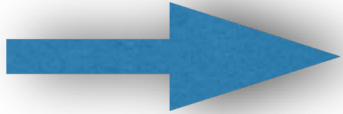


A physically-based random dynamics for gravity waves and currents interaction on the ocean surface

Valentin Resseguier,

Bertrand Chapron, Fabrice Collard

Motivations

- Mimic interaction wave / (small-scale) current
 - Small-scale currents are hardly observed or simulated
 - Waves see them during very short time
- Quantification of associated modelling errors
 -  Ensemble forecasts and data assimilation
- Improve the Scalian's wave simulator
- Study influence of swell / current interactions onto satellite altimetry

Part I

Background

L@b

(~ 15 peoples)

Research, R&T, R&D

Expertise:

- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

CEN « Simulation » (~ 70 people)

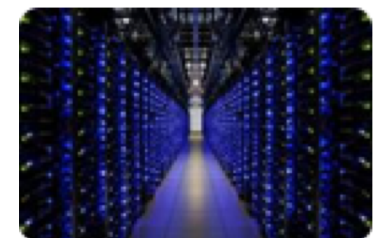
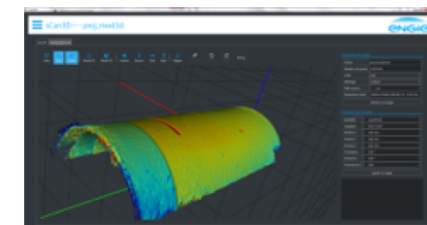
R&D and engineering

Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:

- Scientific softwares
- Simulations, HPC
- VR & AR



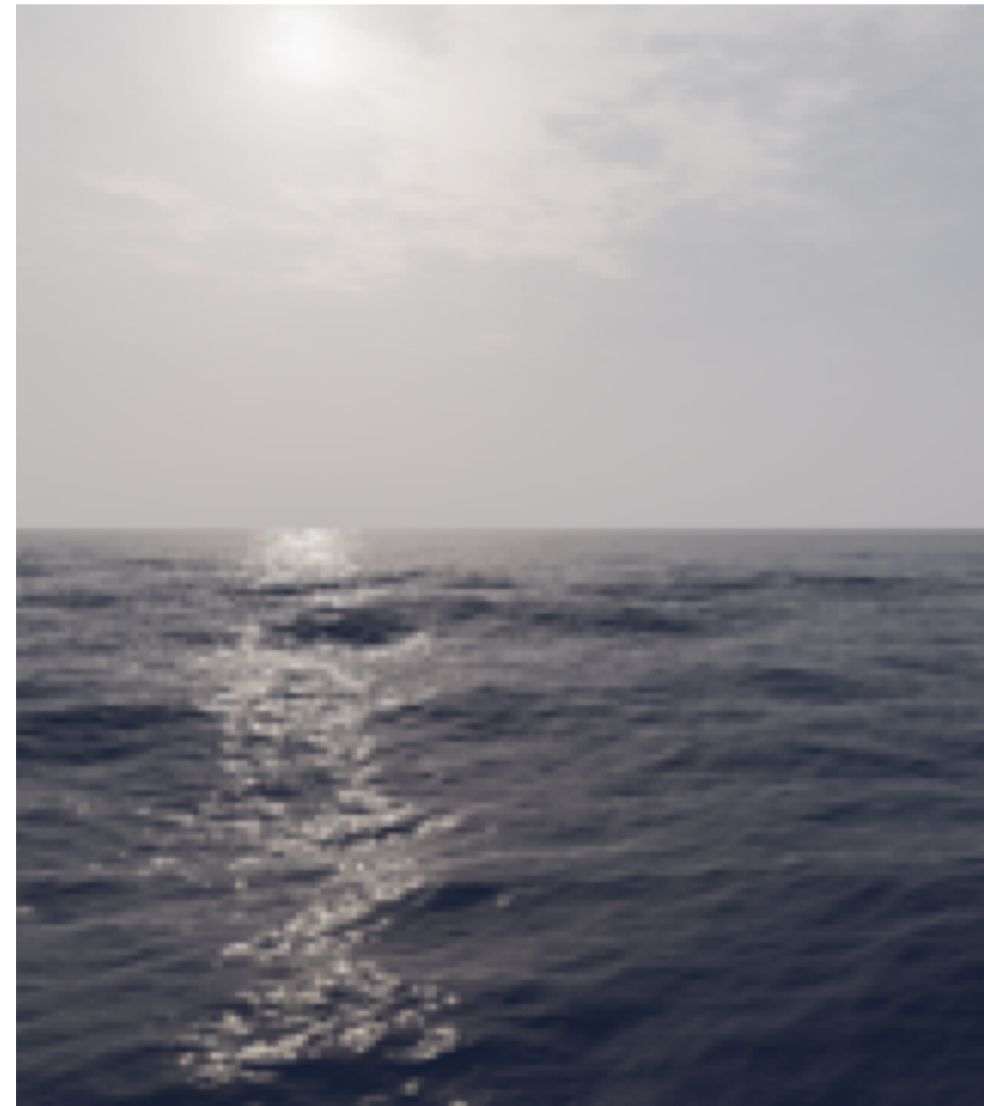
Other Business Units

~ 2400 people



SeaMotion-radar simulator

- Simple and fast simulation of surface gravity waves over very large domain
- Coupled with radar simulation
- Lot of physical phenomena taking into account
- Used to study satellite altimetry measurements (e.g. SKIM)



Large scale
group velocity:

$$\omega + \nabla_k \omega_0$$

Wave:

$$ae^{\frac{i}{\epsilon}\phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g\|k\|}$$

Wave group dynamics in the presence of currents

At the first order in steepness

Random ray

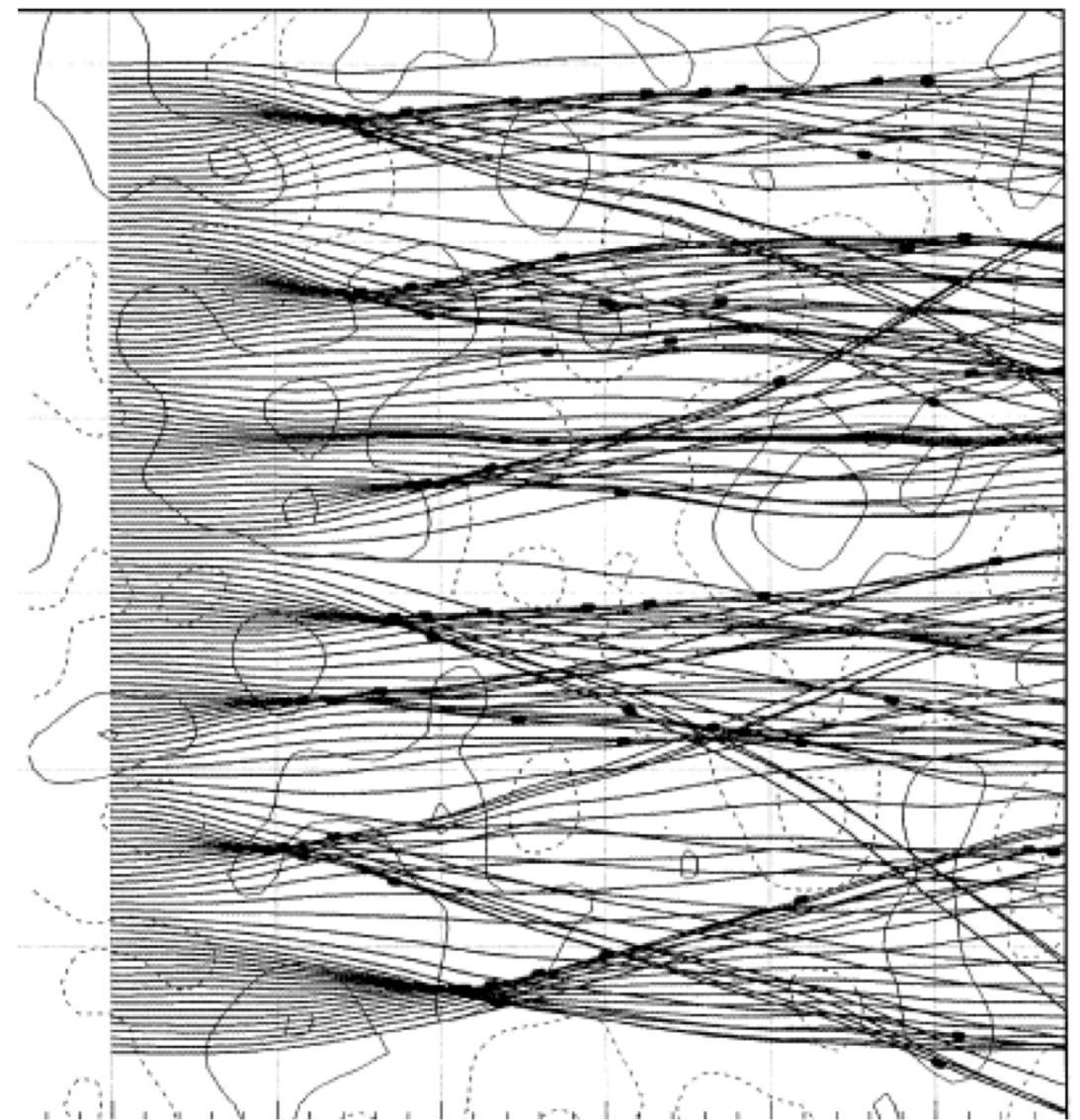
$$\dot{X}_R = u_g$$

Stretching of phase

$$\dot{k} = -\nabla \omega^T k$$

Conservation of action

$$\text{cst.} = A = \frac{a^2}{\omega_0}$$



White & Fornberg 1997

(stationary velocity)

Part II

Waves under location uncertainty

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$

Models under location uncertainty (LU) : Adding random velocity

Resolved large scales

White-in-time small scales

Initially for uncertainty quantification & data assimilation

$$v = w + \sigma \dot{B}$$

References : Mikulevicius & Rozovskii, 2004
 Flandoli, 2011

LU	SALT
Memin, 2014 Resseguier et al. 2017 a, b, c Cai et al. 2017 Chapron et al. 2018 Yang & Memin 2019	Holm, 2015 Holm and Tyranowski, 2016 Arnaudon et al. 2017 Crisan et al., 2017 Gay-Balmaz & Holm 2017 Cotter and al. 2018 a, b Cotter and al. 2019
Cotter and al. 2017 Resseguier et al. 2019 a, b	

Large scale
group velocity:

$$\mathbf{w} + \nabla_k \omega_0$$

Small scale
group velocity:

$$\sigma \dot{B}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

Dispersion ratio

At the first order
in steepness :

Stochastic
transport

$$\frac{D\phi}{Dt}$$

Frequency without currents

$$\omega_0(k) = \sqrt{g \|k\|}$$

Large scale
group velocity:

$$w + \nabla_k \omega_0$$

Small scale
group velocity:

$$\sigma \dot{B}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

General case

Non-linear
coupling

$$\dot{X}_R = \underbrace{u_g}_{\text{Function of } k} \quad \text{Random ray}$$

Stochastic stretching of phase

$$\dot{k} = \underbrace{\left(-\nabla (w + \sigma \dot{B})^T + M_\sigma \right)}_{\text{Function of } x} k \quad \left| \text{Wave- vector} \right.$$

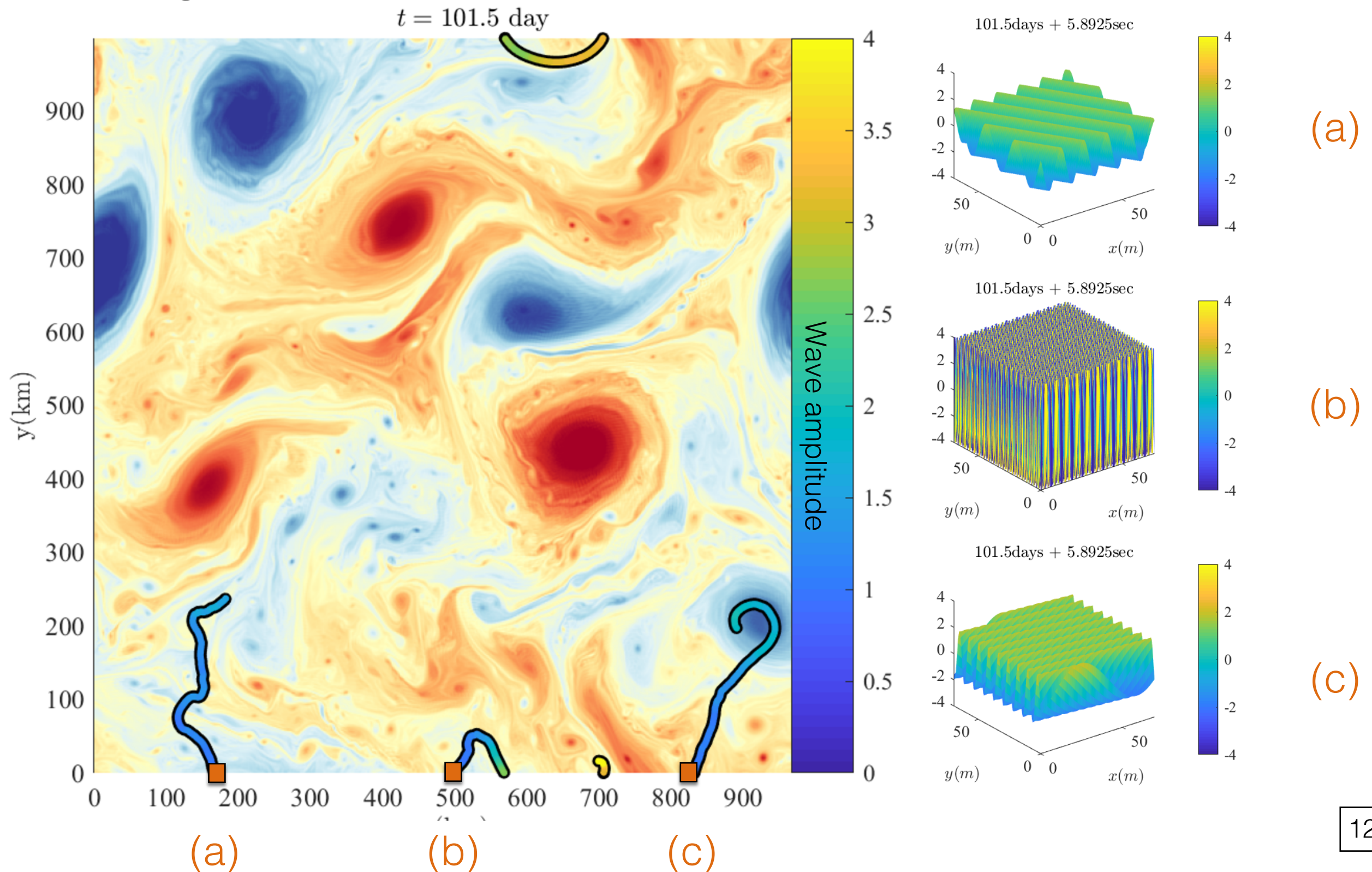
Conservation of action

$$\dot{A} = -\nabla \cdot u_g A \quad \left| \quad A = \frac{a^2}{\omega_0} \quad \text{Amplitude} \right.$$

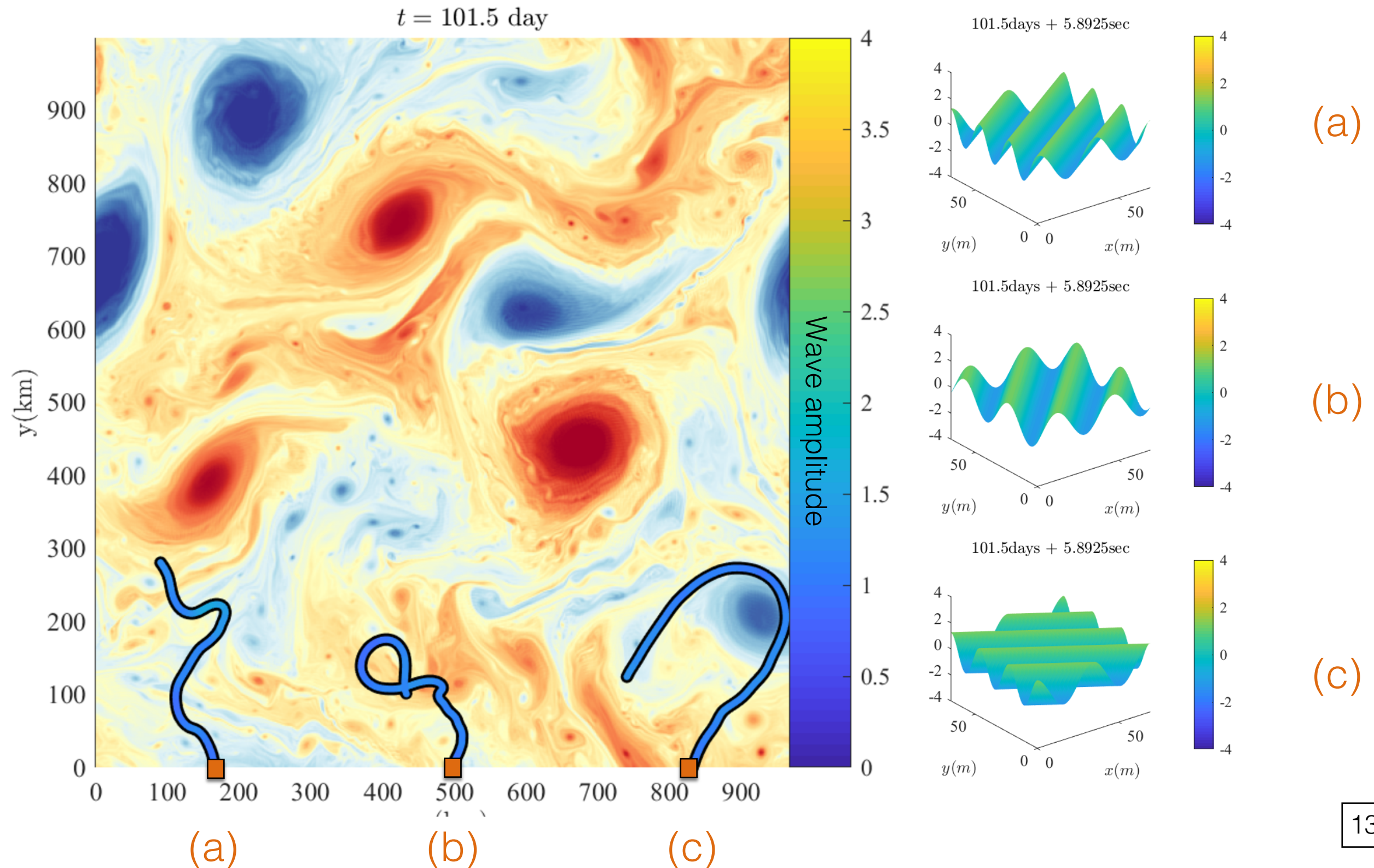
Part III

Some first numerical results

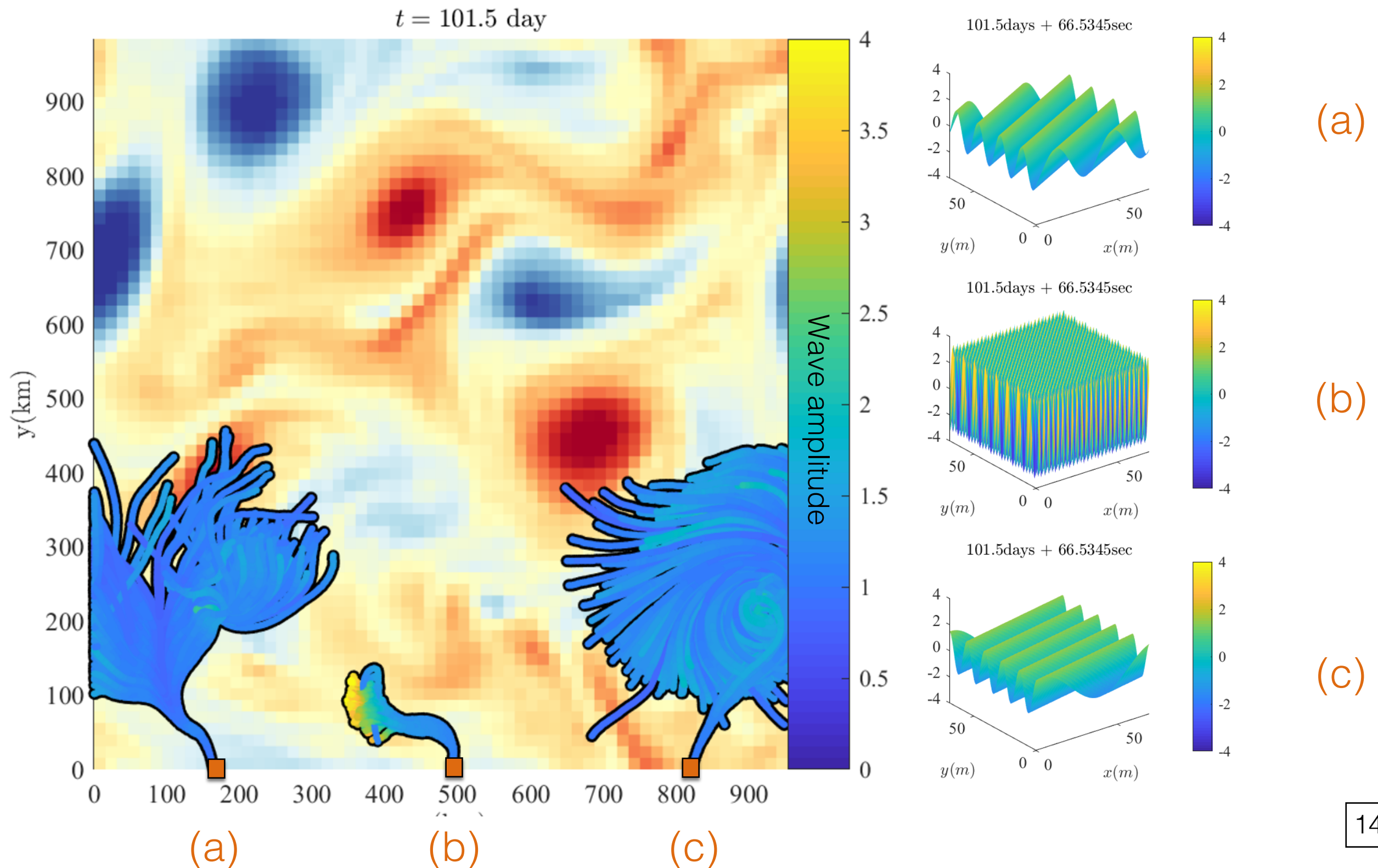
Dynamics of 3 wave group in *high-resolution* SQG currents (512x512)



Dynamics of 3 wave group in *low-resolution* SQG currents (64x64)



Dynamics of 3 wave group in *stochastic low-resolution* SQG currents (64x64)



Conclusion

Conclusion and outlooks

- Method of characteristics : low computational costs
We can simulate a lot of realisations, and initial conditions
- Understanding and simulating the dynamics with increasing complexity
 - Backward integration to obtain wave spectra (Quilfen et al. 2018)
 - Analytic time integration for wave-group subgrid dynamics
 - Heterogeneous small-scale velocity variance and correlation lengths (e.g. Pannekoucke et al. 2018)
- With satellite measurements,
try to predict wave spectra variations due to Agulhas courant
- Integration into the SeaMotion Radar
- Which are the swell / current effects on satellite measurements?
- Outlook : stochastic Wave Watch III

Bonus slides

Large scale
group velocity:

$$\mathbf{w} + \nabla_{\mathbf{k}} \omega_0$$

Wave:

$$a e^{i \frac{\omega}{\epsilon} \phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g \|\mathbf{k}\|}$$

Simple linear case 1:

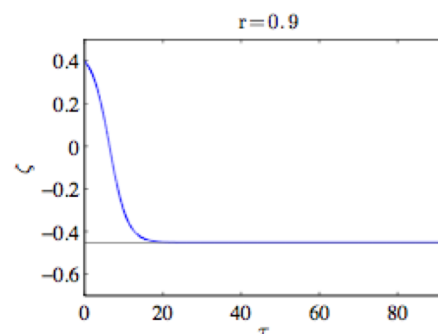
stationary deterministic,
divergence-free and
linear in \mathbf{x} large-scale velocity

Stretching of phase

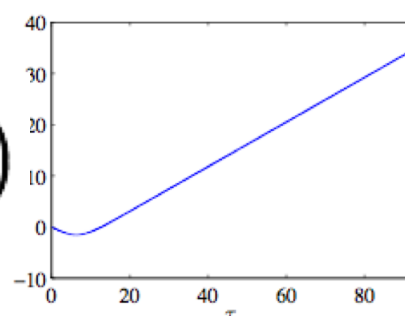
$$\dot{\mathbf{k}} = -\nabla \mathbf{w}^T \mathbf{k}$$

$$|r| < 1$$

Wave shortening



$\ln(\|\mathbf{k}\|)$

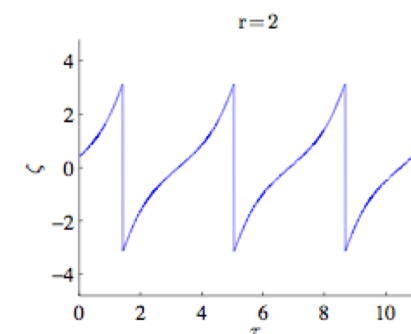


Ratio
vorticity / strain rate

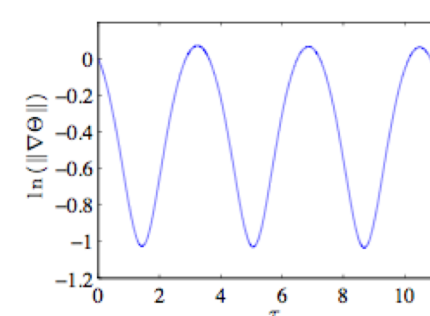
$$r = \frac{\omega}{\sigma_w}$$

$$|r| > 1$$

Wave rotation



$\ln(\|\mathbf{k}\|)$



$$\frac{D\zeta}{D\tau} = r - \cos(\zeta)$$

$$\frac{D}{D\tau} \ln(\|\mathbf{k}\|) = -\sin(\zeta)$$

Large scale
group velocity:

$$+\nabla_k \omega_0$$

Small scale
group velocity:

$$\sigma \dot{B}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

Simple linear case 2:

no large-scale current
+ homogeneous and divergence-free
small-scale velocity

$$\beta(x, t) e^{i(\omega_0 t + k \cdot \sigma B_t)}$$

Wave with
Brownian phase

Stochastic stretching of phase

$$\dot{k} = -\nabla(\sigma \dot{B})^T k$$

Log-normal wave- number

Conservation of action

$$\text{cst.} = A = \frac{a^2}{\omega_0}$$

Log-normal amplitude

Large scale
group velocity:

$$w + \nabla_k \omega_0$$

Small scale
group velocity:

$$\sigma \dot{B}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

Simple linear case 3

divergence-free and linear-in-x large-scale current
+ homogeneous and divergence-free
small-scale velocity

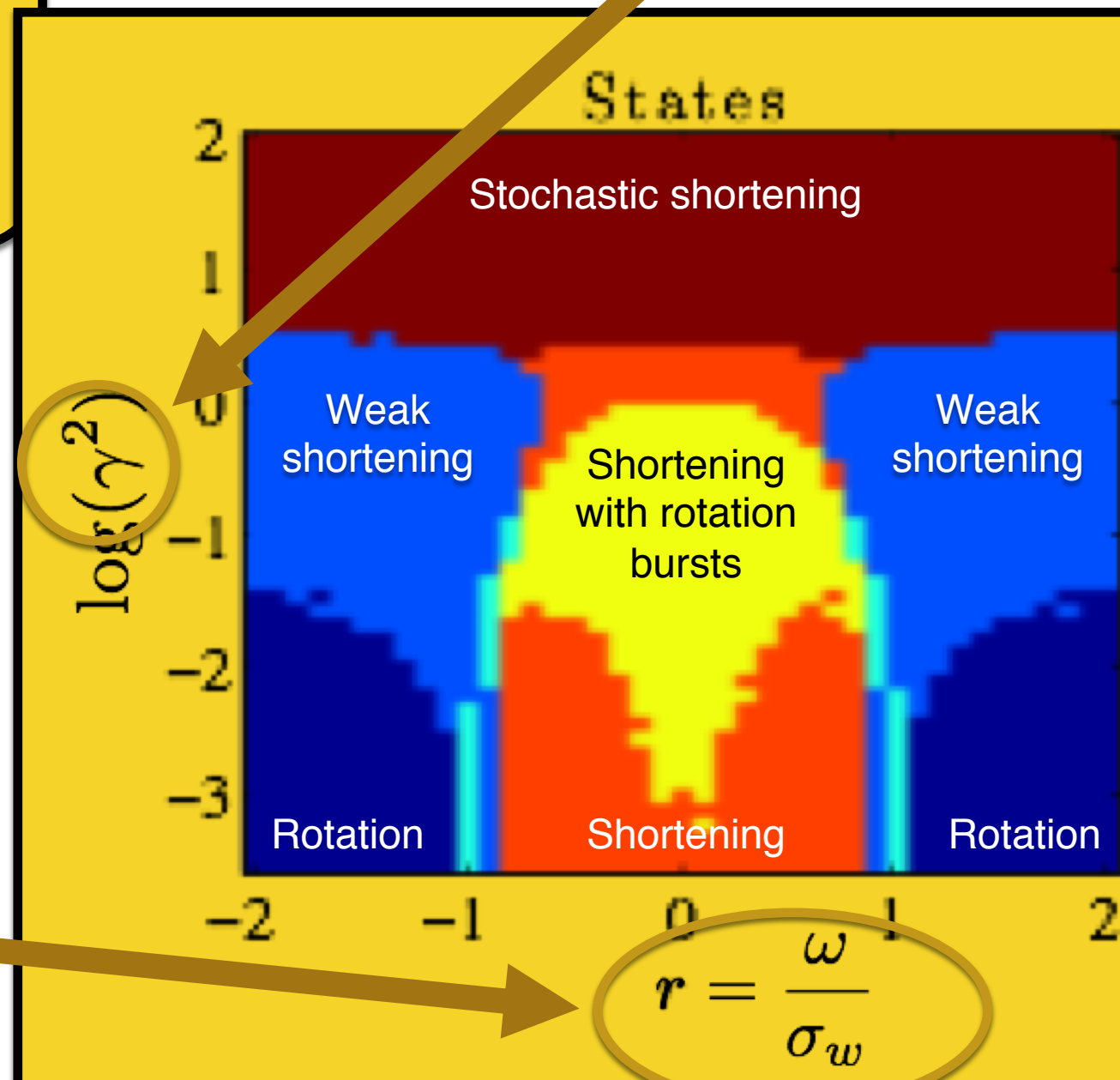
Stochastic stretching of
phase

$$\dot{k} = -\nabla(w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Depends on
small scale's
statistics



Ratio
vorticity / strain rate

$t = 100$ day

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

$$-\frac{1}{\tau} (b - F) \quad \text{Forcing}$$

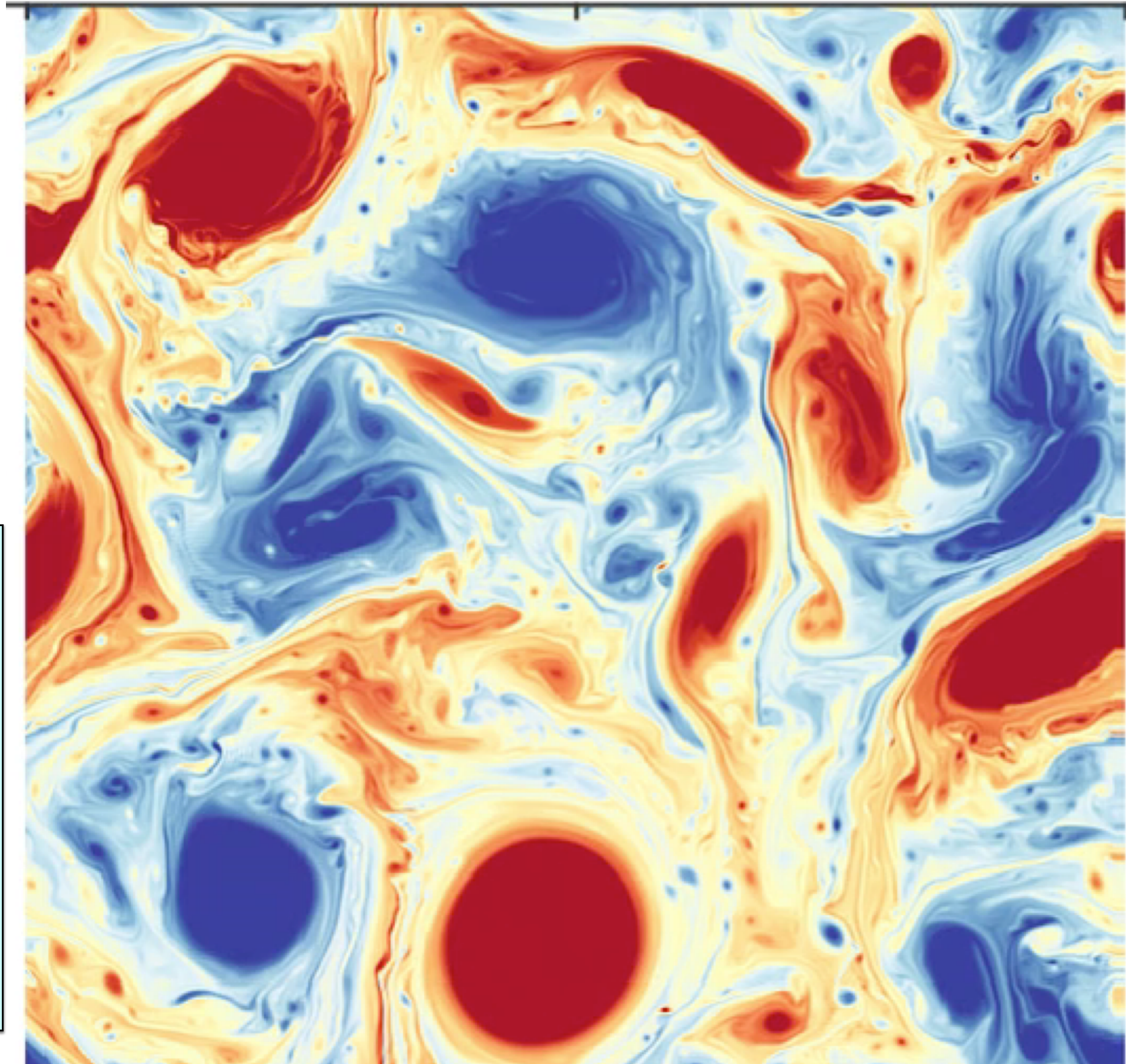
$$u = \left(\text{cst.} \nabla^\perp \Delta^{-1/2} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Absolute Diffusivity Spectral Density

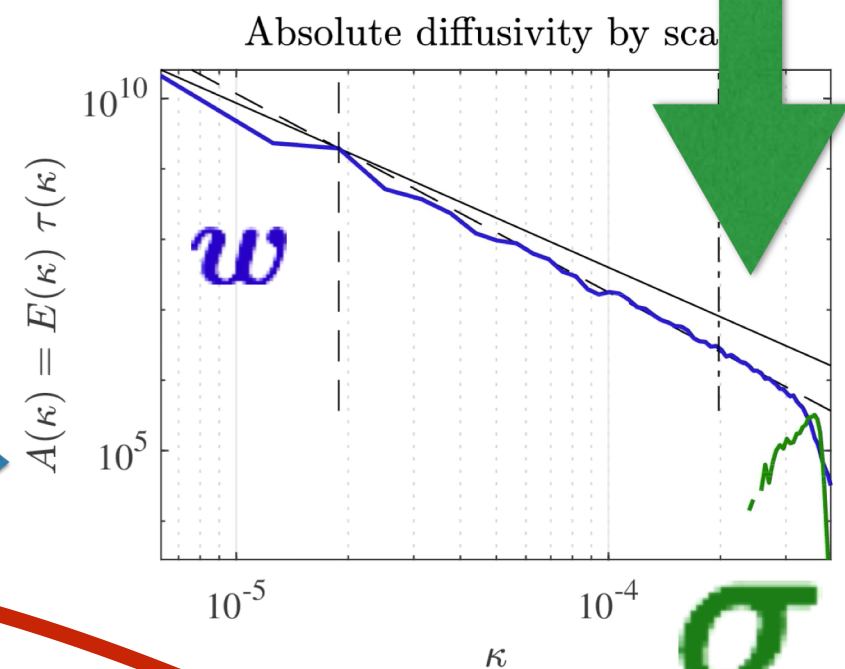
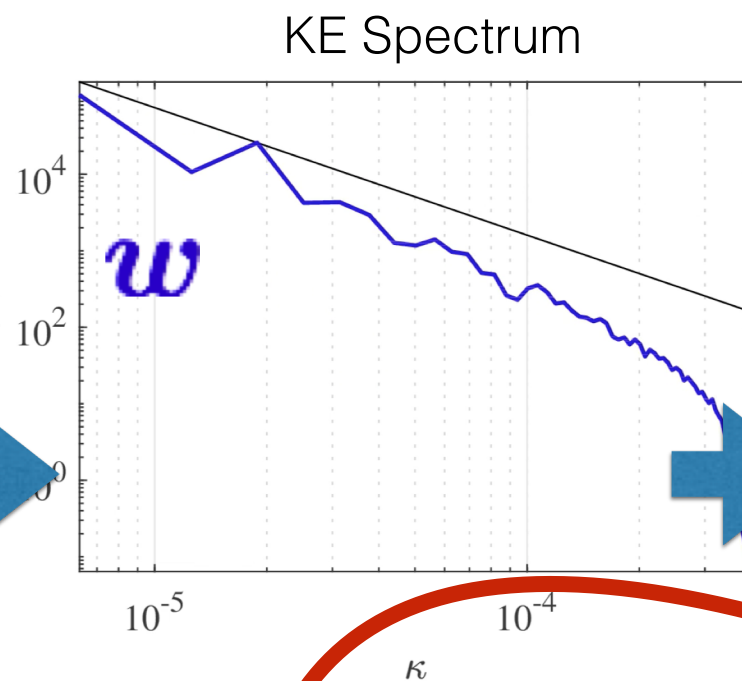
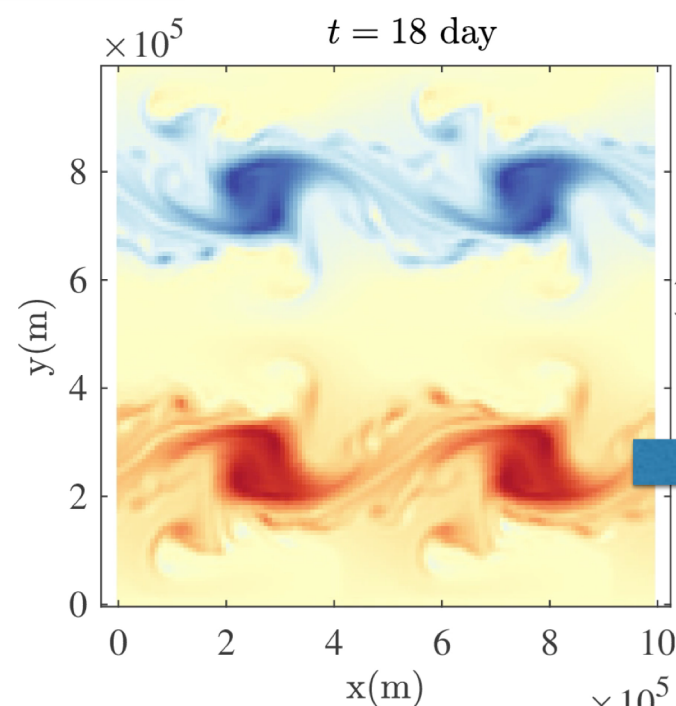
MU ADSD

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa) \tau(\kappa)$$

Residual
non-stationary
ADSD



$$\sigma \dot{B} = (\text{filter}) * (\text{white noise})$$

Reference:
 Resseguier, Pan & Fox-Kemper 2019