

# Block Turbo Codes for Space-Time Systems

K. Cavalec-Amis and R. Pyndiah  
 ENST Bretagne  
 Technopôle Brest Iroise  
 BP 832, 29285 Brest Cedex  
 FRANCE  
 E-mail: Karine.Cavalec@enst-bretagne.fr

**Abstract** - Recent advances in the field of Information Theory have shown that wireless systems using multiple transmitters and multiple receivers can significantly improve spectral efficiencies. Systems using transmit and receive diversity associated with coding such as Space Time Trellis codes have since been developed. More recently, a scheme using Convolutional Turbo Codes (CTC) [1] with two transmitters and two receivers has been proposed [2] and exhibits significant gains over Space Time Trellis codes (up to 6 dB at a BER of  $10^{-5}$ ). Our purpose is to extend that study to Block Turbo Codes (BTC) [3]. We use the  $(32, 26, 4)^2$  extended BCH Product code with two kinds of modulation (QPSK and 16-QAM). Simulations show that BTC associated with transmit and receive diversity perform as well as CTC in terms of coding gains while achieving higher spectral efficiencies.

## I. INTRODUCTION

Capacity of wireless communication systems undergoes limitations due to limited bandwidth, propagation loss, noise, multipath effects of the physical channel ... On the other hand the development of mobile applications requires higher and higher capacities. Recent research in the field of Information Theory showed that systems using multiple transmitters and multiple receivers can achieve very high spectral efficiencies on wireless channels. Several Space-Time architectures [4], [5] have since been proposed and Space-Time codes have been developed [6]. More recently Stefanov and Duman [2] proposed Convolutional Turbo Codes for achieving Transmit and Receive diversity. This Forward Error Correction technique introduced in 1993 by C. Berrou[1] was a real breakthrough in the field of Information Theory as it achieves very low BERs at SNRs close to the theoretical Shannon's limit. In 1994, R. Pyndiah extended the principle to Product codes [3], [7]. Block Turbo Codes (BTC) perform as well as Convolutional Turbo Codes (CTC). In this article, we propose to apply BTC to Space-Time systems.

In Section II we present the considered wireless system and the associated notations. In Section III, after a quick overview of Product Codes, we deal with the interleaving device, the Soft Input Output Decoder computations and the Turbo Decoding algorithm. Finally in Section IV, we give performance of BTC

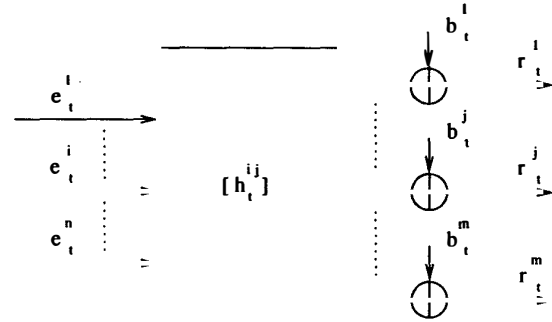


Figure 1: The System Model.

used with a QPSK and a 16-QAM.

## II. THE SYSTEM MODEL

We consider a communication system consisting of  $n$  transmitters and  $m$  receivers. Information data are first encoded, then interleaved with other frames if necessary before passing through a serial to parallel converter. Each converter output is mapped to a given signal constellation. Let us note  $e_t^i$  the signal transmitted from the  $i^{th}$  antenna at time  $t$ . The Space-Time system is shown in Fig.1.

The signal at time  $t$  at the  $j^{th}$  receiver is the sum of the noise with the superposition of the  $n$  attenuated signals  $e_t^i$ . The noise samples  $(b_t^1, \dots, b_t^j, \dots, b_t^m)$  are supposed to be independent, complex, gaussian, of zero mean and variance  $N_0/2$  per dimension. The channel attenuation coefficients  $h_t^{ij}$  are considered complex, gaussian, of zero mean and variance  $1/\sqrt{2}$  per dimension. So as to have a constant Average Signal to Noise Ratio and equal to  $1/N_0$ , we have normalized the coefficients  $h_t^{ij}$  as well as the constellation signals. Channel output  $r_t^j$  can be written as :

$$r_t^j = \sum_{i=1}^n h_t^{ij} \times e_t^i + b_t^j \quad (1)$$

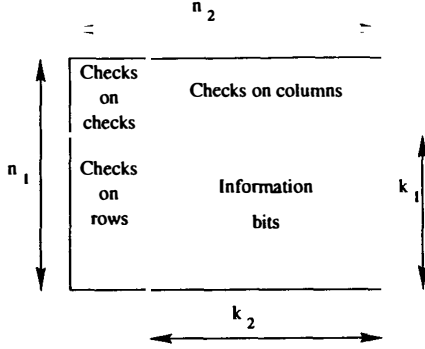


Figure 2: Construction of product code  $\mathcal{P}$ .

### III. BTC AND SPACE-TIME CODING

#### A. Encoding : Product codes

The concept of product codes was first introduced by Elias [8] in 1954. A product code consists of two systematic linear block codes  $\mathcal{C}_1(k_1, n_1, \delta_1)$  and  $\mathcal{C}_2(k_2, n_2, \delta_2)$ , where  $n_i, k_i, \delta_i$  stand for code length, number of information bits and minimum Hamming distance respectively. So as to obtain the product code  $\mathcal{P} = \mathcal{C}_1 \times \mathcal{C}_2$ , we first place information bits in a  $k_1 \times k_2$  matrix (cf. Fig.2). Then we encode the  $k_1$  rows using  $\mathcal{C}_2$  and finally we encode the  $n_2$  columns using  $\mathcal{C}_1$ . The code parameters of  $\mathcal{P}$  are :

$$n = n_1 \times n_2; \quad k = k_1 \times k_2; \quad \delta = \delta_1 \times \delta_2$$

One can show that all the  $n_1$  rows of the matrix in Fig.2 are code words of  $\mathcal{C}_2$  just as the  $n_2$  columns are code words of  $\mathcal{C}_1$ .

#### B. Interleaving

After encoding, the encoded frame  $E_k$  is interleaved diagonalwise giving  $I_k$ . Then, if inter frames interleaving is required, we proceed according to the device illustrated in Fig.3.  $(I_k)_{1 \leq k \leq K}$  are the frames that are to be interleaved and  $D$  the  $n \times (L \times q \times \tau)$  resulting matrix with  $\tau$  the symbol number over which the channel coefficients are considered constant,  $L \times \tau$  the number of symbols transmitted from a given antenna and  $2^q$  the modulation constellation size.

#### C. Decoder input computation

Let us assume that the modulation constellation size is  $2^q$ . We note  $\mathcal{E}$  this constellation,  $(d_1^i, d_2^i, \dots, d_q^i)$  the  $q$ -uple mapped to the constellation signal  $e^i$ . At time  $t$ , we have to extract  $n \times q$  reliability values  $x_p^i$  from  $r^1, \dots, r^m$ . They will be used as inputs of the SISO decoder. As proposed by Stefanov and Duman [2], we have taken the weighted Log Likelihood Ratio

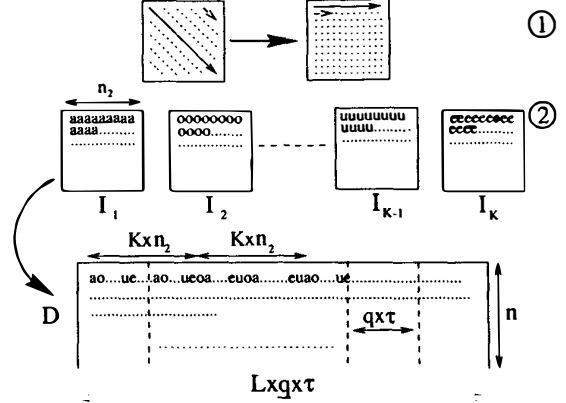


Figure 3: Interleaving.

(LLR) values for  $x_p^i$  (corresponding to  $d_p^i$ ). The LLR of  $d_p^i$  is defined as :

$$\Gamma(d_p^i) = \ln \frac{P(d_p^i = 1 | r^1, \dots, r^m)}{P(d_p^i = 0 | r^1, \dots, r^m)} \quad (2)$$

We note  $r = [r^1 r^2 \dots r^m]^T$  and  $e = [e^1 e^2 \dots e^n]^T$ . Eq.(2) can be written in the following way :

$$\Gamma(d_p^i) = \ln \frac{\sum_{e \in \mathcal{E}^n | d_p^i = 1} P(e|r)}{\sum_{e \in \mathcal{E}^n | d_p^i = 0} P(e|r)} \quad (3)$$

By applying the Bayes' rule and taking into account the fact that the noise is AWGN per dimension, we can write  $\Gamma(d_p^i)$  as follows :

$$\Gamma(d_p^i) = \ln \frac{\sum_{e \in \mathcal{E}^n | d_p^i = 1} e^{-\frac{\|r - he\|^2}{N_0}}}{\sum_{e \in \mathcal{E}^n | d_p^i = 0} e^{-\frac{\|r - he\|^2}{N_0}}} \quad (4)$$

The main drawback of this approach is its computational complexity. A first simplification can be obtained by the approximation given in Eq.(5). Simulations show that this approximation does not introduce any significant degradation.

$$\Gamma(d_p^i) = \min_{e \in \mathcal{E}^n | d_p^i = 0} \frac{\|r - he\|^2}{N_0} - \min_{e \in \mathcal{E}^n | d_p^i = 1} \frac{\|r - he\|^2}{N_0} \quad (5)$$

#### D. Iterative SISO decoding

After the Log Likelihood computations, data are deinterleaved before being decoded. We note  $v_i$  the deinterleaved data. The received matrix (see Fig.2) is formed by correctly placing the data  $v_i$ . The iterative decoding consists in decoding first the rows of the matrix then its columns. This procedure

is iterated  $x$  times. Each elementary decoder uses soft inputs provided by the channel and by the previous decoder which delivers soft outputs called "extrinsic information" [3], [7]. This extrinsic information is added to the channel observation and becomes the next decoder soft input. Each elementary decoder uses the Chase algorithm [9] to find the most likely code word  $D = (d_i)_{1 \leq i \leq n}$  and evaluates the Log Likelihood Ratio (LLR) of each decoded bit  $d_i$  given by :

$$\Gamma(d_j) = \ln \frac{P\{c_j = +1|V\}}{P\{c_j = -1|V\}} \quad (6)$$

Let  $\mathcal{S}^{\epsilon(i)} = \{C \in \mathcal{C} | c_j = \epsilon\}$ . Then (6) becomes :

$$\Gamma(d_j) = \ln \frac{\sum_{C \in \mathcal{S}^{+1(i)}} P\{V|C\}}{\sum_{C \in \mathcal{S}^{-1(i)}} P\{V|C\}} \quad (7)$$

Let us assume that the noise is AWGN of variance  $\sigma^2$ . One can write (7) in the following way :

$$\Gamma(d_j) = \ln \frac{\sum_{C \in \mathcal{S}^{+1(i)}} \exp \frac{-\|V-C\|^2}{2\sigma^2}}{\sum_{C \in \mathcal{S}^{-1(i)}} \exp \frac{-\|V-C\|^2}{2\sigma^2}} \quad (8)$$

One can approximate [3], [7]  $\Gamma(d_j)$  by the following :

$$\Gamma(d_j) = \frac{\|V - C^{-1(j)}\|^2}{2\sigma^2} - \frac{\|V - C^{+1(j)}\|^2}{2\sigma^2} \quad (9)$$

with  $C^{\epsilon(j)} = \operatorname{argmin}\{\|V - C^{\epsilon(j)}\|, C \in \mathcal{S}^{\epsilon(i)}\}$ . Let us note  $D = \operatorname{argmin}\{\|V - C^{+1(j)}\|, \|V - C^{-1(j)}\|\}$ .  $D$  is the most probable code word and  $C = \operatorname{argmax}\{\|V - C^{+1(j)}\|, \|V - C^{-1(j)}\|\}$  is called the competing code word of  $D$ . Eq.(9) can be rewritten as :

$$\Gamma(d_j) = d_j \times \left( \frac{\|V - C\|^2}{2\sigma^2} - \frac{\|V - D\|^2}{2\sigma^2} \right) \quad (10)$$

Eq.(10) can be also expressed as :

$$\Gamma(d_j) = \frac{2}{\sigma^2} \times d_j \times (v_j + w_j) \quad (11)$$

where  $v_j$  is the soft input and  $w_j$  the soft output called "extrinsic information". If  $C$  is not available, then  $w_j = \beta d_j$ , where  $\beta$  is a predefined constant.

#### IV. SIMULATION RESULTS

In all simulations, our Space Time system consists of 2 transmitters and 2 receivers. We consider the  $(32, 26, 4)^2$  extended BCH product code which bears a code rate close to  $2/3$ . We compare several interleaved frame numbers  $K$ . Therefore the total transmitted bit number is equal to  $K \times 1024$ . The channel coefficients are supposed to be constant over  $\tau = 128$  constellation symbols. For each modulation, the ideal interleaving

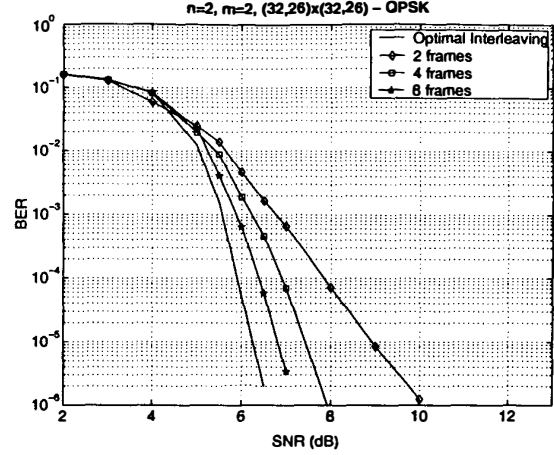


Figure 4: QPSK,  $\eta = 2.66$  bits/s/Hz, two transmit and two receive antennas.

performance in terms of BER are given; ideal interleaving because the channel coefficients are changed for each symbol. By increasing  $K$ , we get closer to these optimal performance. Taking into account that this system is dedicated to high bit rate applications, we have only used 4 iterations. We assume a perfect knowledge of the channel at the receiver.

##### A. 4-PSK modulation

The spectral efficiency is close to 3. We compare different interleaved frame numbers. One of the main features of a Phase Modulation is that the amplitude of the constellation signals is constant. We have considered a Gray mapping. Simulation results are shown in Fig.4. If we compare the curve with 2 interleaved blocks with the optimal curve, we observe a gap of 2.5 dB for a BER of  $10^{-5}$ . But we improve the SNR by 1.5 dB with 4 interleaved blocks and by 2 dB with 8 interleaved blocks. By increasing the number of blocks, we would get closer to the optimal curve. Our results confirm Stefanov and Duman's conclusions [2] : they announced a gain of up to 6 dB with a one half rate Convolutional Turbo Code over the 16 state space-time trellis code [6] at a BER of  $10^{-5}$ , with a QPSK (For an interleaving over 10400 bits, they achieve a BER of  $10^{-5}$  at a SNR of 5 dB). Let us recall that our code rate is nearly  $2/3$  and we use only 4 iterations (instead of 10), which accounts for the 2 dB difference with their results at a BER of  $10^{-5}$ , while our spectral efficiency is higher ( 3 bits/s/Hz instead of 2 bits/s/Hz).

##### B. 16-QAM modulation

The spectral efficiency is greater than 5 bits/s/Hz. We have used a Gray mapping (see Fig.5). If we note  $abcd$  the qua-

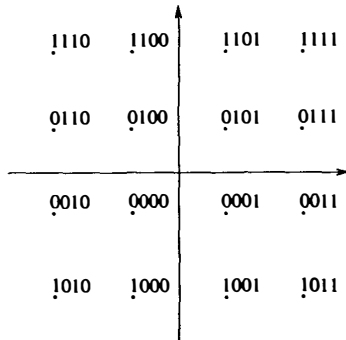


Figure 5: Mapping for the 16-QAM.

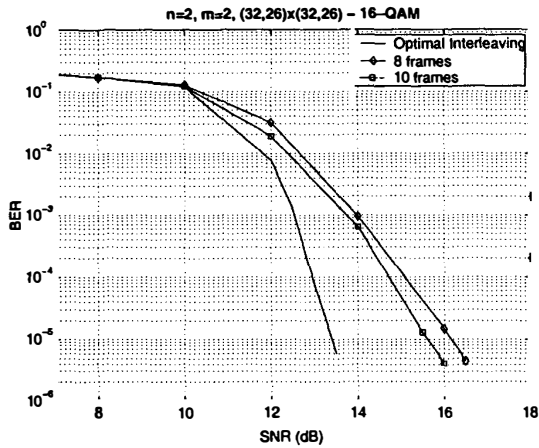


Figure 6: 16-QAM,  $\eta \geq 5.33$  bits/s/Hz, two transmit and two receive antennas.

ternary symbol used for the mapping, it is well-known that the error probability over  $b$  and  $d$  is smaller than the one of  $a$  and  $c$ . The way the mapping is done is therefore crucial. It is important that rows and columns of  $I_k$  (see Fig.3) consist of as many  $a$  as  $b$  (soft decoder inputs for  $b$  are more reliable than those for  $a$ ). Simulation results are shown in Fig.6. We note a difference of 2 dB (resp. 2.5 dB) between the ideal interleaving and the interleaving over 10 frames (resp. 8 frames) at a BER of  $10^{-5}$ . By increasing the number of interleaved frames, performance would be closer to the optimal results. Comparing our results with [2], we observe a difference of 1.75 dB at a BER of  $10^{-5}$  for an equivalent number of interleaved coded bits. However, let us recall that our code rate is  $2/3$  instead of  $1/2$ , we use 4 iterations instead of 10 and we achieve a spectral efficiency of 5.33 bits/s/Hz instead of 4. Nevertheless our results check Stefanov and Duman's conclusions: Turbo Codes used in Space

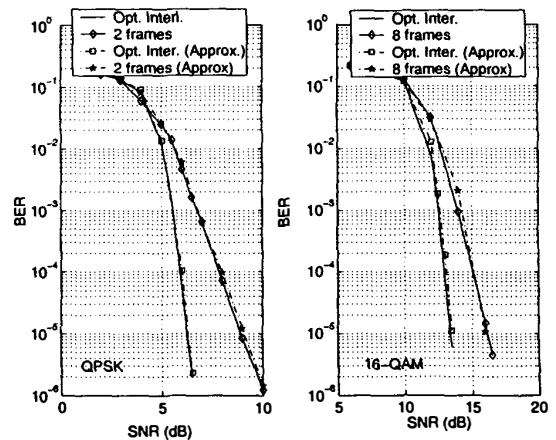


Figure 7: Approximation versus exact computation.

Time systems perform better than space-time trellis codes.

### C. Performance of simplified receiver

In this part we compare previous results with performance obtained when using the approximation given by Eq.(5). When looking at Fig.7, one can notice no significant degradation between performance with the exact computation and those with its approximation. However this approximation bears a lower computational complexity.

## V. CONCLUSION AND PERSPECTIVES

In [2], the authors showed that Convolutional Turbo Codes performed much better than Space-Time Trellis Codes. Our purpose in this paper was to extend this study to Block Turbo Codes. We showed that BTC with multiple transmitters and multiple receivers perform as well as CTC while achieving higher spectral efficiencies within less iterations. The main weak point of the proposed solution is the LLR computation before SISO decoding. Indeed it requires  $(2^q)^n$  metric computations for each bit, which brings a  $(2^q)^n$  order complexity. This complexity becomes prohibitive as  $q$  or  $n$  increases. Therefore future work should consider lower complexity solutions even at the expense of some performance degradation. A simplified solution has been investigated and its performance are equivalent to the optimal solution ones. One interesting feature of Transmit and Receive diversity is the fact that we can use PSK modulations (constant amplitude) with multiple transmitters so as to reach the same spectral efficiency as systems based on QAM modulations. For instance, systems using 16-QAM with  $n$  transmitters could be replaced by 4-QPSK with  $2 \times n$  transmitters. Advantages of such a solution are numerous. First, transmitted power over each transmitter is halved

and two half power transmitters cost less than a full power one. Furthermore, QAMs are very sensitive to AM-AM and AM-PM distortions. They require linear amplification. So as to ensure linear operation, a significant back-off on the output power is to be taken into account, increasing the power requirements. On the other hand QPSK are more robust to non-linearity effects as it is a constant amplitude modulation.

#### REFERENCES

- [1] C. Berrou and A. Glavieux. "Near optimum error correcting coding and decoding : Turbo-codes ". *IEEE Trans. Commun.*, vol. 44, pp. 1261-1271, Oct. 1996.
- [2] A. Stefanov and T. Duman. "Turbo Coded Modulation for Systems with Transmit and Receive Antenna Diversity. ". *Proc. IEEE Globecom '99*, vol. IT-18, Nov.-Dec. 1999.
- [3] R. Pyndiah. "Near-Optimum Decoding of Product Codes: Block Turbo Codes ". *IEEE Trans. Commun.*, vol. 46, pp. 1003-1010, Aug. 1998.
- [4] G. J. Foschini. "Layered Space Time Architecture for Wireless Communication in a Fading Environment when using Multi-Element Antennas. ". *Bell Labs Technical Journal*, Autumn 1996.
- [5] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank. "Combined Array Processing and Space Time Coding. ". *IEEE Trans. Infor. Theory*, vol. 45, pp. 1121-1128, May 1999.
- [6] V. Tarokh, N. Seshadri, and A. R. Calderbank. "Space Time codes for high data rate wireless communication : performance criterion and code construction. ". *IEEE Trans. Infor. Theory*, pp. 744-765, Mar. 1998.
- [7] R. Pyndiah, A. Glavieux, A. Picart, and S. Jacq. "Near-Optimum Decoding of Product Codes ". *Proc. IEEE Globecom '94 Conf.*, vol. 1/3, Nov.-Dec. 1994.
- [8] P. Elias. "Error free coding. ". *IRE Trans. Inform. Theory*, vol. IT-4, pp. 29-37, Sept. 1954.
- [9] D. Chase. "A class of algorithms for decoding block codes with channel measurement information. ". *IEEE Trans. Infor. Theory*, vol. IT-18, Jan. 1972.