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A new efficient method to solve the stream power law model taking into account sediment deposition 2

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Key Points:

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9	• We present an efficient $(O(N))$ and implicit) method to solve a river erosion model	
10	taking into account sediment deposition.	
11	• We show how the foreland stratigraphy is controlled by the efficiency of river ero-	
12	sion and the efficiency of sediment transport by rivers.	
13	• We observe autogenic aggradation and incision cycles in the foreland once the sys-	

y. tem reaches a dynamic steady state. 14

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15 Abstract

The stream power law model has been widely used to represent erosion by rivers, but 16 does not take into account the role played by sediment in modulating erosion and de-17 position rates. Davy and Lague (2009) provide an approach to address this issue, but 18 it is computationally demanding because the local balance between erosion and depo-19 sition depends on sediment flux resulting from net upstream erosion. Here, we propose 20 an efficient (i.e., O(N) and implicit) method to solve their equation. This means that, 21 unlike other methods used to study the complete dynamics of fluvial systems (includ-22 ing the transition from detachment-limited to transport-limited behavior, for example), 23 our method is unconditionally stable even when large time steps are used. We demon-24 strate its applicability by performing a range of simulations based on a simple setup com-25 posed of an uplifting region adjacent to a stable foreland basin. As uplift and erosion 26 progress, the mean elevations of the uplifting relief and the foreland increase, together 27 with the average slope in the foreland. Sediments aggrade in the foreland and prograde 28 to reach the base level where sediments are allowed to leave the system. We show how 29 the topography of the uplifting relief and the stratigraphy of the foreland basin are con-30 trolled by the efficiency of river erosion and the efficiency of sediment transport by rivers. 31 We observe the formation of a steady-state geometry in the uplifting region, and a dy-32 namic steady state (i.e., autocyclic aggradation and incision) in the foreland, with aggra-33 dation and incision thicknesses up to tens of metres. 34

35 1 Introduction



Figure 1. (a) The concept of source to sink with sediment transport and deposition, modified from *Allen and Heller* (2011). (b) Setup for our simulation with uplifting region and foreland basin. (c) Illustration of a simple catchment with normal FastScape stack order (*Braun and Willett*, 2013). In (b), the red lines indicate the closed boundary where sediment flux cannot leave the system, whereas base level (green line) is fixed as an open boundary.

Quantifying the dynamics of river erosion, sediment transport and deposition (Figure 1a) is a fundamental problem in geomorphology that has great relevance for our understanding of landscape evolution in tectonically active areas. Many parameterizations of these processes have been proposed and implemented in numerical landscape evolution models (*Braun and Sambridge*, 1997; *Chase*, 1992; *Crave and Davy*, 2001; *Kooi and Beaumont*, 1994; *Tucker and Slingerland*, 1994).

The Stream Power Law (SPL) model has been widely used to represent erosion by rivers (*Howard and Kerby*, 1983; *Whipple and Tucker*, 1999). In its simplest form, it assumes that erosion rate is proportional to the shear stress exerted by the river on its bed which, in turn, is proportional to net precipitation rate, p, drainage area, A, and local slope, S, according to:

$$\frac{\partial h}{\partial t} = U - K_f \, p^m A^m S^n \,, \tag{1}$$

where h is topographic elevation, t is time, U is uplift rate, K_f is the fluxial erosion co-42 efficient, and m and n are the SPL exponents. An important assumption of the SPL model 43 is that sediments are efficiently transported by rivers and not deposited in the simulated 44 domain. The SPL model has been shown to describe a number of fluvial landscapes and 45 processes. It is for example commonly used to infer uplift pattern from river profiles or 46 to model topographic evolution at the scale of a catchment (Braun and Willett, 2013; 47 Campforts et al., 2017; Lavé and Avouac, 2001). Yet, it is well known that this model 48 might be oversimplified as it does not consider several important processes acting in river 49 channels (Laque, 2014). In particular, it is necessary to take into account the role played 50 by sediment in modulating erosion rate and/or deposition (Whipple and Tucker, 2002), 51 such as a dependence on bedload transport (Davy and Lague, 2009; Kooi and Beaumont, 52 1994) or a bed-cover effect (Cowie et al., 2008; Johnson et al., 2009; Sklar and Dietrich, 53 2001). In fact, transported sediments provide the tools for abrasion and fracturing of rock 54 but also, if overly abundant, they can protect the bedrock from erosion (Sklar and Di-55 etrich, 1998). 56

Several parameterizations have been proposed to adapt the SPL model to incor-57 porate the effects of transported sediments, including the erosion-deposition formulation 58 proposed by Davy and Laque (2009), which differs from models based on the divergence 59 of sediment flux (e.g., Paola and Voller, 2005) in that it conserves mass on the bed and 60 in the water column to treat simultaneous erosion and deposition of a single substrate 61 (Shobe et al., 2017). Based on previous erosion-deposition models (e.g., Beaumont et al., 62 1992; Kooi and Beaumont, 1994), Davy and Laque's (2009) formulation has a limited 63 number of parameters, while attempting to relate these parameters to physical processes 64 and quantities (e.g., saltation length). In addition, their erosion-deposition framework 65 allows the exploration of both detachment-limited and transport-limited models with sim-66 ple parameter changes, and displays a smooth transition between the two types of model 67 behavior. 68

Davy and Lague's (2009) formulation has been used or adapted to obtain simple 69 models (e.g., Carretier et al., 2016; Ganti et al., 2014; Langston and Tucker, 2018; Mouchené 70 et al., 2017; Shobe et al., 2017), which assume that the net rate of topographic change 71 is the sum of the erosion rate (controlled by the SPL model) and of the deposition rate, 72 which is proportional to local suspended sediment flux and to a dimensionless deposi-73 tion coefficient, and inversely proportional to drainage area, a proxy for water discharge. 74 This parameterization is also receiving growing acceptance due to its ability to repro-75 duce many depositional features of fluvial systems (Carretier et al., 2016; Mouchené et al., 76 2017; Shobe et al., 2017). However, because the local balance between erosion and de-77 position depends on sediment flux resulting from net upstream erosion, this parameter-78 ization is computationally demanding. 79

Braun and Willett (2013) have proposed an efficient algorithm for solving the SPL model, which is ideally suited for a large number of model simulations as required for

inverting observational constraints in a Bayesian approach. Here we present an equally 82 efficient (i.e., O(N) and implicit) method to solve the equation proposed by Davy and 83 Laque (2009) that takes into account sediment deposition. O(N) means that the com-84 putational time increases linearly with the number of points used to discretize the land-85 scape. An implicit scheme guarantees unconditionally stable time integration of the land-86 scape evolution equation, which means that large time steps can be used without affect-87 ing numerical stability. This is potentially an important step as it allows one to use sed-88 imentological observations, such as the stratigraphy of foreland basins or the position 89 and thickness of river terraces, to further constrain landscape evolution models. 90

Nonetheless, the novelty of our study is not limited to the description of the com-91 putational efficiency. Based on this algorithm, our model can simulate erosion and de-92 position in fluvial landscapes, at large spatial (up to thousands of kilometers) and tem-93 poral (up to tens of millions of years) scales. Because these two processes are considered 94 in one single equation, deposition can occur anywhere in the domain (i.e., not only in 95 depressed areas but also along channels or in stable continental areas). We therefore use 96 our new algorithm to explore the impact of coupling erosion and deposition in a fluvial 97 landscape, and show that this erosion-deposition relationship, which is often ignored, has 98 a strong impact on relief in the uplifted domain, the concavity of the channels and their 99 steepness index. We also demonstrate that this relatively simple model leads to the cyclic 100 formation and destruction of river banks as the stream continuously migrates and some-101 times erodes into the sediment it has previously deposited. Such autogenic aggradation 102 and incision cycles are currently difficult to simulate in landscape evolution models. The 103 simulations using the new model can thus improve our understanding of the links be-104 tween external forcings, internal processes, and depositional features. 105

In the next section, we first present our implementation of *Davy and Lague*'s (2009) model and the O(N) and implicit numerical scheme. Model implications of our new formulation are shown in Section 3. We then explore the model behavior in Section 4 by performing a range of simulations based on a simple setup composed of an uplifting region adjacent to a stable continental area on which a foreland basin develops. In Section 5, using our model, we observe the formation of autocyclic aggradation and incision in the foreland once the system reaches a dynamic steady state.

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2 Model implementation

2.1 SPL model taking into account sediment deposition

The effect of upstream sediment flux was first incorporated into the SPL model by *Kooi and Beaumont* (1994), by assuming that the rate of topographic change results from the imbalance between a sediment "carrying capacity", q_f^{eqb} , and the upstream sediment yield, q_s , according to:

$$\frac{\partial h}{\partial t} = U - \frac{1}{L_f} \left(q_f^{eqb} - q_s \right) \quad \text{with} \quad q_f^{eqb} = K \, q_w \, S \,, \tag{2}$$

where L_f is a transport length, K is a dimensionless erosion coefficient, $q_w = Q_w/W$ is water discharge per unit width (Q_w , water discharge, and W, river width), and q_s is the sediment flux per unit width obtained by integrating the net upstream erosion rate:

$$q_s = \frac{1}{W} \int_A \left(U - \frac{\partial h}{\partial t} \right) dA \,. \tag{3}$$

¹¹⁵ This sediment flux therefore accounts for the whole solid load (bed, suspended, and wash

load). The transport length L_f in (2) can be regarded as the length scale over which the

¹¹⁷ imbalance between the upstream sediment yield and the river carrying capacity is resolved

either by deposition (in cases where the river is over-capacity) or by erosion (in cases where

the river is under-capacity). Physically, L_f represents the average transport distance of

sediment grains within the flow from the location where they are eroded to the location where they are deposited (*Beaumont et al.*, 1992). L_f thus characterizes the proportion of incoming sediment flux which is deposited (the larger the value of L_f , the lower the rate of deposition).

Combining equations (2) and (3) leads to:

$$\frac{\partial h}{\partial t} = U - \frac{1}{L_f} \left[\frac{K Q_w S}{W} - \frac{1}{W} \int_A \left(U - \frac{\partial h}{\partial t} \right) dA \right].$$
(4)

To derive the SPL model, two major assumptions are commonly made. Firstly, it is assumed that river width W varies as the square root of the water discharge (*Lacey*, 1930; *Leopold and Miller*, 1956):

$$W = c Q_w^{0.5}, (5)$$

where c is an empirical constant, typically of the order of $(0.1-1)\times 10^{-2}$ (Montgomery and Gran, 2001, and references therein). Secondly, water discharge can be expressed as the product of net precipitation rate p and drainage area A:

$$Q_w = pA = p_0 \,\tilde{p}A\,,\tag{6}$$

where p_0 is mean net precipitation rate, and $\tilde{p} = p/p_0$ represents any spatial or temporal variation in precipitation p relative to the mean precipitation p_0 . Combining equations (5) and (6) with equation (4) leads to

$$\frac{\partial h}{\partial t} = U - \frac{1}{L_f} \left[\frac{K p_0^{0.5}}{c} \tilde{p}^{0.5} A^{0.5} S - \frac{1}{c p_0^{0.5} \tilde{p}^{0.5} A^{0.5}} \int_A \left(U - \frac{\partial h}{\partial t} \right) dA \right].$$
(7)

The contribution of *Davy and Lague* (2009) can be regarded as an improvement on *Kooi and Beaumont*'s (1994) method in an attempt to relate the transport length L_f to physical parameters (e.g., water discharge and settling velocity of grains within the flow). According to *Davy and Lague*'s (2009) formulation, the rate of change of topographic elevation is given by:

$$\frac{\partial h}{\partial t} = U - K' q_w^{m'} S^{n'} + \frac{q_s}{L_f} = U - K' q_w^{m'} S^{n'} + \frac{d^* v_s}{q_w} q_s \,, \tag{8}$$

where K' is an erosion efficiency coefficient, m' and n' are two exponents, v_s is the net 124 settling velocity of sediment grains, and d^* is the ratio between the sediment concentra-125 tion near the riverbed interface and the average concentration over the water column. 126 The value of $d^*(\geq 1)$ varies as a function of the Rouse number which defines the rela-127 tive contribution of bed, suspended, and wash loads (Davy and Lague's (2009) Figure 128 4). Davy and Lague (2009) discuss how d^* can be calculated for suspended load and bed 129 load rivers. For small rivers (or large particles), most of the entrainment mechanisms lie 130 in the bed load, d^* is much larger than 1 and the transport length L_f is small. Conversely, 131 for large rivers (or small particles) the Rouse number is small, d^* is close to 1 and L_f 132 is large. Davy and Laque (2009) present such a model based on the relative contribu-133 tions of (i) erosion from the bed into the water column (suspended load) and (ii) depo-134 sition from the water column onto the bed. Thus, the transport length L_f takes into ac-135 count the deposition of the bed load and the suspended load. With this approach, the 136 deposition term is proportional to the ratio between the sediment flux q_s and the wa-137 ter flux q_w . If $q_s \gg q_w$, deposition is high. On the contrary, if $q_s \ll q_w$, the deposi-138 tion term tends toward 0. Note that both terms are proportional to the drainage area 139 of the catchment. 140

The equivalent of Kooi and Beaumont's (1994) transport length L_f in the Davy and Lague's (2009) approach is therefore:

$$L_f = \frac{q_w}{d^* v_s} = \frac{Q_w}{Wd^* v_s} = \frac{pA}{Wd^* v_s} = \frac{p_0 \,\tilde{p}A}{Wd^* v_s} = \frac{\tilde{p}A}{WG} \,, \quad \text{with} \quad G = \frac{d^* v_s}{p_0} \,, \tag{9}$$

where G is a dimensionless deposition coefficient, which is a function of the sediment concentration ratio in transport, the settling velocity of sediment, and the mean precipitation rate. G/\tilde{p} is identical to Θ as defined in *Davy and Lague* (2009). They showed that, for typical values of the various parameters ($p = 10^{-7} \text{ m/s}, v_s \in [10^{-6} - 10^{-1}] \text{ m/s}),$ G/\tilde{p} is of order 1 or greater, in good agreement with estimates from natural sedimen-

tary systems (*Guerit et al.* (2018), pers. comm.).

Equations (3), (5), (6) and (9) can be combined with equation (8) to yield:

$$\frac{\partial h}{\partial t} = U - K' \left(\frac{Q_w}{W}\right)^{m'} S^{n'} + \frac{1}{WL_f} \int_A \left(U - \frac{\partial h}{\partial t}\right) dA
= U - \frac{K' p_0^{0.5m'}}{c^{m'}} \tilde{p}^{0.5m'} A^{0.5m'} S^{n'} + \frac{G}{\tilde{p}A} \int_A \left(U - \frac{\partial h}{\partial t}\right) dA.$$
(10)

In the parametric study of Davy and Lague (2009), m' = n' = 1 and their erosion term $\frac{K' p_0^{0.5}}{c} \tilde{p}^{0.5} A^{0.5} S$ is thus similar to that of Kooi and Beaumont (1994). The main differences between their models are: (i) the depositional term is inversely proportional to either the drainage area (Davy and Lague, 2009) or the square root of drainage area (Kooi and Beaumont, 1994); and (ii) L_f is applied only to q_s (Davy and Lague, 2009), or is applied to both q_f^{eqb} and q_s (Kooi and Beaumont, 1994).

Replacing $K' p_0^{0.5}/c^{m'}$, 0.5m' and n' in (10) by K_f , m and n, respectively, we can make a more direct connection to the SPL model and write that the rate of topographic change $\partial h/\partial t$ in response to tectonic uplift, river erosion and sediment deposition is given by:

$$\frac{\partial h}{\partial t} = U - K_f \,\tilde{p}^m A^m S^n + \frac{G}{\tilde{p}A} \int_A \left(U - \frac{\partial h}{\partial t} \right) dA \,. \tag{11}$$

The modified SPL formulation has only one additional parameter (i.e., the dimension-153 less deposition coefficient G) compared to the classic SPL model. The dimensionless con-154 stant G multiplying the deposition rate in (11) depends on an assumed mean precipi-155 tation rate; any spatial or temporal variation in precipitation rate is introduced through 156 the variable \tilde{p} . At steady state, the catchment area A in the deposition term vanishes, 157 and the deposition term is equal to GU/\tilde{p} . Note that this modified SPL formulation is 158 constructed by considering fluxes and, therefore, it does not specifically consider the ef-159 fect of grain size. However, the mathematical definition of G in (9) makes it related to 160 the size of the sediments in transport through the settling velocity v_s . In this work, we 161 consider different values of $G = d^* v_s / p_0$ as a whole, rather than studying individually 162 different values of d^* , v_s , and p_0 . 163

Fluvial erosion leads to the formation of hillslopes along river channels. Fluvial erosion and hillslope processes are interdependent, therefore hillslope processes need to be included in our model, which are commonly represented by a linear diffusion term (Ah-nert, 1967):

$$\frac{\partial h}{\partial t} = K_d \nabla^2 h \,, \tag{12}$$

where K_d is a hillslope sediment transport coefficient. In our model, the diffusion equation (12) is calculated separately, after solving equation (11). Both equations are applied in every cell of the landscape.

Easily detachable materials such as unconsolidated sediments should be character-167 ized by a larger erosion coefficient K_f than bedrock (Davy and Lague, 2009; Kooi and 168 Beaumont, 1994). Therefore, K_f depends on whether topographic elevation h is higher 169 than basement elevation h_{base} or not. In areas that are in net erosion (i.e., $h \leq h_{base}$), 170 we assume $K_f = K_{fb}$ (subscript b represents bedrock), whereas in areas covered by sed-171 iments (i.e., $h > h_{base}$), we assume $K = K_{fs}$ (subscript s represents sediments). In 172 most of our simulations, we assume $K_{fb} = K_{fs}$ for the sake of simplicity, and we use 173 $K_{fb} \neq K_{fs}$ for our sensitivity analysis in Section 4.2.2. 174

2.2 O(N) and implicit algorithm

The most challenging part is to solve equation (11) in an efficient manner (i.e., in O(N) operations) and using an implicit algorithm that allows for large time steps. For this we first discretize equation (11) using a backward Euler implicit finite difference scheme for each of the $n_x \times n_y$ nodes $(n_x \text{ and } n_y \text{ are the number of nodes to discretize the land$ scape in the x- and y-directions, respectively) as follows:

$$\frac{h_i^{t+\Delta t} - h_i^t}{\Delta t} = U - K_f \, \tilde{p}^m A_i^m \Big(\frac{h_i^{t+\Delta t} - h_{rec(i)}^{t+\Delta t}}{\Delta l_i}\Big)^n + \frac{G}{\tilde{p}\tilde{A}_i} \sum_{j=ups(i)} \Big(U - \frac{h_j^{t+\Delta t} - h_j^t}{\Delta t}\Big) \,, \quad 1 \le i \le n_x \times n_y$$

$$\tag{13}$$

in which, h_i^t and $h_i^{t+\Delta t}$ are the elevations of the *i*-th node at time t and time $t+\Delta t$, respectively, $\dot{h}_{rec(i)}^{t+\Delta t}$ is the elevation of the *i*-th node's receiver (the node in the steepestdescent drainage direction of the *i*-th node) at time $t+\Delta t$, Δl_i is the distance between the *i*-th node and its receiver, and $\sum_{j=ups(i)}$ represents the sum of the *i*-th node's upstream catchment nodes. \tilde{A}_i in equation (13) is a dimensionless catchment area defined as:

$$\tilde{A}_i = A_i / (\Delta x \Delta y) = N_i \,, \tag{14}$$

where $\Delta x, \Delta y$ are the horizontal sizes of the cells, and N_i is simply the number of cells 176 upstream of cell i. To compute the catchment areas A_i in O(N) operations, we use the 177

reverse stack order as defined in the FastScape algorithm (Braun and Willett, 2013). 178

To explain the remaining parts of our proposed numerical scheme, we assume that n = 1. The general case $(n \neq 1)$ is dealt with later. When n = 1, equation (13) can be expressed as

$$-F_{i}h_{rec(i)}^{t+\Delta t} + (1+F_{i})h_{i}^{t+\Delta t} + \frac{G}{\tilde{p}\tilde{A}_{i}}\sum_{j=ups(i)}h_{j}^{t+\Delta t} = b_{i}^{t},$$
with $F_{i} = \frac{K_{f}\tilde{p}^{m}A_{i}^{m}\Delta t}{\Delta l_{i}},$ and $b_{i}^{t} = h_{i}^{t} + U\Delta t + \frac{G}{\tilde{p}\tilde{A}_{i}}\sum_{j=ups(i)}(h_{j}^{t} + U\Delta t).$

$$(15)$$

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The term b_i^t on the right-hand side of the equation is known from the solution at time t, while elevations on the left-hand side are unknown at time $t + \Delta t$. 180

For the nodes at base level (open boundary, Figure 1b), we assume that the elevation is constant through time:

$$h_{base\,level}^{t+\Delta t} = h_{base\,level}^t \,. \tag{16}$$

We also assume that sediment can leave the system from these base level nodes. 181

The above finite difference equations can be expressed in the following matrix form:

$$\underline{B} \cdot \underline{h}^{t+\Delta t} = \underline{b}^t \,. \tag{17}$$

As shown in Appendix A, if we use the FastScape stack order in Figure 1c to solve equation (17), every row i of <u>B</u> has a single non-zero element before the diagonal element that corresponds to the receiver of node i, and many non-zero elements after the diagonal element that correspond to all upstream nodes of i. Solving (17) by factorizing the matrix <u>B</u> (e.g., by Gauss-Jordan elimination) is a problem of complexity of $O(n^3)$. To obtain a greater efficiency, we use a Gauss-Seidel iteration scheme to compute $\underline{h}^{t+\Delta t}$ in equation (17). This iterative algorithm requires to split the matrix <u>B</u> into its lower <u>F</u> and strictly upper triangular matrix \underline{E} as follows:

$$\underline{\underline{B}} \cdot \underline{\underline{h}}^{t+\Delta t} = (\underline{\underline{F}} + \underline{\underline{E}}) \cdot \underline{\underline{h}}^{t+\Delta t} = \underline{\underline{b}}^t , \qquad (18)$$



Figure 2. The number of iterations required for the solution to be convergent as a function of resolution $n_x \times n_y$. Solving equation (19) on a square area of resolution $n_x \times n_y = 961,10000,99856$, and 1000000, using U = 0.2 mm/yr, $K_f = 2 \times 10^{-5} \text{ m}^{1-2m}/\text{yr}$, p = 1 m/yr, and varying the value of G.

where the matrices \underline{F} and \underline{E} are shown in Appendix A.

The Gauss-Seidel iterative process starts with an initial guess $\underline{h}^{t+\Delta t, 0} = \underline{h}^t$ and uses the following recurrence to obtain an improved estimate $\underline{h}^{t+\Delta t, \overline{k}+1}$:

$$\underline{\underline{F}} \cdot \underline{\underline{h}}^{t+\Delta t, \, k+1} = \underline{\underline{b}}^t - \underline{\underline{E}} \cdot \underline{\underline{h}}^{t+\Delta t, \, k} \,, \tag{19}$$

from the value $\underline{h}^{t+\Delta t, k}$ obtained at the previous iteration. Interestingly, equation (19) can be written in a different form for each node *i*:

$$h_{i}^{t+\Delta t, k+1} = \frac{b_{i}^{t} - G/(\tilde{p}\tilde{A}_{i})\sum_{j=ups(i)}h_{j}^{t+\Delta t, k} + F_{i}h_{rec(i)}^{t+\Delta t, k+1}}{1+F_{i}}, \qquad (20)$$

¹⁸³ if the nodes are processed in the FastScape stack order.

The procedure is continued until the maximum difference in node elevation between two successive iterations is below a given tolerance ϵ (expressed in meters) as

$$\max|h_i^{t+\Delta t,k+1} - h_i^{t+\Delta t,k}| < \epsilon \quad \text{for all } i \text{ such that } 1 \le i \le n_x \times n_y \,. \tag{21}$$

The tolerance is taken as a small fraction (10^{-3}) of the increment in topography, $U\Delta t$.

The above procedure based on a Gauss-Seidel iterative scheme is known to converge if the matrix \underline{B} is strictly diagonally dominant: $|-F_i| + |\sum_{j=ups(i)} G/(\tilde{p}\tilde{A}_i)| < |1+F_i|$, thus $G/\tilde{p} < N_i/(N_i-1)$ after some derivations. Therefore, the iterative method is proven to converge unconditionally at least when $G/\tilde{p} \leq 1$, but we show experimentally in section 4 that this method can also converge even if this condition is not satisfied.

As shown in Figure 2, our new implicit method to solve equation (13) is O(N) as the number of iterations required in the Gauss-Siedel scheme depends on the value of *G* but not on the resolution of the model $(n_x \times n_y)$.

We note that the left-hand side of equation (19) is the same as in the FastScape algorithm (*Braun and Willett*, 2013) while the right-hand side only differs by a single term given by: $\underline{\underline{E}} \cdot \underline{\underline{h}}^{t+\Delta t, k}$ at time $t + \Delta t$. The implementation of our new algorithm is therefore a very simple addition to the FastScape algorithm. Note also that, if the value of the deposition coefficient G is null, the right-hand side term simplifies to $\underline{\underline{b}}^t$, and the new algorithm is identical to the basic FastScape algorithm which does not require the

Gauss-Seidel iteration to obtain the elevation at time $t + \Delta t$.

The above Gauss-Seidel iteration algorithm can be extended to consider different values of K_f and G to differentiate between bedrock and previously deposited sediments during the iteration process. Basement elevation $h_{base}^{t+\Delta t}$ at each step is obtained using:

$$h_{base}^{t+\Delta t} = \min(h^{t+\Delta t}, h_{base}^t + U\Delta t), \qquad (22)$$

where $h_{base}^t + U\Delta t$ is the basement elevation resulting from uplift without surface processes.

When the value of n is not equal to 1, equation (13) becomes:

$$h_i^{t+\Delta t,\,k+1} + K_f \,\tilde{p}^m A_i^m \Delta t \left(\frac{h_i^{t+\Delta t,\,k+1} - h_{rec(i)}^{t+\Delta t,\,k+1}}{\Delta l_i}\right)^n = h_i^t + U\Delta t + \frac{G}{\tilde{p}\tilde{A}_i} \sum_{j=ups(i)} (h_j^t + U\Delta t - h_j^{t+\Delta t,\,k}) \,.$$

$$(23)$$

This non-linear equation can be solved by combining the Gauss-Seidel iterations with a local Newton scheme. We solve the diffusion equation (12) using an alternating direction implicit and O(N) scheme (*Peaceman and Rachford*, 1955).

²⁰⁶ 3 Model implications on geomorphological relationships

Before studying the behaviour of the numerical scheme presented above, we first wish to derive several basic geomorphological relationships from the evolution equation (11). These include the steady-state slope-area relationship, the shape of steady-state river profile, and the expression for the response time (i.e., the time necessary to reach steady state). In this section, all relationships are for the uplifting region only, and we neglect the hillslope processes.

3.1 Steady-state slope-area (S - A) relationship

At steady state (i.e., when uplift is balanced by channel incision assumed to be governed by the SPL model), the slope and intercept of the relationship between slope and drainage area provide constraints on the concavity (the ratio m/n) and the steepness index ($k_s = (U/K_f)^{1/n} \tilde{p}^{-m/n}$), respectively (e.g., Wobus et al., 2006, and references therein). When taking deposition into account, the SPL model must be replaced by equation (11), which, under the assumption of steady state (i.e., $\frac{\partial h}{\partial t} = 0$) leads to the following slopearea (S - A) relationship:

$$S = \left[\frac{(1+G/\tilde{p})U}{K_f}\right]^{1/n} A^{-m/n} \tilde{p}^{-m/n} = (1+G/\tilde{p})^{1/n} (U/K_f)^{1/n} A^{-m/n} \tilde{p}^{-m/n} .$$
(24)

From this equation, a new steepness index can be defined:

$$k'_{s} = (1 + G/\tilde{p})^{1/n} (U/K_{f})^{1/n} \tilde{p}^{-m/n} , \qquad (25)$$

that only differs by a factor $(1 + G/\tilde{p})^{1/n}$. For typical values of $G/\tilde{p} = 1$ (Davy and Lague, 2009) and n = 1 (Whipple and Tucker, 1999), the effect of sediment deposition is to increase the steepness index by a factor of 2.

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3.2 Steady-state river profile

At steady state, a power-law relationship is generally observed between the length of a stream, x, and its upstream drainage area (*Hack*, 1957; *Lague et al.*, 2003; *Mont-gomery and Dietrich*, 1992; *Morisawa*, 1962; *Walcott and Summerfield*, 2009) that can

be expressed as:

$$A = k x^b, (26)$$

where k and b, usually close to 0.5 and 2, respectively, are called Hack's law coefficients. Combining equation (26) with equation (24) leads to:

$$S = \frac{dh}{dx} = (1 + G/\tilde{p})^{1/n} U^{1/n} K_f^{-1/n} \tilde{p}^{-m/n} k^{-m/n} x^{-\frac{bm}{n}}.$$
 (27)

By integrating the above equation over the length of the stream, we obtain the steadystate river profile:

$$h(x) = (1 + G/\tilde{p})^{1/n} U^{1/n} K_f^{-1/n} \tilde{p}^{-m/n} k^{-m/n} (1 - \frac{bm}{n})^{-1} x^{1 - \frac{bm}{n}} + C, \quad \text{for} \quad bm \neq n,$$
(28)

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where C = 0 if the elevation at base level is zero (i.e., h(0) = 0). We see that the steadystate elevation of every point along the river profile is $(1 + G/\tilde{p})^{1/n}$ times higher than what is expected from the classic SPL model (i.e., without sediment deposition).

3.3 Response time

The response time of a fluvial landscape can be defined as the time needed for river profiles to reach steady state. In our model, the response time is

$$\tau' = h(L)/U = (1 + G/\tilde{p})^{1/n} U^{1/n-1} K_f^{-1/n} \tilde{p}^{-m/n} k^{-m/n} (1 - \frac{bm}{n})^{-1} L^{1 - \frac{bm}{n}}, \qquad (29)$$

where L is the length of the uplifting region. In the classic SPL form (without the term of deposition), the response time of a fluvial landscape is (*Whipple*, 2001)

$$\tau = U^{1/n-1} K_f^{-1/n} \tilde{p}^{-m/n} k^{-m/n} (1 - \frac{bm}{n})^{-1} L^{1 - \frac{bm}{n}}.$$
(30)

The ratio between these two response times is once again given by:

$$\frac{\tau'}{\tau} = (1 + G/\tilde{p})^{1/n} \,. \tag{31}$$

The impact of continental deposition on the evolution of a fluvial landscape is highlighted by the three geomorphological relationships above: steady-state slope is higher, average topography in the uplifting region is higher, and response time is longer than with the classic SPL model (i.e., without deposition). However, neither the shape of the river profile, nor the dependency of the response time or the steepness index to other parameters (such as K_f , m and n, or the length of the channel, precipitation rate or the uplift rate) are affected by sediment deposition. The dimensionless deposition coefficient G alone controls the difference with respect to the classic SPL model and appears only inside a multiplying factor, $(1 + G/\tilde{p})^{1/n}$. In all three relationships we derived above, the factor $(1 + G/\tilde{p})^{1/n}$ multiplies the other poorly constrain factor $K_f^{-1/n}$; this means that the effect of sediment deposition (on most morphometric measures and scales) can be included in the SPL erosion coefficient K_f by simply redefining this constant in the following way:

$$K'_f = K_f / (1 + G/\tilde{p}).$$
 (32)

This also means that the value of the constant G cannot be easily derived from the concavity of rivers, the total relief of river channels or the response time of fluvial erosion.

4 Model behavior

We now demonstrate the behavior and applicability of our numerical implementation of the modified SPL model by performing a range of simulations. These simulations are based on a simple setup composed of an uplifting region adjacent to a stable

Notation	Definition	Values/Range	Unit
h	elevation of topography		m
h_{base}	elevation of basement		m
t	time	10	Myr
Δt	incremental time	1000	yr
x	horizontal dimension	$105 \ {\rm to} \ 400$	$\rm km$
y	horizontal dimension	100	$\rm km$
$\Delta x, \Delta y$	cell size	1	km
U	uplift rate	0.2, 0.5, 1	$\rm mm/yr$
p	net precipitation rate	0.5, 1, 2	m/yr
p_0	mean net precipitation rate	1	m/yr
$\tilde{p} = p/p_0$	variation ratio of precipitation rate	0.5, 1, 2	-
A	drainage area		m^2
S	surface slope in drainage direction		-
m	SPL exponent	0.4^a	-
n	SPL exponent	1^b	-
K_{fb}	SPL erosion coefficient of bedrock	$2\times 10^{-5\ c}$	m^{1-2m}/yr
K_{fs}	SPL erosion coefficient of sediments	$(2,4,8) \times 10^{-5}$	m^{1-2m}/yr
G	deposition coefficient	$(0, 0.1, 1, 10)^d$	-
K_d	hillslope diffusion coefficient	0.01^{e}	m^2/yr

Table 1. Parameters for the simulations.

^aParameters from Stock and Montgomery (1999) and Perron et al. (2009); ^bparameters from Braun and Willett (2013); Stock and Montgomery (1999); Whipple and Tucker (1999) and Braun and Willett (2013); ^cparameters from Whipple and Tucker (1999); ^dparameters from Davy and Lague (2009); ^eparameters from Densmore et al. (2007) and Armitage et al. (2013).

continental area on which a foreland basin is allowed to develop. Setup for the simula-228 tions is a rectangular area of $(105 \text{ to } 400) \times 100 \text{ km}^2$ (Table 1, Figure 1b). The initial to-229 pography has random noise elevation of up to 1 m. This domain is discretized into a num-230 ber of cells with a cell size of $\Delta x = \Delta y = 1$ km. The left-hand side of the domain (100× 231 100 km²) is uplifted at a constant rate U while the foreland is fixed, and the foreland 232 edge (base level) through which the sediments can leave the system is also fixed. We use 233 a constant diffusion coefficient of $K_d = 0.01 \text{ m}^2/\text{yr}$ in all simulations. The net mean 234 precipitation rate p_0 is set to 1 m/yr. We perform a series of model runs for a total time 235 of 10 Myr (10 000 time steps of duration 1000 yr) (Table 1). 236

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4.1 Simulations without foreland

We first performed a series of simulations without the foreland to better illustrate 238 the effect of sediment deposition. We run a simulation with $U = 0.2 \text{ mm/yr}, K_f = 2 \times$ 239 10^{-5} m^{0.2}/yr, p = 1 m/yr and G = 0 to test the classic SPL model without sediment 240 deposition. During the experiment, topography rises, and after 4 Myr, reaches steady 241 state. The final simulated landscape is shown in Figure 3a and the evolution of the mean 242 elevation is presented in Figure 3b. As observed from the black curve (G = 0, Figure 243 3b), the mean elevation in the uplifting region increases progressively before decreasing, 244 and then reaches a constant value at steady state. The same behavior is observed in Davy245 and Lague's (2009) detachment-limited simulations (their Figure 1). Note that this non-246 monotonous evolution of the mean elevation is probably related to the presence of lo-247 cal minima which are important at the beginning because we start with an initial topog-248 raphy that has random noise. Local minima are known to artificially reduce the erosional 249



Figure 3. (a) Simulated, plan view topography of the uplifting region for four values of G = 0, 0.1, 1, and 10, at 10 Myr. G = 0 corresponds with the classic SPL model without taking into account sediment deposition. (b) Mean elevation of uplifting region as a function of time for the four G values. (c) $\log_{10}(S)$ as a function of $\log_{10}(A)$ in steady-state uplifting region for G = 0 (SPL model) and 1. We observe that the concavity is m/n = 0.4, and the difference in steepness index k_s is by a factor of 2.

efficiency of landscape evolution models by preventing some river discharge to reach downstream locations.

On the contrary, a peak in elevation is not observed in experimental models such 252 as in *Babault et al.* (2005) where they study the development of relief by submitting a 253 thick layer of silica powder to uplift and runoff-driven erosion. In their experiments, the 254 mean elevation increases monotonously before stabilizing around a constant value (their 255 Figure 2A). We suggest that the difference between the two behaviors is due to sediment 256 deposition. To prove this point, we run simulations with G = 0.1, 1, and 10. Physically, 257 the larger G, the more deposition in the river channels. The final landscape for the cases 258 of G = 0.1, 1, and 10 are compared in Figure 3a. The two simulations with G = 0 and 259 G = 0.1 are very similar to each other (Figure 3a), but the topography changes signif-260 icantly if the value of G is ≥ 1 . As demonstrated by Davy and Lague (2009), the value 261 of $G/\tilde{p}=1$ corresponds to the transition from detachment-limited to transported-limited 262 behavior. 263

In terms of average topography, the larger G, the higher the mean elevation (Fig-264 ure 3b). For G = 0.1, only slight differences in mean elevation can be observed with 265 respect to the G = 0 simulation (Figure 3b) and steady-state elevation is reached af-266 ter ~ 4 Myr. On the contrary, the mean elevation for G = 1 increases continuously 267 for $\sim 6-7$ Myr before reaching a constant elevation of ~ 400 m, which is almost twice 268 higher than the classic SPL model simulation (G = 0) (Figure 3b). This results agrees 269 with our estimate of the response time for fluvial systems including the effect of sedi-270 ment deposition. When $G \geq 1$, the evolution of the mean elevation is consistent with 271 the one observed in the experiments by Babault et al. (2005). As expected from equa-272 tions (28) and (31), the average elevation at steady state and the response time of the 273 landscape are $(1 + G/\tilde{p})$ times larger when sediment deposition is considered with re-274 spect to the classic SPL model. For the case of G = 10 (Figure 3b), the mean eleva-275 tion increases substantially but it does not reach steady state over 10 Myr of simulation. 276 If the model run is allowed to last for a longer period, it should reach steady state af-277 ter approximately 40 Myr, based on equation (31). 278

The similarities between laboratory experimental landscapes and our simulations with G > 0 confirm that a term for the effect of sediment is required to accurately describe the evolution of a fluvial landscape: although bedrock erosion is the dominant process in the uplifted region (i.e., no net deposition takes place anywhere), the reduction in erosion rate due to sediment deposition, the so-called "cover effect", is key to capture landscape evolution (*Sklar and Dietrich*, 1998; *Whipple and Tucker*, 2002).

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4.2 Simulations with foreland sedimentation

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4.2.1 Reference simulation

We run a "reference simulation" that includes the presence of a foreland (i.e., a re-287 gion where U = 0). In this reference experiment, the erosion coefficients for bedrock 288 and sediments are first set to the same value for the sake of simplicity (i.e., $K_{fb} = K_{fs} =$ 289 K_f). The value of K_f is set to 2×10^{-5} m^{0.2}/yr, identical to the one used in the pre-290 vious set of experiments without a foreland, and G is set to 1. The uplift rate is set to 291 U = 0.2 mm/yr, the precipitation rate to p = 1 m/yr, and the foreland length to 50 292 km. The landscape evolution through time and the axial stratigraphy of the resulting 293 foreland basin, with a synthetic stratigraphic layer generated each Myr, are presented 294 in Figure 4a. The associated animations Movie S1 and Movie S2 are presented in the 295 296 Supporting Information.

At the outlets of the catchments that drain the uplifting region, we observe the formation of small sedimentary fans that progressively coalesce into larger ones. This fanlike deposition is the result of using the SPL model with sediment deposition. As up-



Figure 4. (a) Map view of landscape evolution and longitudinal profile of the central foreland topography (white dashed line in a), and (b) water discharge (the warmer the color, the larger the water discharge) at 2 Myr, 6 Myr, and 10 Myr. See also Movie S1 for landscape evolution, Movie S2 for foreland central cross-section, and Movie S3 for water discharge (lateral variation of river channels in foreland). In (a), the foreland develops a dynamic steady state with the continuous formation and destruction of the riverbeds.

lift and erosion progress, the fans prograde to reach the base level at the far edge of theforeland where the sediments are allowed to leave the system.

The mean elevation of the uplifting region, the maximum thickness of sediments, and the mean surface slope of the foreland basin as function of time are shown by solid curves in Figures 5a-c. Compared to the equivalent simulation without foreland (red curve in Figure 5a), the mean elevation of the uplifting region with a 50-km-long foreland (solid curve in Figure 5a) is about ~ 70 m higher as it reaches a steady-state elevation of \sim 470 m.

Sediment deposition in the foreland basin results in a progressive increase of its slope 308 up to a threshold, over which the sediment flux coming from the uplifting region by passes 309 the foreland. This steady-state slope varies over the foreland, and the mean slope of the foreland is ~ 0.0032 $\left(\sum_{i=1}^{N_f} S_i/N_f\right)$, where N_f is the number of foreland nodes, solid curve 310 311 in Figure 5c), typical of fans built by shallow to deep braided channelized flows (Stanistreet 312 and McCarthy, 1993). Many studies of sedimentary fans (Guerit et al., 2014; Reitz and 313 Jerolmack, 2012; Van Dijk et al., 2009) have shown that the surface slope stabilizes af-314 ter some time, which is considered as the steady-state morphology of the fan. In many 315 experimental studies, the fan slope stabilizes quickly because its apex is fixed. Stabiliza-316 tion also occurs if the apex is uplifted through time as the mountain front eventually reaches 317 a steady-state elevation (Densmore et al., 2007). In our simulations, the slope tends to 318 stabilize after several Myr when the mean elevation of the uplifting region has reached 319 steady state. 320

4.2.2 Sensitivity analysis

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In the following, we explore the impact of several parameters such as different foreland lengths, uplift rates, precipitation rates, and different erosion coefficients for bedrock and sediments. The values used for the sensitivity analysis are summarized in Table 1.

In Figure 5, we first assess the effect of foreland length by running four simulations 325 with foreland length of 50 km (the reference simulation, solid curve), 100 km (dashed 326 curve), 200 km (dotted curve), and 300 km (dash-dotted curve), respectively. U is set 327 to 0.2 mm/yr, p to 1 m/yr, G to 1, and K_f to 2×10^{-5} m^{0.2}/yr. The maximum thick-328 ness of foreland sediments for larger foreland length is higher than the reference simu-329 lation (Figure 5b). The sediment thickness in foreland imposes the local base level of the 330 mountain range. Thus, the mean elevation of the uplifting region for larger foreland length 331 is higher than the reference simulation (Figure 5a). However, in the foreland, the mean 332 slopes are significantly lower than the reference simulation (Figure 5c), as they are re-333 lated to the length of the foreland basin. In fact, sediments are allowed to deposit ev-334 erywhere until they reach the edge of the domain. Therefore, for a given amount of sed-335 iments, the larger the foreland basin, the lower the slope of the foreland basin. This be-336 havior is observed because in our model, rivers are always able to transport sediments 337 even across a large foreland basin. 338

In Figure 4a, the slope of the fan varies over the 50-km-long foreland, and the fan 339 surface is concave up. We performed several simulations with smaller foreland lengths 340 (5 km, 10 km, and 20 km) in Figure 6. The predicted foreland thickness scales directly 341 with the foreland length (Figure 6). The simulations also show that the fan surface is 342 relatively straight when the foreland length is small (e.g., 5 km). The reduction in fan 343 slope could be due to the reduction in the ratio between the sediment flux q_s and the 344 water flux q_w . The larger the foreland length, the larger the reduction in q_s/q_w down-345 346 stream, and the more curved the fan surface, as observed in km-scale fans (e.g., Bull, 1964; Densmore et al., 2007). 347

We test for the impact of the uplift rate by running three simulations with U = 0.2, 0.5 and 1 mm/yr, respectively. Basin length is set to 50 km, p to 1 m/yr, G to 1,



Figure 5. (a) Mean elevation of the uplifting region without foreland, and with foreland length of 50 km, 100 km, 200 km, and 300 km. (b) Maximum thickness of sediments in foreland, and (c) mean slope of the foreland basin as a function of time for foreland length of 50 km, 100 km, 200 km, and 300 km. U is set to 0.2 mm/yr, p to 1 m/yr, G to 1, and K_f to 2×10^{-5} m^{0.2}/yr.



Figure 6. Longitudinal profile of the central foreland topography for various of foreland lengths. Fan surface is relatively straight when the foreland length is small (e.g., 5 km), and is concave up when the foreland length is large (≥ 10 km), consistent with field observations (*Bull*, 1964; *Densmore et al.*, 2007).



Figure 7. Modelled output for foreland basin showing sensitivity tests on (a) uplift rate U, (b) precipitation rate p, and (c) sediment erosion coefficient K_{fs} . First row shows the standard simulation with the same values throughout. All figures have the same vertical scale.

and K_f to 2×10^{-5} m^{0.2}/yr. The main impact of an increased uplift rate is a higher steady-state elevation in the uplifting region. In fact, according to equation (28), the steadystate elevation in the uplifting region is proportional to $U^{1/n}$. The increase in sediment accumulation is also a direct result of the increase in sediment flux resulting from a higher uplift rate. In consequence, sediments can accumulate more in the foreland, and the outlets of the drainage basins are significantly higher (up to 600 m for U = 1 mm/yr compared to ~ 120 m for the reference simulation, Figure 7a).

We then test for the impact of precipitation rate by running three simulations with 357 p = 0.5, 1, and 2 m/yr, respectively. Basin length is set to 50 km, U to 0.2 mm/yr, G 358 to 1, and K_f to 2×10^{-5} m^{0.2}/yr. We observe that the higher the precipitation rate p, 359 the lower the foreland deposition and the foreland slope (Figure 7b). This is due to the 360 fact that the erosion rate is proportional to \tilde{p}^m in equation (11), but the deposition term 361 is inversely proportional to \tilde{p} , thus, the net deposition rate is proportional to $1/\tilde{p}^{1-m}$. 362 This implies that at higher precipitation rates, the transit of sediments in the foreland 363 is faster (i.e., bypass is more efficient). 364

Finally, we test for the impact of the erosion coefficient, K_f . In these simulations, 365 we consider different values for the bedrock (K_{fb}) and the sediment (K_{fs}) erosion co-efficients. We run three simulations with $K_{fb} = 2 \times 10^{-5} \text{ m}^{0.2}/\text{yr}$ and $K_{fs} = 2 \times 10^{-5}$, 4×10^{-5} and $8 \times 10^{-5} \text{ m}^{0.2}/\text{yr}$, respectively. Basin length is set to 50 km, p to 1 m/yr, 366 367 368 and G to 1. As the erosion coefficient increases, sediments in the foreland are more and 369 more remobilized by river erosion, resulting in lower slopes in the foreland basin (Fig-370 ure 7c). Almost no sediments are preserved in the simulation of $K_{fs} = 8 \times 10^{-5} \text{ m}^{0.2}/\text{yr}$. 371 This means that the reworking (by erosion) of sediments in the foreland is an efficient 372 process that enhances the bypass of sediments from the uplifting region toward the base 373 level. 374

375 5 Discussion

5.1 Slope-area (S - A) relationship from simulations

By deriving equation (24), we have shown that the inclusion of depositional pro-377 cesses should increase the steepness index, k_s , with respect to the classic SPL model (i.e., 378 with G = 0 while the concavity should remain unchanged. This is illustrated by the 379 slope-area analysis of two experiments without foreland with G = 0 and G = 1 (Fig-380 ure 3c). With n = 1, the concavity and the steepness index can be derived from log-log plots, where the slope of the slope-area relationship is the concavity and the intercept 382 with the origin is the \log_{10} of the steepness index. The slope-area analysis of the sim-383 ulations G = 0 and G = 1 reveals a concavity of 0.4 in both experiments, as we ex-384 pect from our m and n values (0.4 and 1, respectively). On the contrary, despite sim-385 ilar U and K_f , the steepness index k_s is increased by a factor 2 when G = 1 (Figure 386 3c). Through these numerical experiments, we confirm that the steepness index, usually 387 used to derive information about uplift and landscape properties, is also sensitive to the 388 amount of deposition within the fluvial network. 389

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5.2 Autocyclic aggradation and incision in the foreland

During a simulation, the landscape evolves until it reaches an equilibrium topography (when the elevation in the mountain range and the slope in the foreland become stable, Figure 5). In our simulations, river channels continuously migrate laterally in the foreland (Figure 4b, see the animation Movie S3 on water discharge in the Supporting Information), similar to observations made in laboratory experiments (*Van Dijk et al.*, 2009) and other numerical simulations (*Croissant et al.*, 2017; *Pepin et al.*, 2010). By considering the axial topography (Figure 4a; the corresponding animation Movie S2 is presented in the Supporting Information), we can indeed observe that the foreland steady-



Figure 8. Foreland cross sections at 10 km, 20 km, 30 km, and 40 km from the mountain front for the uplift rate (a) U = 0.2 mm/yr, (b) U = 0.5 mm/yr, and (c) U = 1 mm/yr. Animation of the simulation Movie S4 for aggradation and incision cycles in foreland (a) is presented in the Supporting Information. Red ellipses show the general locations of autocyclic aggradation and incision around a dynamic steady state.

state configuration is a dynamic steady state with the continuous formation and destruc-399 tion of river banks. The formation and destruction of river banks can be observed from 400 the foreland sections parallel to the mountain front (Figure 8a; the corresponding an-401 imation Movie S4 is presented in the Supporting Information). The successive cycles of aggradation and incision mainly occur in the lower foreland (30 km and 40 km in Fig-403 ure 8a) where the longitudinal slope is low. In the upper foreland, where the slope is higher 404 (10 km and 20 km in Figure 8a), sediments mainly aggrade until reaching steady state. 405 This can be seen, for example, in Figure 4a where sediments aggrade up to a maximum 406 of ~ 120 m at the fan apex (i.e., along the transition between the foreland and the up-407 lifting region). Yet, the amplitude of aggradation and incision is higher close to the moun-408 tain range than in the lower foreland, in good agreement with Bull (1964). 409

The successive aggradation and incision cycles arises from the competition between 410 the SPL erosion term $K_f \tilde{p}^m A^m S^n$ and the deposition term $\frac{G}{\tilde{p}A} \int_A (U - \frac{\partial h}{\partial t}) dA$ in equa-tion (11) where the uplift rate U = 0 mm/yr in the foreland. When the foreland slope S is low, the erosion term $K_f \tilde{p}^m A^m S^n$ is lower than the deposition term $\frac{G}{\tilde{p}A} \int_A (U - \frac{\partial h}{\partial t}) dA$ 411 412 413 $\frac{\partial h}{\partial t}$) dA. Then, sediments accumulate in the foreland (because sediments are deposited 414 more than eroded), resulting in an increase of the foreland slope. However, when the fore-415 land slope S is sufficiently large, the erosion term becomes greater than the deposition 416 term, leading to a decrease in slope. The SPL erosion and deposition terms are never 417 equal to each other in the foreland, and a dynamic steady state develops characterized 418 by autocyclic and alternating phases of aggradation and incision. 419

Note that the node spacing in the simulation is $1 \times 1 \text{ km}^2$ (Figure 8a). To capture morphological features such as valleys and hillslopes that typically occur over shorter space scales, we ran two other simulations with a higher spatial resolution of 100 m and 20 m (Figure S1 in the Supporting Information). The model is smaller $(15 \times 10 \text{ km}^2)$ than the reference simulation, and has an uplifted domain of $10 \times 10 \text{ km}^2$. The processes that lead to the autocyclic aggradation and incision are replicated (Figure S1), and the spacing between valleys and the migration frequency of river channels are not affected by the spatial resolution of the model.

Langston et al.'s (2015) simulations show that the repeated aggradation and aban-428 donment of high surfaces take place in the upper foreland due to modulation of sediment 429 supply associated with climate change. Unlike their simulations, ours show that, with-430 out changing external forcings, streams migrate and sometimes erode into the sediment 431 they have previously deposited, mainly in the lower foreland, around a dynamic steady 432 state. Under a constant forcing, we observe that the upper foreland is quite stable. This 433 could be related to the relatively limited numbers of outlets at the edges of the moun-434 tain range, which limits the possibility of major fluvial reworking close to the range. 435

Riverbed aggradation and incision cycles are often interpreted to reflect changing 436 external forcings including tectonics (Bull, 1991; Yanites et al., 2010), climate (Bridg-437 land and Westaway, 2008; Hancock and Anderson, 2002), and base-level change (Fisk 438 et al., 1945). Yet, without varying tectonics (e.g., the value of U) or climate (e.g., the 439 value of p), our numerical simulations continuously produce lateral migration of river chan-440 nels in the foreland, resulting in autocyclic aggradation and incision around a dynamic 441 steady state. Using different uplift rates (e.g., U = 0.5 mm/yr and 1 mm/yr in Fig-442 ures 8b and 8c, respectively), we observe larger aggradation and incision thicknesses, be-443 cause sediments accumulate more in the foreland for higher uplift rates. 444

5.3 Limitations of the model

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There are several limitations to this model and how it can be used to represent natural processes. First of all, our model of course encompasses some limitations of the classic SPL model from which it is derived and such limitations should be considered to design future studies. For example, we can not account for river width variations or lateral migration in bedrock channel, preventing the simulations of alluvial terraces within
uplifted areas (*Langston and Tucker*, 2018). Channel narrowing and widening also influence the stream power per unit width, and therefore the transport capacity (e.g., *Finnegan et al.*, 2005; *Whittaker et al.*, 2007).

Second, the thickness and location of deposited sediments above bedrock elevation 454 is not recorded, as erosion and deposition are calculated within the same equation and 455 at the same time step. A valuable addition to this work would be to track sediment de-456 position within uplifting regions, either within channels or alongside channels in the form 457 of terraces. The transient sediment storage in inter-montane basins or within uplifting 458 regions could be a potential application to better understand landscape evolution. For 459 this, the thickness of sediments above bedrock elevation should be tracked during land-460 scape evolution (e.g., Shobe et al., 2017). 461

In addition, the model is built in fluxes and it does not account for grain-size dis-462 tribution (i.e., the eroded material behaves as a single grain-size material). Accordingly, 463 we neither distinguish between bedload and suspended load, nor simulate processes such 464 as sediment downstream fining. The size of the sediments is an important parameter that 465 partly controls the shape and the geometry of sedimentary bodies such as alluvial fans 466 (Armitage et al., 2011; Delorme et al., 2016; Paola et al., 1992). Multiple grain sizes can 467 be incorporated into the model framework by splitting the flux into different loads as-468 sociated with different deposition coefficients. This would also require to track grain size, 469 which is not trivial to perform in *Davy and Lague*'s (2009) flux-based formulation. 470

Last, the model did not account for flexural isostasy either in the uplifted domain 471 or in the foreland, mostly for the sake of simplicity, to explain the behavior of the sur-472 face process model, independent of any assumption regarding isostatic response. Flex-473 ural subsidence of foreland basins creates accommodation space and allows sediments 474 supplied from the uplifted relief to be preserved through time. To simulate sediment stratig-475 raphy with the current model, a first-order approximation would be to impose a nega-476 tive uplift in the foreland basin to simulate subsidence and enhance sediment deposition 477 and preservation. 478

479 6 Conclusion

We have developed a new efficient (i.e., O(N) and implicit) method to solve the 480 equations arising from Davy and Laque's (2009) formulation to investigate the role of 481 sediment deposition on fluvial erosion of landscapes. In our implementation, the depo-482 sition rate is assumed to be inversely proportional to drainage area, and proportional 483 to the local suspended sediment flux (estimated from the integrated upstream net ero-484 sion rate) and to a dimensionless deposition coefficient, which we term G. This depo-485 sition coefficient G is a function of the sediment concentration ratio in transport, the set-486 tling velocity of sediment, and the mean precipitation rate. From a macro point of view, 487 G/\tilde{p} in equation (11) is a direct measure of the local deposition rate relative to the mean 488 net erosion rate of upstream drainage area. 489

To study the behavior of this new scheme and its numerical implementation, we 490 performed a series of simple experiments in which a region subject to uplift at a constant 491 rate is adjacent to a stable area. Based on multiple simulations, we demonstrate that 492 the relief in the uplifting area and the stratigraphy in the foreland basin are controlled 493 by the river erosion coefficient K_f and by the value of G. The deposition coefficient G 494 controls the average elevation in the uplifting region and the surface slope in the asso-495 ciated foreland basin, especially when the value of G is ≥ 1 . At steady state, the slope, the average elevation along river profiles and the response time of the uplifted relief are 497 a factor $(1 + G/\tilde{p})^{1/n}$ higher than those derived from the SPL model without a depo-498 sition term. 499

In our simulations, as uplift and erosion progress, sediments aggrade in the fore-500 land basin and prograde toward the foreland edge (the base level). The foreland slope 501 increases until it reaches a threshold over which sediments bypass the foreland. The fore-502 land then develops a dynamic steady state, in which the stream continuously migrates 503 and sometimes cuts into the sediment it has previously deposited. This process leads to 504 the continuous formation and destruction of river banks. This behavior mainly takes place 505 in the lower foreland where the slope is low, whereas in the upper foreland where the slope 506 is relatively high and rivers are less free to migrate, the sediments mainly aggrade un-507 til reaching steady state. This observed dynamic steady state suggests that aggradation 508 and incision cycles can occur in the absence of variations in external forcings such as tec-509 tonics, climate or change of base level. 510

As our method is optimally efficient (i.e., O(N) and implicit), it is therefore highly suitable for performing large numbers of simulations and it can thus be used in optimization procedures that require many simulations to infer the amplitude of variations in tectonic forcings, climatic events or base-level changes in natural examples. In the future, we plan on developing this model to better constrain the nature and timing of erosional events on continents, through an inversion of the stratigraphy of the adjacent foreland basins.

A Matrix of finite difference equations in FastScape stack order

To illustrate our algorithm, we now explicitly write the coefficients of matrix $\underline{\underline{B}}$ and vectors $\underline{\underline{h}}^{t+\Delta t}$ and $\underline{\underline{b}}^{t}$ in equation (17) for a single catchment made of 11 nodes (Figure 1c) using the FastScape stack order. This gives:

$$\underline{h}^{t+\Delta t} = \begin{pmatrix} h_1^{+,-\tau} \\ h_2^{t+\Delta t} \\ h_3^{t+\Delta t} \\ h_4^{t+\Delta t} \\ h_5^{t+\Delta t} \\ h_5^{t+\Delta t} \\ h_6^{t+\Delta t} \\ h_7^{t+\Delta t} \\ h_8^{t+\Delta t} \\ h_1^{t+\Delta t} \\ h_1^{t+\Delta t} \\ h_1^{t+\Delta t} \\ h_1^{t+\Delta t} \end{pmatrix}, \quad \text{and} \quad \underline{b}^t = \begin{pmatrix} h_1^t \\ h_2^t + U\Delta t + G(h_3^t + h_5^t + h_6^t + 4U\Delta t)/(\tilde{p}\tilde{A}_2) \\ h_3^t + U\Delta t + G(h_4^t + h_5^t + 2U\Delta t)/(\tilde{p}\tilde{A}_3) \\ h_4^t + U\Delta t \\ h_6^t + U\Delta t \\ h_6^t + U\Delta t \\ h_6^t + U\Delta t \\ h_7^t + U\Delta t + G(h_8^t + h_9^t + h_{10}^t + h_{11}^t + 4U\Delta t)/(\tilde{p}\tilde{A}_7) \\ h_8^t + U\Delta t \\ h_9^t + U\Delta t + G(h_{10}^t + h_{11}^t + 2U\Delta t)/(\tilde{p}\tilde{A}_9) \\ h_{10}^t + U\Delta t \\ h_{10}^t + U\Delta t \\ h_{11}^t + U\Delta t \end{pmatrix}, \quad (A.2)$$

Let's now take node 2 (the second row in equation A.1) as an example (Figure 1c). Its receiver is node 1 thus there is the term $-F_2 = -K_f \tilde{p}^m A_2^m \Delta t / \Delta l_2$ in the second row of the first column of the matrix <u>B</u>. The node 2 has several upstream nodes (node 3, node 4, node 5, and node 6) which contribute the drainage water, thus, there are dimension-

less terms $G/(\tilde{p}A_2)$ in the second row of the third to the sixth column of the matrix <u>B</u>.

The matrix \underline{B} is split into its lower \underline{F} and strictly upper triangular matrix \underline{E} as:

and

	$\left(0 \right)$	0	0	0	0	0	0	0	0	0	0)
	0	0	$G/(\tilde{p}\tilde{A}_2)$	$G/(\tilde{p}\tilde{A}_2)$	$G/(\tilde{p}\tilde{A}_2)$	$G/(\tilde{p}\tilde{A}_2)$	0	0	0	0	0
	0	0	0	$G/(\tilde{p}\tilde{A}_3)$	$G/(\tilde{p}\tilde{A}_3)$	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
$\underline{E} =$	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	$G/(\tilde{p}\tilde{A}_7)$	$G/(\tilde{p}\tilde{A}_7)$	$G/(\tilde{p}\tilde{A}_7)$	$G/(\tilde{p}\tilde{A}_7)$
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	$G/(ilde{p} ilde{A}_9)$	$G/(\tilde{p}\tilde{A}_9)$
	0	0	0	0	0	0	0	0	0	0	0
	$\left(0 \right)$	0	0	0	0	0	0	0	0	0	0 /
										(.	A.4)

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⁵³⁰ github.com/fastscape-lem/fastscapelib-fortran.

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