force) depends on the position of the droplets.

The conductivity of water is  $10^6$ – $10^{10}$  times as high as that of insulating oils ( $\gamma_2 \gg \gamma_1$ ). If the frequency variations

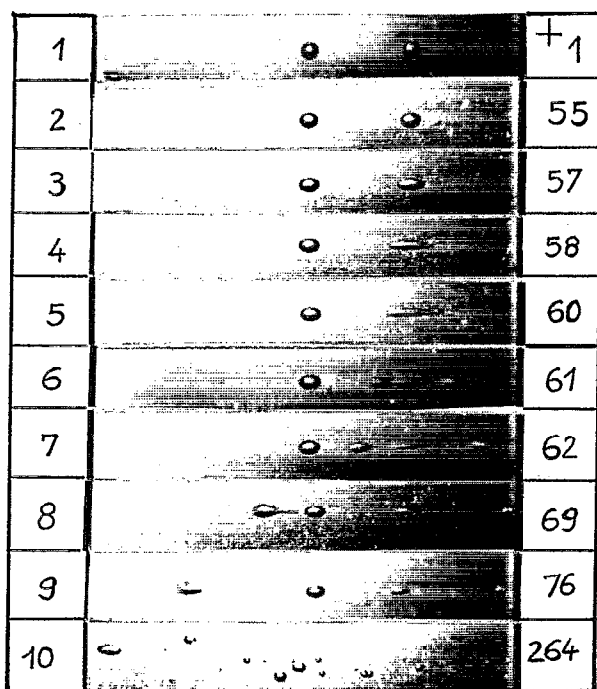


FIG. 4. Influence of the position of two identical water droplets within a silicon oil on their deformation. Electrode gap is 18 mm. Direct voltage is increased from 0 to 20 kV with a duration of 1.5 s. Camera speed is 300 frames/s. The frame number in the film is included on the right-hand side of each photo.

of the voltage are small, the field inside the droplet would be null ( $E_2 = 0$ ). Thus, water can be considered as a perfect conductor. Consider a spherical water droplet of radius  $a$  between two plane electrodes separated by a distance  $L$ . Let  $h$  be the distance from the center of the droplet to one of the electrodes (Fig. 5). Using the spherical approximation, the electric field at  $N$  will be

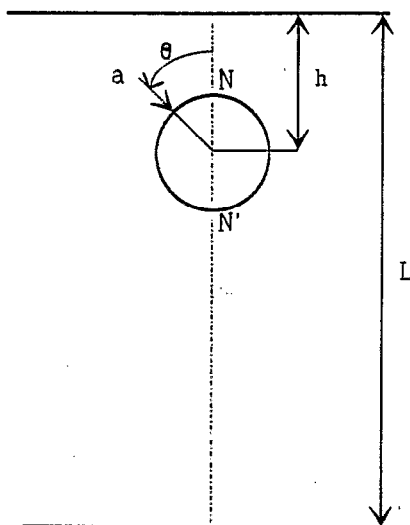


FIG. 5. Schema of a spherical bubble between two plane electrodes.

$$E_N = \frac{U}{a} \frac{h}{(h-a)}$$

From this relation, it is clear that when  $h \rightarrow a$  (i.e., when the droplet is very close to the electrode),  $E_N \rightarrow \infty$ : The closer the droplet is to the electrode, the higher the electric field. Thus, the asymmetry is possible. Indeed

$$E_{N'} = \frac{U}{a} \left( \frac{L-h}{L-h-a} \right),$$

and the ratio  $E_N/E_{N'}$  increases when the droplet draws near the electrode:  $E_N/E_{N'} \rightarrow \infty$  when  $h \rightarrow a$ .

This ratio is equal to 1 when the bubble is situated at the middle of the electrode gap. The instability of the water droplet (bubble) can either result in the continuous elongation of the droplet until it bridges the gap between the electrodes and thus causing total breakdown or the formation of sharp points at the ends of the droplet where disintegration into smaller ones can take place (Fig. 4). The field at the tips of the droplet can reach tremendous values, well in excess of the intrinsic strength of liquid, and this may be the starting point of breakdown.

## V. CONCLUSION

The mathematical expression of the electric traction  $P_e$  acting at every free point on the surface of a gas or liquid bubble immersed in an insulating liquid here established, taking into account the surface charge density  $\sigma$  of the bubble, has allowed us to show that the bubble elongates in the direction of the applied electric field. The shape that it could take is generally dissymmetric. This result has been verified experimentally. The expression obtained for  $P_e$  is more general than that used by Garton and Krasucki,<sup>5</sup> which can be deduced by making  $\sigma = 0$  in  $P_e$ . In this case, the shape of the bubble distorted in a uniform electric field is exactly that of a prolate spheroid, as previously shown by these authors.<sup>5</sup>

As has been earlier stated, this deformation is dependent on the bubble concentrations and their positions in the electrode gap.

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