

# Walls Dynamics in a Ferromagnetic Nanowire

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Partial Differential Equations - Theories and Applications**  
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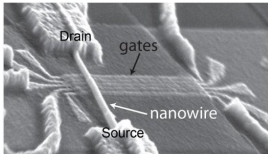
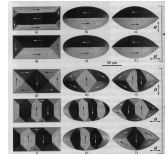
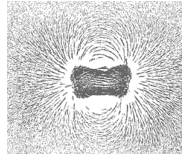
# Plan

- 1 Ferromagnetism
- 2 Domain Walls in Nanowires
- 3 Quasi-Solutions and Metastability
- 4 Stabilization by pinching

# 3d model

## Properties

- Spontaneous magnetization
- Magnetic field generated by the magnetization
- Formation of domains and domain walls



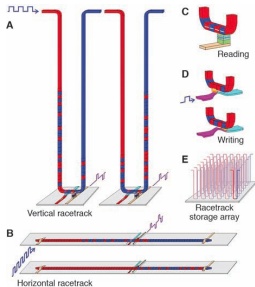
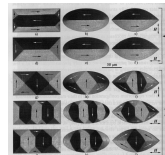
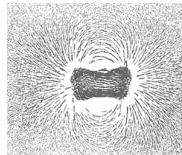
## Industrial Applications

- Electromagnets
- Transformers
- Radar absorbing paints
- Mobile phones
- Modern Recording Devices

## 3d model

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Stuart S. P. Parkin, Masamitsu Hayashi, Luc Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

- 3d storage devices
- Fast access to the information

## Magnetic Moment

- Magnetic domain :  $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment  $m : \mathbb{R}_t^+ \times \Omega_x \rightarrow \mathbb{R}^3$

$$|m(t, x)| = 1 \text{ p. p.}$$

- Constitutive Relation :  $B = H + m$

## Landau-Lifschitz

$$\frac{\partial m}{\partial t} = -m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

- preserves the saturation constraint  $|m| = 1$
- Precession term (makes  $m$  move around  $H_0$ )
- Damping term (tends to align  $m$  with  $H_0$ )
- Energy Dissipation

## Ferromagnetism Energy

$$E(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |H_d(m)|^2 dx - \frac{1}{2} \int_{\Omega} (\xi \cdot m)^2$$

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$$H_{\text{eff}} = A\Delta m + H_d(m) + \Phi_{\text{anis}}(m)$$

## Exchange field

- Nonlinear parabolic equation
- Homogeneous Neumann boundary condition

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## Demagnetizing field

- field generated by the magnetization
- $\text{curl } H_d(m) = 0$
- $\text{div } (H_d(m) + m) = 0$

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## Anisotropic field

- Preferential axis for the magnetization
- $\Phi_{\text{anis}}(m) = (m \cdot \xi)\xi$  (Uniaxial anisotropy)

## Ferromagnetism Energy

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## Applied Magnetic or Electric Field

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## Action of an Applied Magnetic Field

$H_a$  : applied magnetic field

Zeeman Energy

$$- \int_{\Omega} H_{\text{app}} \cdot m \, dx$$

## Action of an Applied Electric Field

$\nu$  : Electric Current

Transport Term

$$\nu \cdot \nu = 0 \text{ on } \partial\Omega$$

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# References

## Modelization

- Landau-Lifschitz (30')
- Miltat, Thiaville (LPS, Orsay)

## Existence Results

- Joly-Métivier-Rauch (LL-Maxwell without exchange field)
- Visintin, Alouges-Soyeur, C.-Fabrie, Ding-Guo (Global in time weak solutions)
- C.- Fabrie-Guès (Local in time strong solutions)
- C.-Fabrie-Efendiev (models of hysteresis and magnetostriction)

## Numerical Simulations

- Joly-Haddad (without exchange field)
- Labbé, Garcia-Cervera, Pröhl (with exchange field)



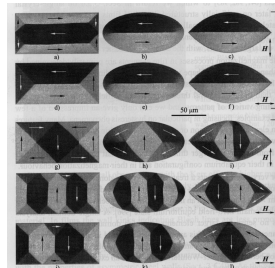
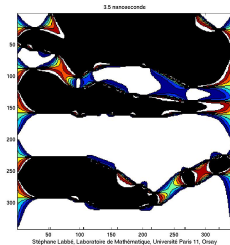
# Plan

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# Domains and Domain Walls

Organization of the magnetization :

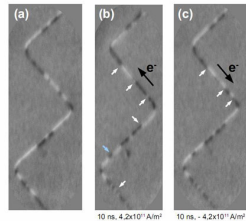
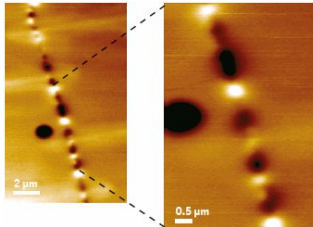
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- domain walls (microstructures in which we observe stiff variations of  $m$ )



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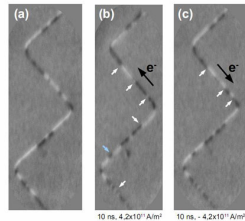
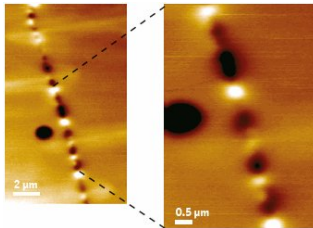


Goal : describe walls dynamics in ferromagnetic nanowires

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# Nanowires model

## Landau-Lifschitz Equation

$$m : \mathbb{R}_t^+ \times \Omega \rightarrow \mathbb{S}^2$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

$$H_{\text{eff}} = A\Delta m + H_d(m)$$

$$\Omega = [0, L] \times B(0, \eta)$$

$$\eta \rightarrow 0$$

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- Fil assimilated to  $[0, L]e_1$
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- Equivalent Demagnetizing Field :  $\mathcal{H}(m) = -m_2\vec{e}_2 - m_3\vec{e}_3$

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$$H_{\text{eff}} = \varepsilon^2 m_{xx} - m_2 \vec{e}_2 - m_3 \vec{e}_3 + H_a$$

- Applied Magnetic Field  $H_a = h_a \vec{e}_1$
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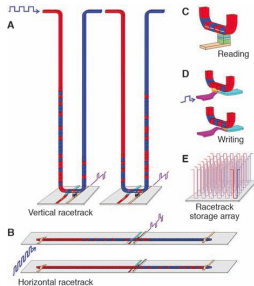
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Goal : describe walls dynamics in this model

## Exact Static Solutions for Infinite Nanowires

## Infinite Nanowire

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## Static Solution

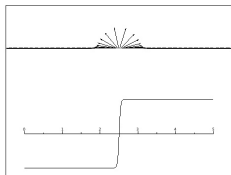
$$M_0(x) = \begin{pmatrix} \text{th } x \\ 1/\text{ch } x \\ 0 \end{pmatrix},$$

$$R_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

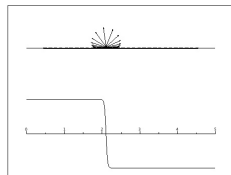
$$\text{-/+ wall : } \mathcal{M}(x) = R_\theta (M_0(x - \sigma))$$

$$\text{+/- wall : } \mathcal{M}(x) = R_\theta (M_0(-(x - \sigma)))$$

-/+ wall



+/- wall





## Exact Solutions with Applied Magnetic Field

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### Moving Solution

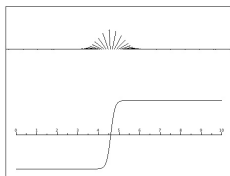
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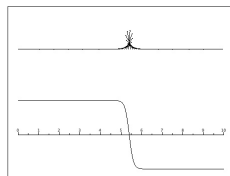
$$\text{-/+ wall : } \mathcal{M}(x) = R_{h_a t} (M_0(x + h_a t))$$

$$\text{+/- wall : } \mathcal{M}(x) = R_{h_a t} (M_0(-(x - h_a t)))$$

-/+ wall, magnetic field along  $\vec{e}_1$



+/- wall, magnetic field along  $\vec{e}_1$



# Exact Solutions with Applied Magnetic Field

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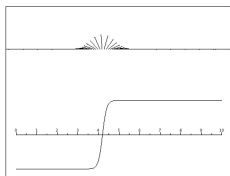
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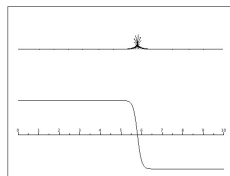
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-/+ wall, magnetic field along  $\vec{e}_1$



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## Moving Solution

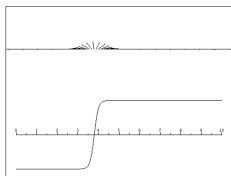
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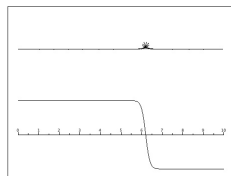
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-/+ wall, magnetic field along  $\vec{e}_1$



+/- wall, magnetic field along  $\vec{e}_1$



# Exact Solutions with Applied Magnetic Field

## Infinite Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{S}^2$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

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## Moving Solution

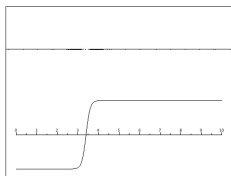
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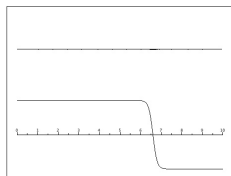
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-/+ wall, magnetic field along  $\vec{e}_1$



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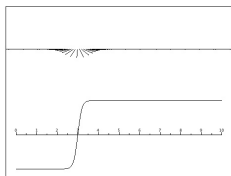
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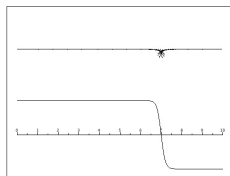
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## Exact Solutions with Applied Magnetic Field

### Infinite Nanowire

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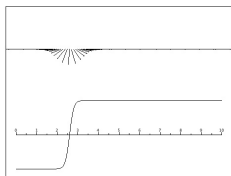
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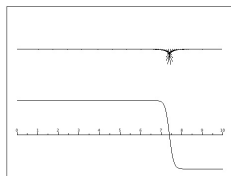
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## Exact Solutions with Applied Magnetic Field

### Infinite Nanowire

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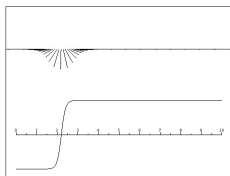
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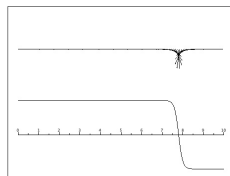
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# Exact Solutions with Applied Magnetic Field

## Infinite Nanowire

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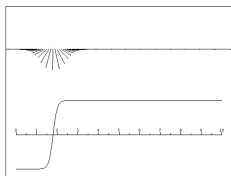
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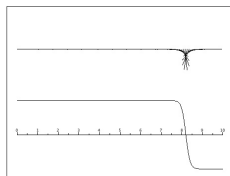
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## Exact Solutions with Applied Magnetic Field

### Infinite Nanowire

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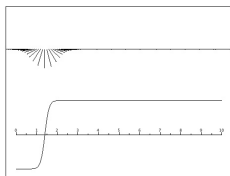
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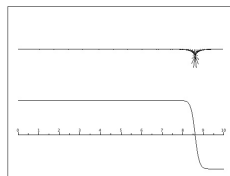
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# Exact Solutions with Applied Magnetic Field

## Infinite Nanowire

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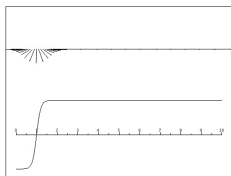
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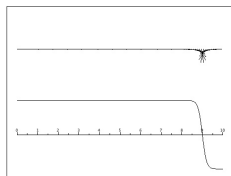
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## Exact Solutions with Applied Magnetic Field

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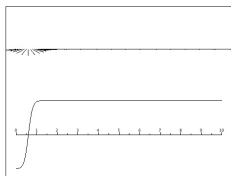
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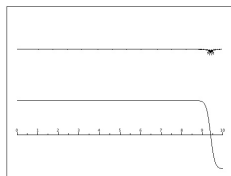
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## Exact Solutions with Applied Electric Field

### Infinite Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{S}^2$$

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### Moving Solution

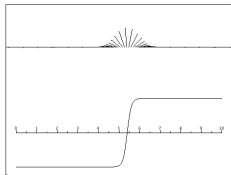
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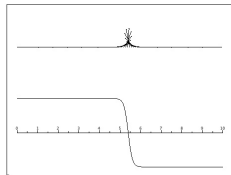
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+/- wall, electric field along  $\vec{e}_1$



## Exact Solutions with Applied Electric Field

### Infinite Nanowire

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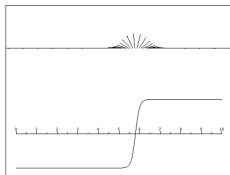
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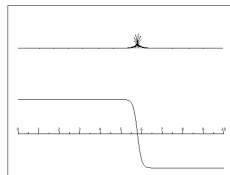
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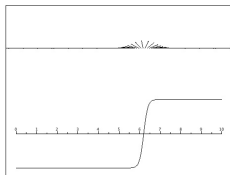
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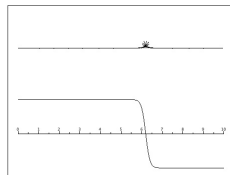
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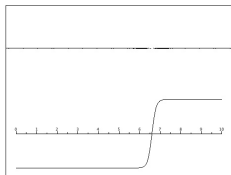
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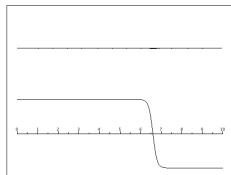
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# Exact Solutions with Applied Electric Field

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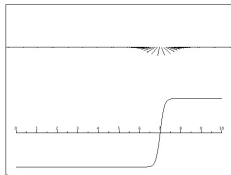
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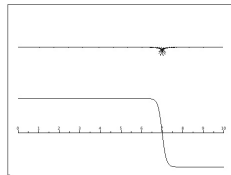
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## Exact Solutions with Applied Electric Field

### Infinite Nanowire

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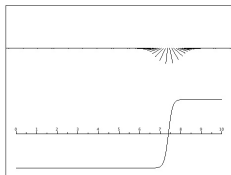
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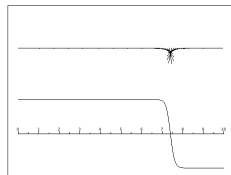
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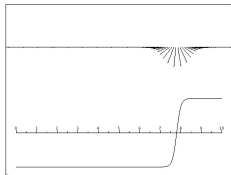
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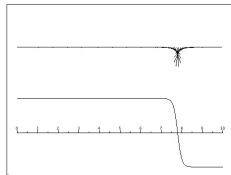
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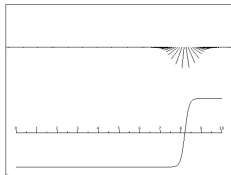
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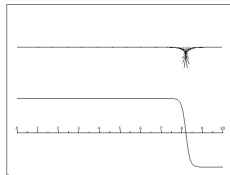
$$\text{-/+ wall : } \mathcal{M}(x) = R_{vt} (M_0(x - vt))$$

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-/+ wall, electric field along  $\vec{e}_1$



+/- wall, electric field along  $\vec{e}_1$



## Exact Solutions with Applied Electric Field

### Infinite Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathbb{S}^2$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}}) \\ -v \partial_x m - m \wedge v \partial_x m$$

$$H_{\text{eff}} = m_{xx} - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

### Moving Solution

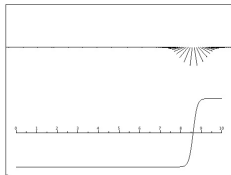
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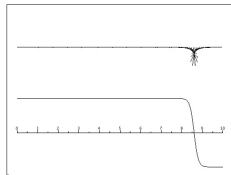
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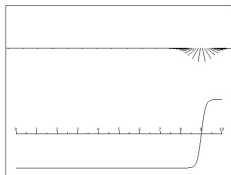
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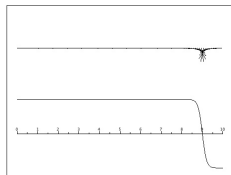
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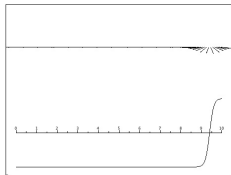
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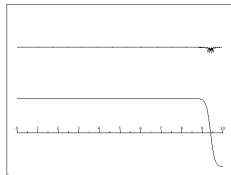
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G. Carbou, S. Labbé, *Stability for Static wall in a ferromagnetic Nanowire*, Discrete Contin. Dyn. Syst. Ser. B 6 (2006)

Stability for wall profiles for small applied field

R. Jizzini, *Optimal stability criterion for a wall in a ferromagnetic wire in a magnetic field*, J. Differential Equations 250 (2011)

Stability Threshold :  $|h_a| < 1$

G. Carbou, S. Labbé, E. Trélat, *Control of travelling walls in a ferromagnetic nanowire*, Discrete Contin. Dyn. Syst. Ser. S 1 (2008), no. 1, 51–59

Controlability of the wall position by the mean of the applied field

# Stability Results

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R. Jizzini, *in preparation*

Stability of the exact solutions

Stability Threshold :  $|v| < 2$



## Drawbacks of these works on exact solutions

G. Carbou, S. Labbé, *Stabilization of walls for nano-wires of finite length*, ESAIM Control Optim. Calc. Var. 18 (2012)

- One wall in a finite wire : centered in the middle of the wire
- This solution is unstable

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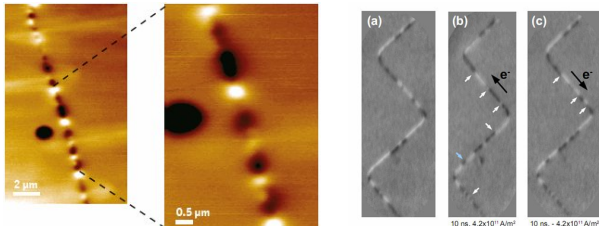
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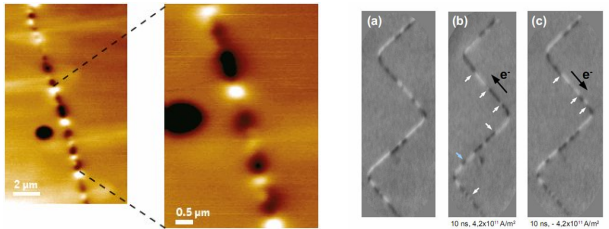
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# Plan

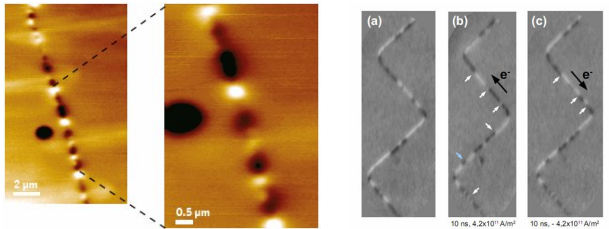
- 1 Ferromagnetism
- 2 Domain Walls in Nanowires
- 3 Quasi-Solutions and Metastability**
- 4 Stabilization by pinching

Goal : description of  $K$  walls complex configurations in finite wire subject to an applied magnetic field or an electric current



Key idea : taking into account the smallness of the exchange coefficient

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$\varepsilon^2$  : small exchange coefficient. Time rescaling : long time behaviour

- We build a  $2K$ -parameters family of quasi-solutions describing all the configurations with  $K$  walls
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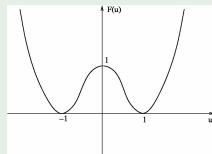


## Related Work

J. Carr, R. L. Pego, *Metastable patterns in solutions of  $u_t = \varepsilon^2 u_{xx} - f(u)$* , Comm. Pure Appl. Math. **42** (1989)

$$f(u) = F'(u)$$

- Description of phase transitions
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We will use Carr and Pego's methods (geometric methods, see also Hale and Fusco)

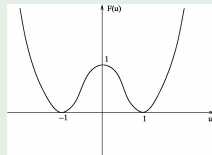
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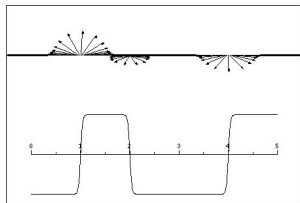
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## Quasi-solutions profiles



- Walls positions :  $\sigma_1, \dots, \sigma_K$   
 $0 < \delta < \sigma_1$   
 $\sigma_1 + \delta \leq \sigma_2 - \delta$   
 $\sigma_K + \delta \leq L$
- Walls angles :  $\theta_1, \dots, \theta_K$

### Definition of $\mathbf{m}_\varepsilon(\theta, \sigma)$

In the domains

$$D_i = [\sigma_{i-1} + \delta, \sigma_i - \delta]$$

$$\mathbf{m}_\varepsilon(\theta, \sigma)(x) = (-1)^i \mathbf{e}_1 \text{ dans } D_i$$

Central Zone of a wall

$$\text{in } \left[ \sigma_i - \frac{\delta}{2}, \sigma_i + \frac{\delta}{2} \right],$$

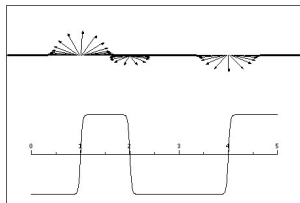
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Smooth connection

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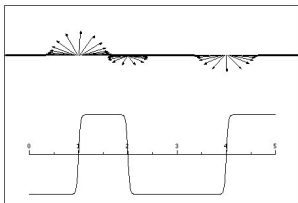
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## Metastability Results

### Finite Nanowire with Small Exchange Length

$$m : \mathbb{R}_t^+ \times [0, L]_x \rightarrow \mathcal{S}^2$$

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$$H_{\text{eff}} = \varepsilon m_{xx} - \frac{1}{\varepsilon} (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

### Metastability of Static solutions

We fix  $\theta$  and  $\sigma$ . If the initial data is closed to  $\mathbf{m}_\varepsilon(\theta, \sigma)$ , then **the solution remains close to  $\mathbf{m}_\varepsilon(\theta, \sigma)$  on a long time interval  $[0, \varepsilon^{\frac{\delta}{4\varepsilon}}$**

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**With applied magnetic field**

Initial data close to  $\mathbf{m}_\varepsilon(\theta^0, \sigma^0)$

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### Meta-stability with general applied field

$$|h| < 1$$

The solution remains close to  $\mathbf{m}_\varepsilon(\theta^{\text{ref}}, \sigma^{\text{ref}})$  on the time interval  $[0, \frac{1}{\varepsilon}]$ .



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## Meta-stability with constant applied field

$$|h| < 1$$

$h$  constant in the walls determined by  $\sigma^{\text{ref}}$

The solution remains close to  $\mathbf{m}_\varepsilon(\theta^{\text{ref}}, \sigma^{\text{ref}})$  on the time interval  $[0, e^{\frac{\delta}{4\varepsilon}}[$

## New Coordinates

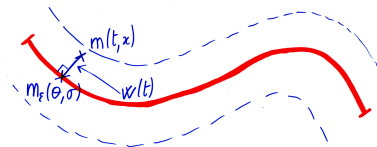
2K-parameters family of static quasisolutions

$$\mathcal{M} = \{ \mathbf{m}_\varepsilon(\theta, \sigma), \sigma_{i+1} - \sigma_i \geq 2\delta \}$$

New coordinates in a neighborhood of  $\mathcal{M}$

$$m = \mathbf{m}_\varepsilon(\theta, \sigma) + w + \nu(w)\mathbf{m}_\varepsilon(\theta, \sigma)$$

- $w \cdot \mathbf{m}_\varepsilon(\theta, \sigma) = 0$
- $\nu(\xi) = \sqrt{1 - |\xi|^2} - 1$
- $\langle w | \partial_{\sigma_i} \mathbf{m}_\varepsilon(\theta, \sigma) \rangle = 0$
- $\langle w | \partial_{\theta_i} \mathbf{m}_\varepsilon(\theta, \sigma) \rangle = 0$



Local inversion Theorem  $\implies$  good parametrization of a neighborhood of  $\mathcal{M}$  which size does not depend on  $\varepsilon$

## Equation in the new coordinates

We describe  $m$  in the new coordinates and we plug it in the Landau-Lifschitz

$$m(t) = \mathbf{m}_\varepsilon(\theta(t), \sigma(t)) + w(t) + \nu(w(t))\mathbf{m}_\varepsilon(\theta(t), \sigma(t))$$

By taking the inner product with  $\partial_{\theta_i} \mathbf{m}_\varepsilon$   
 and  $\partial_{\sigma_j} \mathbf{m}_\varepsilon$  :

Equation for  $\theta$  and  $\sigma$

$$\frac{d\theta_i}{dt} = h(t, \sigma_i) + a_\varepsilon^1 + G_\varepsilon^1(\theta_i, \sigma_i, w)(w)$$

$$\frac{d\sigma_j}{dt} = (-1)^j h(t, \sigma_j) + a_\varepsilon^2 + G_\varepsilon^2(\theta_j, \sigma_j, w)(w)$$

Perturbation of the o.d.e. satisfied by  
 $(\theta^{ref}, \sigma^{ref})$

$$a_\varepsilon^j = \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}})$$

By subtraction : equation on  $w$

Equation for  $w$

$$\partial_t w = a_\varepsilon + \Lambda_\varepsilon w + P_\varepsilon w + I^\varepsilon w + G^\varepsilon(w, \theta, \sigma)$$

- $a_\varepsilon = \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}})$
- $\Lambda_\varepsilon$  linear operator
- $P_\varepsilon$  linear operator coming from the applied magnetic field
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We describe  $m$  in the new coordinates and we plug it in the Landau-Lifschitz

$$m(t) = \mathbf{m}_\varepsilon(\theta(t), \sigma(t)) + w(t) + \nu(w(t))\mathbf{m}_\varepsilon(\theta(t), \sigma(t))$$

By taking the inner product with  $\partial_{\theta_i} \mathbf{m}_\varepsilon$   
 and  $\partial_{\sigma_i} \mathbf{m}_\varepsilon$  :

Equation for  $\theta$  and  $\sigma$

$$\frac{d\theta_i}{dt} = h(t, \sigma_i) + a_\varepsilon^1 + G_\varepsilon^1(\theta_i, \sigma_i, w)(w)$$

$$\frac{d\sigma_i}{dt} = (-1)^i h(t, \sigma_i) + a_\varepsilon^2 + G_\varepsilon^2(\theta_i, \sigma_i, w)(w)$$

Perturbation of the o.d.e. satisfied by  
 $(\theta^{ref}, \sigma^{ref})$

$$a_\varepsilon^j = \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}})$$

By subtraction : equation on  $w$

Equation for  $w$

$$\partial_t w = a_\varepsilon + \Lambda_\varepsilon w + P_\varepsilon w + I^\varepsilon w + G^\varepsilon(w, \theta, \sigma)$$

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- $\Lambda_\varepsilon$  linear operator
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## Sketch of the proof

Equation for  $w$

$$\partial_t w = a_\varepsilon + \Lambda_\varepsilon w + P_\varepsilon w + I^\varepsilon w + G^\varepsilon(w, \theta, \sigma)$$

Coercivity of the linear part

$$\frac{d}{dt} \|w\|_{H_\varepsilon^1}^2 + c \|w\|_{H_\varepsilon^2}^2 \leq \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}}) + C \|w\|_{H_\varepsilon^2}^2 \|w\|_{H_\varepsilon^1}$$

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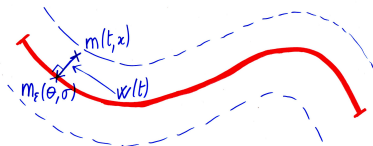
Coercivity of the linear part

$$\frac{d}{dt} \|w\|_{H_\varepsilon^1}^2 + c \|w\|_{H_\varepsilon^2}^2 (1 - C \|w\|_{H_\varepsilon^1}) \leq \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}})$$

While  $\|w\|_{H_\varepsilon^1} \leq \frac{1}{2C}$ ,

$$\frac{d}{dt} \|w\|_{H_\varepsilon^1}^2 + c \|w\|_{H_\varepsilon^1}^2 \leq \mathcal{O}(e^{-\frac{\delta}{4\varepsilon}})$$

$\|w\|$  remains small



## Sketch of the proof

Equation for  $w$

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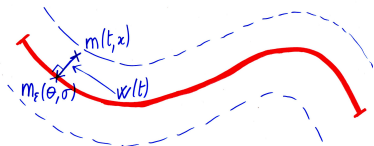
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## Sketch of the proof

### Equation for $w$

$$\partial_t w = a_\varepsilon + \Lambda_\varepsilon w + P_\varepsilon w + I^\varepsilon w + G^\varepsilon(w, \theta, \sigma)$$

$\|w\|$  remains small

### Equation for $\theta$ and $\sigma$

$$\frac{d\theta_i}{dt} = h(t, \sigma_i) + a_\varepsilon^1 + G_\varepsilon^1(\theta_i, \sigma_i, w)(w)$$

The perturbation of the reference equation remains small

$$\frac{d\sigma_i}{dt} = (-1)^i h(t, \sigma_i) + a_\varepsilon^2 + G_\varepsilon^2(\theta_i, \sigma_i, w)(w)$$

## Conclusion for this model

Good understanding of the dynamics of walls in a wire subject to an applied field

Quasi controllability of the walls'positions

Same kind of results with an electric current

It remains to describe :

- the collapse of two consecutive walls
- when a wall goes out the wire
- walls and domains formation

**The walls move slowly but they move ! ! ! !**

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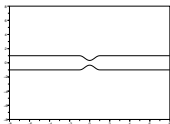
**The walls move slowly but they move!!!!**



# Plan

- 1 Ferromagnetism
- 2 Domain Walls in Nanowires
- 3 Quasi-Solutions and Metastability
- 4 Stabilization by pinching**

## Pinched Wire



Size of the section :  $\alpha$

- $\alpha : \mathbb{R} \rightarrow \mathbb{R}^+, C^\infty$ ,
- $\alpha = 1$  outside  $[-a, a]$ ,
- $\alpha(-x) = \alpha(x)$ ,
- $\alpha$  non decreasing on  $[0, a]$

Ferromagnetism Energy in pinched nanowire

$$E(m) = \frac{A}{2} \int_I \alpha(x) |\partial_x m|^2 dx + \frac{1}{4} \int_I \alpha(x) (|m_2|^2 + |m_3|^2) dx$$

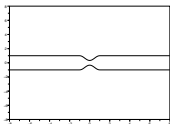
Model of Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow S^2, \quad I = \mathbb{R} \text{ or } I = [-L, L], \quad L > a$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

$$H_{\text{eff}} = \frac{1}{\alpha(x)} (\partial_x(\alpha(x)\partial_x m) - \alpha(x)(m_2 \vec{e}_2 + m_3 \vec{e}_3))$$

## Pinched Wire



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## Infinite Pinched Wire

### Model of Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathcal{S}^2,$$

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G. Carbou, D. Sanchez, *Stability of walls for pinched nano-wires*, in preparation

- Existence of a profile for one wall
- **Asymptotically stable for the position**

## Finite Pinched Wire

### Model of Nanowire

$$m : \mathbb{R}_t^+ \times [-L, L]_x \rightarrow \mathcal{S}^2,$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

$$H_{\text{eff}} = \partial_{xx} m + \frac{\alpha'}{\alpha} \partial_x m - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

$$\partial_x m(t, -L) = \partial_x m(t, L) = 0$$

G. Carbou, D. Sanchez, *Stability of walls for pinched nano-wires*, in preparation

- Existence of a profile for one wall for  $L \geq \pi/2$
- **Asymptotically stable in the position if  $L$  is large enough**

## Existence of profiles

- $M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

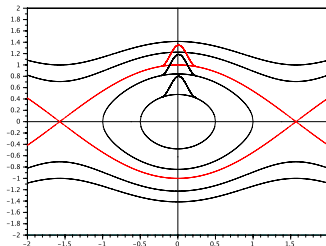
- BC :  $\theta \rightarrow \pm\pi/2$  when  $x \rightarrow \pm\infty$

$$\theta'(-L) = \theta'(L) = 0$$

- $\varphi$  is constant

- $\theta'' + \frac{\alpha'}{\alpha} \partial_x \theta + \cos \theta \sin \theta = 0$

- $((\theta')^2 - \cos^2 \theta)' = -2 \frac{\alpha'}{\alpha} (\theta')^2$



# Stability

## Pinched Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2$$

$$\frac{\partial m}{\partial t} = m \wedge H_{\text{eff}} - m \wedge (m \wedge H_{\text{eff}})$$

$$H_{\text{eff}} = m_{xx} + \frac{\alpha'}{\alpha} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

Boundary Conditions

## Wall Profile

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$$

$$\theta'' + \frac{\alpha'}{\alpha} \theta' + \cos \theta \sin \theta = 0$$

$$\text{BC} : \theta \rightarrow \pm \pi/2 \text{ when } x \rightarrow \pm \infty$$

$$\theta'(-L) = \theta'(L) = 0$$

## Stability of $M_0$ , asymptotic stability for the position of the wall ?

If  $\|m(t=0) - M_0\|_{H^1(\mathbb{R})}$  is small, then

- for all time,  $\|m(t) - M_0\|_{H^1(\mathbb{R})}$  remains small,
- $m(t)$  tends to  $R_{\theta_\infty}(M_0)$  when  $t$  tends to  $+\infty$ .

## Mobile Frame

Difficulty : geometrical constraint  $|m| = 1$  satisfied by the perturbations

We describe the perturbations in the mobile frame

$$m(t, x) = \sqrt{1 - |r(t, x)|^2} M_0(x) + r_1(t, x) M_1(x) + r_2(t, x) M_2$$

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix} \quad M_1(x) = \begin{pmatrix} -\cos \theta(x) \\ \sin \theta(x) \\ 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

### New Equations for $r$

$$r : \mathbb{R}_t^+ \times I_x \rightarrow \mathbb{R}^2$$

$$\partial_t r_1 = -L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$\partial_t r_2 = L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$L_1(r) = -\partial_{xx} r - \frac{\alpha'}{\alpha} \partial_x r + (\sin^2 \theta - \cos^2 \theta) r$$

$$L_2(r) = -\partial_{xx} r - \frac{\alpha'}{\alpha} \partial_x r + (\sin^2 \theta - (\theta')^2) r$$

### Exact Solution

$M_0$  for LL  $\sim 0$  for the new system

Stability of 0 ?



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### Exact Solution

$M_0$  for LL  $\sim 0$  for the new system

Stability of 0 ?

## Properties of the linearized : $L_2$

On  $L^2(\mathbb{R})$ , weighted scalar product :

$$\langle u|v \rangle = \int_{\mathbb{R}} \alpha(x)u(x)v(x) dx,$$

$$L_2 = -\frac{1}{\alpha} \partial_x (\alpha \partial_x) + (\sin^2 \theta_0 - (\theta'_0)^2) \implies L_2 \text{ is self-adjoint for } \langle | \rangle .$$

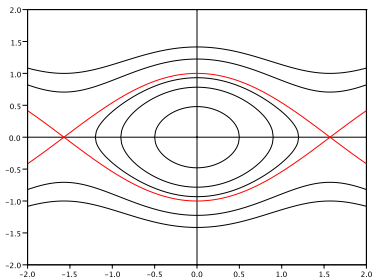
$$\langle L_2 u|v \rangle = \langle \ell u|\ell v \rangle, \quad \text{with } \ell = \partial_x + \theta' \tan \theta$$

$$L_2 \geq 0, \quad \text{Ker } L_2 = \mathbb{R}(\cos \theta)$$

- Valid whatever the configurations of the wire (several pinched zones)
- for one wall, on  $(\cos \theta)^\perp$ ,  $L_2 \geq 1$
- Invariance by rotation  $\implies 0$  is an eigenvalue of the linearized

## Properties of the linearized : $L_1$

$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$

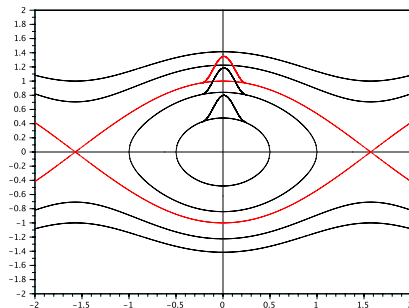


**Key point :  $L_1 \geq 0$  ?**

- Infinite nanowire without pinching :  $L_1 = L_2$ ,  $L_1 \geq 0$ . **Stability**
- Finite nanowire without pinching :  $L_1 = L_2 - c$ . **Bad sign, unstability for the position**

## Properties of the linearized : $L_1$

$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$



- Infinite wire :  $(\theta')^2 - \cos^2 \theta > 0$  on  $\text{Ker } L_2 \implies L_1 > 0$
- Short finite wire :  $(\theta')^2 - \cos^2 \theta \leq 0 \implies$  Unstability
- Long finite wire :  $(\theta')^2 - \cos^2 \theta > 0$  on a sufficiently large zone  $\implies L_1 > 0$

## Conclusion for pinched wires

**Pinching stabilizes the wall position**

Open Questions :

- more general geometry for pinching
- wire with several pinched zones
- effects of a magnetic applied field or an electric current on the walls in narrowing wires

## New unknown

Problem : Zero is an eigenvalue of the linearized

$\theta \mapsto R_\theta M_0(x)$  one parameter family of solutions for LL

$$\mathcal{R}_\theta(x) = \begin{pmatrix} R_\theta M_0(x) \cdot M_1(x) \\ R_\theta M_0(x) \cdot M_2 \end{pmatrix}$$

$r(t,x) = R(\theta(t)) + w(t,x)$   
 $x \mapsto \mathcal{R}(\theta)(x)$  stationary solution  
 $w_2$  in  $(\cos \theta)^\perp$

Equations for  $w$  and  $\theta$

$$\theta \text{ et } \sigma : \mathbb{R}^+ \rightarrow \mathbb{R}, \quad w : \mathbb{R}^+ \rightarrow \mathbb{R} \times (\cos \theta)^\perp$$

$$\partial_t \sigma = A_1(t, \theta, w)(w)$$

$$\partial_t w_1 = -L_1(w_1) - L_2(w_2) + \text{non linear}$$

$$\partial_t w_2 = -L_1(w_1) - L_2(w_2) + \text{non linear}$$

To be proved

$$w \rightarrow 0$$

$\theta$  remain small

$$\theta \rightarrow \theta_\infty$$

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$\theta \rightarrow \theta_\infty$

## Variational Estimates

### Equations for $w, \theta$

$$\theta : \mathbb{R}^+ \rightarrow \mathbb{R}, \quad w : \mathbb{R}^+ \rightarrow \mathbb{R} \times (\cos \theta)^\perp$$

$$\partial_t \theta = A(t, \theta, w)(w)$$

$$\partial_t w_1 = -L_1(w_1) - L_2(w_2) + \text{non linear}$$

$$\partial_t w_2 = L_1(w_1) - L_2(w_2) + \text{non linear}$$

By coercivity of the linearized,  $w$  is exponentially decreasing

$\partial_t \theta$  is integrable

Stability of  $M_0$ , asymptotic stability modulo rotation

## Variational Estimates

### Equations for $w, \theta$

$$\theta : \mathbb{R}^+ \rightarrow \mathbb{R}, \quad w : \mathbb{R}^+ \rightarrow \mathbb{R} \times (\cos \theta)^\perp$$

$$\partial_t \theta = A(t, \theta, w)(w)$$

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## Conclusion : pinching stabilize the position of walls

### Open Questions :

- more general geometry for pinching
- wire with several pinched zones
- effects of a magnetic applied field or an electric current on the walls in narrowing wires