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Noname manuscript No.
(will be inserted by the editor)

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Received: date / Accepted: date

Abstract Urban delivery optimization is mainly based on the classical Traveling Salesman Problem (TSP). Time-Dependent TSP (TD-TSP) is an extension of the TSP wherein the cost of an edge depends on the departure time from its source node. It is particularly relevant in real urban traffic environments, as the actual travel speeds vary according to the time of the day. By decomposing the time horizon into equal-sized time steps, and associating a travel time to each time-step of each edge, we first examine the relationship between the length of the time-step and the spatio-temporal features of the data-set, which describe the amount of information degradation in the data-set along both dimensions. We also study the effect of this spatio-temporal granularity on the quality of the TSP and the TD-TSP solutions. Our benchmark data-set is produced from a realistic traffic flow micro-simulation of the city of Lyon. Four time-step lengths, ranging from six to sixty minutes, and several numbers of deliveries, ranging from ten to thirty, are considered for two exact solvers, namely dynamic programming and an integer linear programming solver.

Keywords Urban freight · Asymmetric Traveling Salesman Problem · Time Granularity · Empirical Experimentation

This work is supported by the LABEX IMU (ANR-10-LABX-0088) of Université de Lyon, within the program "Investissements dAvenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR).

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1 Introduction

Freight tour optimization consists in minimizing the time to visit a given set of locations to pick-up or deliver goods. This problem is usually modeled as a directed graph where vertices are the locations[1] and edges correspond to shortest paths between each pair of vertices. Edges are weighted by travel times between the two incident locations. Then, the optimal tour can be calculated as the shortest route that visits each vertex of the graph exactly once. It corresponds to the classical definition of travelling salesman problem (TSP) and has been widely studied in the past.

Classically, edge weights are assumed to be constant. This means that travel times are constant, whatever the hour of the day. It turns out that this supposition is far from the reality and is a poor representation because it implies that traffic conditions remain also constant even if congestion may appear during peak hours. Consequently, actual travel times between locations vary along the day. Also shortest paths (*i.e.*, successions of road links) between locations may change along the day. To fill this lack of realism in the classical TSP, cost functions that define edge weights must be adapted to traffic dynamics: they become *time-dependent*. Determining the optimal freight tour with a realistic traffic dynamic description is thus related to solve the time-dependent version of the TSP problem (TD-TSP problem), which takes into account variations of travel times during the day [6, 10, 11].

A natural question is: What is the effect of integrating time-dependency to travel between places in the optimization model? Fig. 1 shows through a simple example how a sole change of time-step durations from 6 to 12 minutes unveil different tours with different durations. An issue which is amplified for complex road networks of thousands of links and for whole day time horizons. Fleischmann *et al.* [9] compare tour durations yield by constant travel time to those based on 5 and 10 minutes time-steps, and found that static TSP underestimate the total travel time by over 10%. We further explore the reliability and quality of static TSP based tours, compared and evaluated in finer time granularity costs, addressing the crucial question of when it is necessary to exploit time-dependent costs for urban freight deliveries.

A second question is: Which level of variations must be accounted for in TD-TSP? To the best authors knowledge, only few attempts exist in the literature [8, 9]. These papers aim to compare constant travel time versus piece-wise-constant time-dependent travel times. Moreover, the variations of the travel times may change either the order of the places to visit but also the fastest routes between places. It is important to notice that considering more variations of travel times increases the size of the TSP graphs. Similarly, considering space variations of the speed of the links of a route between two locations to visit also extends the size of the graph. As a consequence, it will increase exponentially the calculation time because TSP is an NP-hard problem. To fix ideas, the size of the problem is slightly different when considering

[1] And the initial and final warehouses.

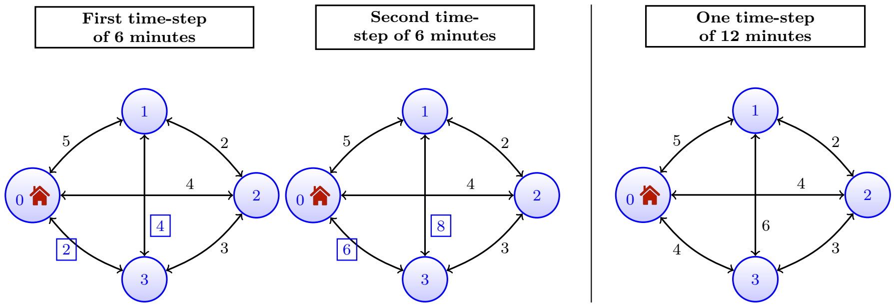

Fig. 1: Illustration of the impact of time-step granularities on the computed tours. Left: With two time-steps of 6 minutes, such that costs of $(0, 3)$ and $(1, 3)$ increase between the first and the second time-step due to congestion, the best tour is $\langle 0, 3, 1, 2, 0\rangle$ and edges $(0, 3)$ and $(3, 1)$ are traversed before becoming congested. Right: With one time-step of 12 minutes, such that costs of edges $(0, 3)$ and $(1, 3)$ are averaged over the two 6-minute time-steps, the best tour is $\langle 0, 1, 2, 3, 0\rangle$.

constant travel times (one value between each location for the whole horizon of optimization) and piece-wise constant travel times (one value per time period between each location for the whole horizon of optimization). The same remark holds for the space scale: the size of the problem changes if one speed value per link is considered or if links with the same speed are gathered. Besides, there is also major differences between considering a priori travel time estimations (TSP and TD-TSP) or real-time estimations (D-TSP).

Consequently, the way of modeling and estimating travel times is crucial to determine optimal urban freight tour. With this in mind, it is thus appealing to determine the convenient and efficient space and temporal scales of travel times modeling. A trade-off between accounting for the traffic dynamics, guaranteeing an efficient computational time, and finding a satisfactory optimal solution must be found.

In this context, the main contribution of the paper is to focus on a crucial question, that is rarely addressed by the literature: What is the effect of integrating travel time variations in the optimization model? To this end, we propose to use a micro-simulation software as a proxy of the reality and to compare the quality of solutions found with a TSP model and a TD-TSP model. It makes it possible to provide to the community a benchmark data-set of time-dependent TSP graphs to test and compare algorithms and calculated optimal tours. The associated instances correspond to a real transportation network where travel times and shortest paths between locations to visit are time-dependent. Different time-scale and spatial coverage of the traffic conditions can be defined, leading to different estimations of the travel times. Effects of the spatio-temporal granularity of the observations can thus be analyzed.

The organization of the paper is as follows. The next section describes the TD-TSP, and the exact solving approaches used in the paper. In Section 3, we introduce a new benchmark for the TD-TSP that has been built by using recent techniques and simulation models originating from traffic flow theory. Finally, Section 4 reports the first experimental results involving both optimal tours of static and TD versions of the TSP.

2 The TD-TSP problem

Description. Introduced in 1992 by Malandraki and Daskin [16], the TD-TSP aims to find the least-duration Hamiltonian circuit, *i.e.*, visiting each node exactly once, similarly as the TSP. The difference is that the traversal cost c_{ij} of each edge (i, j) varies over time, and is a function of the departure time t from i. Thus, it is denoted c^t_{ij}. Time-dependant TSP is a generalization of the TSP, which is NP-hard [12]. In this study, we consider complete directed graphs of delivery addresses, which are built from the road transportation network. The traversal cost c^t_{ij} in our case is modeled as a step-wise function, which assumes that travel times are constant during each time-step. This model is actually well suited for traffic data, since it fits the usual scheme of travel time estimation.

We denote n the number of vertices, V the set of all vertices, and $0 \in V$ the depot from which the tour begins. Given a path $P = \langle v_1, \ldots, v_k \rangle$, a starting time t_0, and a vertex $v_i \in P$, the arrival time on v_i is denoted $at(v_i, t_0, P)$ and is recursively defined by:

$$\begin{aligned} at(v_1, t_0, P) &= t_0 \\ \forall i \in [2, k],\, at(v_i, t_0, P) &= at(v_{i-1}, t_0, P) + d(v_{i-1}) + c^{at(v_{i-1}, t_0, P) + d(v_{i-1})}_{v_{i-1}, v_i}, \end{aligned}$$

where $d(v_i)$ is the duration associated to vertex v_i. The time-span associated with the path P and a start time t_0 is denoted $ts(t_0, P)$ and is given by :

$$ts(t_0, P) = at(v_k, t_0, P) - \sum_{i=1}^{k} d(v_i) - t_0.$$

The goal of the TD-TSP is to find a tour $P = \langle v_0, \ldots, v_n \rangle$ such that (i) P starts from and ends on vertex 0 (*i.e.*, $v_0 = v_n = 0$); (ii) $\{v_0, \ldots, v_{n-1}\}$ is a permutation of V; and (iii) the time-span of P when leaving from v_0 at time t_0 (*i.e.*, $ts(t_0, P)$) is minimal. Finally, the following three steps are required to determine the optimal freight tour:

(i) Estimation of the road travel times $f(t, k)$ for each road link k, and each time-step Δt starting at t (see Section 3 for more details).
(ii) Calculation of the shortest path for each couple of locations i and j and every time-step Δt, using a time-dependent variation of the Dijkstra algorithm. c^t_{ij} will take the value of the associated duration.
(iii) Determination of the optimal freight tours using an optimization algorithm (see the next paragraph).

Solving approaches. For solving the TD-TSP, we focus on exact methods, *i.e.*, Integer Linear Programming (ILP), Constraint Programming (CP) and Dynamic Programming (DP). When there are no time-window constraints (*i.e.*, the vehicle is not constrained to arrive to each location i within a pre-defined interval $[l_i,\ u_i]$), which is our case, existing ILP [1, 2, 5, 20] and CP [17] approaches do not scale well, and none of them is able to solve our instances with $n = 30$ vertices within a reasonable amount of time when the number of time-steps exceeds 3[2]. DP on the other hand does find the optimal solution (and prove optimality), in less than 0.5 second for $n \leq 20$ and less than 1000 seconds for $n = 30$ vertices[3]. The DP approach is a straightforward extension of the DP algorithm for the TSP, and it is based on the following Bellman equation :

$$\begin{cases} p(v_i, S) = c_{0v_i}^{t_0},\ S = \emptyset, \\ p(v_i, S) = \min_{v_j \in S} p(v_j, S \setminus \{v_j\}) + d(v_j) + c_{v_j v_i}^{p(v_j, S \setminus \{v_j\}) + d(v_j)},\ S \neq \emptyset, \end{cases}$$

where $p(v_i, S)$ is the earliest arrival time of a path that starts from vertex 0 at time t_0, visits each vertex of $S \subseteq V$ exactly once, and finishes on vertex v_i. The time complexity of this algorithm is $\mathcal{O}(n^2 \cdot 2^n)$, and its space complexity is $\mathcal{O}(n \cdot 2^n)$. We use DP for solving TD-TSP. For the static case however, we use an efficient and basic ILP approach, *i.e.*, a Branch&Cut procedure wherein sub-tours are eliminated starting with those with the smallest cardinality [19]. This approach is able to find the optimal solution in less than 0.4 second for $n = 30$.

3 Description of the benchmark

Motivation. One of the main objectives of this study is to provide to the research community a benchmark data-set with the following goals in mind: (i) a data-set designed to test, evaluate and compare performance of various TSP solvers, (ii) accounting for different spatio-temporal granularities from the perfect knowledge to a realistic time and space coverage, and (iii) accounting also for different levels of travel time variations, *i.e.*, different time-steps to be considered.

Spatio-temporal features of the benchmark. To obtain full access of ground data, we decided to use a dynamic microscopic simulator of traffic flows, called SYMUVIA [3], on a sub-part of the Lyon transportation network, shown in Fig. 2. This software can simulate the whole complexity of the urban traffic flow by taking into account different classes of vehicles, individual driving behaviors, lane-changing phenomenons, intersections, etc. It uses also a car-following law based on Newell's model [18] and its extensions [13, 14]. Even if

[2] We thank Vu *et al.* [20] for sharing their source code.

[3] All experiments were performed on an `Intel(R) Xeon(R) Platinum 8175M CPU @ 2.50GHz` processor with `32 GB` memory machine.

		Spatial information (-)	Spatial information (+)	
temporal information	(-)	**Data-set D1** Actual sensor cover of the network Average of 10 traffic scenario days Temporal trend filtering	**Data-set D2** 100% sensor cover of the network Average of 10 traffic scenario days Temporal trend filtering	partial
	(+)	**Data-set D3** Actual sensor cover of the network One traffic scenario day No temporal trend filtering	**Data-set D4** 100% sensor cover of the network One traffic scenario day No temporal trend filtering	complete
		partial	complete	

Table 1: Description of the four data-sets of the benchmark.

simulation is only a proxy of the real world, it makes it possible to have access to the finest level of details and to emulate every possible measurements of traffic dynamic: individual travel times, link speeds, loop detector data, etc. Several traffic scenarios of the day can be replicated simply by changing the random seed input of the simulator.

Table 1 describes the four data-sets generated in our benchmark. The data-set $D4$ has been obtained according to the previous paragraph procedure, when considering a sensor placed on each link of the network. The data-set $D3$ follows similar pattern as $D4$ with the same random seed input of $D4$, except that sensors in this case only cover the actual spatial position found in the urban transportation network of Lyon city, which are placed by the agglomeration of Lyon city [4]. This partial spatial cover only concerns 7.35% links of the target network. It is also shown in Fig. 2. The travel times of the other links are generated by interpolation, taking the value of the closest (Euclidean distance) sensor. On other hand, $D1$ and $D2$ data-sets tend to capture the temporal trend linked to the daily demand, thus travel-time costs of several traffic scenarios are averaged, exactly 10. Then, we perform a temporal trend filtering via an adequate moving average filter. The spatial sensor cover of the data-set $D1$ (resp. $D2$) is similar to the data-set $D3$ (resp. $D4$).

Fig. 3 shows a view of the 4 data-sets when considering an instance of four Origin-Destination (O-D) addresses, while Table 2 displays the Euclidean distances between the data-sets Di (with $i \in \{1, 2, 3, 4\}$) and $D4$ for the time-step $l = 6mn$. Therefore, $D1$ is the farthest from $D4$, followed by $D3$ then $D2$.

Estimation of time-dependent travel times of roads. We use the following consistent spatio-temporal mean formulation to calculate the travel time $f(k, t)$ for every road link k of the network and every time-step Δt starting at t:

$$f(k,t) = \frac{l_k}{V_k(t)} \qquad \text{and} \qquad V_k(t) = \frac{Q_k(t)}{K_k(t)},$$

Fig. 2: The Lyon road network considered in the study, and the actual positions of sensors placed by the agglomeration of Lyon city (in yellow).

	$D1$	$D2$	$D3$	$D4$
$n = 10$	9 072.71	3 546.0	8 228.26	0.0
$n = 20$	18 120.9	7 270.3	16 180.9	0.0
$n = 30$	27 675.6	11 110.6	25 282.4	0.0

Table 2: Euclidean distances between the data-sets Di and $D4$ given for each n and Di by $\sum_{\text{instance}} \left(\sum_s \sum_d \sum_k \left(c^k_{sd}(Di) - c^k_{sd}(D4) \right)^2 \right) /(\text{number instances})$.

where l_k is the length of link k, Q_k is the spatio-temporal mean of the flow in link k at time t and K_k is the spatio-temporal mean of the density in link k at time t, calculated according to definitions of [7] :

$$Q_k(t) = \frac{\sum_r d_r}{l_k \Delta t}, \qquad K_k(t) = \frac{\sum_r \tau_r}{l_k \Delta t},$$

where d_r and τ_r are respectively the distance traveled and the time spent by vehicle r within the link k during time t and $t + \Delta t$. It is important to notice that those definitions are fully consistent with the dynamic of traffic flow because it weights accurately the different traffic conditions that can be observed within a road link [15], on the contrary of classical loop detector data. Moreover, outputs of micro-simulation give easily access to the values of d_r and τ_r.

Description of the benchmark instances. Three increasing numbers of deliveries $n \in \{10, 20, 30\}$ are considered. For each value of n, we have generated

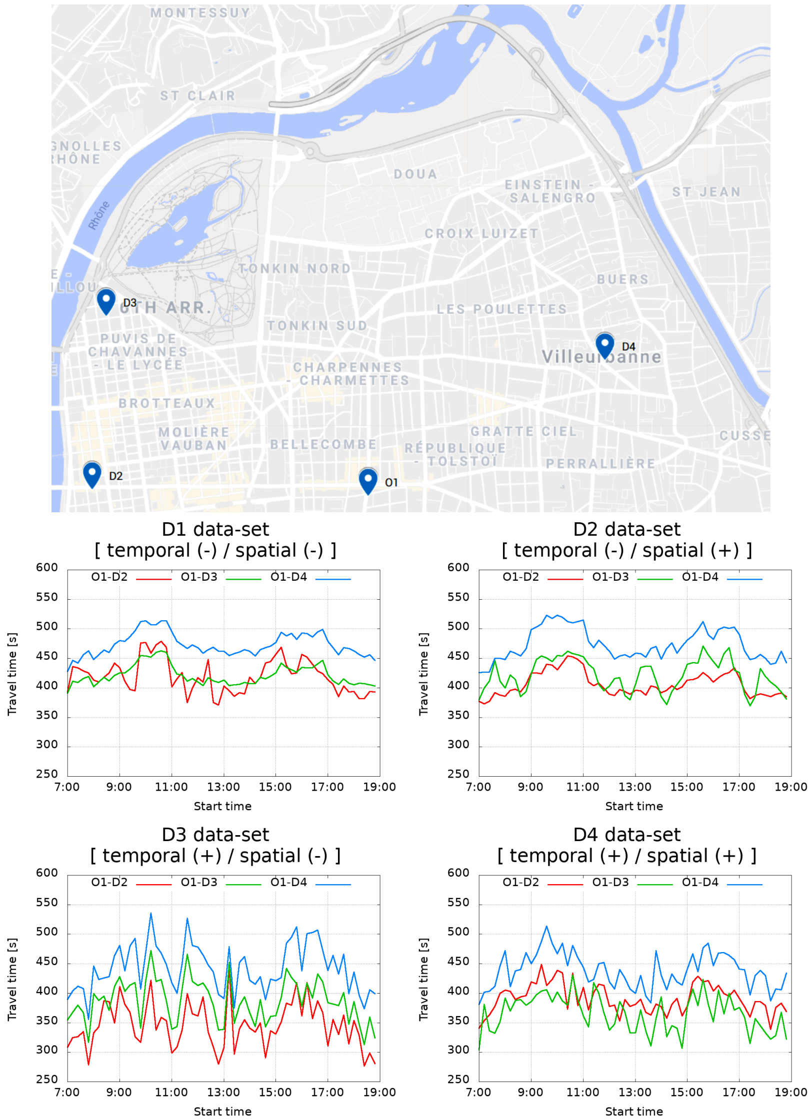

Fig. 3: An instance of travel costs between four O-D delivery addresses, which are shown in the top map, and computed according to D1, D2, D3 and D4 data-sets.

30 instances by randomly selecting n urban addresses from the city of Lyon. For each set of addresses, we have build 5 time-dependent cost functions corresponding to 5 time-step lengths $\Delta t = l \in \{6, 12, 24, 60, 720\}$ expressed in minutes. $l = 720$ corresponds to the constant costs, since our time horizon [7:00, 19:00] is of 12 hours. We consider that all tours start at time $t_0 = $ 7:00

and cannot finish after 19:00. A fixed stop duration of 6 minutes is associated to each address. The same sets of addresses are used for the 4 data-sets of the benchmark. Hence, our new benchmark is composed of $3 * 30 * 5 * 4 = 1800$ instances that will be publicly available.

4 Experimental analysis: First results

In this section, we present an overview of the first results obtained when comparing static TSP and TD-TSP yield solutions. Let T^{DiSl} denote the optimal tour calculated based on travel-time costs of the data-set Di, with $i \in \{1, 2, 3, 4\}$, and the time-step $l \in \{6, 12, 24, 60, 720\}$. To allow a fair and realistic comparison, we evaluate all optimal tours using $D4S6$ costs. Once T^{DiSl} is found, its time-span using $D4S6$ costs, denoted as $ts^{D4S6}(T^{DiSl})$, is computed. This will allow us to compare our delivery tours in the same basis and in the best approximation of real traffic conditions, since $D4$ is considered to be the ground true data, and $l = 6$mn is the smallest time-step we consider.

Temporal degradation and the size of time steps. The gap in percentage between the static T^{DiS720} and the time-dependent T^{DiSl} optimal tours, which is defined as:

$$\frac{ts^{D4S6}(T^{DiS720}) - ts^{D4S6}(T^{DiSl})}{ts^{D4S6}(T^{DiSl})} \times 100,$$

for $l \in \{6, 12, 24, 60\}$ and $i \in \{1, 2, 3, 4\}$, is evaluated in Fig. 4. The distributions of Fig. 4 show that when the costs of evaluation are the same as those used for optimization, *i.e.*, the case of $D4S6$ costs, the gap is always positive, which is indeed expected since $ts^{D4S6}(T^{D4S6})$ is the optimization's evaluation. The crucial observation here is that this gap increases with larger values of n. When enlarging to the cases of $D4Sl$ costs, we observe that the gaps tend to decrease more, the wider the time-step l is. Hence, it is worth optimizing tours with time-dependent costs, since the evaluation $ts^{D4S6}(T^{DiSl})$ is getting lower for finer time-steps l.

Comparing cases of temporal degradation amounts to compare tours based on $D1$ to $D3$ data-set, and those based on $D2$ to $D4$ data-set. In the case of a partial spatial network cover ($D1$ and $D3$ data-sets), the observed trend is that having a full temporal visibility ($D3$ data-set) contributes to slightly lowering the gap in the negative direction, thus favoring static tours, as to the partial temporal visibility situation ($D1$ data-set). However, the opposite happens in case of a full spatial cover of the network. $D4$ leads to larger gaps, thus it suggests a more accurate tour evaluation than that of $D2$.

Fig. 5 further examines the quality of static and time-dependent optimal tours using $ts^{D4S6}(\,.\,)$ evaluation function. It is worth to notice that, when $n = 10$, static tours are better than or equivalent to optimal TD-TSP tours, for over 60% of the cases, in every $DiSl$ costs, with $i \in \{1, 2, 3, 4\}$ and $l \in \{6, 12, 24, 60\}$, except for the case of $D4S6$ (where evaluation and optimization costs correspond). Thus, tours yield by constant travel costs are more

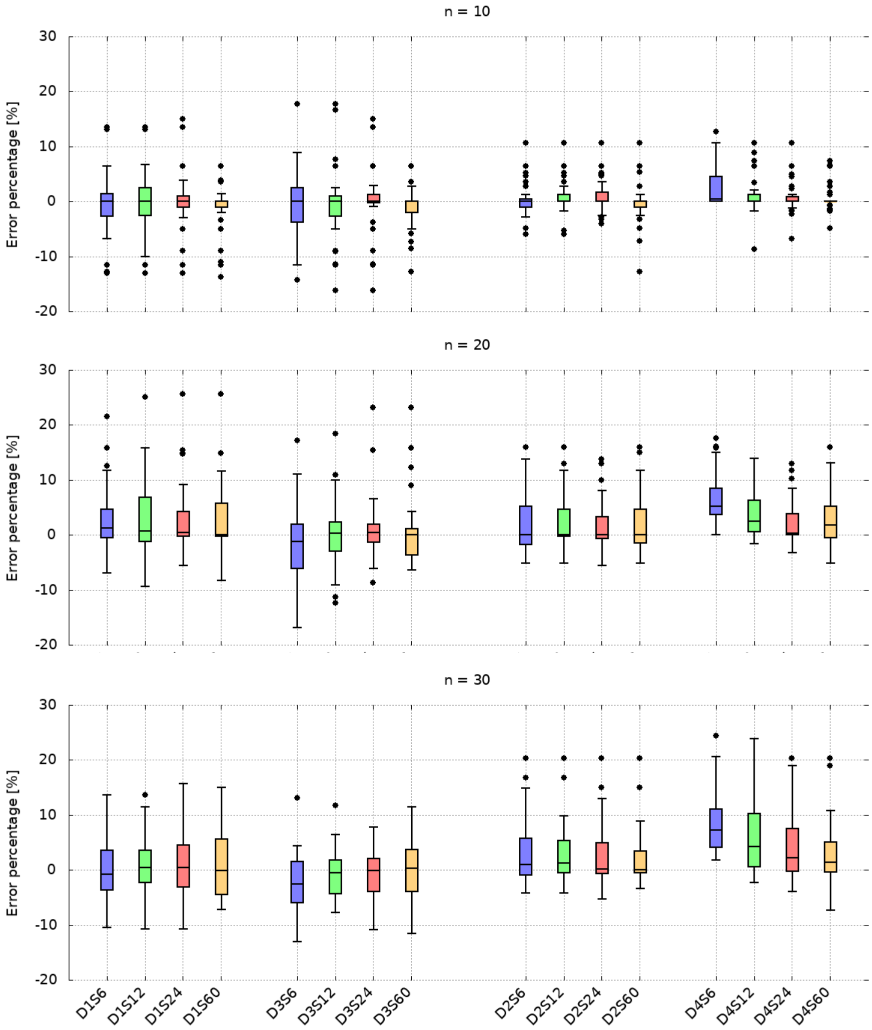

Fig. 4: Distribution of the gap (in percentage) between the static $ts^{D4S6}(T^{DiS720})$ and the time-dependent $ts^{D4S6}(T^{DiSl})$ optimal tours for $l \in \{6, 12, 24, 60\}$, $i \in \{1, 2, 3, 4\}$, and $n = \{10, 20, 30\}$.

appealing for small deliveries, especially when we don't have a full temporal and spatial visibility of the network (data-sets $D1$, $D2$ and $D3$).

Spatial degradation and the size of time steps. In Fig. 6, we focus on data-sets wherein a full temporal predictive model is assumed, *i.e.*, $D3$ and $D4$. Fig. 6 shows that accounting only for the actual cover of sensors ($D3$ data-set) induces an over-estimation of the time-spans for the optimal tours, in such a manner that static tours of $D4$ are way better than the $l = 6$mn time-step

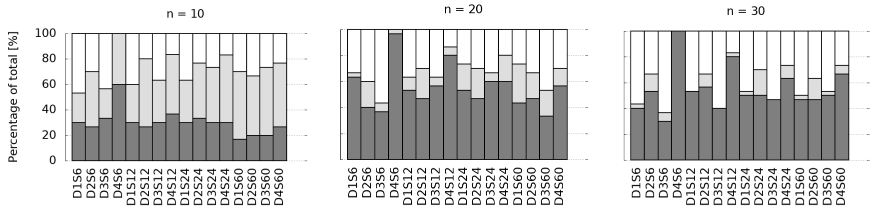

Fig. 5: Allocation of the following cases: in dark (resp. white) color, the TD optimal tour $ts^{D4S6}(T^{DiSl})$ is better (resp. worse) than the static optimal tour $ts^{D4S6}(T^{DiS720})$, for $i \in \{1, 2, 3, 4\}$ and $l \in \{6, 12, 24, 60\}$, while in gray color both tours correspond. The result is shown for n={10, 20, 30}.

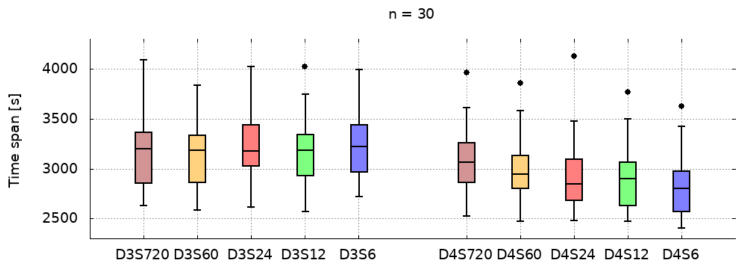

Fig. 6: Distributions of $ts^{D4S6}(T^{D3Sl})$ and $ts^{D4S6}(T^{D4Sl})$, for $l \in \{6, 12, 24, 60, 720\}$ and $n = 30$.

based tours of $D3$: $ts^{D4S6}(T^{D4S720}) < ts^{D4S6}(T^{D3S6})$. Already in Fig. 3, we have noticed that $D3$ data-set displays high variations in terms of travel times, which is suggesting a more congested network than what the actual $D4$ costs would imply. Actually, sensors are much often placed in congested areas of urban networks, which is leading to a biased and rather a worst-case scenario when estimating travel times (provided that the estimation method of travel times is exact). This prompts us to ask the following question of how to better place sensors in urban road networks with the aim of capturing a truthful picture of the traffic flow.

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